GCE FURTHER MATHEMATICS

AdvancedSubsidiary— Paper1: CorePureMathematics(8FM0/01)

Mark Scheme — June 2026

Question 1 (Total: 9 Marks) — Cubic Equations & Roots

Given equation: $2x^3 - 3x^2 + 5x + 7 = 0$ with roots α, β, γ.

Par	Scheme / Working	Marks	AO
(i)	State or use standar d relationships: $\Sigma\alpha = -(-3)/2 = 3/2$ and $\Sigma\alpha\beta = 5/2$.	M1	1.1b
	Use $\alpha^2 + \beta^2 + \gamma^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta = (3/2)^2 - 2(5/2) = 9/4 - 5 = -11/4$ (or -2.75)	A1 A1	1.1b 2.1
(ii)	State or use product of roots: $\alpha\beta\gamma = -7/2$. Express $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = (\Sigma\alpha\beta)/(\alpha\beta\gamma)$. Substitute values: $(5/2) / (-7/2) = -5/7$	M1	1.1b
		A1 A1	1.1b 1.1b
(iii)	Expand product: $(1-\alpha)(1-\beta)(1-\gamma) = 1 - \Sigma\alpha + \Sigma\alpha\beta - \alpha\beta\gamma$	M1	2.1
	Substitute values: $1-(3/2) + (5/2) - (-7/2) = 1 - 1.5 + 2.5 + 3.5 = 5.5 = 11/2$ *Alternative:* Substitute $x = 1$ into $f(x) / 2 = (2(1)^3 - 3(1)^2 + 5(1) + 7) / 2 = 11 / 2$.	A1 A1	1.1b 3.1a
	Total for Question 1	**9 Marks**	

Question 2 (Total: 8 Marks) — Summations

Part	Scheme / Working	Marks	AO
(a)	Split summation and apply formulas: $\Sigma\_\{r=1\}^n(2r - 1)^2=\Sigma(4r^2 - 4r+1)= 4\Sigma r^2 - 4\Sigma r + \Sigma 1$	M1	2.1
	Substitute standard results: $=4*[n(n+1)(2n+1)/6] - 4*[n(n+1)/2] + n$	M1 A1	1.1b 1.1b
	Factorise out n: $= (n/3) * [2(n+1)(2n+1) - 6(n+1) + 3] = (n/3)(4n^2 + 6n + 2 - 6n - 6 + 3) = (n/3)(4n^2 - 1)$ Hence $a = 4, b = -1$.	M1 A1	2.1 1.1b
(b)	Set up equation: $(k/3)(4k^2 - 1) = 3k^2$ *ightarrow* $4k^3 - 1k = 9k^2$ *ightarrow* $k(4k^2 - 9k - 1) = 0$	M1	3.1a
	Since k is a positive integer, *keq* 0. Solve $4k^2 - 9k - 1 = 0$: Discriminant |*Delta*$=(-9)^2-4(4)(-1) = 81 + 16 = 97$. Since 97 is not a perfect square, k cannot be an integer. Thus, no such integer values exist.	M1 A1	2.4 2.2a
	Total for Question 2	**8 Marks**	

Question 3 (Total: 8 Marks) — Complex Roots

Given: $f(z) = 3z^3 + az^2 + bz + 17 = 0$, one root is $4 - i$.

Part	Scheme / Working	Marks	AO
(a)	State complex conjugate root: $4 + i$	B1	1.2
	Find quadratic factor: $(z-(4-i))(z - (4+i)) = z^2 - 8z + 17$	M1 A1	1.1b 1.1b
	Perform division or match constants: $(3z + c)(z^2 - 8z +17)$. Product of constants $17c = 17$ *ightarrow* $c = 1$. Third root is found from $3z + 1 = 0$ *ightarrow* $z = -1/3$. Full solution set: $4 - i, 4 + i, -1/3$.	M1 A1	3.1a 1.1b
(b)	Expand product $(3z + 1)(z^2 - 8z + 17) = 3z^3 - 24z^2 + 51z + z^2 - 8z + 17 = 3z^3 - 23z^2 + 43z + 17$	M1	1.1b
	Compare coefficients: $a = -23, b = 43$	A1	1.1
(c)	Roots of $f(z+2) = 0$ are shifted by -2 from original roots: $z_1 = (4 - i) - 2 = 2 - i$ $z_2 = (4 + i) - 2 = 2 + i$ $z_3 = -1/3 - 2 = -7/3$	B1	b 2.2a
	Total for Question 3	**8 Marks**	

Question 4 (Total: 7 Marks) — Invariant Lines & Points

Matrix: *Q = egin{pmatrix} 0 & 1 \ 5 & 4 \end{pmatrix}*

Part	Scheme / Working	Marks	AO
(a)	Let invariant line be $y = mx$. Set up system: *egin{pmatrix} 0 & 1 \ 5 & 4 \end{pmatrix}egin{pmatrix} x \ mx \end{pmatrix} = egin{pmatrix} x' \ y' \end{pmatrix}* where $y' = mx'$. $x' = mx$ and $y' = 5x + 4mx$.	M1	3.1a
	Substitute equations: *5x + 4mx = m(mx) ightarrow m² - 4m - 5 = 0*	M1 A1	1.1b 1.1b
	Factorise: *(m - 5)(m + 1) = 0 ightarrow m = 5, m = -1*. Equations of invariant lines are $y = 5x$ and $y = -x$.	M1 A2	1.1b 2.2a
(b)	For line of invariant points, solve *Q\mathbf{x} = \mathbf{x} ightarrow y = x* and *5x + 4y = y ightarrow 5x + 3y = 0*. The only simultaneous solution is the point $(0,0)$, so no *line* of invariant points exists except the trivial point. Therefore, equation does not exist (or line is non-existent).	B1	2.4
	Total for Question 4	**7 Marks**	

Question 5 (Total: 6 Marks) — Proof by Induction

Prove that matrix summation / sequence formula holds for all positive integers n.

Part	Scheme / Working	Marks	AO
–	**Basis Step:** Prove true for $n = 1$ by substituting and checking equality.	B1	1.1b
	Assumption Step: Assume statement is true for $n = k$. Write expression clearly.	M1	2.4
	Inductive Step: Add the $(k+1)$-th term to the assumption formula and use algebraic manipulation to transform it into the required format for $n = k + 1$.	M1 A1 A1	2.1 1.1b 1.1b
	Conclusion: State that since it is true for $n = 1$, and true for $n = k + 1$ when assumed true for $n = k$, it is true for all positive integers n by induction.	B1	2.4
	Total for Question 5	**6 Marks**	

Question 6 (Total: 9 Marks) — Argand Diagrams & Loci

Part	Scheme / Working	Marks	AO
(a)	From Figure 1, the center of circle C is at $(3, -2)$. Locus format: $\|z - (a + bi)\| = 1$ ightarrow $\|z - (3 - 2i)\| = 1$. Therefore, $a = 3$ and $b = -2$. Half-line l starts at $(1, 0)$ and passes through $(0, 1)$. Gradient = -1. Angle with positive real axis is 3\pi / 4.	B1 B1	1.2 1.1b
(b)	Equation: rg(z - 1) = rac{3\pi}{4} Line equation in Cartesian form: $y - 0 = -1(x - 1)$ ightarrow $y = -x + 1$.	M1 A1	1.1b 2.2a
(c)	Circle equation: $(x - 3)^2 + (y + 2)^2 = 1$. Substitute $y = -x + 1$: $(x - 3)^2 + (-x + 3)^2 = 1$ ightarrow $2(x - 3)^2 = 1$ ightarrow $(x - 3)^2 = 1/2$. x = 3 - rac{1}{\sqrt{2}} (choosing smaller x value from graph). Then y = -(3 - rac{1}{\sqrt{2}}) + 1 = -2 + rac{1}{\sqrt{2}}. Complex number P = (3 - rac{\sqrt{2}}{2}) + (-2 + rac{\sqrt{2}}{2})i. Region constraints:	M1 A1	3.1a 1.1b
(d)	1. Inside/on the circle: |z - (3 - 2i)| \le 1 2. Bound by half-line l and horizontal line m (which has rg(z - (3-2i)) = 0): Inequality: \pi \le rg(z - (3-2i)) \le rac{7\pi}{4} (or equivalent boundary metrics).	M1 A2	2.1 2.5
	Total for Question 6	**9 Marks**	

Question 7 (Total: 12 Marks) — Matrices & Simultaneous Equations

<table>
<tr><th>Par</th><th>Scheme / Working</th><th>Marks</th><th>AO</th></tr>
<tr><td>(a)</td><td>Matrix M = egin{pmatrix} 6 & -5 & 3 | 5 & 1 & -1 | 1 & -3 & 2 |end{pmatrix}. (With variable k variation if applicable).
Find determinant: |det(M) = 6(2 - 3) - (-5)(10 - (-1)) + 3(-15 - 1) = -6 + 55 - 48 = 1.
If M is singular, det = 0. Shows constant holds or find k = ext{value}.

Find matrix of cofactors, transpose to find adjugate, and divide by determinant to obtain M^{-1}.</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td>(b)</td><td>Multiply M^{-1} by the column matrix of constants containing p to isolate x, y, z solutions in terms of p.</td><td>M1
A3</td><td>1.1b
1.1b</td></tr>
<tr><td>(c)</td><td>Set up equation compatibility check via row reduction / elimination.</td><td>M1
A1</td><td>3.1a
1.1b</td></tr>
<tr><td>(d)</td><td>Find condition for consistency: p = 3 (or equivalent determined value).

Geometrical interpretation: The three planes do not meet at a</td><td>M1
A1</td><td>2.1
1.1b</td></tr>
<tr><td>(e)</td><td>unique point.
Since the equations are inconsistent when p is not equal to the specific value, they form a triangular prism.</td><td>B1
B1</td><td>2.4
2.2a</td></tr>
<tr><td colspan="2">Total for Question 7</td><td colspan="2">12 Marks</td></tr>
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Question 8 (Total: 10 Marks) — Vectors & 3D Geometry

<table>
<tr><th>Part</th><th>Scheme / Working</th><th>Marks</th><th>AO</th></tr>
<tr><td>(a)</td><td>Substitute parametric form of line l_1 into plane equation 2x + y - 2z = 4 to find the scalar parameter value, then substitute back to get the unique point coordinates.
Use scalar product formula: \sin(heta) = rac{|\mathbf{d} \cdot</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td>(b)</td><td>\mathbf{n}|}{|\mathbf{d}||\mathbf{n}|}, where \mathbf{d} is the line direction vector and \mathbf{n} = egin{pmatrix} 2 \ 1 \ -2 \end{pmatrix}.
Calculate exact angle to one decimal place.

Construct line direction vector \mathbf{b} using the cross product</td><td>M1
M1
A1</td><td>1.1b
2.1
1.1b</td></tr>
<tr><td>(c)</td><td>or angle constraints such that it lies within plane \Pi and fulfills the target angle requirement. State final vector equation in form \mathbf{r} = \mathbf{a} + \mu \mathbf{b}.</td><td>M1
A2</td><td>3.1
a
2.2
a</td></tr>
<tr><td colspan="2">Total for Question 8</td><td colspan="2">10 Marks</td></tr>
</table>

Question 9 (Total: 11 Marks) — Volumes of Revolution & Modeling

Part	Scheme / Working	Marks	AO
(a)	Substitute coordinates of $C(1, k)$ into curve equation $x^2 + 9(y - 4)^2 = 9$: $1^2 + 9(k - 4)^2=9ightarrow 9(k - 4)^2=8ightarrow(k - 4)^2 = 8/9$ $k = 4 - rac{\sqrt{8}}{3}= 4 - rac{2\sqrt{2}}{3}$ (since $k < 4$ from the diagram).	M1 A1	3.3 1.1b
(b)	Volume of cylinder component from $y = 0$ to $y = k$: $V_1 = \pi \int_0^k 1^2 dy = \pi k$. Volume of curved head component: $V_2 = \pi \int_k^4 x^2 dy = \pi \int_k^4 [9 - 9(y - 4)^2] dy$.	M1	3.1a
	Integrate curved expression: $\int [9 - 9(y - 4)^2] dy = \left[9y - 3(y - 4)^3 ight]_k^4$. Evaluate limits accurately to find exact terms.	M1 A1	1.1b 1.1b
	Combine volumes: $Total Volume = V_1 + V_2$. Substitute numerical values and round to nearest whole number: $pprox 51 ext{ cm}^3$.	M1 A1	2.1 3.2a
(c)	Limitation statement: The model assumes the door handle profile perfectly matches an ellipse shape, ignoring any micro-manufacturing variations or internal hollowness.	B1	3.5b
(d)	Compare values: Calculated value ($51 ext{ cm}^3$) is close to actual value ($48 ext{ cm}^3$) with a percentage error of approx 6.25%. Conclude that the model is reliable and valid.	B1	3.5a
(e)	Refinement: Change the curve equation component from an elliptical form to a standard circle equation form ($x^2 + (y - y_0)^2 = r^2$) to represent a true spherical cross section.	B1	3.5c
	Total for Question 9	**11 Marks**	