



EXAM PAPERS PRACTICE

GCSE OCR Math J560

Equation of a circle

Answers

*"We will help you to
achieve A Star "*



Answer 1

The equation of a curve is $y = a^x$
 A is the point where the curve intersects the y -axis.

(a) State the coordinates of A .

ON y -AXIS, $x = 0$
WHEN $x = 0$, $y = a^0 = 1$

So $A(0, 1)$

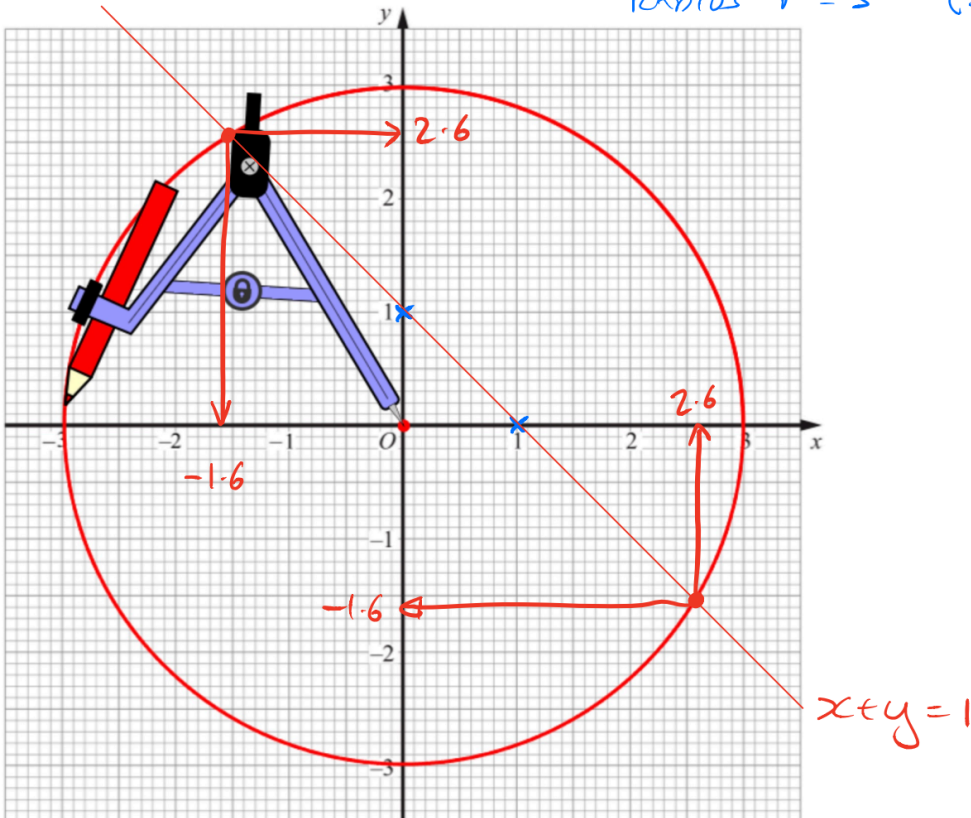
$r^2 = 16$
 $r = 4$

INDICES (POWER)

Answer 2

(a) Construct the graph of $x^2 + y^2 = 9$

CIRCLE, CENTRE ORIGIN
RADIUS $r = 3$ ($3^2 = 9$)





Answer 3

Describe fully the graph which has the equation $x^2 + y^2 = 9$.

$r^2 = 9$ so $r = 3$

$x^2 + y^2 = 9$ IS A CIRCLE
CENTRE $(0,0)$, RADIUS 3.

EQUATION OF A CIRCLE
 $x^2 + y^2 = r^2$
CENTRE $(0,0)$, RADIUS r

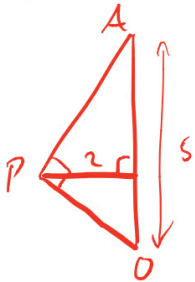
Answer 4

(b) The tangent to the circle at the point P $(-2, 1)$ intersects the y-axis at A.

Show that the area of the triangle APO is 5 square units.

$GRAD_{OP} = \frac{RISE}{RUN} = \frac{-1}{2}$ so $GRAD_{AP} = 2$
(since $-\frac{1}{2} \times 2 = -1$)

So A is 4 UNITS UP from P, so $A(0,5)$



AREA = $\frac{1}{2} \times 5 \times 2$
 $= 5$ SQ UNITS

PERPENDICULAR
GRADIENTS
MULTIPLY TO
GIVE -1

AREA OF TRIANGLE
 $A = \frac{1}{2}bh$

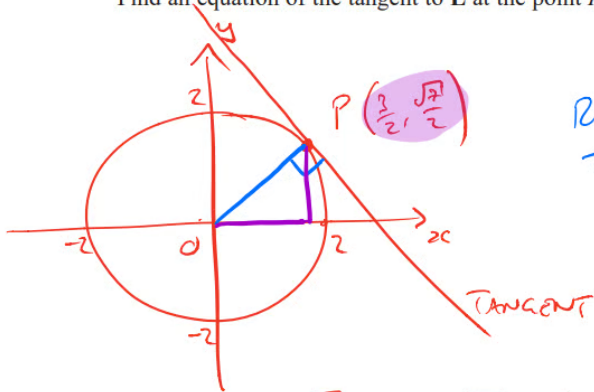


Answer 5

L is the circle with equation $x^2 + y^2 = 4$ $r^2 = 4$
 $r = 2$

$P\left(\frac{3}{2}, \frac{\sqrt{7}}{2}\right)$ is a point on L.

Find an equation of the tangent to L at the point P.



DRAW IT!

RADIUS AND TANGENT MEET AT 90°

EQUATION OF A STRAIGHT LINE
 $y = mx + c$
↑ ↑
GRADIENT y-INTERCEPT

GRADIENT
 $m = \frac{\text{RISE}}{\text{RUN}}$
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
FOR TWO POINTS (x_1, y_1) AND (x_2, y_2)

PERPENDICULAR GRADIENTS MULTIPLY TO GIVE -1

$$m_{OP} = \frac{\text{RISE}}{\text{RUN}} = \frac{\frac{\sqrt{7}}{2}}{\frac{3}{2}} = \frac{\sqrt{7}}{3} \times \frac{2}{3} = \frac{\sqrt{7}}{3}$$

$$m \times m_{OP} = -1 \quad \text{so} \quad m \times \frac{\sqrt{7}}{3} = -1 \times 3$$

$$\frac{m \times \sqrt{7}}{\sqrt{7}} = \frac{-3}{\sqrt{7}}$$

$$m = \underline{\underline{-\frac{3}{\sqrt{7}}}}$$

EQUATION OF TANGENT: $y = -\frac{3}{\sqrt{7}}x + c$

SUB IN $\left(\frac{3}{2}, \frac{\sqrt{7}}{2}\right)$: $\frac{\sqrt{7}}{2} = -\frac{3}{\sqrt{7}} \times \frac{3}{2} + c$

$$\Rightarrow c = \frac{\sqrt{7}}{2} + \frac{9}{2\sqrt{7}}$$

EQUATION: $y = \underline{\underline{-\frac{3}{\sqrt{7}}x + \frac{\sqrt{7}}{2} + \frac{9}{2\sqrt{7}}}}$ (NO FURTHER SIMPLIFICATION REQUIRED)



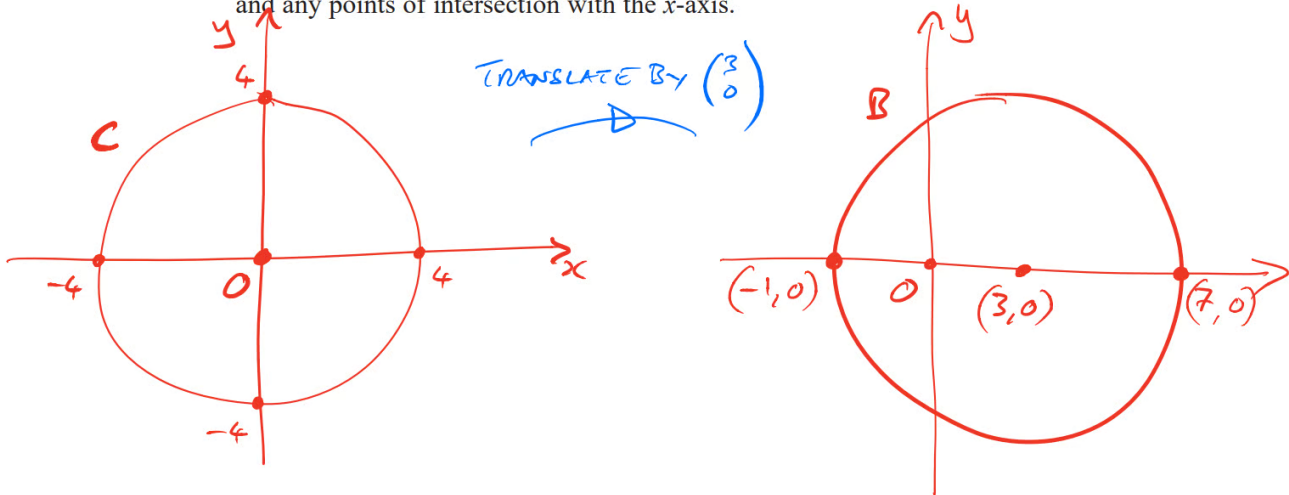
Answer 6

The equation of circle **C** is $x^2 + y^2 = 16$

The circle **C** is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to give circle **B**.

(b) Draw a sketch of circle **B**.

Label with coordinates
the centre of circle **B**
and any points of intersection with the x -axis.



Answer 7

(b) By drawing the line $x + y = 1$ on the grid, solve the equations $x^2 + y^2 = 9$
 $x + y = 1$

$x = 0 : y = 1$

$y = 0 : x = 1$

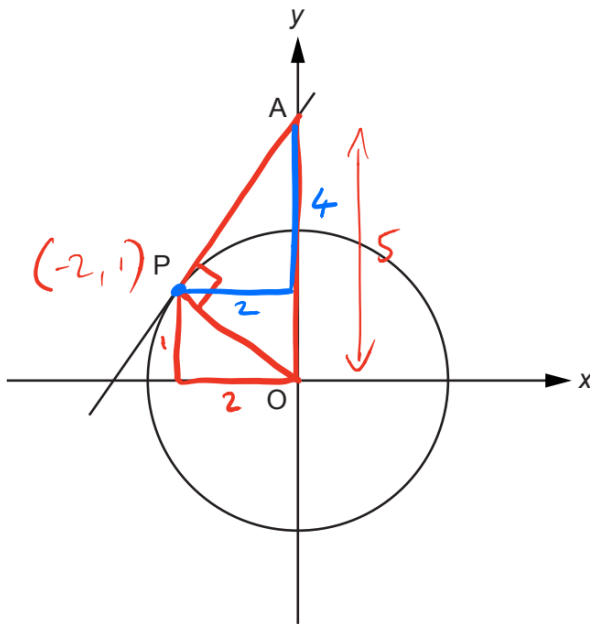
$x = 2.6, y = -1.6$

OR $x = -1.6, y = 2.6$



Answer 8

The diagram shows the circle $x^2 + y^2 = 5$.



EQUATION OF A CIRCLE
 $x^2 + y^2 = r^2$
CENTRE $(0,0)$, RADIUS r

Not to scale

- (a) Mandy says that the point $(2, 1.5)$ lies inside the circle.

Is she correct?
Show how you decide.

$$2^2 + 1.5^2 = 4 + 2.25 \\ = 6.25 > 5$$

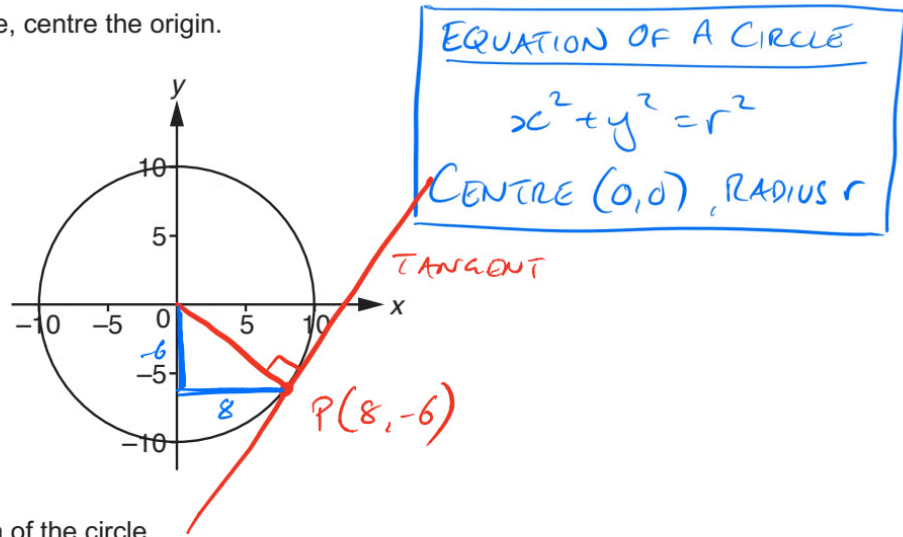
SO MANDY IS WRONG

GRADIENT
 $m = \frac{\text{RISE}}{\text{RUN}}$
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
FOR TWO POINTS
 (x_1, y_1) AND (x_2, y_2)



Answer 9

The diagram shows a circle, centre the origin.



(a) Write down the equation of the circle.

$x^2 + y^2 = 10^2$



Answer 10

(c) Find the equation of the tangent to the circle at point P.

$$\text{GRAD}_{OP} = \frac{\text{RISE}}{\text{RUN}} = \frac{-6}{8} = -\frac{3}{4}$$

$$\text{SO GRAD}_{\text{TANG}} = \frac{4}{3} \quad \left(\text{SINCE } -\frac{3}{4} \times \frac{4}{3} = -1 \right)$$

$$\text{EQN OF TANGENT: } y = \frac{4}{3}x + c$$

$$\text{SUB IN } (8, -6): -6 = \frac{4}{3} \times 8 + c$$

$\frac{-32}{3} \qquad \frac{-32}{3}$

$$-\frac{18}{3} - \frac{32}{3} = c \quad \text{SO } c = -\frac{50}{3}$$

$$\text{SO EQN IS } \underline{y = \frac{4}{3}x - \frac{50}{3}}$$

EQUATION OF A STRAIGHT LINE

$$y = mx + c$$

GRADIENT \uparrow \uparrow y-INTERCEPT

GRADIENT

$$m = \frac{\text{RISE}}{\text{RUN}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

FOR TWO POINTS (x_1, y_1) AND (x_2, y_2)

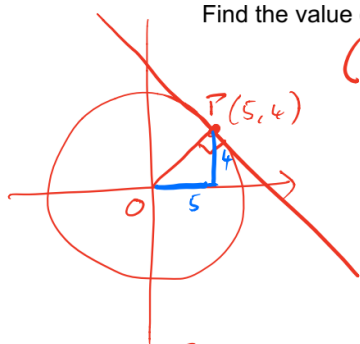
PERPENDICULAR GRADIENTS MULTIPLY TO GIVE -1



Answer 11

- (b) The line $10x + py = q$ is a tangent at the point $(5, 4)$ in another circle with centre $(0, 0)$.

Find the value of p and the value of q .



$$(5, 4) \rightarrow 10x + py = q$$

$$50 + 4p = q$$

$$10x + py = q$$
$$\begin{matrix} -10x & & -10x \end{matrix}$$

$$\frac{py}{p} = \frac{-10x + q}{p}$$

$$y = \frac{-10}{p}x + \frac{q}{p}$$

$$\text{So } -\frac{10}{p} = -\frac{5}{4} \text{ so } \underline{p = 8}$$

$$\text{So } 50 + 4 \times 8 = q \text{ so } \underline{q = 82}$$

GRADIENT

$$m = \frac{\text{RISE}}{\text{RUN}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

FOR TWO POINTS
 (x_1, y_1) AND (x_2, y_2)

EQUATION OF A
STRAIGHT LINE

$$y = mx + c$$

↑ ↑
GRADIENT y-INTERCEPT

PERPENDICULAR
GRADIENTS
MULTIPLY TO
GIVE -1

$$\text{GRAD}_{OP} = \frac{\text{RISE}}{\text{RUN}} = \frac{4}{5}$$

$$\text{GRAD}_{TANG} = \frac{-5}{4}$$

$$\text{(As } -\frac{5}{4} \times \frac{4}{5} \times -1)$$

Answer 12

- (b) Colin says that the point $(5, 7)$ lies outside the circle.

Is Colin correct?
Show your reasoning.

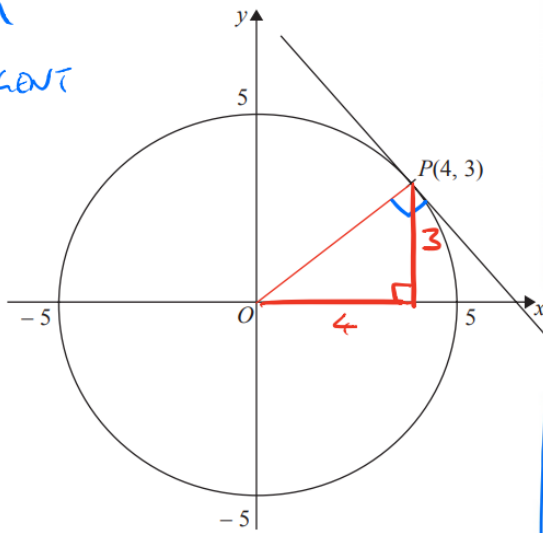
$$\underline{5^2 + 7^2 = 25 + 49 = 74 < 80 \text{ So Colin is incorrect}}$$



Answer 13

Here is a circle, centre O , and the tangent to the circle at the point $P(4, 3)$ on the circle.

CIRCLE THEOREM
RADIUS AND TANGENT
MEET AT 90°



EQUATION OF A STRAIGHT LINE
 $y = mx + c$
↑ ↑
GRADIENT y-INTERCEPT

GRADIENT
 $m = \frac{\text{RISE}}{\text{RUN}}$
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
FOR TWO POINTS
 (x_1, y_1) AND (x_2, y_2)

PERPENDICULAR GRADIENTS MULTIPLY TO GIVE -1

Find an equation of the tangent at the point P .

RADIUS: GRADIENT = $\frac{\text{RISE}}{\text{RUN}} = \frac{3}{4}$

TANGENT: $m \times \frac{3}{4} = -1 \times \frac{4}{3}$

$m = -\frac{4}{3}$

EQN: $y = -\frac{4}{3}x + c$

SUB IN $x=4$ } $\rightarrow 3 = -\frac{4}{3} \times 4 + c$
 $y=3$ }

$3 = -\frac{16}{3} + c$
 $\frac{9}{3} + \frac{16}{3} = c$
 $\Rightarrow c = \frac{25}{3}$

TANGENT IS

$y = -\frac{4}{3}x + \frac{25}{3}$



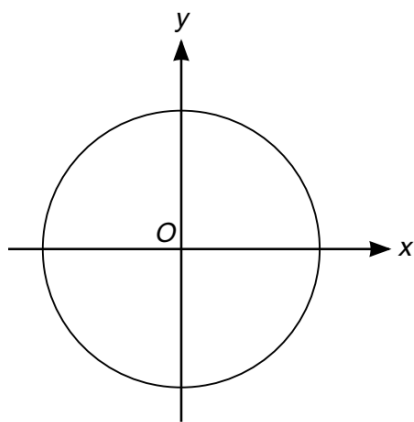
Answer 14

- (b) Point P has coordinates (8, -6).
Show that point P lies on the circle.

$$\begin{aligned}8^2 + (-6)^2 &= 64 + 36 \\ &= 100 \\ &= 10^2 \quad \checkmark\end{aligned}$$

Answer 15

- (a) The diagram shows a circle, centre O.



EQUATION OF A CIRCLE
 $x^2 + y^2 = r^2$
CENTRE (0,0), RADIUS r

CIRCLE
CIRCUMFERENCE
 $C = \pi d (= 2\pi r)$

The circumference of the circle is 20π cm.

Find the equation of the circle.

$$C = \frac{2\pi r}{2} = \frac{20\pi}{2}$$

$$r = 10 \text{ cm}$$

So EQN is $x^2 + y^2 = 10^2$