Question 1

The number of seats a row has at a comedy festival follows a regular pattern where the first row has u_1 seats and the number of seats in each successive row increases by d seats. In the fourth row there are 25 seats and in the sixteenth row there are 49 seats.

$u_4 = 25$ $u_{16} = 49$

(a) Write down an equation, in terms of u_1 and d, for the number of seats

(i) in the fourth row

(ii) in the sixteenth row.

[2]

(b) Find the value of u_1 and the value of d.

[2]

The festival has 18 rows of seats in total.

(c) Calculate the total number of seats.

[3]

The price for a seat in the first row is \$22 and the price decreases by 5% each successive row.

(d) (i) Find the row in which the price of a seat first falls below \$10.

(ii) Find the total revenue the comedy festival generates if 22 tickets are sold for every row. Give your answer rounded to the nearest dollar.

a) $u_n = u_1 + (n-1)d$ } n^{th} term of an arithmetic sequence

(i) $u_4 = u_1 + (4-1)d = u_1 + 3d = 25$

$$\boxed{u_1 + 3d = 25}$$

(ii) $u_{16} = u_1 + (16-1)d = u_1 + 15d = 49$

$$\boxed{u_1 + 15d = 49}$$

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(i) in the fourth row

(ii) in the sixteenth row.

$\boxed{u_1 + 3d = 25}$ ①

$\boxed{u_1 + 15d = 49}$ ②

} simultaneous equations

[2]

(b) Find the value of u_1 and the value of d.

[2]

The festival has 18 rows of seats in total.

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(d) (i) Find the row in which the price of a seat first falls below \$10.

(ii) Find the total revenue the comedy festival generates if 22 tickets are sold for every row. Give your answer rounded to the nearest dollar.

b) Subtract ① from ②

$$u_1 + 15d = 49$$
$$-(u_1 + 3d = 25)$$
$$12d = 24 \Rightarrow \boxed{d = 2}$$

$$\underbrace{u_1 = 25 - 3d}_{\text{from ①}} = 25 - 3(2) = 25 - 6$$

$$\boxed{u_1 = 19}$$

You could also solve simultaneous equations with your GDC.

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(i) in the fourth row

(ii) in the sixteenth row.

[2]

(b) Find the value of u_1 and the value of d.

$\boxed{u_1 = 19}$ $\boxed{d = 2}$

[2]

$n = 18$

The festival has 18 rows of seats in total.

(c) Calculate the total number of seats.

[3]

c) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$, $S_n = \frac{n}{2}(u_1 + u_n)$ } Sum of n terms of an arithmetic sequence

$$S_{18} = \frac{18}{2}\left(2(19) + (18-1)(2)\right)$$

$$= 9(38 + 34) = 9(72) = 648$$

$\boxed{\text{There are 648 seats.}}$

The price for a seat in the first row is \$22 and the price decreases by 5% each successive row.

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[2]

(b) Find the value of u_1 and the value of d.

[2]

The festival has 18 rows of seats in total.

(c) Calculate the total number of seats.

[3]

$u_1 = 22$ $r = 1 - 0.05 = 0.95$

The price for a seat in the first row is \$22 and the price decreases by 5% each successive row.

(d) (i) Find the row in which the price of a seat first falls below \$10.

(ii) Find the total revenue the comedy festival generates if 22 tickets are sold for every row. Give your answer rounded to the nearest dollar.

d) (i) $u_n = u_1 r^{n-1}$ } n^{th} term of a geometric sequence

$$22(0.95)^{n-1} = 10 \Rightarrow (0.95)^{n-1} = \frac{10}{22}$$

$$\Rightarrow n - 1 = \log_{0.95}\left(\frac{10}{22}\right) \qquad a^x = b \Longleftrightarrow x = \log_a b$$

$$\Rightarrow n = 1 + \log_{0.95}\left(\frac{10}{22}\right) = 16.371548\ldots$$

round to next higher integer

$\boxed{n = 17 \quad (17^{th} \text{ row})}$

(ii) $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$ } Sum of n terms of a geometric sequence $(r \neq 1)$

$$\text{Total} = 22 \times \underbrace{\left(\frac{22(1 - 0.95^{18})}{1 - 0.95}\right)}_{= S_{18}} = 5834.96539\ldots$$

$= S_{18}$ (sum of prices for one ticket from each row)

$\boxed{\$5835}$

Question 2

A study was conducted on 6 participants, measuring their body fat percentage (%) and their resting heart rate in beats per minute (BPM). The results are shown in the table below.

Body fat percentage (x)	22.0	14.2	15.5	12.6	29.8	10.1
Resting heart rate (y)	65	59	54	68	74	51

(a) Use your graphic display calculator to find

(i) $\bar{x}$, the mean body fat percentage

(ii) $\bar{y}$, the mean resting heart rate

(iii) r, the Pearson's product-moment correlation coefficient.

[3]

(b) (i) Write down the equation of the regression line of y on x for this data, giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(ii) Show that the point A$(\bar{x}, \bar{y})$ lies on the regression line of y on x.

[4]

A seventh participant, John, has a resting heart rate of 60 BPM.

(c) (i) Use the regression line equation to estimate John's body fat percentage.

(ii) Justify whether it is valid to use the regression line of y on x to estimate John's body fat percentage.

[4]

John's body fat percentage is 13.5%.

(d) Calculate the percentage error in John's estimated body fat percentage from part (c).

[2]

a) (i) $\boxed{\bar{x} = 17.3666666... = 17.4\% \ (3 \text{ s.f.})}$

(ii) $\boxed{\bar{y} = 61.8333333... = 61.8 \text{ bpm} \ (3 \text{ s.f.})}$

(iii) $\boxed{r = 0.74981007 = 0.750 \ (3 \text{ s.f.})}$

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$\boxed{\bar{x} = 17.3\dot{6} = 17.4\% \ (3 \text{ s.f.})}$

$\boxed{\bar{y} = 61.8\dot{3} = 61.8 \text{ bpm} \ (3 \text{ s.f.})}$

b) (i) $m = 0.90106629 = 0.901 \ (3 \text{ s.f.})$

$c = 46.1848154 = 46.2 \ (3 \text{ s.f.})$

$$\boxed{y = 0.901x + 46.2}$$

(ii) Find y when $x = \bar{x}$, using exact values

When $x = \bar{x}$,

$$y = 0.90106629\left(\overbrace{17.3\dot{6}}^{=\frac{521}{30}}\right) + 46.1848154$$

$$= 61.8333333 = \bar{y}$$

So $(\bar{x}, \bar{y})$ is on the regression line.

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$\boxed{y = 0.901x + 46.2}$

[3]

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(ii) Show that the point A$(\bar{x}, \bar{y})$ lies on the regression line of y on x.

$y = 60$

[4]

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John's body fat percentage is 13.5%.

(d) Calculate the percentage error in John's estimated body fat percentage from part (c).

[2]

c) (i) $y = 0.901x + 46.2 \implies x = \dfrac{y - 46.2}{0.901}$

When $y = 60$, $x = \dfrac{60 - 46.2}{0.901} = 15.316315\ldots$

$\boxed{15.3\% \text{ (3 s.f.)}}$

(ii) In general, the regression line of y on x should not be used to predict a value of x from a value of y. So the estimate of 15.3 % may not be reliable.

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Percentage error

$\varepsilon = \left| \dfrac{V_A - V_E}{V_E} \right| \times 100\%$

V_A is the approximate value

V_E is the exact value

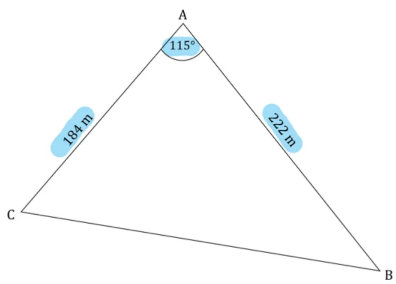
d) $\varepsilon = \left| \dfrac{15.3 - 13.5}{13.5} \right| \times 100$

$= \dfrac{40}{3} = 13.333333\ldots$

$\boxed{13.3\% \text{ (3 s.f.)}}$

Question 3

A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point B to point C, and the farmhouse is located at point A. AB and AC form the other two sides of the farm, and the distances from the farmhouse to points B and C are 222 m and 184 m respectively. Angle $C\hat{A}B$ is 115°, and points A, B and C lie in a horizontal plane.



(a) Calculate the distance along the motorway from B to C.

[2]

The cost of fencing in US dollars (USD) is \$89.99 per metre.

(b) Calculate the total cost of fencing the whole perimeter of the farm. Give your answer to 2 decimal places.

[2]

(c) Calculate the area of the farm.

[2]

a) $c^2 = a^2 + b^2 - 2ab\cos C$ } Cosine rule

$(BC)^2 = 184^2 + 222^2 - 2(184)(222)\cos 115$

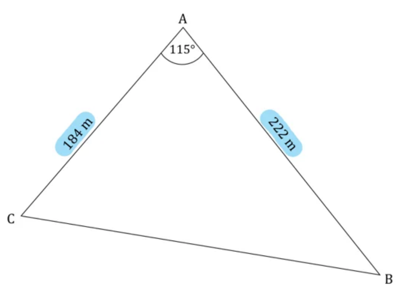
$= 83140 - 81696\cos 115$

$BC = \sqrt{83140 - 81696\cos 115}$

$= 343.025103\ldots$

$\boxed{343 \text{ m } (3 \text{ s.f.})}$

A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point B to point C, and the farmhouse is located at point A. AB and AC form the other two sides of the farm, and the distances from the farmhouse to points B and C are 222 m and 184 m respectively. Angle $C\hat{A}B$ is 115°, and points A, B and C lie in a horizontal plane.



(a) Calculate the distance along the motorway from B to C.

$BC = \sqrt{83140 - 81696\cos 115} = \boxed{343 \text{ m } (3 \text{ s.f.})}$ [2]

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(b) Calculate the total cost of fencing the whole perimeter of the farm. Give your answer to 2 decimal places.

[2]

(c) Calculate the area of the farm.

[2]

b) $89.99 \times \left(184 + 222 + \overbrace{\sqrt{83140 - 81696\cos 115}}^{\text{exact value of BC}}\right)$

$= 67404.7690\ldots$

$\boxed{\$\,67404.77 \ (2 \text{ d.p.})}$

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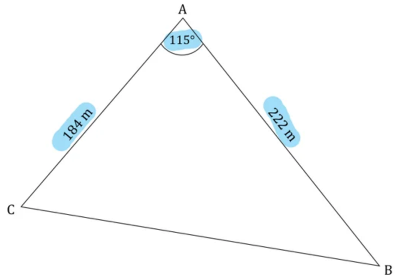

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[2]

(c) Calculate the area of the farm.

[2]

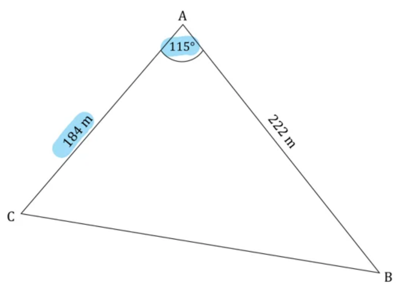
c) $\text{Area} = \frac{1}{2} ab \sin C$ } area of a triangle

$\text{Area} = \frac{1}{2}(184)(222)\sin 115$

$= 18510.4302\ldots$

$\boxed{18510 \text{ m}^2 \text{ (to nearest m}^2\text{)}}$

A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point B to point C, and the farmhouse is located at point A. AB and AC form the other two sides of the farm, and the distances from the farmhouse to points B and C are 222 m and 184 m respectively. Angle CÂB is 115°, and points A, B and C lie in a horizontal plane.



(a) Calculate the distance along the motorway from B to C.

$BC = \sqrt{83140 - 81696 \cos 115} = \boxed{343 \text{ m (3 s.f.)}}$

[2]

(d) Find the sizes of angles AB̂C and AĈB.

[2]

(e) Calculate the shortest distance from the farmhouse to the motorway.

[3]

d) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ } Sine rule

$\frac{BC}{\sin 115} = \frac{184}{\sin A\hat{B}C} \Rightarrow \sin A\hat{B}C = \frac{184}{BC} \sin 115$

$A\hat{B}C = \sin^{-1}\left(\frac{184}{\sqrt{83140 - 81696 \cos 115}} \times \sin 115\right)$

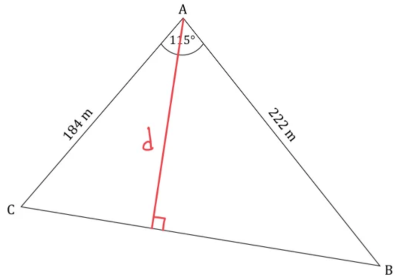
$= 29.087649\ldots$

$A\hat{C}B = 180 - 115 - A\hat{B}C = 35.912350\ldots$

To 1 d.p.,

$A\hat{B}C = 29.1°$ $A\hat{C}B = 35.9°$

A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point B to point C, and the farmhouse is located at point A. AB and AC form the other two sides of the farm, and the distances from the farmhouse to points B and C are 222 m and 184 m respectively. Angle CÂB is 115°, and points A, B and C lie in a horizontal plane.



(a) Calculate the distance along the motorway from B to C.

$BC = \sqrt{83140 - 81696 \cos 115} = \boxed{343 \text{ m } (3 \text{ s.f.})}$ [2]

(d) Find the sizes of angles AB̂C and AĈB.

$\hat{ABC} = 29.087649... = 29.1° \ (1 \text{ d.p.})$

$\hat{ACB} = 35.912350... = 35.9° \ (1 \text{ d.p.})$ [2]

(e) Calculate the shortest distance from the farmhouse to the motorway.

→ i.e., the perpendicular distance [3]

$\sin \hat{ABC} = \frac{184}{BC} \sin 115$ } from part (d) working

e) $\sin \hat{ABC} = \frac{d}{222}$ SOHCAHTOA

$d = 222 \sin \hat{ABC}$

$= 222 \left(\frac{184}{\sqrt{83140 - 81696 \cos 115}} \times \sin 115 \right)$

$= 107.924639...$

$\boxed{108 \text{ m } (3 \text{ s.f.})}$

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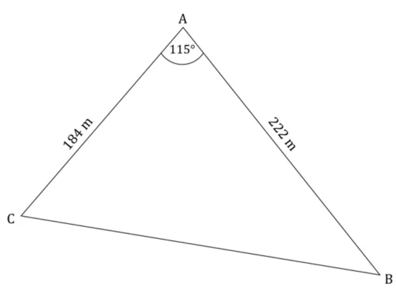


(a) Calculate the distance along the motorway from B to C.

$BC = \sqrt{83140 - 81696 \cos 115} = \boxed{343 \text{ m } (3 \text{ s.f.})}$ [2]

A vertical signpost is located at point C, and the top of the signpost is designated as point D. The angle of elevation to the top of the signpost from point B is measured to be 1.4°.

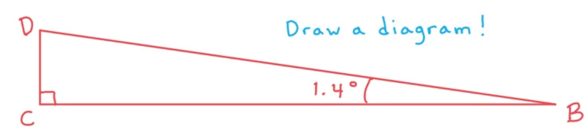
(f) Calculate the distance CD, the vertical height of the signpost.

[2]

(g) Calculate the distance between the top of the signpost, D, and point A.

[2]

f) Draw a diagram!



$\tan(1.4) = \frac{CD}{BC}$ SOHCAHTOA

$CD = BC \tan(1.4)$

$= \left(\sqrt{83140 - 81696 \cos 115}\right) \tan(1.4)$

$= 8.383352...$

$\boxed{8.38 \text{ m } (3 \text{ s.f.})}$

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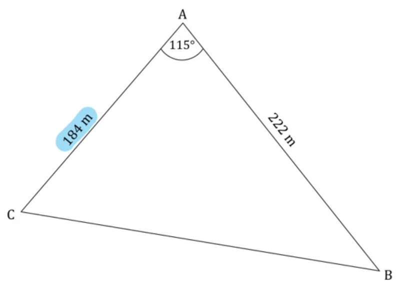

(a) Calculate the distance along the motorway from B to C.

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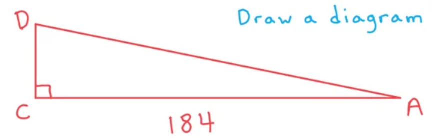
(f) Calculate the distance CD, the vertical height of the signpost.

$\boxed{CD = BC \tan(1.4) = 8.38 \text{ m } (3 \text{ s.f.})}$ [2]

(g) Calculate the distance between the top of the signpost, D, and point A.

[2]

g)



Draw a diagram!

$AD = \sqrt{AC^2 + CD^2}$ Pythagoras

$= \sqrt{184^2 + \left(\underbrace{\sqrt{83140 - 81696 \cos 115} \tan(1.4)}_{\text{exact value of CD}}\right)^2}$

$= 184.190880...$

$\boxed{184.2 \text{ m } (1 \text{ d.p.})}$

The rate of growth of the grass on the farm, G, in inches per month, can be modelled by the function

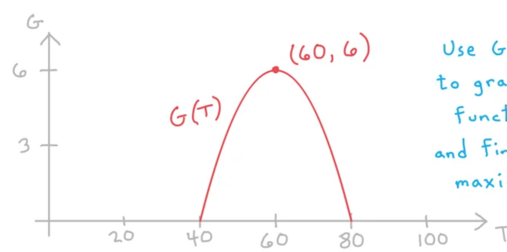
$$G(T) = -0.015(T - 40)(T - 80)$$

where T is the temperature in degrees Fahrenheit.

(h) Find the maximum rate of grass growth on the farm and the temperature required.

[3]

h)



Use GDC to graph function and find the maximum

The maximum rate of growth is 6 inches per month, when the temperature is 60° F.

Question 4

The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

use this as 'X List' in GDC

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

use this as 'Freq List' in GDC

(a) Calculate

(i) the mean number of baskets scored by a player

(ii) the standard deviation.

[2]

(b) Find the median number of baskets scored.

[1]

(c) Find the interquartile range.

[2]

(d) Determine if a player who scored 8 baskets would be considered an outlier.

[2]

Two players are randomly chosen.

(e) Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

[4]

The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.

(f) (i) Calculate the probability that a player trains less than 6 hours a week.

(ii) Calculate the probability that a player trains less than 4 hours a week.

(iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3]

a) (i) $\bar{x} = 2.03333333$ from GDC

$\boxed{\bar{x} = 2.03 \text{ baskets } (3 \text{ s.f.})}$

(ii) $\sigma_x = 1.67099438$ from GDC

$\boxed{\sigma_x = 1.67 \text{ baskets } (3 \text{ s.f.})}$

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(iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3]

b) $\boxed{\text{Median} = 2 \text{ baskets}}$ from GDC

EXAM PAPERS PRACTICE

The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

use this as 'X List' in GDC

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

use this as 'Freq List' in GDC

(a) Calculate

(i) the mean number of baskets scored by a player

(ii) the standard deviation.

[2]

(b) Find the median number of baskets scored.

[1]

(c) Find the interquartile range.

[2]

(d) Determine if a player who scored 8 baskets would be considered an outlier.

[2]

Two players are randomly chosen.

(e) Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

[4]

The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.

(f) (i) Calculate the probability that a player trains less than 6 hours a week.

(ii) Calculate the probability that a player trains less than 4 hours a week.

(iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3]

c) $IQR = Q_3 - Q_1$ } Interquartile range

$Q_1 = 0$ $\quad Q_3 = 3$ $\quad$ from GDC

$Q_3 - Q_1 = 3 - 0 = 3$

$\boxed{IQR = 3 \text{ baskets}}$

The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

(a) Calculate

(i) the mean number of baskets scored by a player

(ii) the standard deviation.

[2]

(b) Find the median number of baskets scored.

[1]

(c) Find the interquartile range.

$Q_1 = 0$ $\quad Q_3 = 3$

$\boxed{IQR = 3 \text{ baskets}}$

[2]

(d) Determine if a player who scored 8 baskets would be considered an outlier.

[2]

Two players are randomly chosen.

(e) Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

[4]

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(f) (i) Calculate the probability that a player trains less than 6 hours a week.

(ii) Calculate the probability that a player trains less than 4 hours a week.

(iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3]

d) The upper outlier boundary is

$Q_3 + 1.5 \times IQR$

$3 + 1.5 \times 3 = 3 + 4.5 = 7.5$

$8 > 7.5$

A player scoring 8 baskets would be an outlier.

The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

$\underbrace{41+17+34}_{} = 92$

(a) Calculate

(i) the mean number of baskets scored by a player

(ii) the standard deviation.

[2]

(b) Find the median number of baskets scored.

[1]

(c) Find the interquartile range.

[2]

(d) Determine if a player who scored 8 baskets would be considered an outlier.

[2]

Two players are randomly chosen.

(e) Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

[4]

The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.

(f) (i) Calculate the probability that a player trains less than 6 hours a week.

(ii) Calculate the probability that a player trains less than 4 hours a week.

(iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3]

e) For that to be true...

... the first player must be one of the 17 players who scored one basket, out of the 92 who scored two or less...

$$\frac{17}{92} \times \frac{16}{149} = \frac{68}{3427} = 0.0198424...$$

... and then of the remaining 149 players, the second player must be one of the remaining 16 players who scored one basket.

The probability is $\boxed{\frac{68}{3427} = 0.0198 \text{ (4 d.p.)}}$

The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

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Frequency	41	17	34	31	10	15	2

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(b) Find the median number of baskets scored.

[1]

(c) Find the interquartile range.

[2]

(d) Determine if a player who scored 8 baskets would be considered an outlier.

[2]

Two players are randomly chosen.

(e) Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

[4]

The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.

$\sigma = 0.8$ $\mu = 5$

(f) (i) Calculate the probability that a player trains less than 6 hours a week.

(ii) Calculate the probability that a player trains less than 4 hours a week.

(iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3]

f) $X \sim N(5, 0.8^2)$

(i) $P(X < 6) = 0.89435022$ from GDC

$\boxed{0.8944 \text{ (4 d.p.)}}$

(ii) $P(X < 4) = 0.10564977$ from GDC

$\boxed{0.1056 \text{ (4 d.p.)}}$

(iii) $P(4 \le X \le 6) = 0.78870045$ from GDC

$0.78870045 \times 150 = 118.305067...$

$\boxed{118 \text{ players}}$

Question 5

Best Beans is a New Zealand-based company that sells baked beans packaged in cylindrical cans.

(a) Given that their cans have a height of 15 cm and a diameter of 8 cm, calculate

(i) the volume of the can

(ii) the surface area of the can.

$h = 15$ $\quad r = \frac{8}{2} = 4$

[4]

Every month, Best Beans expects to sell x thousand cans of baked beans. It is known that

$$\frac{dP}{dx} = -2x + 472, \qquad x \geq 0$$

where P is the monthly profit, in New Zealand dollars (NZD), from the sale of x thousand cans of baked beans. It is also known that Best Beans makes a profit of 2450 NZD in a month where it sells 8000 cans of baked beans.

(b) Find $P(x)$.

[5]

(c) Find the least number of cans which must be sold each month in order to make a profit.

[3]

(d) Find the monthly sales level that will maximise profit, and the expected profit at this level.

[3]

a) (i) $V = \pi r^2 h$ } Volume of a cylinder

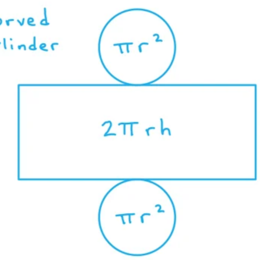
$V = \pi(4)^2(15) = 240\pi = 753.982236...$

$$\boxed{V = 240\pi \text{ cm}^3 = 754 \text{ cm}^3 \text{ (3 s.f.)}}$$

exact answer

(ii) $A = 2\pi rh$ } Area of the curved surface of a cylinder

$A = \pi r^2$ } Area of a circle



$A = 2\pi(4)(15) + 2 \times \pi(4)^2$

$= 152\pi = 477.522083...$

$$\boxed{A = 152\pi \text{ cm}^2 = 478 \text{ cm}^2 \text{ (3 s.f.)}}$$

exact answer

Best Beans is a New Zealand-based company that sells baked beans packaged in cylindrical cans.

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(i) the volume of the can

(ii) the surface area of the can.

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So when $x = 8$, $P(x) = 2450$

(b) Find $P(x)$.

[5]

(c) Find the least number of cans which must be sold each month in order to make a profit.

[3]

(d) Find the monthly sales level that will maximise profit, and the expected profit at this level.

[3]

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ } Integral of x^n $(n \neq -1)$

b) $P(x) = \int \frac{dP}{dx} \, dx$

$P(x) = \int (-2x + 472) \, dx$

$= -2\left(\frac{x^2}{2}\right) + 472x + c = -x^2 + 472x + c$

$\frac{8000}{1000} = 8$ $\quad$ Don't forget that x is thousands of cans

So $P(8) = 2450$

$-(8)^2 + 472(8) + c = 2450$

$c + 3712 = 2450 \Rightarrow c = -1262$

$$\boxed{P(x) = -x^2 + 472x - 1262}$$

Best Beans is a New Zealand-based company that sells baked beans packaged in cylindrical cans.

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Every month, Best Beans expects to sell x thousand cans of baked beans. It is known that

$$\frac{dP}{dx} = -2x + 472, \qquad x \geq 0$$

where P is the monthly profit, in New Zealand dollars (NZD), from the sale of x thousand cans of baked beans. It is also known that Best Beans makes a profit of 2450 NZD in a month where it sells 8000 cans of baked beans.

(b) Find $P(x)$.

$$\boxed{P(x) = -x^2 + 472x - 1262}$$

[5]

(c) Find the least number of cans which must be sold each month in order to make a profit.

[3]

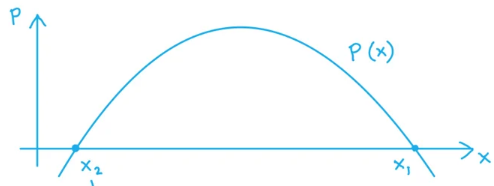
(d) Find the monthly sales level that will maximise profit, and the expected profit at this level.

[3]

c) We need to find when $P(x) = 0$

$-x^2 + 472x - 1262 = 0$ Solve with GDC

$x_1 = 469.3109513 \qquad x_2 = 2.689048693$



$\rightarrow x_2$ is the 'break even' point

$1000x_2 = 2689.048693$

Don't forget that x is thousands of cans

We need the smallest integer bigger than this

$$\boxed{2690 \text{ cans}}$$

Best Beans is a New Zealand-based company that sells baked beans packaged in cylindrical cans.

(a) Given that their cans have a height of 15 cm and a diameter of 8 cm, calculate

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$$\boxed{P(x) = -x^2 + 472x - 1262}$$

[5]

(c) Find the least number of cans which must be sold each month in order to make a profit.

[3]

(d) Find the monthly sales level that will maximise profit, and the expected profit at this level.

[3]

d) $P(x)$ has a maximum when $\frac{dP}{dx} = 0$

$-2x + 472 = 0$

$2x = 472 \implies x = 236$

Selling 236000 cans will maximise profit.

Don't forget that x is thousands of cans

So the max profit is $P(236)$

$-(236)^2 + 472(236) - 1262 = 54434$

Maximum profit is 54434 NZD

* You could also find these values by graphing $P(x)$ on your GDC and finding the max.

Best Beans wants to buy a new factory at a cost of 800 000 NZD. The CEO decides to invest 60% of the company's monthly profit into a savings account paying a nominal annual interest rate of 5.5%, **compounded monthly**.

(e) Under the assumption that the company's monthly profit will attain its maximum value every month throughout the period, determine whether Best Beans will have saved enough to buy the factory by the end of two years.

[4]

$\boxed{\text{Maximum profit is } 54434 \text{ NZD}}$ } from part (d)

e) Find 60% of the maximum profit

$0.6 \times 54434 = 32660.4$

Use the compound interest function on your GDC

$n = 24$ 24 months in two years $I\% = 5.5$

$PV = 0$ $PMT = -32660.40$

$P/Y = 12$ $C/Y = 12$

$\Rightarrow FV = 826587.6989$

$\boxed{FV > 800000, \text{ so yes they will have enough.}}$

Question 6

85 people are asked if they like juice (J), tea (T) and/or coffee (C) for breakfast.

12 like all three
16 like coffee and tea
14 like coffee and juice
5 like juice only
27 like coffee only
14 like tea only

(a) (i) Draw a Venn diagram to represent the information provided.

(ii) Write down the number of people who like coffee but not tea.

[5]

There are 31 people in total who like tea.

(b) (i) Calculate the number of people who like tea or juice.

(ii) Find the number of people who like none of the drinks for breakfast.

[4]

A person is chosen at random from the 85 people.

(c) Find the probability that this person

(i) likes coffee

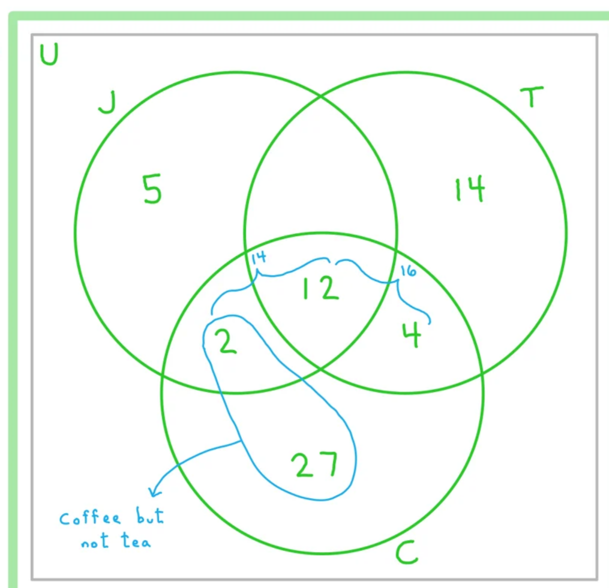
(ii) likes coffee and tea but not juice

(iii) does not like either tea or juice

(iv) does not like coffee given that the person does not like tea.

[5]

a) (i)



(ii) $2 + 27 = \boxed{29}$

85 people are asked if they like juice (J), tea (T) and/or coffee (C) for breakfast.

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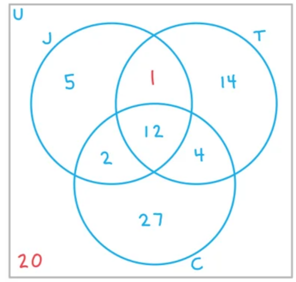

$12 + 4 + 14 + \underline{1} = 31$

There are 31 people in total who like tea.

(b) (i) Calculate the number of people who like tea or juice.

(ii) Find the number of people who like none of the drinks for breakfast.

[4]

b) (i) This is the total number inside the 'T' and 'J' circles.

$5 + 2 + 1 + 12 + 14 + 4 = \boxed{38}$

(ii) This is the number out of 85 not in any of the circles.

$85 - (5 + 2 + 1 + 12 + 14 + 4 + 27) = \boxed{20}$

A person is chosen at random from the 85 people.

(c) Find the probability that this person

(i) likes coffee

(ii) likes coffee and tea but not juice

(iii) does not like either tea or juice

(iv) does not like coffee given that the person does not like tea.

[5]

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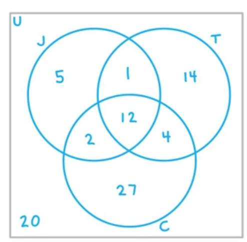

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[5]

c) (i) $\frac{2 + 12 + 4 + 27}{85} = \frac{45}{85} = \boxed{\frac{9}{17}}$

(ii) $\boxed{\frac{4}{85}}$

(iii) $\frac{27 + 20}{85} = \boxed{\frac{47}{85}}$

(iv) $\frac{5 + 20}{5 + 20 + 2 + 27} = \boxed{\frac{25}{54}}$

$\frac{\text{number that don't like coffee } \underline{\text{and}} \text{ don't like tea}}{\text{number that don't like tea}}$