Question 1

Let $\log_2 16 = \log_2 a^b$, where a and b are integers and $a < b$.

(a) (i) Find the values of a and b.

(ii) Hence, or otherwise, find the value of $\log_2 16$.

[3]

Let $\log 25 + \log 4 = \log c$.

(b) (i) Find the value of c.

(ii) Hence, or otherwise, find the value of $\log 25 + \log 4$.

[2]

Let $\log_5 500 - \log_5 4 = \log_5 d$.

(c) (i) Find the value of d.

(ii) Hence, or otherwise, find the value of $\log_5 500 - \log_5 4$.

[2]

(a) (i) $16 = a^b$

$16 = 4^2$ or 2^4

$a = 2, \quad b = 4$

(ii) $\log_2 16 = \log_2 2^4$ ← Formula booklet: $\log_a x^m = m \log_a x$

$= 4 \log_2 2$

$= 4 \times 1$

$\log_2 16 = 4$

Let $\log_2 16 = \log_2 a^b$, where a and b are integers and $a < b$.

(a) (i) Find the values of a and b.

(ii) Hence, or otherwise, find the value of $\log_2 16$.

[3]

Let $\log 25 + \log 4 = \log c$.

(b) (i) Find the value of c.

(ii) Hence, or otherwise, find the value of $\log 25 + \log 4$.

[2]

Let $\log_5 500 - \log_5 4 = \log_5 d$.

(c) (i) Find the value of d.

(ii) Hence, or otherwise, find the value of $\log_5 500 - \log_5 4$.

[2]

(b) (i) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 25 + \log 4 = \log (25 \times 4)$

$= \log 100$

$c = 100$

(ii) $a^x = b \Leftrightarrow x = \log_a b \qquad a > 0, b > 0, a \neq 1$ ← Formula booklet

$\log 100 = x$

$100 = 10^x$

$x = 2$

Let $\log_2 16 = \log_2 a^b$, where a and b are integers and $a < b$.

(a) (i) Find the values of a and b.

(ii) Hence, or otherwise, find the value of $\log_2 16$.

[3]

Let $\log 25 + \log 4 = \log c$.

(b) (i) Find the value of c.

(ii) Hence, or otherwise, find the value of $\log 25 + \log 4$.

[2]

Let $\log_5 500 - \log_5 4 = \log_5 d$.

(c) (i) Find the value of d.

(ii) Hence, or otherwise, find the value of $\log_5 500 - \log_5 4$.

[2]

(c) (i) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log_5 500 - \log_5 4 = \log_5 \left(\frac{500}{4}\right)$

$= \log_5 125$

$d = 125$

(ii) $a^x = b \Leftrightarrow x = \log_a b \qquad a > 0, b > 0, a \neq 1$ ← Formula booklet

$\log_5 125 = x$

$125 = 5^x$

$x = 3$

Question 2

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$.

[2]

(b) $\ln 45$.

[2]

(c) $\ln 135$.

[3]

(a) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\ln 5 = \ln 15 - \ln 3$

$\ln 5 = x - y$

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$.

[2]

(b) $\ln 45$.

[2]

(c) $\ln 135$.

[3]

(b) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\ln 45 = \ln 15 + \ln 3$

$\ln 45 = x + y$

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$.

(b) $\ln 45$.

(c) $\ln 135$.

[2]

[2]

[3]

$\ln 45 = x + y$

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\ln 135 = \ln 45 + \ln 3$

$= (x + y) + y$

$\ln 135 = x + 2y$

Question 3

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$.

(b) $\log 3$.

(c) $\log 72$.

[2]

[3]

[3]

(a) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 24 = \log 2 + \log 12$

$\log 24 = r + s$

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$.

(b) $\log 3$.

(c) $\log 72$.

[2]

[3]

[3]

(b) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 4 = \log 2 + \log 2$

$\log 4 = 2r$

$\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log 3 = \log 12 - \log 4$

$\log 3 = s - 2r$

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$.

[2]

(b) $\log 3$.

[3]

(c) $\log 72$.

[3]

(c) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log 6 = \log 12 - \log 2$

$= s - r$

$\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 72 = \log 6 + \log 12$

$= (s - r) + s$

$\log 72 = 2s - r$

Question 4

Simplify the following equations:

(a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$.

[2]

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$.

[2]

(c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$.

[2]

a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$ ↓ expand numerator

$\frac{-48x^{-3}y^{10}}{6x^2y}$ ↓ cancelling

$\frac{-48^{-8}x^{-3}y^{10}y^{9}}{6x^2y}$

x^5

$\frac{-8y^9}{x^5}$

Simplify the following equations:

(a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$.

[2]

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$.

[2]

(c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$.

[2]

b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$ ↓ rewrite as fraction

$\frac{(4x^2y^3)^4}{(2x^{-1}y^{-2})^3}$ ↓ expand numerator and denominator

$\frac{256x^8y^{12}}{8x^{-3}y^{-6}}$ ↓ cancelling

$\frac{256^{32}x^8x^{11}y^{12}y^{18}}{8x^{-3}y^{-6}}$

$32x^{11}y^{18}$

Simplify the following equations:

(a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$.

[2]

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$.

[2]

(c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$.

[2]

c) $\sqrt{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$

rewrite as a fraction and use indice laws

$\frac{(9x^6y^{-2}z^4)^{\frac{3}{2}}}{(3xyz)^2}$

expand numerator and denominator

$\frac{27x^9y^{-3}z^6}{9x^2y^2z^2}$

cancelling

$\frac{3^{1}27x^{7}y^{-3}z^{6}z^{4}}{9x^{2}y^{2}z^{2}}$

y^5

$\frac{3x^7z^4}{y^5}$

Question 5

Solve the equation $2 - x\sqrt{3} = \frac{7x}{\sqrt{3}}$, giving your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are integers.
State the values of a and b.

[5]

$2 - x\sqrt{3} = \frac{7x}{\sqrt{3}}$

$2\sqrt{3} - 3x = 7x$

$2\sqrt{3} = 10x$

$\frac{2\sqrt{3}}{10} = x$

$\frac{\sqrt{3}}{5} = x$

$a = 3 \quad b = 5$

Question 6

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$.

[2]

(b) Find the value of a.

[2]

(c) Find the value of $\log_{a^2} 8$.

[3]

(a) $\log_a x^m = m\log_a x$ ← Formula booklet

$\log_a 64 = \log_a 8^2$

$= 2\log_a 8$

$= 2 \times 3$

$\log_a 64 = 6$

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$.

[2]

(b) Find the value of a.

[2]

(c) Find the value of $\log_{a^2} 8$.

[3]

(b) $a^x = b \iff x = \log_a b$ ← Formula booklet

$\log_a 8 = 3 \Rightarrow a^3 = 8$

$a = \sqrt[3]{8}$

$a = 2$

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$.

[2]

(b) Find the value of a.

[2]

(c) Find the value of $\log_{a^2} 8$.

[3]

(c) $a^x = b \iff x = \log_a b$ ← Formula booklet

$\log_{2^2} 8 = x \Rightarrow (2^2)^x = 8$

$2^{2x} = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

Question 7

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x.

[2]

(b) Find an expression for $\log_b 4$ in terms of y.

[2]

(c) Find an expression for $\log_b 48$ in terms of x and y.

[3]

(a) $\log_b 9 = \log_b 3^2$

$= 2\log_b 3$ ← $\log_a x^m = m\log_a x$ ← Formula booklet

$= 2x$

$\log_b 9 = 2x$

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x.

[2]

(b) Find an expression for $\log_b 4$ in terms of y.

[2]

(c) Find an expression for $\log_b 48$ in terms of x and y.

[3]

(b) $\log_b 4 = \log_b \sqrt{16}$

$= \log_b 16^{\frac{1}{2}}$

$= \frac{1}{2} \log_b 16$

$= \frac{1}{2} y$

$\log_b 4 = \frac{1}{2} y$

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x.

[2]

(b) Find an expression for $\log_b 4$ in terms of y.

[2]

(c) Find an expression for $\log_b 48$ in terms of x and y.

[3]

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log_b 48 = \log_b 3 + \log_b 16$

$\log_b 48 = x + y$

Question 8

(a) Show that $\frac{(4-2\sqrt{x})^2}{8x}$ can be written as $2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$.

[2]

(b) Given that $8\sqrt{2} = 2^a$, find the value of a.

[2]

(c) Show that $\frac{x(2x^4-\sqrt{x})}{x^2}$ can be written as $2x^a - x^b$, where a and b are rational numbers. State the value of a and b.

[2]

(a) Expand the numerator

$\frac{16 - 16\sqrt{x} + 4x}{8x}$

Split into 3 separate terms and cancel

$\frac{\cancel{16}^{2}}{\cancel{8x}_{1}} - \frac{\cancel{16}^{2}\sqrt{x}}{\cancel{8x}_{1}} + \frac{\cancel{4x}^{1}}{\cancel{8x}_{2}}$

$\frac{2}{x} - \frac{2\sqrt{x}}{x} + \frac{1}{2}$

Rewrite powers of x

$2x^{-1} - 2x^{\frac{1}{2}}x^{-1} + \frac{1}{2}$

$2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$

$2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$

(a) Show that $\frac{(4-2\sqrt{x})^2}{8x}$ can be written as $2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$.

[2]

(b) Given that $8\sqrt{2} = 2^a$, find the value of a.

[2]

(c) Show that $\frac{x(2x^4-\sqrt{x})}{x^2}$ can be written as $2x^a - x^b$, where a and b are rational numbers. State the value of a and b.

[2]

(b) $8 = 2^3$, $\sqrt{2} = 2^{1/2}$

$8\sqrt{2} = 2^3 \times 2^{1/2}$

$= 2^{7/2}$

$a = \frac{7}{2}$

(a) Show that $\frac{(4-2\sqrt{x})^2}{8x}$ can be written as $2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$.

[2]

(b) Given that $8\sqrt{2} = 2^a$, find the value of a.

[2]

(c) Show that $\frac{x(2x^4-\sqrt{x})}{x^2}$ can be written as $2x^a - x^b$, where a and b are rational numbers. State the value of a and b.

[2]

(c) Expand the numerator

$\frac{2x^5 - x^{3/2}}{x^2}$

Simplify the powers of x

$2x^3 - x^{-1/2}$

$a = 3 \quad b = -\frac{1}{2}$

Question 9

Solve the equation $16^x - 3(4^{x+1}) = 28$. Write your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers.

[5]

Rewrite expression using powers of 4

$(4^2)^x - 3(4^x)(4^1) = 28$

$(4^x)^2 - 12(4^x) = 28$

Let $m = 4^x$

$m^2 - 12m - 28 = 0$

Solve the quadratic by hand or using the GDC

$(m - 14)(m + 2) = 0$

$m = 14$ or $m = -2$ ← m cannot be negative because you can't take a log of a negative

$4^x = 14$

Take ln of both sides

$\ln 4^x = \ln 14$

$x \ln 4 = \ln 14$ ← $\log_a x^m = m \log_a x$ Formula booklet

$x = \frac{\ln 14}{\ln 4}$

Question 10

$\sqrt{425}$ can be written in the form $a\sqrt{b}$. Find the values of a and b. Show all of your working.

[5]

Find the largest square number that goes into 425

$\sqrt{425} = \sqrt{25 \times 17}$

$= 5\sqrt{17}$

$a = 5 \quad b = 17$

Question 11

The expression $a^{\frac{1}{5}} \times a^{\frac{2}{5}}$ can be expressed in the form a^p.

(a) Find the value of p.

[1]

(a) $a^{1/5} \times a^{2/5} = a^{3/5}$

$p = \frac{3}{5}$

Let $a^{\frac{1}{5}} \times a^{\frac{2}{5}} = 8$.

(b) Find the value of a.

[2]

The expression $b \times b^{-\frac{3}{2}}$ can be written in the form $\frac{1}{b^q}$.

(c) Find the value of q.

[1]

Let $b \times b^{-\frac{3}{2}} = \sqrt{2}$

(d) Find the value of b.

[2]

The expression $a^{\frac{1}{5}} \times a^{\frac{2}{5}}$ can be expressed in the form a^p.

(a) Find the value of p.

$p = \frac{3}{5}$

[1]

Let $a^{\frac{1}{5}} \times a^{\frac{2}{5}} = 8$.

(b) Find the value of a.

[2]

(b) $a^{3/5} = 8$

$a = \sqrt[3]{8^5}$

$a = 32$

The expression $b \times b^{-\frac{3}{2}}$ can be written in the form $\frac{1}{b^q}$.

(c) Find the value of q.

[1]

Let $b \times b^{-\frac{3}{2}} = \sqrt{2}$

(d) Find the value of b.

[2]

The expression $a^{\frac{1}{5}} \times a^{\frac{2}{5}}$ can be expressed in the form a^p.

(a) Find the value of p.

[1]

Let $a^{\frac{1}{5}} \times a^{\frac{2}{5}} = 8$.

(b) Find the value of a.

[2]

The expression $b \times b^{-\frac{3}{2}}$ can be written in the form $\frac{1}{b^q}$.

(c) Find the value of q.

[1]

Let $b \times b^{-\frac{3}{2}} = \sqrt{2}$

(d) Find the value of b.

[2]

(c) $b \times b^{-3/2} = b^{-1/2}$

$= \frac{1}{b^{1/2}}$

$q = \frac{1}{2}$

(d) $\frac{1}{b^{1/2}} = \sqrt{2}$

$b^{1/2} = \frac{1}{\sqrt{2}}$

$b = \frac{1}{2}$

The expression $a^{\frac{1}{5}} \times a^{\frac{2}{5}}$ can be expressed in the form a^p.

(a) Find the value of p.

[1]

Let $a^{\frac{1}{5}} \times a^{\frac{2}{5}} = 8$.

(b) Find the value of a.

[2]

The expression $b \times b^{-\frac{3}{2}}$ can be written in the form $\frac{1}{b^q}$.

(c) Find the value of q.

$q = \frac{1}{2}$

[1]

Let $b \times b^{-\frac{3}{2}} = \sqrt{2}$

(d) Find the value of b.

[2]