Question 1

Find the value of each of the following, giving your answer as an integer.

(a) $\ln e$. [2]

(b) $\log_2 16$. [2]

(c) $\log 25 + \log 4$. [2]

(d) $\log_5 500 - \log_5 4$. [2]

(a) ln and e cancel each other out

$\boxed{\ln e = 1}$

Find the value of each of the following, giving your answer as an integer.

(a) $\ln e$. [2]

(b) $\log_2 16$. [2]

(c) $\log 25 + \log 4$. [2]

(d) $\log_5 500 - \log_5 4$. [2]

(b) Write 16 as a power with the same base number as the log so they cancel out

$2^4 = 16$

$\log_2 2^4 = 4$

$\boxed{\log_2 16 = 4}$

Find the value of each of the following, giving your answer as an integer.

(a) $\ln e$. [2]

(b) $\log_2 16$. [2]

(c) $\log 25 + \log 4$. [2]

(d) $\log_5 500 - \log_5 4$. [2]

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 25 + \log 4 = \log 100$

$= \log 10^2$ ← The bases are the same so they cancel

$= 2$

$\boxed{\log 25 + \log 4 = 2}$

EXAM PAPERS PRACTICE

Find the value of each of the following, giving your answer as an integer.

(a) $\ln e$. [2]

(b) $\log_2 16$. [2]

(c) $\log 25 + \log 4$. [2]

(d) $\log_5 500 - \log_5 4$. [2]

(d) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log_5 500 - \log_5 4 = \log_5 125$ ← Rewrite 125 as a power of 5

$= \log_5 5^3$ ← The bases are the same so they cancel

$= 3$

$\boxed{\log_5 500 - \log_5 4 = 3}$

Question 2

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$. [2]

(b) $\ln 45$. [2]

(c) $\ln 135$. [3]

(a) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\ln 5 = \ln 15 - \ln 3$

$\boxed{\ln 5 = x - y}$

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$. [2]

(b) $\ln 45$. [2]

(c) $\ln 135$. [3]

(b) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\ln 45 = \ln 15 + \ln 3$

$\boxed{\ln 45 = x + y}$

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$. [2]

(b) $\ln 45$. [2]

$\boxed{\ln 45 = x + y}$

(c) $\ln 135$. [3]

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\ln 135 = \ln 45 + \ln 3$

$= (x + y) + y$

$\boxed{\ln 135 = x + 2y}$

Question 3

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$. [2]

(b) $\log 3$. [3]

(c) $\log 72$. [3]

(a) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 24 = \log 2 + \log 12$

$\boxed{\log 24 = r + s}$

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$. [2]

(b) $\log 3$. [3]

(c) $\log 72$. [3]

(b) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 4 = \log 2 + \log 2$

$\log 4 = 2r$

$\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log 3 = \log 12 - \log 4$

$\boxed{\log 3 = s - 2r}$

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$. [2]

(b) $\log 3$. [3]

(c) $\log 72$. [3]

(c) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log 6 = \log 12 - \log 2$

$= s - r$

$\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 72 = \log 6 + \log 12$

$= (s-r) + s$

$\boxed{\log 72 = 2s - r}$

Question 4

Simplify the following equations:

(a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$. [2]

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$. [2]

(c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$. [2]

a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$

expand numerator

$\frac{-48x^{-3}y^{10}}{6x^2y}$

cancelling

$\frac{\cancel{-48}^{-8}\,\cancel{x^{-3}}\,\cancel{y^{10}}^{y^9}}{\cancel{6}\,\cancel{x^2}\,\cancel{y}_{x^5}}$

$\boxed{\frac{-8y^9}{x^5}}$

Simplify the following equations:

(a) $\dfrac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$. [2]

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$. [2]

(c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$. [2]

b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$

rewrite as fraction

$$\frac{(4x^2y^3)^4}{(2x^{-1}y^{-2})^3}$$

expand numerator and denominator

$$\frac{256x^8y^{12}}{8x^{-3}y^{-6}}$$

cancelling

$$\frac{\cancel{256}^{32}\cancel{x^8}^{x^{11}}\cancel{y^{12}}^{y^{18}}}{\cancel{8}\cancel{x^{-3}}\cancel{y^{-6}}}$$

$$\boxed{32x^{11}y^{18}}$$

Simplify the following equations:

(a) $\dfrac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$. [2]

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$. [2]

(c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$. [2]

c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$

rewrite as a fraction and use indice laws

$$\frac{(9x^6y^{-2}z^4)^{\frac{3}{2}}}{(3xyz)^2}$$

expand numerator and denominator

$$\frac{27x^9y^{-3}z^6}{9x^2y^2z^2}$$

cancelling

$$\frac{\cancel{27}^{3}\cancel{x^9}^{x^7}\cancel{y^{-3}}\cancel{z^6}^{z^4}}{\cancel{9}\cancel{x^2}\cancel{y^2}\cancel{z^2}_{y^5}}$$

$$\boxed{\frac{3x^7z^4}{y^5}}$$

Question 5

Solve the equation $2 - x\sqrt{3} = \frac{7x}{\sqrt{3}}$, giving your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are integers.
State the values of a and b.

[5]

$$2 - x\sqrt{3} = \frac{7x}{\sqrt{3}}$$

$$2\sqrt{3} - 3x = 7x$$

$$2\sqrt{3} = 10x$$

$$\frac{2\sqrt{3}}{10} = x$$

$$\frac{\sqrt{3}}{5} = x$$

$$\boxed{a = 3 \quad b = 5}$$

Question 6

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$.

[2]

(b) Find the value of a.

[2]

(c) Find the value of $\log_{a^2} 8$.

[3]

(a) $\log_a x^m = m \log_a x$ ← Formula booklet

$$\log_a 64 = \log_a 8^2$$

$$= 2\log_a 8$$

$$= 2 \times 3$$

$$\boxed{\log_a 64 = 6}$$

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$. [2]

(b) Find the value of a. [2]

(c) Find the value of $\log_{a^2} 8$. [3]

(b) $a^x = b \iff x = \log_a b$ ↶ Formula booklet

$\log_a 8 = 3 \Rightarrow a^3 = 8$

$a = \sqrt[3]{8}$

$\boxed{a = 2}$

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$. [2]

(b) Find the value of a. [2]

(c) Find the value of $\log_{a^2} 8$. [3]

(c) $a^x = b \iff x = \log_a b$ ↶ Formula booklet

$\log_{2^2} 8 = x \Rightarrow (2^2)^x = 8$

$2^{2x} = 8$

$2^{2x} = 2^3$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

Question 7

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x. [2]

(b) Find an expression for $\log_b 4$ in terms of y. [2]

(c) Find an expression for $\log_b 48$ in terms of x and y. [3]

(a) $\log_b 9 = \log_b 3^2$

$= 2\log_b 3$

$= 2x$

$\log_a x^m = m\log_a x$ ↶ Formula booklet

$\boxed{\log_b 9 = 2x}$

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x.

[2]

(b) Find an expression for $\log_b 4$ in terms of y.

[2]

(c) Find an expression for $\log_b 48$ in terms of x and y.

[3]

(b) $\log_b 4 = \log_b \sqrt{16}$

$= \log_b 16^{1/2}$

$= \frac{1}{2}\log_b 16$

$= \frac{1}{2} y$

$\boxed{\log_b 4 = \frac{1}{2} y}$

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x.

[2]

(b) Find an expression for $\log_b 4$ in terms of y.

[2]

(c) Find an expression for $\log_b 48$ in terms of x and y.

[3]

(c) $\log_a xy = \log_a x + \log_a y$ Formula booklet

$\log_b 48 = \log_b 3 + \log_b 16$

$\boxed{\log_b 48 = x + y}$

Question 8

(a) Show that $\frac{(4-2\sqrt{x})^2}{8x}$ can be written as $2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$.

[2]

(b) Given that $8\sqrt{2} = 2^a$, find the value of a.

[2]

(c) Show that $\frac{x(2x^4-\sqrt{x})}{x^2}$ can be written as $2x^a - x^b$, where a and b are rational numbers. State the value of a and b.

[2]

(a) Expand the numerator

$$\frac{16 - 16\sqrt{x} + 4x}{8x}$$

Split into 3 separate terms and cancel

$$\frac{\cancel{16}^{2}}{\cancel{8}_{1}x} - \frac{\cancel{16}^{2}\sqrt{x}}{\cancel{8}_{1}x} + \frac{\cancel{4}^{1}\cancel{x}}{\cancel{8}_{2}\cancel{x}}$$

$$\frac{2}{x} - \frac{2\sqrt{x}}{x} + \frac{1}{2}$$

Rewrite powers of x

$$2x^{-1} - 2x^{1/2}x^{-1} + \frac{1}{2}$$

$$2x^{-1} - 2x^{-1/2} + \frac{1}{2}$$

$$\boxed{2x^{-1} - 2x^{-1/2} + \frac{1}{2}}$$

(a) Show that $\frac{(4-2\sqrt{x})^2}{8x}$ can be written as $2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$.

[2]

(b) Given that $8\sqrt{2} = 2^a$, find the value of a.

[2]

(c) Show that $\frac{x(2x^4-\sqrt{x})}{x^2}$ can be written as $2x^a - x^b$, where a and b are rational numbers. State the value of a and b.

[2]

(b) $8 = 2^3$, $\sqrt{2} = 2^{1/2}$

$$8\sqrt{2} = 2^3 \times 2^{1/2}$$

$$= 2^{7/2}$$

$$\boxed{a = \frac{7}{2}}$$

(a) Show that $\frac{(4-2\sqrt{x})^2}{8x}$ can be written as $2x^{-1}-2x^{-\frac{1}{2}}+\frac{1}{2}$.

[2]

(b) Given that $8\sqrt{2}=2^a$, find the value of a.

[2]

(c) Show that $\frac{x(2x^4-\sqrt{x})}{x^2}$ can be written as $2x^a-x^b$, where a and b are rational numbers. State the value of a and b.

[2]

(c) Expand the numerator

$$\frac{2x^5 - x^{3/2}}{x^2}$$

Simplify the powers of x

$$2x^3 - x^{-1/2}$$

$$\boxed{a = 3 \quad b = -\frac{1}{2}}$$

Question 9

Solve the equation $16^x - 3(4^{x+1}) = 28$. Write your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers.

[5]

Rewrite expression using powers of 4

$(4^2)^x - 3(4^x)(4^1) = 28$

$(4^x)^2 - 12(4^x) = 28$

Let $m = 4^x$

$m^2 - 12m - 28 = 0$

Solve the quadratic by hand or using the GDC

$(m-14)(m+2) = 0$

$m = 14$ or $m = -2$

m cannot be negative because you can't take a log of a negative

$4^x = 14$

Take ln of both sides

$\ln 4^x = \ln 14$

$x \ln 4 = \ln 14$

$\log_a x^m = m \log_a x$ Formula booklet

$$\boxed{x = \frac{\ln 14}{\ln 4}}$$

EXAM PAPERS PRACTICE

Question 10

$\sqrt{425}$ can be written in the form $a\sqrt{b}$. Find the values of a and b. Show all of your working.

[5]

Find the largest square number that goes into 425

$\sqrt{425} = \sqrt{25 \times 17}$

$= 5\sqrt{17}$

$\boxed{a = 5 \quad b = 17}$