EXAM PAPERS PRACTICE

Factor theorem

Name: ____________________

Class: ____________________

Date: ____________________

Time: **125 min.**

Marks: **105 marks**

Comments:

Q1.

The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + ax^2 - x - 21$$

where a is a constant.

The remainder when $\mathrm{p}(x)$ is divided by $(x + 2)$ is -7

(a) Use the Remainder Theorem to show that $a = 5$

(2)

(b) Use the Factor Theorem to show that $(x + 3)$ is a factor of $\mathrm{p}(x)$.

(2)

(c) Given that $x > 0$, use your answer to part (b) to find

$$\int \frac{x^3 + 5x^2 - x - 21}{\sqrt{x}(x+3)}\,\mathrm{d}x$$

(6)

(Total 10 marks)

Q2.

(a) The expressions $x^2 + bx + c$ and $x^2 - 3mx + 2n$ have a common factor of $(x - p)$, where b, c, m and n are positive.

Show that $p = \dfrac{2n - c}{b + 3m}$

(3)

(b) The equation $x^2 + (3k + 1)x + 3(k + 3) = 0$ has no real roots.

Find the range of possible values of k.

Show clearly each step of your working.

(6)

(Total 9 marks)

Mark schemes

Q1.

	Answer	Mark	Comments
(a)	(p(−2) =) $(-2)^3 + a(-2)^2 - (-2) - 21 = -7$ or $-8 + 4a + 2 - 21 = -7$	M1	oe. Uses Remainder Theorem with $x = -2$ to form equation in a, simplified or unsimplified. Allow one error.
	$a = 5$	A1	CSO. Be convinced. Must follow from completely correct working.

(b)	(p(−3) =) $(-3)^3 + 5(-3)^2 - (-3) - 21$	M1	p(−3) attempted. Must use Factor Theorem.
	(p(−3) =) $-27 + 45 + 3 - 21 = 0$	A1	CSO. Correctly shows p(−3) = 0.

(c)	Quadratic factor $(x^2 + 2x - 7)$	M1	Attempt to express numerator as product of $x + 3$ and a quadratic factor. Allow one error in quadratic factor. Can use inspection, long division or compare coefficients.
	$(x + 3)(x^2 + 2x - 7)$	A1	Numerator correctly expressed as product of $x + 3$ and a quadratic factor. May be implied by further correct working.
	$x^{\frac{3}{2}} + bx^{\frac{1}{2}} + dx^{-\frac{1}{2}}$ or $x\sqrt{x} + b\sqrt{x} + \frac{d}{\sqrt{x}}$	M1	Attempt to write their quotient as the sum of powers of x. $x + 3$ cancelled and division by $\sqrt{x}$ attempted. Allow one error in dividing quadratic term by $\sqrt{x}$. Allow sum of powers of x given in surd form with final term $\frac{d}{\sqrt{x}}$. Sum must be of three powers of x. ft if numerator was expressed in the form $(x + 3)(x^2 + bx + d)$
	$x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 7x^{-\frac{1}{2}}$ or	A1	Correct expression for quotient as the sum of powers of x.

$x\sqrt{x} + 2\sqrt{x} - \frac{7}{\sqrt{x}}$		Allow answer given in surd form.
$\frac{2}{5} \times x^{\frac{5}{2}} + 2 \times \frac{2}{3} \times x^{\frac{3}{2}} - 7 \times 2 \times x^{\frac{1}{2}} + c$ or $\frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} - 14x^{\frac{1}{2}} + c$ or $\frac{2}{5}x^2\sqrt{x} + \frac{4}{3}x\sqrt{x} - 14\sqrt{x} + c$	B2ft	Correct answer simplified or unsimplified. Can be given in index or surd form. ft their integral provided previous M1 scored. B2 for fully correct answer. B1B0 for two terms correct including signs Condone omission of + c.
		Total 10 marks

Q2.

	Answer	Mark	Comments
(a)	$p^2 + bp + c = p^2 - 3mp + 2n$ or $p^2 + bp + c = 0$ and $p^2 - 3mp + 2n = 0$	B1	Application of the Factor Theorem to show either two correct quadratic expressions equated, or two correct equations in p set to equal zero
	$p(b + 3m) = 2n - c$	M1	Correct manipulation of their $p^2 + bp + c = p^2 - 3mp + 2n$ isolating p on one side and factorised
	$p = \frac{2n - c}{b + 3m}$	A1	CSO. Dependent upon first two marks awarded
(b)	No real roots $\Rightarrow b^2 - 4ac < 0$	B1	Condition for no real roots stated or used
	$(3k + 1)^2 - 4(3(k + 3)) < 0$ or $9k^2 + 6k + 1 - 12k - 36 < 0$	M1	Correct inequality unsimplified. Condone +36. Must include < 0
	$9k^2 - 6k - 35 < 0$	A1	Correct inequality with collected terms
	$(3k + 5)(3k - 7)$	M1	Correct factorisation of the quadratic expression or correct unsimplified quadratic equation formula $k = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 9 \times (-35)}}{2 \times 9}$
	$k = \frac{-5}{3}$ and $k = \frac{7}{3}$	A1	oe. For correct critical values. Fractions must be fully simplified, or decimal equivalents must be exact.

$\frac{-5}{3} < k < \frac{7}{3}$	A1	oe. For correct inequality. Fractions must be fully simplified, or decimal equivalents must be exact.
		Total 9 marks

Examiner reports

Q1.

(a) The vast majority of students showed a good knowledge of the Remainder Theorem and gained both marks with correct substitutions and working leading to the required result.

The question required the use of the Remainder Theorem, hence the few students who used polynomial division to get the remainder in terms of a, set it equal to -7 and then solve for a gained no credit.

(b) The vast majority of students also showed a good knowledge of the Factor Theorem and gained both marks with the correct substitution shown and correct working leading to a value of zero. As in the previous part, the small number of students who used polynomial division scored zero.

(c) Just over a third of the students formed a perfectly correct solution. However, another third failed to gain any marks at all as they did not attempt to use the information in the previous part to express the numerator as the product of x +3 and a quadratic factor, which was vital for carrying on with the solution. Instead, a significant number of students tried to multiply the numerator by $\left(x^{-\frac{3}{2}} + 3x^{-\frac{1}{2}}\right)$ or similar.

It was also relatively common to see students correctly cancel the factor x + 3 in both numerator and denominator only to multiply the numerator by $x^{\frac{1}{2}}$. This was insufficient for the second method mark to be awarded, and any further marks were dependent on this second method mark, so these students scored 2 marks in total.

A significant minority lost the final mark by making a mistake while integrating one of the terms in their integral.

Q2.

In part (a) the majority of students showed a good knowledge of the Factor Theorem and used it to form proofs as outlined in the mark scheme. Students correctly manipulated the equation and nearly all showed the factorisation $p(b + 3m)$ which was a necessary justification if they were to gain full marks. A small number of students made the proof more difficult by using polynomial division. Two thirds of the students produced a thoroughly correct solution.

A quarter of the students, however, showed no knowledge of the Factor Theorem and made little if any attempt.

Part (b) gave another example where a common technique had been well-practised by the students. Very few of the students who attempted this question made an error when stating the condition on the discriminant for no real roots, and just over a half of students gained full marks.

Students failed to gain marks through incorrect algebraic manipulation or algebraic errors when finding the critical values. However it was pleasing to see students not being tempted into converting fractional values into inexact decimal equivalents and thus losing marks for accuracy.