YOUR NOTES
⬇

1. Number

CONTENTS

1.1 ARITHMETIC

1.1.1 MULTIPLICATION (NON-CALC)

(Non-calculator) multiplication – why so many methods?

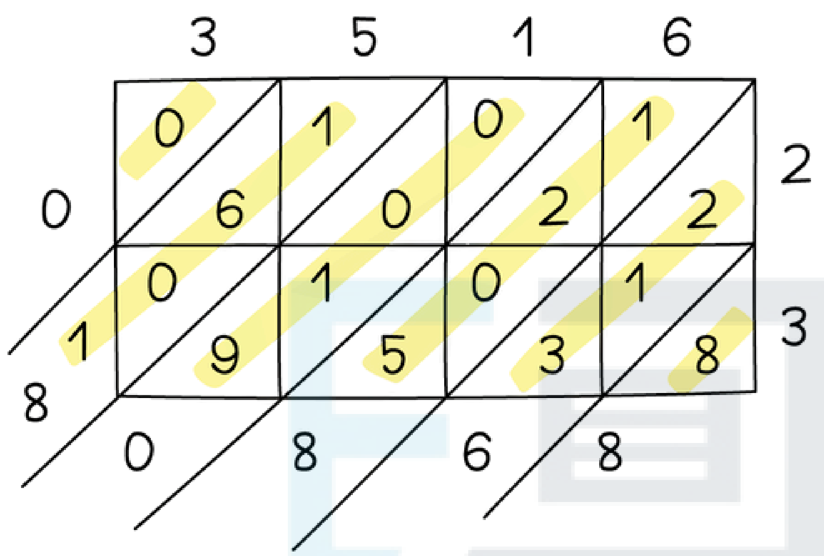
- Different methods work for different people, and some are better depending on the size of number you are dealing with
- We recommend the following 3 methods depending on the size of number you are dealing with
  (If in doubt all methods will work for all numbers!)

1. Number

1. Lattice method

(Best for numbers with two or more digits)

- This method allows you to work with digits
- So in the number 3 516 you would only need to work with the digits 3, 5, 1 and 6
- So if you can multiply up to 9×9 you can't go wrong!



So, 3516 × 23 = 80 868

2. Partition method

(Best when one number has just one digit)

- This method keeps the value of the larger number intact
- So with 3 516 you would use 3000, 500, 10 and 6
- This method is not suitable for two larger numbers as you can end up with a lot of zero digits that are hard to keep track of

	3000	500	10	6
7	21 000	3500	70	42

$$\begin{array}{r} 21\,000 \\ 3500 \\ 70 \\ +\quad 42 \\ \hline 24612 \end{array}$$

So, 3516 × 7 = 24 612

3. Repeated addition method

(Best for smaller, simpler cases)

- You may have seen this called ‘chunking’
- It is a way of building up to the answer using simple multiplication facts that can be worked out easily

eg. 13×23
$1 \times 23 = 23$
$2 \times 23 = 46$
$4 \times 23 = 92$
$8 \times 23 = 184$

So, $13 \times 23 = 1 \times 23 + 4 \times 23 + 8 \times 23 = 23 + 92 + 184 = 299$

Decimals

- These 3 methods can easily be adapted for use with decimal numbers
- You ignore the decimal point whilst multiplying but put it back in the correct place in order to reach a final answer

eg. 1.3×2.3
Ignoring the decimals this is 13×23, which from above is 299
There are two decimal places in total in the question, so there will be two decimal places in the answer

So, $1.3 \times 2.3 = 2.99$

Exam Tip

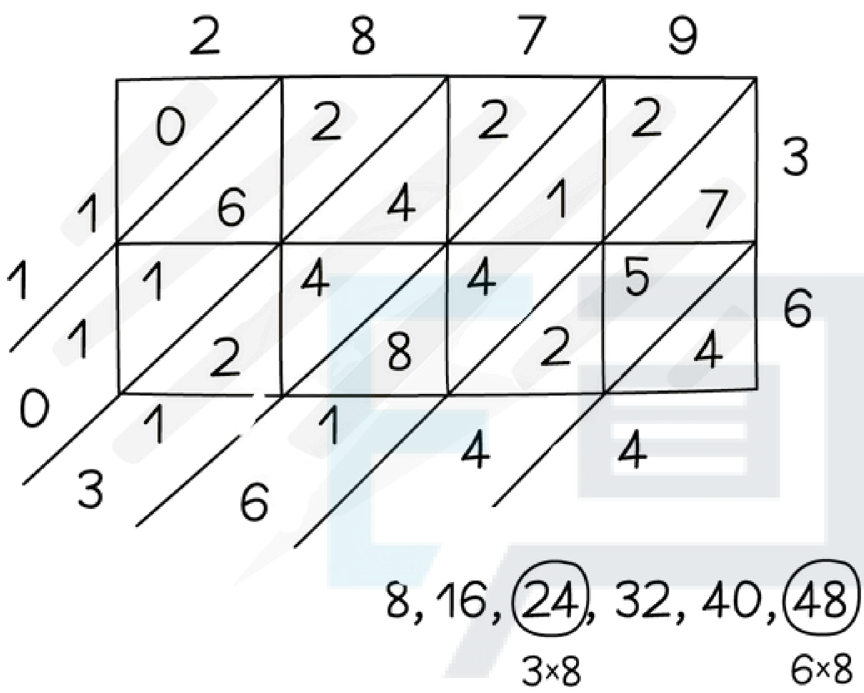
If you do forget your times tables then in the exam write a list out of the table you need as you do a question.
So for example, if you need to multiply by 8, and you’ve forgotten your 8 times tables, write it down: 8, 16, 24, 32, 40, 48, etc. as far as you need to.

1. Number

Worked Example

1. Multiply 2879 by 36

- As you have a 4-digit number multiplied by a 2-digit number then the lattice method (1) is the best choice
- Start with a 4×2 grid....



- Notice the use of listing the 8 times table at the bottom to help with any you may have forgotten
  2879 × 36 = 103 644
- Note that the method would still work if you had set it up as a 2×4 grid

1. Number

2. Pencils are sold in boxes. Each box costs £1.25 and each box contains 15 pencils.

Tyler buys 35 boxes of pencils.

(a) Work out how many pencils Tyler has in total.

(b) Work out the total cost for all the boxes Tyler buys.

(a)

This is a roundabout way of asking you to work out 15×35
As this is a simpl-ish case (3) you should use the repeated addition method

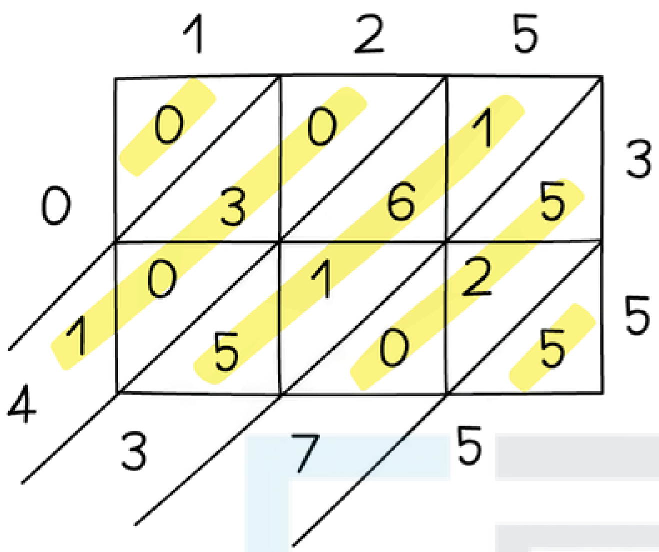
1 × 35 = 35 2
× 35 = 70 4 ×
35 = 140 8 ×
35 = 280 16 ×
35 = 560

It doesn’t matter if you go past 15 ...

15 × 35 = 16 × 35 – 1 × 35 = 560 – 35
15 × 35 = 525

(b)

This question is 1.25×35 so involves decimals (4)
Ignoring the decimals it becomes 125×35 and so the lattice method is best



125 × 35 = 4375

Now count the decimal places from the question and put the decimal point back in the correct place

£1.25 × 35 = £43.75

Exam Tip

Okay, getting the highlighter out during an exam may be a touch excessive! But do use your grid/diagram to help you answer the question – the highlighter in the example above makes it clear which digits to add up at each stage. You can do this in pen or pencil but do make sure you can still read the digits underneath as it is all part of your method/working.

1.1.2 DIVISION (NON-CALC)

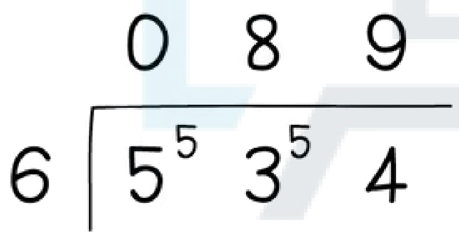
(Non-calculator) division – more methods

- Most students will have seen short division (bus stop method) and long division and there is often confusion between the two
- Fortunately, you only need one – so use short division
- While short division is best when dividing by a single digit, for bigger numbers you need a different approach
- You can use other areas of maths that you know to help – eg. cancelling fractions, “shortcuts” for dividing by 2 and 10, and the repeated addition (“chunking”) method covered in (Non-Calculator) Multiplication

1. Short division (bus stop method)

- Apart from where you can use shortcuts such as dividing by 2 or by 5, this method is best used when dividing by a single digit

eg. 534 ÷ 6



So, 534 ÷ 6 = 89

2. Factoring & cancelling

- This involves treating division as you would if you were asked to cancel fractions
- You can use the fact that with division, most non-calculator questions will have only number answers
- The only thing to be aware of is that this might not be the case if you’ve been asked to write a fraction as a mixed number (but if you are asked to do that it should be obvious from the question)

  eg. 1008 ÷ 28

  1008 ÷ 28 = 504 ÷ 14 = 252 ÷ 7 = 36

- You may have spotted the first two values (1008 and 28) are both divisible by 4 which is fine but if not, divide top and bottom by any number you can
- To do the last part (252 ÷ 7) you can use the short division method above

3. Intelligent repeated addition

- This is virtually identical to the version for multiplication – the process stops when the number dividing into is reachedeg 1674 ÷ 27This is the same as saying ? × 27 = 1674So we can build up in “chunks” of 27 until we get to 16741 × 27 = 27 10 × 27 = 270 20 × 27 = 540 40 × 27 = 1080 60 × 27 = 1620 … by using the last two results added together.Now you are close we can add on 27 one at a time again. 61 × 27 = 1647 62 × 27 = 1674 So 1674 ÷ 27 = 62

4. Dividing by 10, 100, 1000, … (Powers of 10)

- This is a case of moving digits (or decimal points) or knocking off zeros
  eg. 380 ÷ 10 = 38
  45 ÷ 100 = 0.45

5. Dividing by 2, 4, 8, 16, 32, … (Powers of 2)

- This time it is a matter of repeatedly halving
  eg. 280 ÷ 8 = 140 ÷ 4= 70 ÷ 2 = 35
  1504 ÷ 32 = 752 ÷ 16 = 376 ÷ 8 = 188 ÷ 4 = 94 ÷ 2 = 47

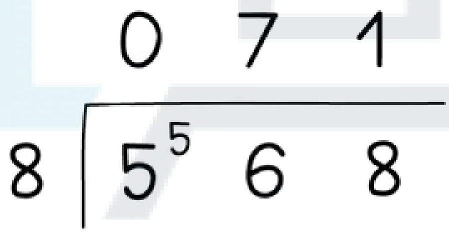
Exam Tip

On the non-calculator paper, division is very likely to have a whole number (exact) answer. So if, when using the repeated addition method, you do not reach this figure then it is likely you’ve made an error in your calculations somewhere.

Worked Example

1. After a fundraising event, the organiser wishes to split the £568 raised between 8 charities. How much will each charity get?

- This is division by a single digit so short division would be an appropriate method
- If you spot it though, 8 is also a power of 2 so you could just halve three times
- Method 1 – short division:



568 ÷ 8 = 71

- Method 2 – powers of 2:

568 ÷ 2 = 284
284 ÷ 2 = 142
142 ÷ 2 = 71
568 ÷ 8 = 71

568 ÷ 8 = 71

You know to halve three times since
2 × 2 × 2 = 8

1. Number

2. A robot packs tins of soup into boxes. Each box holds 24 cans of soup.
The robot has 1824 cans of soup to pack into boxes.
How many boxes will be produced by the robot?

1824 ÷ 24

Both numbers are large so intelligent repeated addition is the best approach

1 × 24 = 24 10 × 24 = 240 20 × 24 = 480 40 × 24 = 960 80 × 24 = 1920 … going too far doesn't matter as you can subtract … 79 × 24 = 1896 78 × 24 = 1872 77 × 24 = 1848 76 × 24 = 1824

- Although this may at first look like a trial and improvement method it is important to show logic throughout as you build the number up – that's why we call it INTELLIGENT repeated addition here at SME!

1824 ÷ 24 = 76

The robot will produce 76 boxes of cans of soup

Exam Question: Easy

Here is part of Gary's electricity bill.

Electricity bill

New reading	7155 units
Old reading	7095 units

Price per unit 15p

Work out how much Gary has to pay for the units of electricity he used.

1. Number

Exam Question: Medium

One sheet of paper is 9×10^{-3} cm thick.

Mark wants to put 500 sheets of paper into the paper tray of his printer.
The paper tray is 4 cm deep.

Is the paper tray deep enough for 500 sheets of paper?
You must explain your answer.

Exam Question: Hard

Each day a company posts some small letters and some large letters.

The company posts all the letters by first class post.

The tables show information about the cost of sending a small letter by first class post and the cost of sending a large letter by first class post.

Small Letter

Weight	First Class Post
0–100 g	60p

Large Letter

Weight	First Class Post
0–100 g	£1.00
101–250 g	£1.50
251–500 g	£1.70
501–750 g	£2.50

One day the company wants to post 200 letters.

The ratio of the number of small letters to the number of large letters is 3 : 2

70% of the large letters weigh 0–100 g.

The rest of the large letters weigh 101–250 g.

Work out the total cost of posting the 200 letters by first class post.

1. Number

1.2 FRACTIONS

1.2.1 MIXED NUMBERS & TOP HEAVY FRACTIONS

What are mixed numbers & top heavy fractions?

- A mixed number has a whole number (integer) part and a fraction
  eg. 3 3/4 means "three and three quarters"
- A top heavy fraction – also called an improper fraction – is one with the top (numerator) bigger than the bottom (denominator)
  eg. 15/4 means "fifteen quarters"

Turning mixed numbers into top heavy fractions

1. Multiply the big number by the bottom (denominator)
2. Add that to the top (numerator)
3. Write as top heavy fraction

Turning top heavy fractions into mixed numbers

- Divide the top by the bottom (to get a whole number and a remainder)
- The whole number is the big number
- The remainder goes over the bottom

Worked Example

1. *Write* $3\frac{3}{4}$ *as a top heavy fraction.*

$3 \times 4 = 12$ — 1 – *multiply the big number by the bottom*

$12 + 3 = 15$ — 2 – *add to the top*

$3\frac{3}{4} = \frac{15}{4}$ — 3 – *your final answer should be top heavy*

2. *Write* $\frac{17}{5}$ *as a mixed number.*

$17 \div 5 = 3$ *remainder* 2 — 4 – *divide the top by the bottom*

$\frac{17}{5} = 3\frac{2}{5}$ — 5 – *final answer is a mixed number*

1. Number

1.2.2 ADDING & SUBTRACTING FRACTIONS

Dealing with mixed numbers

- Always turn Mixed Numbers into Top Heavy Fractions before doing calculations

Adding & Subtracting

- Adding and subtracting are treated in exactly the same way:

1. Find the lowest common bottom (denominator)
2. Write fractions with the new bottoms
3. Multiply tops by same as bottoms
4. Write as a single fraction (take care if subtracting)
5. Simplify the top
6. Turn Top Heavy Fractions back into Mixed Numbers (if necessary)

Worked Example

1. *Work out* $3\frac{3}{4} + \frac{3}{8}$, *giving your answer as a mixed number.*

$3\frac{3}{4} = \frac{3\times4+3}{4} = \frac{15}{4}$ — *First turn the mixed number* $3\frac{3}{4}$ *into a top heavy fraction*

1 – *Spot that the LOWEST common denominator is 8*

$\frac{15}{4} + \frac{3}{8} = \frac{15\times2}{8} + \frac{3}{8}$ — 2, 3 – *Note in this case* $\frac{3}{8}$ *remains unchanged*

$= \frac{15\times2+3}{8}$ — 4

$= \frac{33}{8}$ — 5 – *Be careful, this is NOT your final answer!*

$3\frac{3}{4} + \frac{3}{8} = 4\frac{1}{8}$ — 6 – *In this case the answer has to be a mixed number*

1. Number

1.2.3 MULTIPLYING & DIVIDING FRACTIONS

Dealing with mixed numbers

- Always turn Mixed Numbers into Top Heavy Fractions before doing calculations

Dividing fractions

- Never try to divide fractions
- Instead "flip'n'times"
- So "÷a/b" becomes "×b/a"
- And follow the rules for multiplying…

Multiplying fractions

- Simplify by factorising and cancelling (ignore the × between the fractions)
- Multiply the tops
- Multiply the bottoms
- Simplify by factorising and cancelling (if you missed something earlier)
- Turn Top Heavy Fractions back into Mixed Numbers (if necessary)

Worked Example

1. *Divide* $3\frac{1}{4}$ *by* $\frac{3}{8}$*, giving your answer as a mixed number.*

$3\frac{1}{4} = \frac{3\times4+1}{4} = \frac{13}{4}$	*First turn the mixed number* $3\frac{1}{4}$ *into a top heavy fraction*
$\frac{13}{4} \div \frac{3}{8} = \frac{13}{4} \times \frac{8}{3}$	*A division question – so "flip 'n' times"*
	Now follow the rules for multiplying
$\frac{13}{4} \times \frac{8}{3} = \frac{13}{4} \times \frac{4\times2}{3}$	1. *It is not essential to write 8 as 4 × 2 but you should spot it*
$= \frac{13}{1} \times \frac{2}{3}$	*Cancel the 4's*
$= \frac{26}{3}$	2, 3
	No need to factorise and cancel again (all was done in stage 1)
$3\frac{1}{4} \div \frac{3}{8} = 8\frac{2}{3}$	*In this case the answer has to be a mixed number*

1. Number

Exam Question: Easy

(a) Work out $\frac{1}{7} \times \frac{2}{3}$

(b) Work out $\frac{3}{5} - \frac{1}{3}$

Exam Question: Medium

(a) Work out $1\frac{1}{5} \times 2\frac{1}{3}$

Give your answer as a mixed number in its simplest form.

(b) Work out $2\frac{7}{15} - 1\frac{2}{3}$

1. Number

YOUR NOTES

Exam Question: Hard

(a) Work out $2\frac{1}{4} \times 3\frac{1}{3}$

Give your answer as a mixed number in its simplest form.

(b) Write the numbers 3, 4, 5 and 6 in the boxes to give the greatest possible total.
You may write each number only once.

$$\square\frac{1}{\square} + \square\frac{2}{\square}$$

1. Number

YOUR NOTES
⬇

1.3 BASIC PERCENTAGES

1.3.1 BASIC PERCENTAGES

What is a percentage?

- "Per-cent" simply means " ÷ 100" (or "out of 100")
- You can think of a percentage as a standardised way of expressing a fraction – by always expressing is "out of 100"
- That means it is a useful way of comparing fractions.
- Eg. ½ = 50% (50% means 50 ÷ 100)
  ⅖ = 40% ¾ = 75%

Things to remember:

- Decimal equivalent = percentage ÷ 100
- Percentage = decimal equivalent × 100
- To find "a percentage of A": multiply by the decimal equivalent
- To find "A as a percentage of B": do A ÷ B to get decimal equivalent

Exam Tip

You can always use the decimal equivalent instead of doing a more traditional percentage calculation:
For example, to find 35% of 80 you can just do 80 × 0.35 = 28
(rather than doing the more complicated calulation 80 × 35 ÷ 100)

1. Number

Worked Example

Jamal earns £1200 for a job he does and pays his agent £150 in commission.

Express his agent's commission as a percentage of Jamal's earnings.

1. $Commission \div Earnings = 150 \div 1200$

   $= 0.125$

2. *You should recognise this as the decimal equivalent of* 12.5%

   (*or do* $0.125 \times 100 = 12.5\%$ *if not so confident*)

1. Number

YOUR NOTES
⬇

1.3.2 PERCENTAGE INCREASES & DECREASES

How to increase or decrease by a percentage

- Identify "before" & "after" quantities
- Find percentage of "before" that we want:
    - Increase – add percentage to 100
    - Decrease – subtract percentage from 100
- Write down a statement connecting "before" and "after":
    - "after is a percentage of before"
- Write down the statement as an equation using decimal equivalent
    - remember "is" means "="
- Substitute and solve

Worked Example

Jennie earns £1200 per week in her job.

She is to receive a 5% pay rise.

Find her new weekly pay.

We can call the "before" Old Pay and the "after" New Pay

1. *We want* $100 + 5 = 105\%$

2. *New Pay is 105% of Old Pay*

   The decimal equivalent of 105% is 1.05

3. $\text{New Pay} = 1.05 \times \text{Old Pay}$

4. $\text{New Pay} = 1.05 \times 1200$

   $\text{New Pay} = £1260 \text{ per week}$

1. Number

Exam Question: Easy

Bill's weight decreases from 64.8 kg to 59.3 kg.

Calculate the percentage decrease in Bill's weight.
Give your answer correct to 3 significant figures.

Exam Question: Medium

Railtickets and Cheaptrains are two websites selling train tickets.

Each of the websites adds a credit card charge and a booking fee to the ticket price.

Railtickets	Cheaptrains
Credit card charge: 2.25% of ticket price	Credit card charge: 1.5% of ticket price
Booking fee: 80 pence	Booking fee: £1.90

Nadia wants to buy a train ticket.
The ticket price is £60 on each website.
Nadia will pay by credit card.

Will it be cheaper for Nadia to buy the train ticket from Railtickets or from Cheaptrains?

1. Number

YOUR NOTES
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1.4 REVERSE PERCENTAGES

1.4.1 REVERSE PERCENTAGES

What is a reverse percentage?

- A reverse percentage question is one where we are given the value after a percentage increase or decrease and asked to find the value before the change

How to do reverse percentage questions

- You should do these in exactly the same way as percentage increase & decrease questions!
- Identify "before" & "after" quantities
- FIND percentage we want:
  - Increase – ADD percentage to 100
  - Decrease – SUBTRACT from 100
- Write down a STATEMENT connecting "before" and "after":
  - "after is a percentage of before"
- Write down the statement as an EQUATION using decimal equivalent
  - remember "is" means "="
- SUBSTITUTE and SOLVE

Worked Example

1. Number

Jennie now earns £31500 per year in her job.

She has recently had a 5% pay rise.

Find her annual pay before the pay rise.

We can call the "before" Old Pay and the "after" New Pay

1. *We want* $100 + 5 = 105\%$

2. *New Pay is 105% of Old Pay*

   The decimal equivalent of 105% is 1.05

3. $New\ Pay = 1.05 \times Old\ Pay$

4. $31500 = 1.05 \times Old\ Pay$

Divide by 1.05 : $Old\ Pay = 31500 \div 1.05$

$$Old\ Pay = £30000\ per\ year$$

Exam Question: Medium

The normal price of a television is reduced by 30% in a sale.

The sale price of the television is £350

Work out the normal price of the television.

1. Number

YOUR NOTES
⬇

Exam Question: Hard

In a sale normal prices are reduced by 20%.

A washing machine has a sale price of £464

By how much money is the normal price of the washing machine reduced?

1. Number

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1.5 COMPOUND INTEREST

1.5.1 COMPOUND INTEREST

What is compound interest?

- Compound interest is where interest is paid on the interest from the year (or whatever time frame is being used) before as well as on the original amount
- This is different from simple interest where interest is only paid on the original amount

How do you work with compound interest?

- For COMPOUND changes (can be a decrease as well as an increase):
  - Keep multiplying by the decimal equivalent of the percentage you want
- Otherwise do the same as normal:
- Identify "before" & "after" quantities
- FIND percentage we want:
  - Increase – ADD percentage to 100
  - Decrease – SUBTRACT from 100
- Write down a STATEMENT connecting "before" and "after":
  - "after is a percentage of before"
- Write down the statement as an EQUATION using decimal equivalent
  - remember "is" means "="
- SUBSTITUTE and SOLVE

Exam Tip

This method works for any Compound Change – increase or decrease.

Remembering "× m × m × m = × m3″ can make life a lot quicker:
It is usually much easier to multiply by decimal equivalent raised to a power than to multiply by the decimal equivalent several times in a row.

1. Number

Worked Example

Jasmina invests £1200 in a Savings Account which pays Compound Interest at the rate of 2% per year for 7 years.

To the nearest pound, what is her investment worth at the end of the 7 years?

We can call the "before" Investment and the "after" Final Value

1. *We want* $100 + 2 = 102\%$ *EACH YEAR*

   The decimal equivalent of 102% is 1.02 and we want to apply it 7 times so our multiplier is 1.02^7

2. *Final Value is 102% (applied for 7 years) of Investment*

3. $\text{Final Value} = 1.02^7 \times \text{Investment}$

4. $\text{Final Value} = 1.02^7 \times 1200 = 1378.4228...$

   $\text{Final Value} = £1378$ *(to the nearest £)*

Exam Question: Medium

Liam invests £6200 for 3 years in a savings account.

He gets 2.5% per annum compound interest.

How much money will Liam have in his savings account at the end of 3 years?

EXAM PAPERS PRACTICE

1. Number

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Exam Question: Hard

Viv wants to invest £2000 for 2 years in the same bank.

The International Bank	The Friendly Bank
Compound Interest	Compound Interest
4% for the first year 1% for each extra year	5% for the first year 0.5% for each extra year

At the end of 2 years, Viv wants to have as much money as possible.

Which bank should she invest her £2000 in?

1. Number

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1.6 LCM / HCF / PRIME FACTORS

1.6.1 PRIME FACTORS

What are prime factors?

- Factors are things that are multiplied together
- Prime numbers are numbers which can only be divided by themselves and 1
- The prime factors of a number are therefore all the prime numbers which multiply to give that number
- You should remember the first few prime numbers:
  2, 3, 5, 7, 11, 13, 17, 19, …

How to find prime factors

- Use a FACTOR TREE to find prime factors
- Write the prime factors IN ASCENDING ORDER with × between
- Write with POWERS if asked

Language

This is one of those topics where questions can use different phrases that all mean the same thing …

- Express … as the product of prime factors
- Find the prime factor decomposition of …
- Find the prime factorisation of …

1. Number

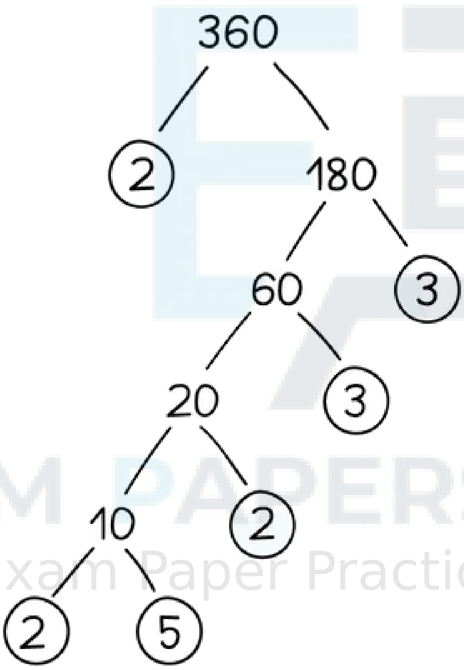
YOUR NOTES

Worked Example

1. *Find the prime factors of 360.*

Give your answer in the form $2^p \times 3^q \times 5^r$ where p, q and r are integers to be found.

1 − For each number find any two numbers which are factors (not 1 ...) and write those as the next pair of numbers in the tree. If a number is prime, put a circle round it. When all the end numbers are circled you're done!



$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$ *2 − Be careful to get them all, especially those repeated ones*

$360 = 2^3 \times 3^2 \times 5^1$ *3 − Write using powers as the question asks for this*

The 1 as a power of 5 isn't really necessary but the question asked for it...

(So $p = 3$, $q = 2$, $r = 1$)

1. Number

YOUR NOTES
⬇

1.6.2 HCFS & LCMS

What are HCFs & LCMs?

- HCF is Highest Common Factor
- This is the biggest number which is a factor of (divides into) two numbers
- LCM is Lowest Common Multiple
- This is the smallest number which two numbers divide into (are factors of)
- You should remember the first few Prime Numbers:
  2, 3, 5, 7, 11, 13, 17, 19, …

How to find HCFs & LCMs

1. Find PRIME FACTORS
2. Create a VENN DIAGRAM
3. ∩ is HIGHEST COMMON FACTOR (Overlap)
4. ∪ is LOWEST COMMON MULTIPLE (Union)

1. Number

YOUR NOTES
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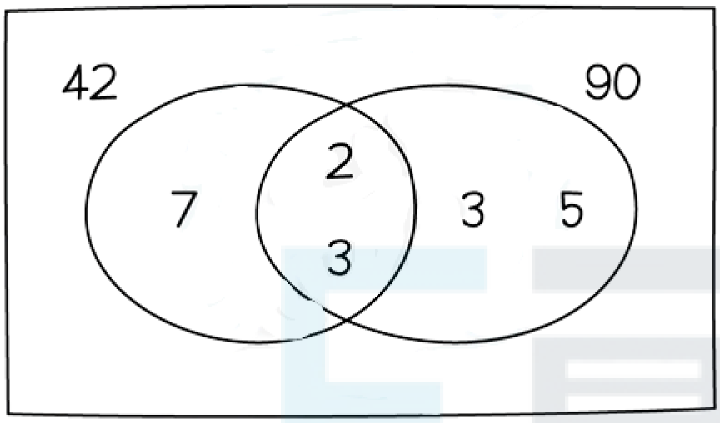
Worked Example

1. *Find the HCF and LCM of* 42 *and* 90.

$42 = 2 \times 3 \times 7$
$90 = 2 \times 3 \times 3 \times 5$

1 – *See the separate notes for finding Prime Factors*



2 – *Box is not essential for our purposes here*

$HCF = 2 \times 3$

3 – ∩, *intersection/overlap*

$HCF = 6$

$LCM = 7 \times 2 \times 3 \times 3 \times 5$

4 – ∪, *union/'or'*

$LCM = 630$

1. Number

Exam Question: Easy

Buses to Acton leave a bus station every 24 minutes.
Buses to Barton leave the same bus station every 20 minutes.

A bus to Acton and a bus to Barton both leave the bus station at 9 00 am.

When will a bus to Acton and a bus to Barton next leave the bus station at the same time?

Exam Question: Medium

Matt and Dan cycle around a cycle track.

Each lap Matt cycles takes him 50 seconds.
Each lap Dan cycles takes him 80 seconds.

Dan and Matt start cycling at the same time at the start line.

Work out how many laps they will each have cycled when they are next at the start line together.

1. Number

1.7 ROOTS & INDICES

1.7.1 ROOTS & INDICES - BASICS

What are indices?

- An Index (plural = indices) is just a power that a number (called the base) is raised to:

Laws of indices – what you need to know

- There are lots of very important laws (or rules)
- It is important that you know and can apply these
- Understanding the explanations will help you remember them:

1. Number

YOUR NOTES

Laws	Explanations
$a^1 = a$	
$a^p \times a^q = a^{p+q}$	$a^3 \times a^2 = (a \times a \times a) \times (a \times a) = a^5$
$a^p \div a^q = a^{p-q}$	$a^5 \div a^3 = \frac{a \times a \times a \times a \times a}{a \times a \times a} = a^2$
$(a^p)^q = a^{p \times q}$	$(a^3)^2 = (a \times a \times a) \times (a \times a \times a) = a^6$
$a^0 = 1$	$a^0 = a^{2-2} = a^2 \div a^2 = \frac{a^2}{a^2} = 1$
$a^{-p} = \frac{1}{a^p}$	$a^{-3} = a^{0-3} = a^0 \div a^3 = \frac{a^0}{a^3} = \frac{1}{a^3}$
$a^{\frac{1}{n}} = \sqrt[n]{a}$	$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 = a = \sqrt{a} \times \sqrt{a}$
$a^{\frac{p}{q}} = (\sqrt[q]{a})^p = \sqrt[q]{(a)^p}$	$a^{\frac{3}{2}} = a^{\frac{1}{2} \times 3} = (a^{\frac{1}{2}})^3 = (\sqrt{a})^3$ $a^{\frac{3}{2}} = a^{3 \times \frac{1}{2}} = (a^3)^{\frac{1}{2}} = \sqrt{(a)^3}$

Exam Tip

Take it slowly and apply the laws one at a time.

Worked Example

Simplify $\sqrt{\dfrac{p^3 \times p^7}{p^6}}$

Use the second law from above on the top of the fraction :

$$\sqrt{\frac{p^3 \times p^7}{p^6}} = \sqrt{\frac{p^{3+7}}{p^6}} = \sqrt{\frac{p^{10}}{p^6}}$$

Use the third law from above on the whole fraction :

$$\sqrt{\frac{p^{10}}{p^6}} = \sqrt{p^{10-6}} = \sqrt{p^4}$$

Use the seventh law from above to change the square root into a power :

$$\sqrt{p^4} = (p^4)^{\frac{1}{2}}$$

Use the fourth law from above to finish :

$$(p^4)^{\frac{1}{2}} = p^{4 \times \frac{1}{2}} = p^2$$

1. Number

YOUR NOTES
⬇

1.7.2 ROOTS & INDICES - HARDER

What are indices?

- An Index (plural = indices) is just a power that a number (called the base) is raised to:

Laws of indices – what you need to know

- There are lots of very important laws (or rules)
- It is important that you know and can apply these
- Understanding the explanations will help you remember them:

1. Number

YOUR NOTES
⬇

Laws	Explanations
$a^1 = a$	
$a^p \times a^q = a^{p+q}$	$a^3 \times a^2$ $= (a \times a \times a) \times (a \times a)$ $= a^5$
$a^p \div a^q = a^{p-q}$	$a^5 \div a^3$ $= \frac{a \times a \times a \times a \times a}{a \times a \times a}$ $= a^2$
$(a^p)^q = a^{p \times q}$	$(a^3)^2$ $= (a \times a \times a) \times (a \times a \times a)$ $= a^6$
$a^0 = 1$	$a^0 = a^{2-2} = a^2 \div a^2 = \frac{a^2}{a^2} = 1$
$a^{-p} = \frac{1}{a^p}$	$a^{-3} = a^{0-3} = a^0 \div a^3 = \frac{a^0}{a^3} = \frac{1}{a^3}$
$a^{\frac{1}{n}} = \sqrt[n]{a}$	$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 = a = \sqrt{a} \times \sqrt{a}$
$a^{\frac{p}{q}} = (\sqrt[q]{a})^p = \sqrt[q]{(a)^p}$	$a^{\frac{3}{2}} = a^{\frac{1}{2} \times 3} = (a^{\frac{1}{2}})^3 = (\sqrt{a})^3$ $a^{\frac{3}{2}} = a^{3 \times \frac{1}{2}} = (a^3)^{\frac{1}{2}} = \sqrt{(a)^3}$

Exam Tip

Write numbers in the question with the same BASE if possible.

Worked Example

Express $\sqrt{\dfrac{8^2 \times 2^7}{4^3}}$ *as a single power of* 2.

Express 8 *as* 2^3 *and* 4 *as* 2^2 :

$$\sqrt{\frac{8^2 \times 2^7}{4^3}} = \sqrt{\frac{\left(2^3\right)^2 \times 2^7}{\left(2^2\right)^3}}$$

Use the fourth law from above to simplify :

$$\sqrt{\frac{\left(2^3\right)^2 \times 2^7}{\left(2^2\right)^3}} = \sqrt{\frac{2^{3\times2} \times 2^7}{2^{2\times3}}} = \sqrt{\frac{2^6 \times 2^7}{2^6}}$$

Use the second law from above on the top of the fraction :

$$\sqrt{\frac{2^6 \times 2^7}{2^6}} = \sqrt{\frac{2^{6+7}}{2^6}} = \sqrt{\frac{2^{13}}{2^6}}$$

Use the third law from above on the whole fraction :

$$\sqrt{\frac{2^{13}}{2^6}} = \sqrt{2^{13-6}} = \sqrt{2^7}$$

Use the seventh law from above to change the square root into a power :

$$\sqrt{2^7} = \left(2^7\right)^{\frac{1}{2}}$$

Use the fourth law (again) from above to finish :

$$\left(2^7\right)^{\frac{1}{2}} = 2^{7\times\frac{1}{2}} = 2^{\frac{7}{2}}$$

1. Number

YOUR NOTES

Exam Question: Easy

Simplify $(m^{-2})^5$

Exam Question: Medium

Simplify $(9x^8y^3)^{\frac{1}{2}}$

Exam Question: Hard

(a) Find the value of 2^{-3}

$5\sqrt{5}$ can be written in the form 5^k

(b) Find the value of k.

1. Number

1.8 ROUNDING & ESTIMATION

1.8.1 ROUNDING & ESTIMATION

Why use estimation?

- We estimate to find approximations for difficult sums
- Or to check our answers are about the right size (right order of magnitude)

How to estimate

- We round numbers to something sensible before calculating
- GENERAL RULE:
  Round numbers to 1 significant figure
  - 7.8 → 8
  - 18 → 20
  - 3.65 × 10-4 → 4 × 10-4
  - 1080 → 1000
- EXCEPTIONS:
  It can be more sensible (or easier) to round to something convenient
  - 16.2 → 15
  - 9.1 → 10
  - 1180 → 1200

  It wouldn’t usually make sense to round a number to zero
- FRACTIONS get bigger when the top is bigger and/or the bottom is smaller and vice versa

Worked Example

1. *Calculate an estimate for* $\frac{15.9\times3.87}{18.7}$.

$3.87 \rightarrow 4$ — *1 − Round both to 1 significant figure*

$18.7 \rightarrow 20$

$15.9 \rightarrow 15$ — *2 − An exception as 15.9 is quite a long way from*

$\frac{15.9\times3.87}{18.7} \approx \frac{15\times4}{20}$ — *Now use the rounded figures to find an approximation*

$\approx \frac{60}{20}$

≈ 3

2. (*a*) *Use your calculator to work out Calculate an estimate for* 108.6×27.3.

(*b*) *Show an estimate to verify your answer to* (*a*) *is of the right order of magnitude.*

(*a*) 2964.78 — *Write down all the digits from your calculator display*

(*b*) $108.6 \rightarrow 100$ — *Round both numbers*

$27.3 \rightarrow 30$

$108.6 \times 27.3 \approx 100 \times 30$ — *Use the rounded figures to find an approximation*

≈ 3000 — *Check both answers are the same order of magnitude*

Exam Question: Medium

Work out an estimate for $\dfrac{31 \times 9.87}{0.509}$

1. Number

Exam Question: Hard

Sanders has a water tank for storing rainwater.

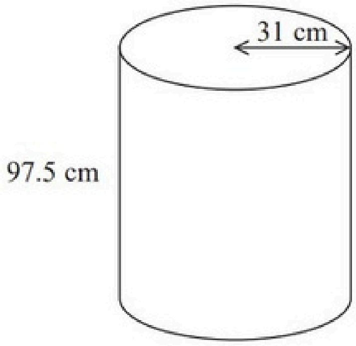


Diagram **NOT** accurately drawn

The tank is in the shape of a cylinder.
The radius of the cylinder is 31 cm.
The height of the cylinder is 97.5 cm.

The tank is full of water.

Work out an estimate for the volume of water in the tank.
Give your answer in litres.
You must show your working.

Use $1000 \text{ cm}^3 = 1$ litre.

1. Number

1.9 STANDARD FORM

1.9.1 STANDARD FORM - BASICS

What is standard form?

- Standard Form (sometimes called Standard Index Form) is a way of writing very big and very small numbers using powers of 10

Why do we use standard form?

- Writing big (and small) numbers in Standard Form allows us to:
  - write them more neatly
  - compare them more easily
  - and it makes things easier when doing calculations

How do we use standard form?

- Using Standard Form numbers are always written in the form:

  $a \times 10n$

- The rules:
  - $1 \leq a < 10$ so there is one non-zero digit before the decimal point
  - $n > 0$ for LARGE numbers – how many times a is multiplied by 10
  - $n < 0$ for SMALL numbers – how many times a is divided by 10
  - Do calculations on a calculator (if allowed)
    Otherwise follow normal rules (including indices) but adjust answer to fit Standard Form (move decimal point and change n

Worked Example

1. *Without using a calculator, multiply* 5×10^{18} *by* 7×10^{-4}.

Give your answer in standard form.

$5 \times 10^{18} \times 7 \times 10^{-4} = 5 \times 7 \times 10^{18} \times 10^{-4}$ *Separate into numbers and powers of 10*

$= 35 \times 10^{18+(-4)}$ *Use Laws of Indices on the powers of 10*

$= 35 \times 10^{14}$

$= 3.5 \times 10 \times 10^{14}$ *Write in standard form (this isn't as 35 > 10)*

$= 3.5 \times 10^{15}$

2. *Use your calculator to find* $\frac{1.275 \times 10^{6}}{3.4 \times 10^{-2}}$.

Write your answer in the form $A \times 10^{n}$, *where* $1 \le A < 10$ *and n is an integer.*

$\frac{1.275 \times 10^{6}}{3.4 \times 10^{-2}} = 37\,500\,000$

Your calculator will not necessarily give the answer in standard form. Copy the digits, especially those zeros, carefully!

$= 3.75 \times 10^{7}$

1. Number

YOUR NOTES
⬇

1.9.2 STANDARD FORM - HARDER

Standard form – harder questions

- Make sure you are familiar with:
  Roots & Indices – Basics
  Standard Form – Basics
- Harder problems often combine algebra and laws of indices with numbers written in standard form
- Other areas of mathematics may be used and questions are often in context
- Units may be muddled up to make things trickier

Worked Example

1. *Given that* $x = 25 \times 10^{4n}$ *write* $x^{\frac{3}{2}}$ *in standard form.*

$x^{\frac{3}{2}} = \left(25 \times 10^{4n}\right)^{\frac{3}{2}}$

$= 25^{\frac{3}{2}} \times \left(10^{4n}\right)^{\frac{3}{2}}$ *Apply the power to both factors in the bracket*

$= 25^{\frac{3}{2}} \times 10^{4n \times \frac{3}{2}}$ *Use the Laws of Indices on the power of 10*

$= 125 \times 10^{6n}$ *Simplify*

$= 1.25 \times 10^{2} \times 10^{6n}$

Write in Standard Form (this isn't as $125 > 10$*)*

$= 1.25 \times 10^{6n+2}$

2. *The diameter of a hydrogen atom is* 1.06×10^{-10} *m.*

The nucleus of a hydrogen atom has diameter 2.40×10^{-13} *cm.*

The diameter of a hydrogen atom is k times the size of the nucleus of a hydrogen atom.

Find the value of k correct to three significant figures.

First spot that we have a mixture of m and cm!

$2.40 \times 10^{-13}\ cm = 2.40 \times 10^{-13} \div 100\ m$

Change one (doesn't matter which) so units match

$= 2.40 \times 10^{-15}\ m$

$k = 1.06 \times 10^{-10} \div 2.40 \times 10^{-15} = 44\ 166.6666\ ...$

To find k divide the (diameter of) the atom by the nucleus

$k = 44\ 200$ *Final answer for k rounded to 3 significant figures*

1. Number

Exam Question: Easy

(a) Write 7.8×10^{-4} as an ordinary number.

(b) Write 95 600 000 as a number in standard form.

Exam Question: Medium

Write 6.7×10^{-5} as an ordinary number.

Work out the value of $(3 \times 10^{7}) \times (9 \times 10^{6})$
Give your answer in standard form.

1. Number

YOUR NOTES
⬇

1.10 BOUNDS & ERROR INTERVALS

1.10.1 BOUNDS & ERROR INTERVALS - BASICS

What are bounds?

- Bounds are the smallest – the Lower Bound (LB) – and largest – the Upper Bound (UB) – numbers that a rounded number can lie between

How do we find bounds?

- The basic rule is "Half Up, Half Down"
- More formally:
  - UPPER BOUND – add on half the degree of accuracy
  - LOWER BOUND – take off half the degree of accuracy
  - ERROR INTERVAL: LB ≤ x < UB
- Note:
  It is very tempting to think that the Upper Bound should end in a 9, or 99, etc. but if you look at the Error Interval – LB ≤ x < UB – it does NOT INCLUDE the Upper Bound so all is well

Worked Example

1\. *The length of a road,* l*, is given as* $l = 3.6$ *km, correct to* 1 *decimal place.*

Find the Lower and Upper Bounds for l*.*

$UB \text{ for } l = 3.6 + 0.05$

$= 3.65$

1 − *The degree of accuracy is* 1 *decimal place (or* 0.1*)* − *half is* 0.05*.*

$LB \text{ for } l = 3.6 - 0.05 = 3.55$

$= 3.55$

2 − *As for the upper bound half the degree of accuracy is* 0.05*.*

3 − *This question doesn't require it but if the error interval had been asked for the final answer would be* $3.55 \le l < 3.65$*.*

1. Number

1.10.2 CALCULATIONS USING BOUNDS

What are bounds?

- Bounds are the smallest – the Lower Bound (LB) – and largest – the Upper Bound (UB) – numbers that a rounded number can lie between

How do we find bounds?

- The basic rule is "Half Up, Half Down"
- More formally:
  - UPPER BOUND – add on half the degree of accuracy
  - LOWER BOUND – take off half the degree of accuracy
  - ERROR INTERVAL: LB ≤ x < UB

Calculations using bounds

- Find bounds before calculating and then:
  - For FRACTIONS/DIVISION:
    UB = UB ÷ LB and
    LB = LB ÷ UB
  - Otherwise:
    UB = UB × UB etc.

Worked Example

1. Number

YOUR NOTES
⬇

1. *A room measures 4m by 7m, where each measurement is made to the nearest metre.*
Find Upper and Lower Bounds for the area of the room.

$3.5 \leq 4 < 4.5$
$6.5 \leq 7 < 7.5$

1, 2 − *First find the bounds for each dimension*
It is not essential to write these as an error intervals, stating LB = 3.5, UB = 4.5, etc is fine

$Area\ LB = 3.5 \times 6.5$
We do not have fractions so 5 − LB = LB × LB

$Area\ LB = 22.75\ m^2$

$Area\ UB = 4.5 \times 7.5$
5 − *UB = UB × UB*

$Area\ UB = 33.75\ m^2$

2. *David is trying to work out how many slabs he needs to buy in order to lay a garden path.*
Slabs are 50 cm long, measured to the nearest 10 cm.
The length of the path is 6 m, measured to the nearest 10 cm.
Find the maximum number of slabs David will need to buy.

$45 \leq 50 < 55$
1, 2 − *First find the bounds for each dimension*

$0.45 \leq 0.5 < 0.55$
Change the units if necessary but do as a separate step

$5.95 \leq 6 < 6.05$

$6.05 \div 0.45 = 13.44...$
The maximum number of slabs will be the Upper Bound
4 − *This is a division calculation so use UB = UB ÷ LB*

Max no. of slabs = 14
The context of the question means it is sensible for the final answer to be a whole number − but more than 13.44...

Exam Question: Easy

A number, n, is rounded to 2 decimal places.
The result is 4.76

Using inequalities, write down the error interval for n.

1. Number

Exam Question: Medium

A train travelled along a track in 110 minutes, correct to the nearest 5 minutes.

Jake finds out that the track is 270 km long.
He assumes that the track has been measured correct to the nearest 10 km.

(a) Could the average speed of the train have been greater than 160 km/h?
You must show how you get your answer.

Jake's assumption was wrong.
The track was measured correct to the nearest 5 km.

(b) Explain how this could affect your decision in part (a).

Exam Question: Hard

Dan does an experiment to find the value of π.
He measures the circumference and the diameter of a circle.

He measures the circumference, C, as 170 mm to the nearest millimetre.
He measures the diameter, d, as 54 mm to the nearest millimetre.

Dan uses $\pi = \frac{C}{d}$ to find the value of π.

Calculate the upper bound and the lower bound for Dan's value of π.

1. Number

YOUR NOTES

Exam Question: V. Hard

$$m = \frac{\sqrt{s}}{t}$$

$s = 3.47$ correct to 2 decimal places
$t = 8.132$ correct to 3 decimal places

By considering bounds, work out the value of m to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

1. Number

YOUR NOTES
⬇

1.11 RECURRING DECIMALS

1.11.1 RECURRING DECIMALS

What are recurring decimals?

- A rational number is any number that can be written as an integer (whole number) divided by another integer
- When you write a rational number as a decimal you either get a decimal that stops (eg ¼ = 0.25) or one that recurs (eg ⅓ = 0.333333…)
- The recurring part can be written with a dot (or dots) over it instead as in the example below

What do we do with recurring decimals?

- Normally, you will be asked to write a recurring decimal as a fraction in its lowest terms
- To do this:
  Write out a few decimal places… …and then:

1. Write as f = …
2. Multiply by 10 repeatedly until two lines have the same decimal part
3. Subtract those two lines
4. DIVIDE to get f = … (and cancel if necessary to get fraction in lowest terms)

eg Write $0.\dot{3}\dot{7}$ as a fraction in its lowest terms.

Write out a few decimal places :

$0.\dot{3}\dot{7} = 0.3737373737...$

1. $f = 0.3737373737...$

2. $100f = 37.37373737...$

3. *$f = ...$ and $100f = ...$ have the same decimal part so we subtract those :*

$100f - f = 37.37373737... - 0.3737373737..$

$99f = 37$

4. $\div 99 : \quad f = \frac{37}{99}$

(No cancelling is necessary in this case as it is already in its lowest terms)

1. Number

YOUR NOTES

Worked Example

Write $0.4\dot{2}\dot{7}$ *as a fraction in its lowest terms.*

Write out a few decimal places :

$$0.4\dot{2}\dot{7} = 0.42727272727\ldots$$

1. $f == 0.42727272727\ldots$

2. $10f = 4.2727272727\ldots$

$$100f = 42.727272727\ldots$$

$$1000f = 427.27272727\ldots$$

3. *Here* $10f = \ldots$ *and* $1000f = \ldots$ *have the same decimal part so we subtract those :*

$$1000f - 10f = 427.27272727\ldots - 4.2727272727..$$

$$990f = 423$$

4. $\div 990: \quad f = \frac{423}{990}$

Both 423 *and* 990 *are multiples of* 9, *so cancel to get the fraction into its lowest terms :*

$$f = \frac{423}{990} = \frac{423 \div 9}{990 \div 9} = \frac{47}{110}$$

Exam Question: Easy

Prove algebraically that the recurring decimal $0.2\dot{5}$ has the value $\frac{23}{90}$

Exam Question: Medium

Write these numbers in order of size.
Start with the smallest number.

$0.2\dot{4}\dot{6}$ $\qquad$ $0.24\dot{6}$ $\qquad$ $0.\dot{2}4\dot{6}$ $\qquad$ 0.246

1. Number

YOUR NOTES
⬇

Exam Question: Hard

Express the recurring decimal $0.2\dot{8}\dot{1}$ as a fraction in its simplest form.

1. Number

YOUR NOTES
⬇

1.12 SURDS

1.12.1 SURDS - BASICS

What is a surd?

- A surd is the square root of a non-square integer

What can we do with surds?

1. Multiplying surds – you can multiply numbers under square roots
eg. $\sqrt{3} \times \sqrt{5} = \sqrt{3\times5} = \sqrt{15}$

2. Dividing surds – you can divide numbers under square roots
eg. $\sqrt{21} \div \sqrt{7} = \sqrt{21 \div 7} = \sqrt{3}$

3. Factorising surds – you can factorise numbers under square roots
eg. $\sqrt{35} = \sqrt{5 \times 7} = \sqrt{5} \times \sqrt{7}$

4. Simplifying surds – separate out a square factor and square root it!
eg. $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4 \times \sqrt{3} = 4\sqrt{3}$

5. Adding or subtracting surds is very like adding or subtracting letters in algebra – you can only add or subtract multiples of “like” surds
eg. $3\sqrt{5} + 8\sqrt{5} = 11\sqrt{5}$ or $7\sqrt{3} - 4\sqrt{3} = 3\sqrt{3}$
Be very careful here! You can not add or subtract numbers under square roots. Think about $\sqrt{9} + \sqrt{4} = 3 + 2 = 5$. It is not equal to $\sqrt{9 + 4} = \sqrt{13} = 3.60555\ldots$

6. All other algebraic rules apply – surds can be treated like letters (as in 5. above) and like numbers (as in 1. and 2. above)

Worked Example

Write $\sqrt{54} - \sqrt{24}$ *in the form* $p\sqrt{q}$ *where p and q are integers and q has no square factors.*

4. $$\sqrt{54} - \sqrt{24} = \sqrt{9 \times 6} - \sqrt{4 \times 6}$$

$$= \sqrt{9} \times \sqrt{6} - \sqrt{4} \times \sqrt{6}$$

$$= 3 \times \sqrt{6} - 2 \times \sqrt{6}$$

5. $$= 3\sqrt{6} - 2\sqrt{6}$$

$$= \sqrt{6}$$

$$so\ p = 1\ and\ q = 6$$

1.12.2 SURDS - RATIONALISING DENOMINATORS

What is a surd?

- A surd is the square root of a non-square integer

How do we rationalise the denominator of a surd?

1. Identify **TYPE** of denominator (bottom):
   - **Type 1:** One term eg $7 / 2\sqrt{3}$
   - **Type 2:** Two terms eg $7 / 2 + \sqrt{3}$
2. **MULTIPLY** top & bottom by:
   - **Type 1:** surd bit of the **BOTTOM**

     eg $\frac{7}{2\sqrt{3}} = \frac{7}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{2\times 3} = \frac{7\sqrt{3}}{6}$ (or $\frac{7}{6}\sqrt{3}$)

   - **Type 2:** bottom with **DIFFERENT SIGN** in the middle

     eg $\frac{7}{2+\sqrt{3}} = \frac{7}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{7(2-\sqrt{3})}{2^2-3} = 14 - 7\sqrt{3}$

Worked Example

1. Number

Write $\frac{4}{\sqrt{6}-2}$ *in the form* $p+q\sqrt{r}$ *where* p, q *and* r *are integers and* r *has no square factors.*

1. *The bottom has two terms so this Type 2*
2. *Multiply top and bottom by* $\sqrt{6}+2$:

$$\frac{4}{\sqrt{6}-2} = \frac{4}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2}$$

Spot the Difference of Two Squares on the bottom :

$$= \frac{4(\sqrt{6}+2)}{6-2^2}$$

Multiply out the top and simplify :

$$= \frac{4\sqrt{6}+8}{2}$$

$$= 2\sqrt{6}+4$$

$$= 4+2\sqrt{6}$$

so $p=4$, $q=2$ *and* $r=6$

Exam Question: Medium

(a) Rationalise the denominator of $\frac{12}{\sqrt{3}}$

(b) Work out the value of $(\sqrt{2}+\sqrt{8})^2$

1. Number

Exam Question: Hard

(a) Rationalise the denominator of $\frac{5}{\sqrt{2}}$

(b) Expand and simplify $(2+\sqrt{3})^2 - (2-\sqrt{3})^2$

1. Number

1.13 USING A CALCULATOR

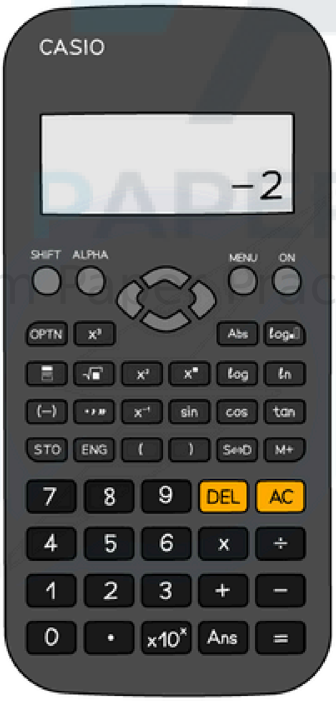
1.13.1 USING A CALCULATOR

Why the fuss about using a calculator?

- GCSE Mathematics goes beyond using the basic features of a calculator and explores many of the special functions of a scientific calculator
- It is important to get to know your calculator, the earlier you get one and learn about the scientific functions the better you will be at using them
- It's not just maths that uses these, some of the scientific functions can be used in science exams too

What do I need to know?

- The notes below apply to most if not all scientific calculators but the images are based on the Casio fx-83GTX
- The Casio fx-85GTX is the same model but also has solar power. Both are labelled "Classwiz" too but be careful here at there is a more advanced "Classwiz" calculator that is used at A level (fx-991EX)



1. Number

YOUR NOTES
⬇

The Casio fx-83GTX Classwiz

- Be aware if you have an old or very basic scientific calculator that they may work backwards
- For example, if you wanted to find sin (57) you would type 57 then press the sin button
- Modern calculators tend to work in the order in which we write things

1. Mode/setup

- Make sure you know how to change the mode of your calculator, especially if someone else has used it
- The “Angle Unit” needs to be degrees – normally indicated by a “D” symbol across the top of the display
- Make sure you can switch between “exact” answers (fractions, surds, in terms of π, etc) and “approximate” answers (decimals)
- Most calculators default to “Math” mode with the word Math written across the top of the display or using a symbol
- When in “Math” mode you can switch whatever is on the answer line between exact and decimals by pressing the “S-D” button

2. Templates

- These are largely the shortcut buttons – the fraction button, the square, cube and power buttons, square roots

Calculator shortcut buttons

3. Trigonometry (sin/cos/tan)

- Remember to use SHIFT (sometimes called 2nd or INV button) when finding angles
- When using these buttons you will find that before you type the angle the calculator automatically gives you an open bracket "(". You should get into the habit of making sure you use a closed bracket ")" after typing the angle in
- This is very important if there is something else to type in that comes after sin/cos/tan

4. Standard Form and π

- Find the ×10x button and know how to use it
- Modern calculators display standard form in the way it is written
- Older models may use a small capital letEt er ""inplaceof×1o0nx t he display line
- π is often near or under SHIFT with the standard form button

5. Memory

- The ANS (answer) button is very useful – especially when working with decimals in the middle of solutions that you should avoid rounding until your final answer
- ANS recalls the last answer the calculator calculated

6.Table

- If your calculator has a table function or mode, use it
- This can be extremely useful in those "complete the table of values and draw the graph" type questions

7. Brackets and negative numbers

- Use as you would in written mathematics
- Remember to use the (-) button for a negative number, not the subtract button

8. Judgement and special features

- The rule of thumb is to use your calculator to do one calculation at a time
- However, you can also make a judgement call on this as to how many marks are available in the question and whether a question asks you to "write down all the digits on your calculator display"
- You are better off writing too much down than not enough!

9. Practise!

- This is a long list but we will finish by going back to the start – there is nothing better you can do than getting a calculator early and learning how to use it by practising the varying types of questions you are likely to come across

Exam Tip

Always put negative numbers in brackets. For a quick example, try using your calculator to

work out -32 and then (-3)2.
In working out always write down more digits than the final answer requires and don't round them (write something like 9.3564… using the three dots shows you haven't rounded). Use the ANS button when you next need that number on your calculator.

Worked Example

3. Complete the table of values for $y = x^3 - 6x + 1$

x	-3	-2	-1	0	1	2	3
y		5					10

$(-3)^3 - 6 \times (-3) + 1 = -8$ — Use brackets around negatives and (-) key

$(-1)^3 - 6 \times (-1) + 1 = 6$ — Use arrow keys and change "3"s to "1"s

$0^3 - 6 \times 0 + 1 = 1$

$1^3 - 6 \times 1 + 1 = -4$ — You can use the TABLE mode/feature

$2^3 - 6 \times 2 + 1 = -3$ — of your calculator if it has one

x	-3	-2	-1	0	1	2	3
y	-8	5	6	1	-4	-3	10

4. Solve the quadratic equation $2x^2 + 6x + 3 = 0$, giving your answers in the form $\frac{a \pm b\sqrt{3}}{2}$

"a" $= 2$, "b" $= 6$, "c" $= 3$ — a, b, (c) from quadratic formula have nothing to do with the a and b mentioned in question

"$b^2 - 4ac$" $= 6^2 4 \times 2 \times 3 = 12$ — Find the discriminant first (the bit under square root)

$x = \frac{-6 \pm \sqrt{12}}{2 \times 2}$ — Now put into full quadratic formula

$x = \frac{-3 \pm \sqrt{3}}{2}$ — ... and your calculator will simplify for you

Note: You'll have to choose "+" or "-" when You type it into your calculator.

1. Use your calculator to work out

$$\frac{\sqrt{4.69}}{0.34^3 + sin\,(45°)}$$

Give your answer as a decimal.

Write down all the figures on your calculator display.

$\sqrt{4.69} = 2.16564\ ...$ To show your working write down the top

$0.34^3 + sin(45) = 0.746410\ ...$ and bottom separately ...

2.901406085 ... but you can type it all in one go for the final answer using the fraction button, etc

2. $a^5 = \frac{p+q}{p^2 q}$

Find the value of a when $p = 1.2 \times 10^{-4}$ and $q = 7.83 \times 10^5$

Give your answer to 3 decimal places.

$p + q = 783000.0001$ Show each stage as working

$p^2 q = 0.0112752$ Use brackets when it gets long or awkward

$a^5 = 69444444.46$ Write down all digits at these stages

$\boldsymbol{a = 37.01071\ ... = 37.011}$ Write more digits than you need, then round

1. Number

Exam Question: Easy

(a) Use your calculator to work out $\frac{38.5 \times 14.2}{18.4 - 5.9}$

Write down all the figures on your calculator display.
You must give your answer as a decimal.

(b) Write your answer to part (a) correct to 1 significant figure.

Exam Question: Medium

Calculate the value of $\sqrt{\frac{\tan 60° + 1}{\tan 60° - 1}}$

Write down all the figures on your calculator display.
You must give your answer as a decimal.

Exam Question: Hard

$$p^2 = \frac{x - y}{xy}$$

$x = 8.5 \times 10^9$
$y = 4 \times 10^8$

Find the value of p.
Give your answer in standard form correct to 2 significant figures.

YOUR NOTES
⬇

1.14 COUNTING

1.14.1 COUNTING

How to count combinations of things

- When you have a question like “How many ways...?”

1.Systematic listing can help in simple cases

2.Write down the thing you are counting with “AND”s and “OR”s

3.Remember that (like in Probability problems):
“AND means ×”
“OR means +”

4.Beware of repetitions and whether they are allowed or not
(If they are not we reduce the number of options by one)

Worked Example

1. *John is choosing an ice cream flavour and a topping.*

   He has a choice of three flavours – vanilla (V), strawberry (S) and raspberry ripple (R).

   He also has a choice of two toppings – nuts (N) and chocolate sauce (C).

   How many different combinations of ice cream can John choose?

   VN, VC
   SN, SC
   RN, RC

   1 – Be systematic – stick with one flavour at a time and work through each of the toppings in turn
   Abbreviations are fine – the question even suggests them!
   We could have used 2 and 3 – flavour AND topping – 3×2
   4 – There are no repetitions to consider here

   No of ice cream combinations is 6

2. *Jamil is out for lunch.*

   The menu has 5 starters, 6 main courses and 4 puddings.

   How many different two course meals (Starter & Main or Main & Pudding) could Jamil have?

   1 – There would be far too many combinations to list

   Starter AND Main OR Main AND Pudding

   2 – Use "AND"s and "OR"s to state what Jamil could have
   4 – There are no repetitions to consider here
   3 – "AND means ×" and "OR means +"

   No of meals $= 5 \times 6 + 6 \times 4$
   $= 30 + 24$

   No of meals $= 54$

3. *Lauren is generating a 4 – digit passcode using the digits 0 – 9.*

   She will pick each digit at random but does not want any digit to be repeated.

   How many different passcodes will Lauren be able to generate?

1. Number

Exam Question: Medium

Jeff is choosing a shrub and a rose tree for his garden.
At the garden centre there are 17 different types of shrubs and some rose trees.

Jeff says,

"There are 215 different ways to choose one shrub and one rose tree."

Could Jeff be correct?
You must show how you get your answer.

1. Number

YOUR NOTES

1.15 BEST BUY

1.15.1 BEST BUY

How do you tackle "Best Buy" problems?

- You may need a range of skills…

1.Decide on a suitable comparison – eg total cost, cost/year, cost/m2 etc)

2.Be consistent with units – convert if necessary

3.Do the calculations – showing working carefully

4.Make sure you answer the question, with reasons

Worked Example

1. *Donna is taking 5 friends out for lunch.*
There are two different offers she could use :
Offer A : 15% off the total bill for parties of 6 or more
Offer B : 25% off main courses
The total bill, before any offer is applied, comes to £147.80
Two people had main courses costing £14.50 and 4 had main courses costing £13.
Given that she can only use one offer, which should she use?

1 − Compare the total cost of the meal for each offer
2 − Units are not an issue here, everything is in pounds

Offer A :
Final cost $= 147.80 \times 0.85 = £125.63$
3 − Offer A, this reduces the total by 15%, so we find 85% of it

Offer B :
Original Cost of Main Courses $= 2 \times 14.50 + 4 \times 13 = £81$
3 − Offer B discount is only on the main courses

Reduced Cost of Main Courses $= 81 \times 0.75 = £60.75$
3 − Main courses cost is reduced by 25%, so we find 75%

Cost of Rest of Meal $= 147.80 - 81 = £66.80$
3 − We need to know the cost of the rest of the meal

Final cost $= 66.80 + 60.75 = £127.55$
3 − Find the discounted total cost

Since £125.63 < £127.55 Donna should choose Offer A
4 − Ensure you answer the question, stating a reason

1. Number

Exam Question: Easy

Plants are sold in three different sizes of tray.

A small tray of 30 plants costs £6.50
A medium tray of 40 plants costs £8.95
A large tray of 50 plants costs £10.99

Kaz wants to buy the tray of plants that is the best value for money.

Which size tray of plants should she buy?
You must show all your working.

Exam Question: Medium

Henry is thinking about having a water meter.

These are the two ways he can pay for the water he uses.

Water Meter	No Water Meter
A charge of £28.20 per year	A charge of £107 per year
plus	
91.22p for every cubic metre of water used	
1 cubic metre = 1000 litres	

Henry uses an average of 180 litres of water each day.

Henry wants to pay as little as possible for the water he uses.
Should Henry have a water meter?

1.16 EXCHANGE RATES

1.16.1 EXCHANGE RATES

Simplifying exchange rate questions

Use ratios!

1. Put exchange rates in ratio form (use more than one line if necessary)
2. Add lines for prices/costs
3. Use scale factors to complete lines
4. Pick out the answer!

It can be that simple!

Worked Example

1. €1 (*Euro*) *is worth* \$21.48 (*Mexican Peso*).
   ฿1 (*Bitcoin*) *is worth* €6882.55 (*Euro*).
   A vintage car costs \$1 000 000 (*Mexican Peso*).
   What is the cost of the car in Bitcoins?

1 − Using ratios
2 − Add a line for each rate/cost
Use unknowns (x, b) *for values to find*

	€ EURO	:	\$ MP	:	฿ BITCOIN
RATE €/\$	1	:	21.48	:	
RATE ฿/€	6882.55	:	x	:	1
RATE \$/฿		:	1000 000	:	b

$x = 21.48 \times 6882.55$ 3 – *Looking at the Euro column, the Scale Factor is 6882.55*
$x = \$147\,837.174$

$Scale\ Factor = 1\,000\,000 \div 147837.174$
3 – *Looking at the MP column, we can find the Scale Factor to convert between the rate for ฿/€ and cost.*
$= 6.764$
$Cost = 1 \times 6.764 =$ ฿6.764 (*Bitcoins*) 4 – *We can now pick out the answer in Bitcoins*

Exam Question: Medium

Linda is going on holiday to the Czech Republic.
She needs to change some money into koruna.

She can only change her money into 100 koruna notes.

Linda only wants to change up to £200 into koruna.
She wants as many 100 koruna notes as possible.

The exchange rate is £1 = 25.82 koruna.

How many 100 koruna notes should she get?

1. Number

Exam Question: Hard

In the UK, petrol cost £1.24 per litre.
In the USA, petrol cost 3.15 dollars per US gallon.

1 US gallon = 3.79 litres
£1 = 1.47 dollars

Was petrol cheaper in the UK or in the USA?