Exponential Growth

Model Answers

Question 1

Alex invests $200 for 2 years at a rate of 2% per year simple interest.
Chris invests $200 for 2 years at a rate of 2% per year compound interest.
Calculate how much more interest Chris has thanAlex.

Answer:

*First, we calculate the interest Alex earns. Simple interest is calculated using the formula I = PRT, where P is the principal amount, R is the rate of interest, and T is the time in years. For Alex, P = $200, R = 2% = 0.02, and T = 2 years. So, I = 200 * 0.02 * 2 = $8.*

*Next, we calculate the interest Chris earns. Compound interest is calculated using the formula A = P(1 + r/n)^(nt), where A is the amount of money accumulated after n years, including interest, P is the principal amount, r is the annual interest rate (in decimal), n is the number of times that interest is compounded per year, and t is the time the money is invested for in years. For Chris, P = $200, r = 2% = 0.02, n = 1 (since it's compounded annually), and t = 2 years. So, A = 200(1 + 0.02/1)^(1*2) = $204.08. The interest Chris earns is the total amount minus the principal, which is $204.08 - $200 = $4.08. Therefore, Chris has $4.08 - $8 = -$3.92 less interest than Alex. So, Alex actually earns more interest than Chris in this case.*

Question 2

The population of Olton is decreasing at a rate of 3% per year.
In 2013, the population was 50000.
Calculate the population after 4 years.
Give your answer correct to the nearest hundred.

Answer:

First, we need to understand that a decrease of 3% per year means that each year, the population is 97% of what it was the previous year (since 100% - 3% = 97%).

*Next, we need to apply this decrease over 4 years. We do this by multiplying the population by 0.97 (which is the decimal equivalent of 97%) four times. So, the calculation would be: 50000 * 0.97 * 0.97 * 0.97 * 0.97 = 44177.13 Rounding to the nearest hundred gives us 44200. So, the population of Olton after 4 years would be approximately 44200.*

Question 3

The value of a motorbike is $12400.
Each year, the value of the motorbike decreases exponentially by 15%.
Calculate the value of the motorbike after 3 years.

Answer:

The value of the motorbike after 3 years can be calculated using the formula:
*V = $12400 * (0.85) ** 3*
where V is the value of the motorbike after 3 years.

Therefore, the value of the motorbike after 3 years is $7812.50.

Question 4

Maryah borrows $12 000 to start abusiness.
The loan is for 3 years at a rate of 5% per year compound interest.
The loan has to be paid back at the end of the 3 years.

Calculate the total amount to be paid back.

Answer:

To calculate the total amount to be paid back, we need to find the future value of the loan. The formula for calculating the future value of a loan is:

*FV = P * (1 + r/n)^(n*t)*

*FV = $12,000 * (1 + 0.05/1)^(1*3)*
*= $12,000 * (1.05)^3*
*= $12,000 * 1.157625*
= $13,891.50

Therefore, Maryah will have to pay back a total of $13,891.50 at the end of the 3 years

Question 5

Bruce invested $420 at a rate of 4% per year compound interest.
Calculate the total amount Bruce has after 2 years.
Give your answer correct to 2 decimal places

Answer:

First, we need to understand that compound interest is calculated annually on the initial principal as well as the accumulated interest of previous periods. The formula for compound interest is A = P(1 + r/n)^(nt), where: - A is the amount of money accumulated after n years, including interest. - P is the principal amount (the initial amount of money). - r is the annual interest rate (in decimal). - n is the number of times that interest is compounded per year. - t is the time the money is invested for in years. In this case, Bruce invested $420 (P = 420), the annual interest rate is 4% or 0.04 (r = 0.04), the interest is compounded once per year (n = 1), and the money is invested for 2 years (t = 2).

*Substituting these values into the formula, we get: A = 420(1 + 0.04/1)^(1*2) A = 420(1 + 0.04)^2 A = 420(1.04)^2 A = 420*1.0816 A = $453.87 So, after 2 years, Bruce will have $453.87.*

Question 6

Carol invests $6250 at a rate of 2% per year compound interest.
Calculate the total amount Carol has after 3 years

Answer:

If Carol invests $6250 at a rate of 2% per year compound interest, she will have a total of $6,816.51 after 3 years

The formula for calculating compound interest is:
A = P(1 + r/n)^(nt)

Using this formula, we can calculate that Carol's investment will grow to:
*A = 6250(1 + 0.02/1)^(1*3)*
A = $6,816.51

Question 7

Acri invested $500 for 3 years at a rate of 2.8% per year compound interest.
Calculate the final amount he has after 3 years

Answer:

First, we need to understand the formula for compound interest, which is A = P(1 + r/n)^(nt), where: - A is the amount of money accumulated after n years, including interest. - P is the principal amount (the initial amount of money). - r is the annual interest rate (in decimal form). - n is the number of times that interest is compounded per year. - t is the time the money is invested for in years. In this case, Acri invested $500 (P) for 3 years (t) at a rate of 2.8% per year (r) compounded annually (n=1).

*So, we can plug these values into the formula: A = 500(1 + 0.028/1)^(1*3) A = 500(1 + 0.028)^3 A = 500(1.028)^3 A = 500 * 1.086144 A = $543.07 So, the final amount Acri has after 3 years is approximately $543.07.*

Question 8

Pedro invested $800 at a rate of 5% per year compound interest.
Calculate the total amount he has after 2 years.

Answer:

After 2 years, Pedro's investment of $800 at a rate of 5% per year compound interest would have grown to $882.021.

The formula for calculating compound interest is:
A = P(1 + r/n)^(nt)
where:

- *A is the amount after t years.*
- *P is the principal amount.*
- *r is the annual interest rate.*
- *n is the number of times the interest is compounded per year.*

Using this formula, we can calculate that Pedro's investment would have grown to:
*A = 800(1 + 0.05/1)^(1*2)*
A = $882.02

Question 9

Eva invests $120 at a rate of 3% per year compound interest.
Calculate the total amount Eva has after 2 years.
Give your answer correct to 2 decimal places

Answer:

The formula for compound interest is: A = P (1 + r/n)^(nt) where: A = the amount of money accumulated after n years, including interest. P = the principal amount (the initial amount of money) r = annual interest rate (in decimal) n = number of times that interest is compounded per year t = time the money is invested for in years

*Eva's principal amount (P) is $120, the annual interest rate (r) is 3% or 0.03 in decimal form, the number of times the interest is compounded per year (n) is 1 (since it doesn't specify otherwise, we assume it's compounded annually), and the time (t) is 2 years. So, we substitute these values into the formula: A = 120 (1 + 0.03/1)^(1*2) A = 120 (1 + 0.03)^2 A = 120 * 1.03^2 A = 120 * 1.0609 A = $127.30 So, after 2 years, Eva will have $127.30.*

Question 10

Johan invested $600 for 3 years at 4% per year compound interest.
Calculate the final amount he had after three year

Answer:

You can use the following formula to calculate the final amount of an investment with compound interest:
*A = P * (1 + r/n)^(n*t)*
Where:
A is the final amount
P is the principal amount
r is the annual interest rate
n is the number of times the interest is compounded per year
t is the number of years
In this case, we have:

Substituting these values into the formula, we get:

*A = $600 * (1 + 0.04/1)^(1*3) = $674.92*

Therefore, Johan's final amount would be $674.92

Question 11

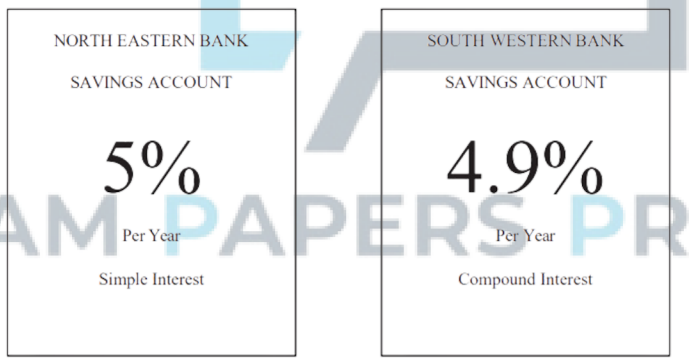
Nikhil invests $200 for 2 years at 4% per year compound interest.
Calculate the exact amount Nikhil has after 2 years

Answer:

The formula for compound interest is A = P(1 + r/n)^(nt), where: - A is the amount of money accumulated after n years, including interest. - P is the principal amount (the initial amount of money). - r is the annual interest rate (in decimal). - n is the number of times that interest is compounded per year. - t is the time the money is invested for in years. In this case, Nikhil's principal amount (P) is $200, the annual interest rate (r) is 4% or 0.04 in decimal, the number of times interest is compounded per year (n) is 1 (since it's not mentioned, we assume it's compounded annually), and the time (t) is 2 years.

*Substituting these values into the formula, we get: A = 200(1 + 0.04/1)^(1*2) A = 200(1 + 0.04)^2 A = 200(1.04)^2 A = 200 * 1.0816 A = $216.32 So, Nikhil will have $216.32 after 2 years.b*

Question 12



Kalid and his brother have $2000 each to invest for 3 years.
(a) North Eastern Bank advertises savings with simple interest at 5% per year.
Kalid invests his money in this bank.
How much money will he have at the end of 3 years

Answer:

*First, we need to calculate the simple interest that Kalid will earn in 3 years. The formula for simple interest is: Interest = Principal * Rate * Time*
*2000 * 0.05 * 3 = 300*
300 is the interest over 3 years. Since he started with 2000 + 300 = 2300 at the end of 3 years.

(b.) South Western Bank advertises savings with compound interest at 4.9% per year.
Kalid's brother invests his money in this bank.
At the end of 3 years, how much more money will he have than Kalid?

Answer:

8.64

Question 12

Marcel invests $2500 for 3 years at a rate of 1.6% per year simple interest.
Jacques invests $2000 for 3 years at a rate of x% per year compound interest.
At the end of the 3 years Marcel and Jacques receive the same amount of interest.
Calculate the value of x correct to 3 significant figures.

Answer:

*First, we calculate the amount of interest Marcel receives. Simple interest is calculated using the formula I = PRT, where I is the interest, P is the principal amount, R is the rate of interest, and T is the time. So, Marcel's interest is I = 2500 * 0.016 * 3 = $120. Now, we know that Jacques also receives $120 in interest. However, his interest is compounded annually. The formula for compound interest is A = P(1 + r/n)^(nt), where A is the amount of money accumulated after n years, including interest, P is the principal amount, r is the annual interest rate, n is the number of times that interest is compounded per year, and t is the time the money is invested for. In Jacques' case, we know that A = P + I = 2000 + 120 = $2120, P = $2000, n = 1 (since the interest is compounded annually), and t = 3. We need to find r, the annual interest rate. So, we can set up the equation 2120 = 2000(1 + r/1)^(1*3) and solve for r. First, divide both sides by 2000 to get 1.06 = (1 + r)^3. Then, take the cube root of both sides to get 1.0198 = 1 + r.*

Finally, subtract 1 from both sides to get r = 0.0198, or 1.98%. So, the value of x is approximately 1.98% to three significant figures.

Question 13

The population of the world grows exponentially at a rate of 1.1% per year.
Find the number of years it takes for the population to grow from 7 billion to 7.31 billion.
Give your answer correct to the nearest whole number.

Answer:

According to the United Nations Department of Economic and Social Affairs, the global population grows at a rate of 1.1% per year

Question 14

The population of the world grows exponentially at a rate of 1.1% per year.
Find the number of years it takes for the population to grow from 7 billion to 7.31 billion.
Give your answer correct to the nearest whole number.

Answer:

First, we need to find out how much the population has grown. We do this by dividing the final population by the initial population. 7.31 billion / 7 billion = 1.04428571429 This means the population has grown by approximately 4.43%.

Next, we need to find out how many years it takes for the population to grow by this amount at a rate of 1.1% per year. We do this by dividing the total growth by the annual growth rate. 4.43% / 1.1% = 4.03 years So, it takes approximately 4 years for the population to grow from 7 billion to 7.31 billion.

Question 15

It is estimated that the world's population is growing at a rate of 1.14% per year.
On January 1st 2014 the population was 7.23 billion.

(a) Find the expected population on January 1st 2020.

Answer:

*We need to calculate the population growth for each year. Since the population is growing at a rate of 1.14% per year, we can use the formula for exponential growth: P = P0 * (1 + r)^t where: P is the final population, P0 is the initial population, r is the growth rate (in decimal form), and t is the time in years. Substituting the given values into the formula, we get: P = 7.23 billion * (1 + 0.0114)^6 Calculating the above expression, we get: P ≈ 7.23 billion * 1.0718 P ≈ 7.75 billion So, the expected population on January 1st 2020 is approximately 7.75 billion.*

(b) Find the year when the population is expected to reach 10 billion

Answer:

To calculate the year when the population is expected to reach 10 billion, we can use the following formula:

t = (ln(N/P)) / (ln(1 + r/100))

where:
t is the number of years it will take for the population to reach N.
P is the initial population.
r is the annual growth rate.
Substituting the given values, we get:

t = (ln(10/7.23)) / (ln(1 + 1.14))
t ≈ 38.6

Therefore, the world population is expected to reach 10 billion in 2052

Question 16

At the start of an experiment there are 20000 bacteria.
The number of bacteria increases at a rate of 30% per hour

(a) Work out the number of bacteria after 4 hours.

Answer:

*First, we need to calculate how many bacteria increase per hour. To do this, we multiply the initial number of bacteria (20000) by the rate of increase (30% or 0.30). 20000 * 0.30 = 6000 bacteria increase per hour. However, this increase is compounded each hour. This means that each hour, the number of bacteria increases by 30% of the current total, not the original total.*

*So, after the first hour, there are 20000 + 6000 = 26000 bacteria. After the second hour, there are 26000 + (26000 * 0.30) = 33800 bacteria. After the third hour, there are 33800 + (33800 * 0.30) = 43940 bacteria. After the fourth hour, there are 43940 + (43940 * 0.30) = 57122 bacteria. So, after 4 hours, there are approximately 57122 bacteria.*

(b) After how many whole hours, from the start of the experiment, will the number of bacteria be greater than one million?

Answer:

We want to find out after how many whole hours, from the start of the experiment, will the number of bacteria be greater than one million. Therefore, we need to solve for h in the following inequality:

*20000 * (1 + 0.3) ^ h > 1000000*

Taking logarithm base 1.3 on both sides, we get:

*log(20000) + h * log(1.3) > log(1000000)*

Solving for h, we get:

h > (log(1000000) - log(20000)) / log(1.3)

Using a calculator, we get that h > 5.6. Therefore, after 6 whole hours, from the start of the experiment, the number of bacteria will be greater than one million.

Question 17

Boris invests $280 for 2 years at a rate of 3% per year compound interest.
Calculate the interest Boris receives at the end of the 2 years.
Give your answer correct to 2 decimal places

Answer:

First, we need to understand the formula for compound interest, which is A = P(1 + r/n)^(nt), where: A = the amount of money accumulated after n years, including interest. P = the principal amount (the initial amount of money) r = annual interest rate (in decimal) n = number of times that interest is compounded per year t = time the money is invested for in years In this case, Boris invests $280 (P) for 2 years (t) at a rate of 3% per year (r) compounded annually (n=1).

*So, we can plug these values into the formula: A = 280(1 + 0.03/1)^(1*2) A = 280(1 + 0.03)^2 A = 280(1.03)^2 A = 280*1.0609 A = $297.05 The interest Boris receives at the end of the 2 years is the total amount minus the initial investment, which is $297.05 - $280 = $17.05. So, Boris receives $17.05 in interest.*

Question 17

Zainab borrows $198 from a bank to pay for a new bed.
The bank charges compound interest at 1.9 % per month.
Calculate how much interest she owes at the end of 3 months.
Give your answer correct to 2 decimal places

Answer:

A = P (1 + r/n)^nt

where A is the amount of money accumulated after n years, including interest. P is the principal amount (the initial amount of money), r is the annual interest rate (in decimal), n is the number of times that interest is compounded per year, and t is the time the money is invested in years.

First, we need to convert the monthly interest rate to a decimal:

r = 1.9/100 = 0.019

Next, we need to calculate the total amount of interest Zainab owes after 3 months:

t = 3/12 = 0.25 years n = 12 (since interest is compounded monthly)

*A = $198 (1 + 0.019/12)^(12*0.25) A = $200.92*

Therefore, Zainab owes $2.92 in interest at the end of 3 months