



A Level Physics Edexcel

9. Thermodynamics

CONTENTS

Thermal Energy Transfer

- 9.1 Specific Heat Capacity & Latent Heat
- 9.2 Core Practical 12: Calibrating a Thermistor
- 9.3 Core Practical 13: Investigating Specific Latent Heat

Kinetic Theory & Ideal Gases

- 9.4 Internal Energy
- 9.5 Temperature & Absolute Zero
- 9.6 Kinetic Theory of Gases Equation
- 9.7 Ideal Gas Equation
- 9.8 Core Practical 14: Investigating Gas Pressure & Volume
- 9.9 Average Molecular Kinetic Energy

Black Body Radiation

- 9.10 Black Body Radiation
- 9.11 Stefan-Boltzmann Law
- 9.12 Wien's Law

Thermal Energy Transfer

9.1 Specific Heat Capacity & Latent Heat

YOUR NOTES

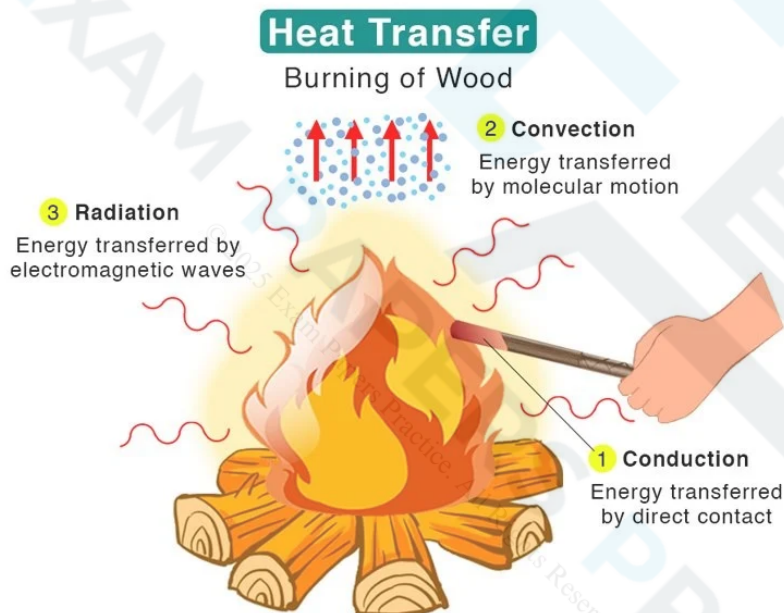


Specific Heat Capacity

- Specific heat capacity is defined as:

the energy required to raise the temperature of one kilogram of a substance by one kelvin

- The increase in temperature of an object depends on:
 - The amount of heat energy transferred
 - The mass of the object
 - The specific heat capacity of the material from which the object is made



Explanation of Heat Energy

- The energy input is in the form of **heat** energy
- The amount of **heat** energy needed is given by the equation:

$$\Delta E = mc\Delta\theta$$

- Where:
 - ΔE = change in heat energy, in joules (J)
 - m = mass, in kilograms (kg)
 - c = specific heat capacity, in joules per kilogram per degree Kelvin (J/kg K **or** J/kg °C)
 - $\Delta\theta$ = change in temperature, in Kelvin or Celsius

- (The symbol Δ in Maths is used to denote a change in value)



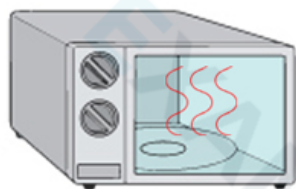
Stove



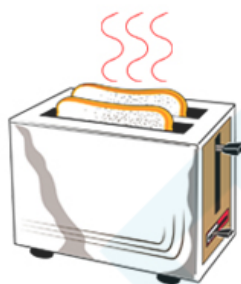
Iron



Heater



Microwave oven



Toaster



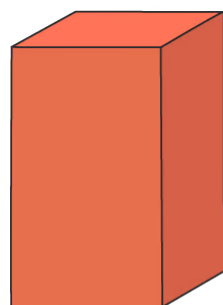
Fireplace

Examples of heat energy

- The thermodynamic **Kelvin temperature scale** is defined as:

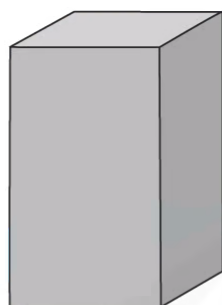
An absolute temperature scale where each degree is the same size as those on the Celsius scale

- Different materials:
 - Have different specific heat capacities because of their molecular structure
 - Have different rises in temperature for the same change in heat energy
- Specific heat capacity is mainly used for **liquids** and **solids**
- Good electrical conductors, such as copper and lead, are **excellent conductors of heat** due to their **low** specific heat capacity
 - On the other hand, **water** has a **very high** specific heat capacity, making it ideal for **heating** homes as the water remains hot in a radiator for a long time
- The specific heat capacity of different substances determines how **useful** they would be for a specific purpose eg. choosing the best material for kitchen appliances



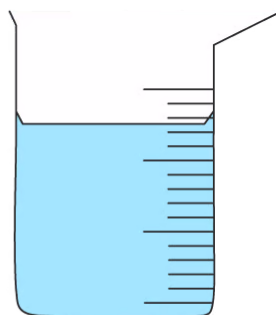
COPPER BLOCK

SPECIFIC HEAT CAPACITY
OF COPPER
= 390 J/kg°C



ALUMINIUM BLOCK

SPECIFIC HEAT CAPACITY
OF ALUMINIUM
= 910 J/kg°C



WATER

SPECIFIC HEAT CAPACITY
OF WATER
= 4200 J/kg°C



LOWER SPECIFIC HEAT CAPACITY –
WARMS UP AND COOLS DOWN QUICKLY
AS IT TAKES MUCH LESS ENERGY TO
CHANGE ITS TEMPERATURE

HIGHER SPECIFIC HEAT CAPACITY –
WARMS UP AND COOLS DOWN SLOWLY
AS IT TAKES MUCH MORE ENERGY
TO CHANGE ITS TEMPERATURE

Low v high specific heat capacity

- The specific heat capacity of some substances are given in the table below as examples:

Table of values of specific heat capacity for various substances

Substance	Specific Heat Capacity (J kg ⁻¹ K ⁻¹)
Water	4200
Ice	2200
Aluminium	900
Copper	390
Gold	130

Copyright © Save My Exams. All Rights Reserved

? Worked Example

Water of mass 0.48 kg is increased in temperature by 0.7 K. The specific heat capacity of water is 4200 J kg⁻¹ K⁻¹. Calculate the amount of energy transferred to the water.

Step 1: Write down the known quantities

- Mass, $m = 0.48$ kg
- Change in temperature, $\Delta\theta = 0.7$ K
- Specific heat capacity, $c = 4200$ J kg⁻¹ K⁻¹

Step 2: Write down the relevant equation

$$\Delta E = mc\Delta\theta$$

Step 3: Calculate the energy transferred by substituting in the values

$$\Delta E = (0.48) \times (4200) \times (0.7) = 1411.2$$

Step 4: Round the answer to 2 significant figures

$$\Delta E = 1400 \text{ J}$$

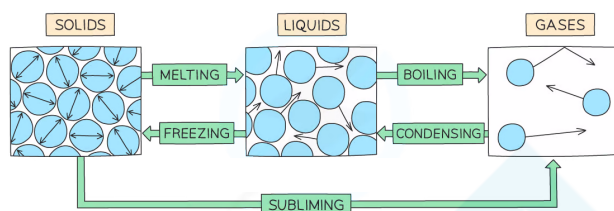


Exam Tip

You will always be given the specific heat capacity of a substance, so you do not need to memorise any values. Make sure that $\Delta\theta$ is the **change** in temperature, therefore, it can be in K or $^{\circ}\text{C}$.

Specific Latent Heat

- Energy is required to change the **state** of substance
- Examples of changes of state are:
 - Melting = solid to liquid
 - Evaporation/vaporisation/boiling = liquid to gas
 - Sublimation = solid to gas
 - Freezing = liquid to solid
 - Condensation = gas to liquid

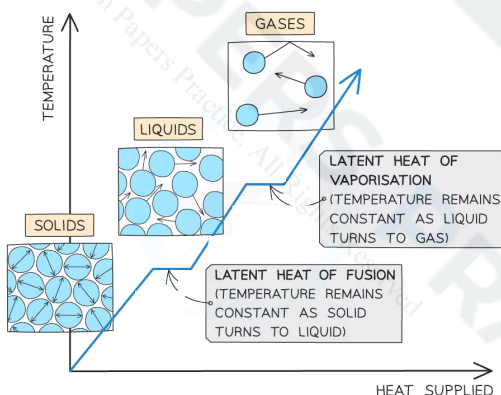


The example of changes of state between solids, liquids and gases

- When a substance changes state, there is **no temperature change**
- The energy supplied to change the state is called the **latent heat** and is defined as:

The thermal energy required to change the state of one kilogram of a substance without any change of temperature

- There are two types of latent heat:
 - Specific latent heat of **fusion** (melting)
 - Specific latent heat of **vaporisation** (boiling)



The changes of state with heat supplied against temperature. There is no change in temperature during changes of state

- The specific latent heat of **fusion** is defined as:

The thermal energy required to convert one kilogram of solid to liquid with no change in temperature

- This is used when melting a solid or freezing a liquid
- The specific latent heat of **vaporisation** is defined as:

The thermal energy required to convert one kilogram of liquid to gas with no change in temperature

- This is used when vaporising a liquid or condensing a gas

Calculating Specific Latent Heat

- The amount of energy ΔE required to melt or vaporise a mass of m with latent heat L is:

$$\Delta E = L\Delta m$$

- Where:
 - ΔE = amount of heat energy to change the state (J)
 - L = latent heat of fusion or vaporisation (J kg^{-1})
 - Δm = change in mass of the substance changing state (kg)
- The values of latent heat for water are:
 - Specific latent heat of fusion = 330 kJ kg^{-1}
 - Specific latent heat of vaporisation = 2.26 MJ kg^{-1}
- Therefore, evaporating 1 kg of water requires roughly **seven times** more energy than melting the same amount of ice to form water
- The reason for this is to do with intermolecular forces:
 - **When ice melts:** energy is required to just increase the molecular separation until molecules can flow freely over each other
 - **When water boils:** energy is required to completely separate the molecules until there are no longer forces of attraction between them, hence this requires much more energy



Worked Example

The energy needed to boil a mass of 530 g of a liquid is 0.6 MJ. Calculate the specific latent heat of the liquid and state whether it is the latent heat of vaporisation or fusion.

Step 1: Substitute in the values

$$\Delta m = 530 \text{ g} = 530 \times 10^{-3} \text{ kg}$$

$$\Delta E = 0.6 \text{ MJ} = 0.6 \times 10^6 \text{ J}$$

$$L = \frac{0.6 \times 10^6}{530 \times 10^{-3}} = 1.132 \times 10^6 \text{ J kg}^{-1} = 1.1 \text{ MJ kg}^{-1} \text{ (2 s.f.)}$$

L is the latent heat of vaporisation because the change in state is from liquid to gas (boiling)



Exam Tip

Use these reminders to help you remember which type of latent heat is being referred to:

- Latent heat of fusion = imagine 'fusing' the liquid molecules together to become a solid
- Latent heat of vaporisation = "water vapour" is steam, so imagine vaporising the liquid molecules into a gas

9.2 Core Practical 12: Calibrating a Thermistor

Core Practical 12: Calibrating a Thermistor

Aim of the Experiment

- To calibrate a thermistor so it can be used as a thermometer

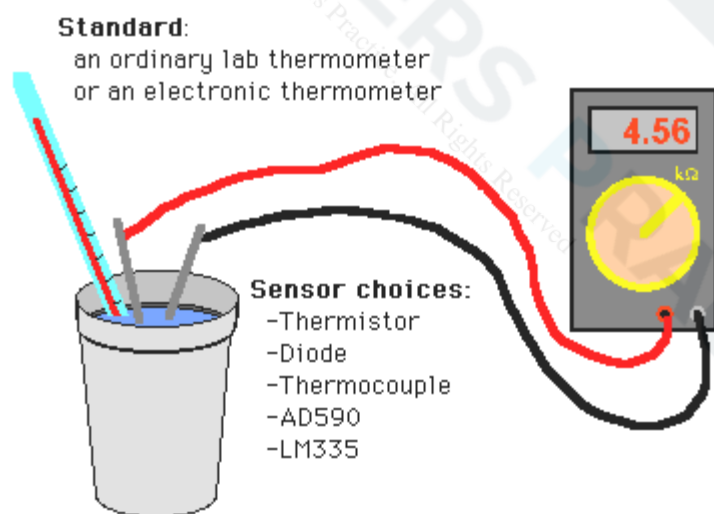
Variables

- Independent variable = temperature ($^{\circ}\text{C}$)
- Dependent variable = resistance of thermistor (Ω)
- Control variables:
 - Temperature gradient controlled by stirring

Equipment

- Power source
- Thermistor
- Fixed resistor
- Ohm-meter
- Bunsen burner, tripod and gauze
- Beaker filled with crushed ice
- Stirring rod
- Liquid in glass or mercury thermometer with range $-10 - 100^{\circ}\text{C}$
- Resolution of measuring equipment:
 - Thermometer = 1°C
 - Ohm-meter = 0.01Ω

Method



An example:

Pairs of readings at
different temperatures:

Standard	Sensor
12.3 $^{\circ}\text{C}$	245.3 $\text{k}\Omega$
23.2 $^{\circ}\text{C}$	133.6 $\text{k}\Omega$
34.7 $^{\circ}\text{C}$	89.4 $\text{k}\Omega$

- Set up the equipment with the thermistor immersed in the ice, the ohm-meter connected to record the resistance of the thermistor by placing it in parallel across it, and the fixed resistor in series with the thermistor
- Place the beaker of ice onto the tripod, without lighting the Bunsen burner
- Measure and record the temperature and reading on the ohm-meter
- Light the Bunsen burner and keep to a gentle flame
- Stir the ice / water gently at all times to keep the temperature as even as possible throughout the beaker
- At approximately 5°C intervals record the new temperature and resistance reading
- Continue until the water is boiling

Analysis

- Plot a graph of resistance against temperature
- Use the temperature graph to find the resistance at a given temperature

Evaluating the Experiment

Systematic Errors:

- Read the thermometer at eye level
- Check the zero error on the ohm-meter by connecting the leads across the terminals

Random Errors:

- Allow time for the temperature to reach equilibrium
- Stir the water before readings
- Ensure thermometer bulb is completely submerged in the water and level with the thermistor
- Turn off current between readings to avoid heating in the wires

Safety Considerations

- Risk of instability, use a stand and clamp to support leads
- Hot equipment and boiling water must be handled with care and allowed to cool down where possible
- Keep plastic cables / leads away from hot metal
- Check the voltage limit of the thermistor and stay within guidelines

9.3 Core Practical 13: Investigating Specific Latent Heat

Core Practical 13: Investigating Specific Latent Heat

Aims of the Experiment

- To determine the specific latent heat of ice

Variables:

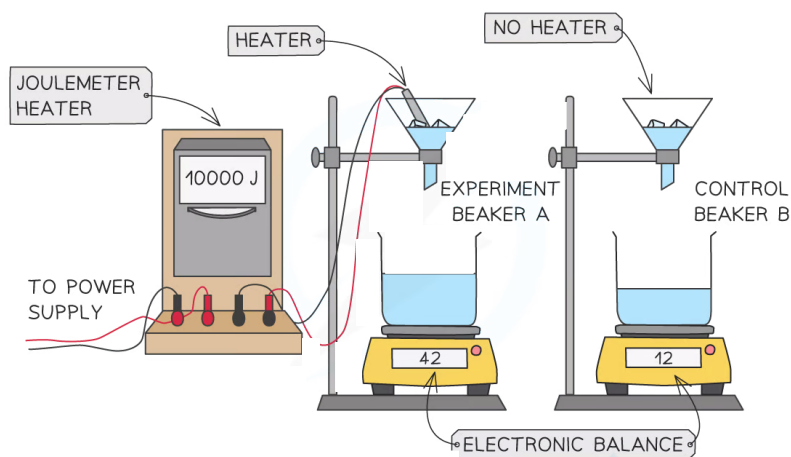
- Independent variable = Energy of the heater (Joulemeter) (J)
- Dependent variable = The temperature, T of the ice/water ($^{\circ}\text{C}$)
- Control variables:
 - Repeat readings with same energy supplied by heater
 - Mass of ice in each set up
 - Time for experiment in each set up

Equipment List

Equipment	Purpose
2 x funnels with filter paper	Hold the ice so when it melts it drips through the funnel opening
3 x retort stands	To hold the funnels and heater in place above the beakers
300 g of crushed ice and spoon	The heater will melt the ice
2 x 250 cm ³ beakers	To collect the water from the melted ice
2 x thermometer	To measure the temperature of the melting ice
2 x electronic balances	To measure the mass of the melted ice
1 x joulemeter and power supply	To heat the ice

- Resolution of measuring equipment:
 - Joulemeter = 1 J
 - Electronic balance = 0.1 g
 - Thermometer = 0.1 $^{\circ}\text{C}$

Method



1. Set up the experiment and the control
 - Attach the funnels to retort stands, place the filter paper inside and add a heater (also on a retort stand) inside one of the funnels – ensure the heater is not touching the funnel
 - Use a spoon and an electronic balance to measure out 50 g of crushed ice into a beaker and pour into each funnel
 - Add a beaker below each funnel and place on top of an electronic balance
2. Wait until the ice reaches 0 °C.
 - This is when it starts to melt and water starts to drip out of both funnels into the beakers below.
 - Check the temperature with a thermometer in each funnel
3. Turn on the heater
 - Set the heater to supply 10, 000 Joules of energy to the experiment funnel
4. Wait until the reading on the Joulemeter says 10, 000 J
 - Turn the heater off
5. Read and record the mass of each beaker of water on each electronic balance
6. Repeat the experiment at least 3 times and calculate the average mass, m for the water in each beaker
 - m_A = average mass of water in beaker A
 - m_B = average mass of water in beaker B

7. Calculate the mass of the melted ice and convert into kg
 - $\Delta m = m_A - m_B$
 - Mass in g $\div 1000$ = Mass in kg
8. Calculate the specific latent heat of fusion of ice to water using the equation $\Delta E = L\Delta m$
 - ΔE = Energy supplied by the heater = 10 000 J
 - L = Specific latent heat of fusion
 - Δm = mass of ice

$$L = \frac{\Delta E}{\Delta m}$$

Sample Results Table:

	Mass of water in Beaker A/g	Mass of water in Beaker B/g
Experiment 1	42	12
Experiment 2	43	12
Experiment 3	41	11
Average mass/g	42	11.6
Average mass/kg	0.042	0.0116

Analysis of Results

- The results obtained a latent heat of fusion of 330 000 J
- The actual value of the latent heat of fusion for ice is 334 000 J
- The percentage error in this value is $\frac{334\,000 - 330\,000}{334\,000} \times 100 = 1.2\%$

Evaluating the Experiment

Systematic Errors:

- Make sure you zero the electronic balances when the beakers are empty
- Always check that the ice has reached 0 °C by reading the thermometer at eye level
- This experiment requires accurate determination of energy transfers
 - To improve the accuracy, consider applying lagging or insulation to the funnels and beakers – this will reduce the amount of energy lost to the surroundings

Random Errors:

- The heater should be switched off and allowed to cool between readings
 - So the rate of heating and the starting temperature of the heater is the same
- Calculate the average mass of the water
 - This will reduce random errors in the reading
 - Repeat the experiment at least 3 times

Safety Considerations

- Ensure no water gets on the electronic balance

- Wipe up any spillages immediately and turn off the balance
- Do not touch the heating element with your fingers, as it could be hot and burn your skin
- Do not handle ice with your bare hands, use a spoon to measure it into the beaker

? Worked Example

A student conducts an experiment to find the latent heat of fusion for ice

They obtain the following table of results:

	Mass of water in Beaker A/g	Mass of water in Beaker B/g
Experiment 1	42	12
Experiment 2	41	13
Experiment 3	42	13
Average mass/g		
Average mass/kg		

Copyright © Save My Exams. All Rights Reserved

Calculate the latent heat of fusion.

Step 1: Complete the average mass row in the table

- The average mass is calculated by adding all 3 masses together and then dividing by 3

	Mass of water in Beaker A/g	Mass of water in Beaker B/g
Experiment 1	42	12
Experiment 2	41	13
Experiment 3	42	13
Average mass/g	42	12.6
Average mass/kg	0.042	0.0126

Step 2: Calculate the average mass in kg

- $\text{Mass in g} \div 1000 = \text{Mass in kg}$

Step 3: Calculate the latent heat of fusion

- Calculate the specific latent heat of fusion of ice to water using the equation $\Delta E = L\Delta m$
 - ΔE = Energy supplied by the heater = 10,000 J
 - L = Specific latent heat of fusion
 - Δm = mass of ice

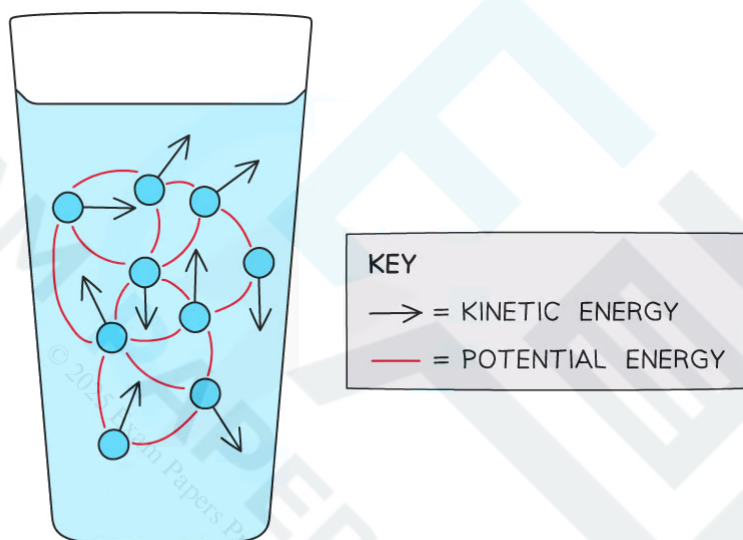
- Rearrange the equation $L = \frac{\Delta E}{\Delta m}$
- So, $L = \frac{10\,000}{0.03} = 340\,136 \text{ J kg}^{-1} = 340\,000 \text{ J}$

Kinetic Theory & Ideal Gases

9.4 Internal Energy

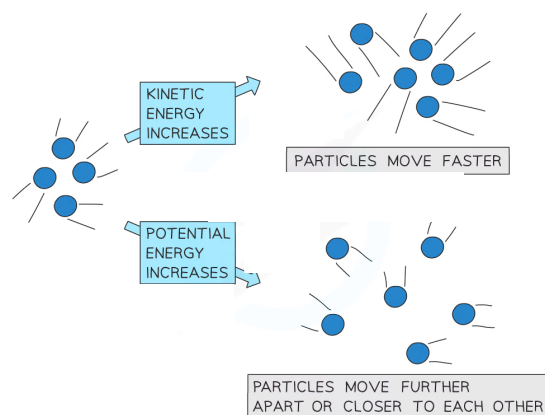
Internal Energy

- Energy can be classified into two forms: kinetic or potential energy
- The molecules of all substances contain both kinetic and potential energies
 - Kinetic energy is due to the speed of the molecules and gives the material its temperature
 - Potential energy is due to the separation between the molecules and their position within the structure



- The amount of kinetic and potential energy a substance contains depends on its phase of matter (solid, liquid or gas)
 - This is known as the **internal energy**
- The internal energy of a substance is defined as:

The sum of the kinetic and potential energies of all the molecules within a given mass of a substance



- The symbol for internal energy is U , with units of **Joules (J)**
- Particles are randomly distributed, meaning they all have different speeds and separations
- The internal energy of a system is determined by:
 - **Temperature** (higher temperature, higher kinetic energy and vice versa)
 - The **random motion** of molecules
 - The **phase of matter**: gases have the highest internal energy, solids have the lowest
 - **Intermolecular forces** between the particles (greater intermolecular forces, higher potential energy and vice versa) – this is linked to the phase (solid, liquid, gas) that the matter is in
- The internal energy of a system can increase by:
 - **Doing work** on it
 - **Adding heat** to it
- The internal energy of a system can decrease by:
 - **Losing heat** to its surroundings
 - **Changing state** from a solid to a liquid or liquid to a gas



Exam Tip

Always remember internal energy is made up of both the kinetic and potential energy of the particles in a substance.

9.5 Temperature & Absolute Zero

Absolute Zero

- On the thermodynamic (Kelvin) temperature scale, absolute zero is defined as:

The lowest temperature possible. Equal to 0 K or -273.15°C

- It is not possible to have a temperature lower than 0 K
 - This means a temperature in Kelvin will **never** be a negative value
- Absolute zero is defined as:

The temperature at which the molecules in a substance have zero kinetic energy

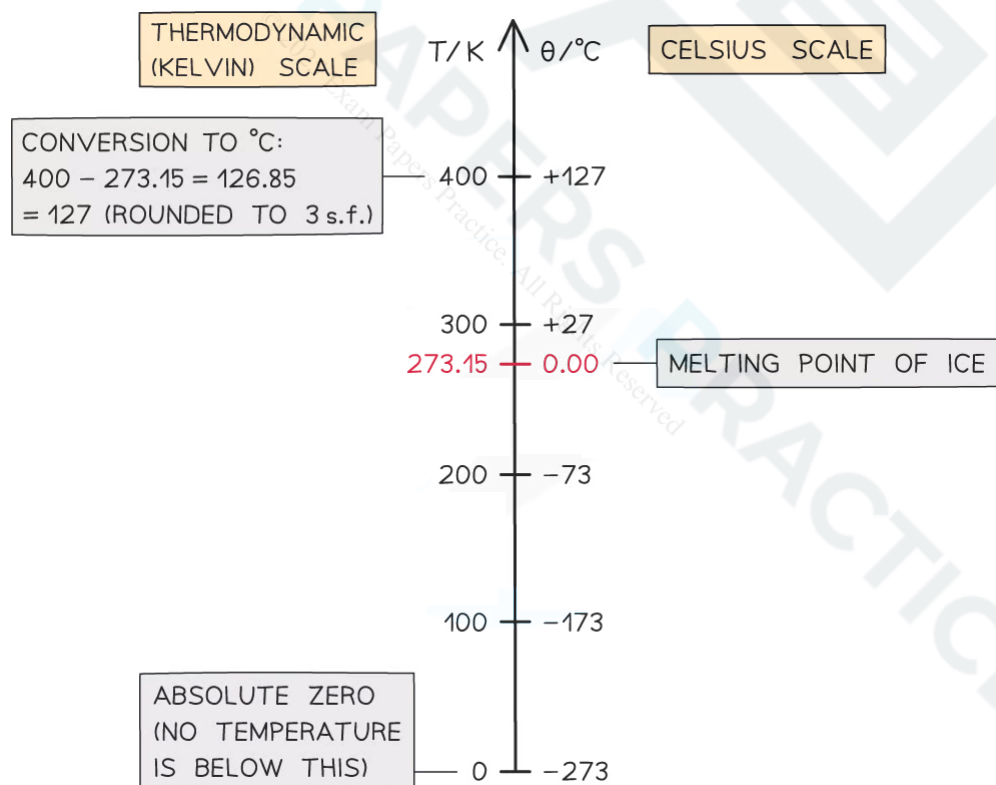
- This means for a system at 0 K, it is not possible to remove any more energy from it
- Even in space, the temperature is roughly 2.7 K, just above absolute zero

Using the Kelvin Scale

- To convert between temperatures θ in the Celsius scale, and T in the Kelvin scale, use the following conversion:

$$\theta/^{\circ}\text{C} = T/\text{K} - 273.15$$

$$T/\text{K} = \theta/^{\circ}\text{C} + 273.15$$



Conversion chart relating the temperature on the Kelvin and Celsius scales

- The divisions on both scales are equal. This means:

A change in a temperature of 1 K is equal to a change in temperature of 1 °C

- This is why when using the specific heat capacity equation

$$\Delta E = mc\Delta\theta$$

- $\Delta\theta$ does not require the temperature to be in either unit
 - This is because the difference in temperature between two values whether in Kelvin or Celsius will be exactly the same

? Worked Example

In many ideal gas problems, room temperature is considered to be 300 K. What is this temperature in Celsius?

Step 1: Kelvin to Celsius equation

$$\theta / ^\circ\text{C} = T / \text{K} - 273.15$$

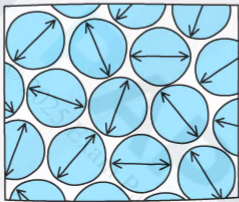
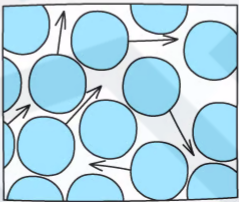
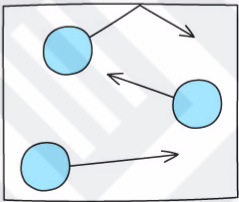
Step 2: Substitute in value of 300 K

$$300 \text{ K} - 273.15 = 26.85 ^\circ\text{C}$$

Kinetic Energy & Temperature

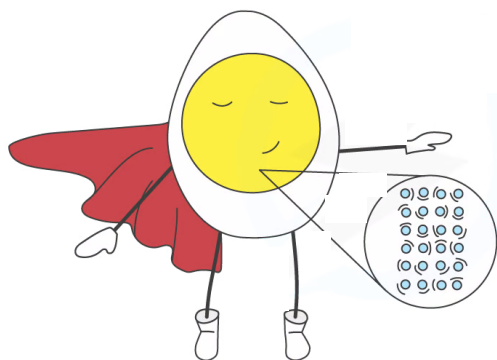
- Heating an object causes its temperature to rise
- According to **kinetic theory** when energy is supplied to an object the molecules in that object receive the energy as kinetic energy and move faster
 - In solids this is in the form of **vibrations**
 - In a gas, molecules move quickly around their container
 - This kinetic energy determines the temperature of the substance.

Solid, Liquid & Gas Summary Table

State	Solid	Liquid	Gas
Density	High	Medium	Low
Arrangement of particles	Regular pattern	Randomly arranged	Randomly arranged
Movement of particles	Vibrate around a fixed position	Move around each other	Move quickly in all directions
Energy of particles	Low energy	Greater energy	Highest energy
2D diagram			

- The internal kinetic energy on the atomic scale is separate from the overall movement of an object
- Molecules vibrate within the material and do not cause it to move
 - For example, an egg being thrown through the air has kinetic energy because of the movement of the whole egg and not because of the movement of the individual molecules inside it.

EGG HAS KINETIC ENERGY AS IT FLIES THROUGH THE AIR



INTERNAL ENERGY
COMES FROM
VIBRATION OF
MOLECULES INSIDE
THE EGG

- Taking energy away from the molecules of a substance causes its temperature to become lower
 - In a situation where kinetic energy is continuously being taken away from a collection of molecules, theoretically there would be a moment where all the kinetic energy has been removed from the substance
 - This is when the substance reaches absolute zero, which remains theoretical point, having never been achieved by scientists in a laboratory



Exam Tip

If you forget in the exam whether it's $+273.15$ or -273.15 , just remember that $0^{\circ}\text{C} = 273.15\text{ K}$. This way, when you know that you need to $+273.15$ to a temperature in degrees to get a temperature in Kelvin. For example: $0^{\circ}\text{C} + 273.15 = 273.15\text{ K}$.

Also remember that C is nearer the beginning of the alphabet to K. So to go back from K to C we must subtract.

9.6 Kinetic Theory of Gases Equation

Kinetic Theory of Gases Equation

- Gases consist of atoms or molecules randomly moving around at high speeds
 - The kinetic theory of gases models the thermodynamic behaviour of gases by linking the **microscopic properties** of particles (mass and speed) to **macroscopic properties** of particles (pressure and volume)
- The theory is based on the following assumptions:
 - Molecules of gas behave as identical, hard, perfectly elastic spheres
 - The volume of the molecules is negligible compared to the volume of the container
 - The time of a collision is negligible compared to the time between collisions
 - There are no forces of attraction or repulsion between the molecules
 - The molecules are in continuous random motion
- The number of molecules of gas in a container is very large, therefore the **average** behaviour (eg. speed) is usually considered

Root-Mean-Square Speed

- The pressure of an ideal gas equation includes the **mean square** speed of the particles:

$$\langle c^2 \rangle$$

- Where
 - c = **average** speed of the gas particles
 - $\langle c^2 \rangle$ has the units $\text{m}^2 \text{s}^{-2}$
- Since particles travel in all directions in 3D space and velocity is a vector, some particles will have a negative direction and others a positive direction
 - When there are a large number of particles, the total positive and negative velocity values will cancel out, giving a net zero value overall
 - In order to find the pressure of the gas, the **velocities must be squared** meaning that all the values end up **positive**
- To calculate the **average speed** of the particles in a gas, take the square root of the mean square speed:

$$\sqrt{\langle c^2 \rangle} = c_{rms}$$

- $c_{r.m.s}$ is known as the **root-mean-square** speed and still has the units of m s^{-1}



Worked Example

An ideal gas has a density of 4.5 kg m^{-3} at a pressure of $9.3 \times 10^5 \text{ Pa}$ and a temperature of 504 K .

Determine the root-mean-square (r.m.s.) speed of the gas atoms at 504 K .

Step 1: Write out the equation for the pressure of an ideal gas with density

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

Step 2: Rearrange for mean square speed

$$\langle c^2 \rangle = \frac{3p}{\rho}$$

Step 3: Substitute in values

$$\langle c^2 \rangle = \frac{3 \times (9.3 \times 10^5)}{4.5} = 6.2 \times 10^5 \text{ m}^2 \text{ s}^{-2}$$

Step 4: To find the r.m.s value, take the square root of the mean square speed

$$c_{rms} = \sqrt{\langle c^2 \rangle} = \sqrt{6.2 \times 10^5 \text{ m}^2 \text{ s}^{-2}} = 787.4 \text{ m s}^{-1}$$

Step 5: Write the answer to the correct significant figures and include units

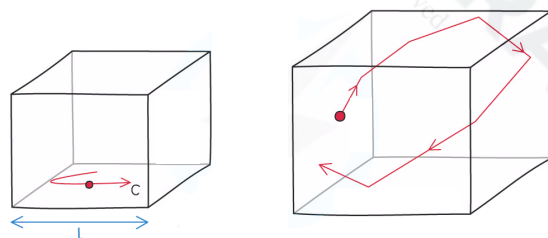
$$c_{rms} = 790 \text{ ms}^{-1} \text{ (2 sig figs)}$$

Deriving the Equation for Kinetic Theory

- When molecules rebound from a wall in a container, the change in momentum gives rise to a force exerted by the particle on the wall
 - Many molecules moving in random motion exert forces on the walls which create an average overall **pressure**, since pressure is the force per unit area

The Situation for the Derivation

- Picture a single molecule in a cube-shaped box with sides of equal length l
- The molecule has a mass m and moves with speed c , parallel to one side of the box
- It collides at regular intervals with the ends of the box, exerting a force and contributing to the pressure of the gas
- By calculating the pressure this one molecule exerts on one end of the box, the total pressure produced by all the molecules can be deduced



A single molecule in a box collides with the walls and exerts a pressure

Five-Step Derivation

Step 1: Find the change in momentum as a single molecule hits a wall perpendicularly

- One assumption of the kinetic theory is that molecules **rebound elastically**

- This means there is no kinetic energy lost in the collision
- If they rebound in the opposite direction to their initial velocity, their final velocity is $-c$
- The change in momentum is therefore:

$$\Delta p = -mc - (+mc) = -mc - mc = -2mc$$

Step 2: Calculate the number of collisions per second by the molecule on a wall

- The time between collisions of the molecule travelling to one wall and back is calculated by travelling a distance of $2l$ with speed c :

$$\text{time between collisions} = \frac{\text{distance}}{\text{speed}} = \frac{2l}{c}$$

- **Note:** c is **not** taken as the speed of light in this scenario

Step 3: Find the change in momentum per second

- The force the molecule exerts on one wall is found using Newton's second law of motion:

$$\text{Force} = \text{rate of change of momentum} = \frac{\Delta p}{\Delta t} = \frac{2mc}{2l/c} = \frac{mc^2}{l}$$

- The change in momentum is $+2mc$ since the force on the molecule from the wall is in the opposite direction to its change in momentum

Step 4: Calculate the total pressure from N molecules

- The area of one wall is l^2
- The pressure is defined using the force and area:

$$\text{pressure, } p = \frac{\text{force}}{\text{area}} = \frac{mc^2/l}{l^2} = \frac{mc^2}{l^3}$$

- This is the pressure **exerted from one molecule**
- To account for the large number of N molecules, the pressure can now be written as:

$$p = \frac{Nmc^2}{l^3}$$

- Each molecule has a different velocity and they all contribute to the pressure
- The mean squared speed of c^2 is written with left and right-angled brackets $\langle c^2 \rangle$
- The pressure is now defined as:

$$p = \frac{Nm \langle c^2 \rangle}{l^3}$$

Step 5: Consider the effect of the molecule moving in 3D space

- The pressure equation still assumes all the molecules are travelling in the same direction and colliding with the same pair of opposite faces of the cube
- In reality, all molecules will be moving in three dimensions equally

- Splitting the velocity into its components c_x , c_y and c_z to denote the amount in the x, y and z directions, c^2 can be defined using pythagoras' theorem in 3D:

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

- Since there is nothing special about any particular direction, it can be determined that:

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$$

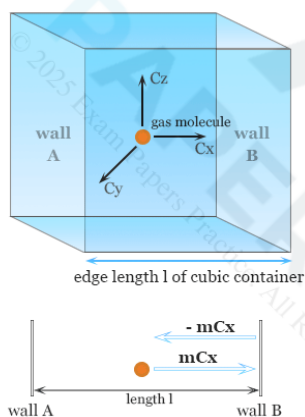
- Therefore, $\langle c_x^2 \rangle$ can be defined as:

$$\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$$

- The box is a cube and all the sides are of length l
 - This means l^3 is equal to the volume of the cube, V
- Substituting the new values for $\langle c^2 \rangle$ and l^3 back into the pressure equation obtains the final equation:

$$pV = \frac{1}{3} Nm \langle c^2 \rangle$$

- This is known as the **Kinetic Theory of Gases equation**:



- It can also be written using the density ρ of the gas:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{Nm}{V}$$

- Rearranging the pressure equation for p and substituting the density ρ :

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$



Exam Tip

Make sure to revise and understand each step for the whole of the derivation, as you may be asked to derive all, or part, of the equation in an exam question.

9.7 Ideal Gas Equation

Ideal Gas Equation

- When calculating for gases, assume that the gas is an ideal gas
- The **three gas laws** (explained below) can be combined to create one equation in terms of pressure, volume, temperature and amount of gas.

The Boltzmann Constant, k

- The Boltzmann constant k is used in the ideal gas equation and is defined as:

$$k = \frac{R}{N_A}$$

- Where:
 - R = molar gas constant
 - N_A = Avogadro's constant
- Boltzmann's constant therefore has a value of

$$k = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

- The Boltzmann constant relates the properties of microscopic particles (e.g. kinetic energy of gas molecules) to their macroscopic properties (e.g. temperature)
 - This is why the units are J K^{-1}
- Its value is very small because the increase in kinetic energy of a molecule is very small for every incremental increase in temperature

The Gas Laws

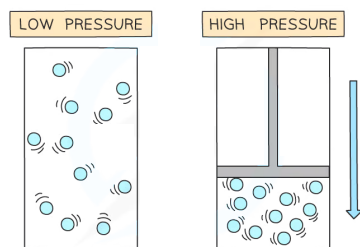
- The ideal gas laws are the experimental relationships between pressure (P), volume (V) and temperature (T) of an ideal gas
- The mass and the number of molecules of the gas is assumed to be constant for each of these quantities

Boyle's Law

- If the temperature T of an ideal gas is constant, then **Boyle's Law** is given by:

$$P \propto \frac{1}{V}$$

- This means the pressure is **inversely proportional** to the volume of a gas



Pressure increases when a gas is compressed

- The relationship between the pressure and volume for a fixed mass of gas at constant temperature can also be written as:

$$P_1 V_1 = P_2 V_2$$

- Where:
 - P_1 = initial pressure (Pa)
 - P_2 = final pressure (Pa)
 - V_1 = initial volume (m^3)
 - V_2 = final volume (m^3)

Charles's Law

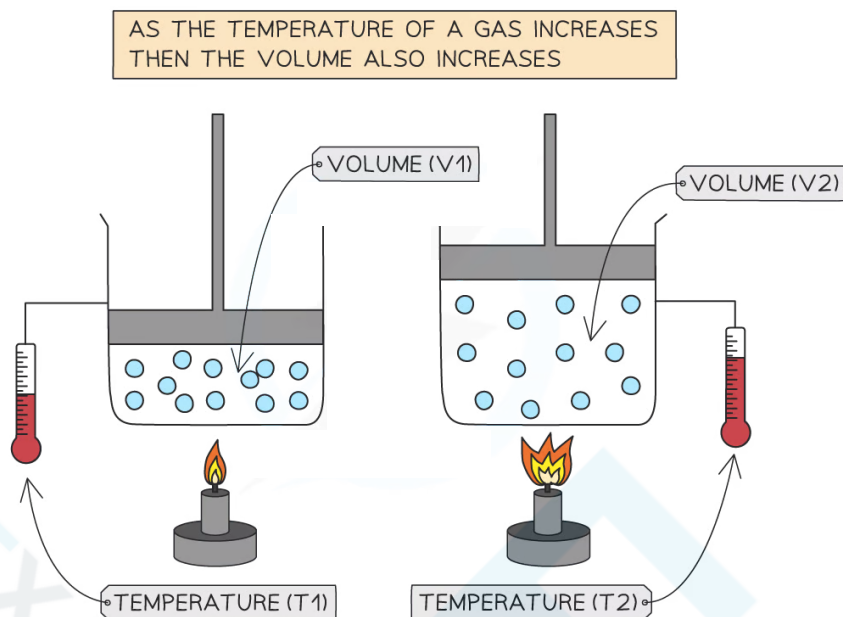
- If the pressure P of an ideal gas is constant, then **Charles's law** is given by:

$$V \propto T$$

- This means the volume is **proportional** to the temperature of a gas
 - The relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure can also be written as:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- Where:
 - V_1 = initial volume (m^3)
 - V_2 = final volume (m^3)
 - T_1 = initial temperature (K)
 - T_2 = final temperature (K)



Pressure Law

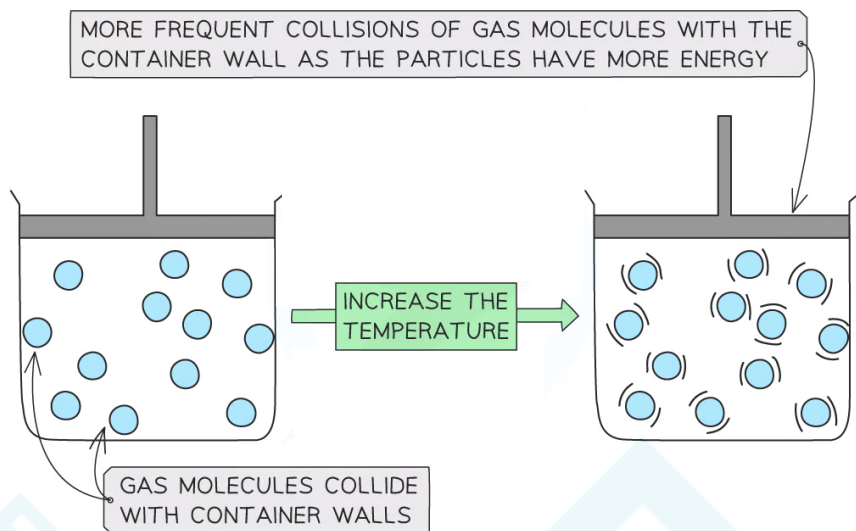
- If the volume V of an ideal gas is constant, the **Pressure law** is given by:

$$P \propto T$$

- This means the pressure is **proportional** to the temperature
 - The relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume can also be written as:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- Where:
 - P_1 = initial pressure (Pa)
 - P_2 = final pressure (Pa)
 - T_1 = initial temperature (K)
 - T_2 = final temperature (K)



Worked Example

A storage cylinder of an ideal gas has a volume of $8.3 \times 10^3 \text{ cm}^3$. The gas is at a temperature of 15°C and a pressure of $4.5 \times 10^7 \text{ Pa}$. Calculate the amount of gas in the cylinder, in moles.

Step 1: Write down the ideal gas equation

Since the number of moles (n) is required, use the equation:

$$pV = nRT$$

Step 2: Rearrange the equation for the number of moles n

$$n = \frac{pV}{RT}$$

Step 3: Substitute in values

$$V = 8.3 \times 10^3 \text{ cm}^3 = (8.3 \times 10^3) \times 10^{-6} = 8.3 \times 10^{-3} \text{ m}^3$$

$$T = 15^\circ\text{C} + 273.15 = 288.15 \text{ K}$$

$$n = \frac{(4.5 \times 10^7) \times (8.3 \times 10^{-3})}{8.31 \times 288.15} = 155.98 = 160 \text{ mol (2 sf)}$$



Exam Tip

After you solve a problem using any of the gas laws (or all of them combined), always check whether your final result makes physical sense – e.g. if you are asked to calculate the final pressure of a fixed mass of gas being heated at constant volume, your result must be greater than the initial pressure given in the problem (since Gay-Lussac's law states that pressure and absolute temperature are directly proportional at constant volume).

9.8 Core Practical 14: Investigating Gas Pressure & Volume

Investigating Gas Pressure and Volume

- Boyle's Law relates pressure and volume at constant temperature
 - This investigation is one valid example of how this required practical might be tackled, others do exist

Variables

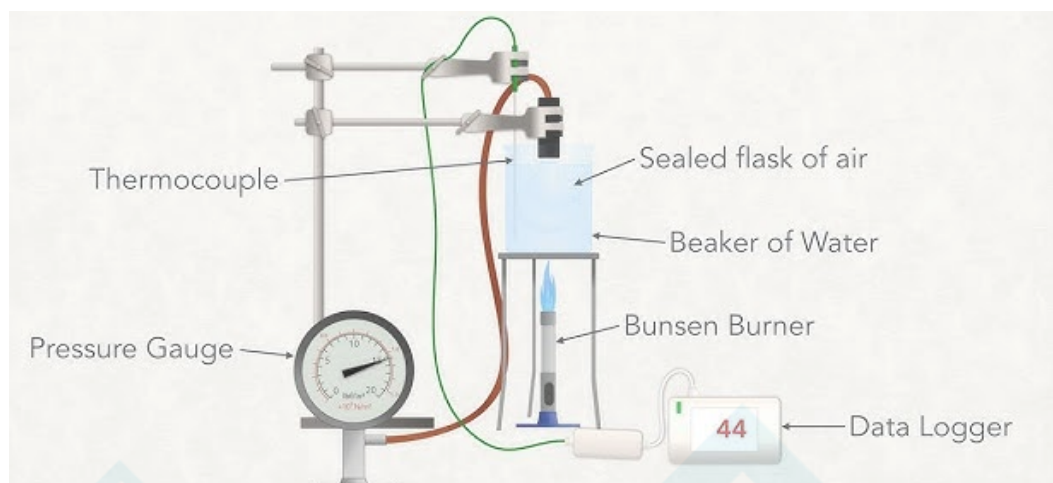
- Independent variable = Mass, m (kg)
- Dependent variable = Volume, V (m³)
- Control variables:
 - Temperature
 - Cross-sectional area of the syringe

Equipment List

Apparatus	Purpose
Clamp Stand	To hold the equipment
Syringe (10 ml)	To measure the volume of the air inside the tube
Rubber Tubing	To stop air getting into the syringe
Pinch Clip	To hold the rubber tubing in place
String	To hang the masses
100g masses with 100g holder	Used to change the volume of the air
Counterweight (or G clamp)	To keep the stand and clamp secure and rigid on the flat surface
Vernier Caliper	To calculate the diameter of the syringe

- Resolution of measuring equipment:
 - Pressure gauge = 0.02×10^5 Pa
 - Volume = 0.2 cm^3
 - Vernier Caliper = 0.02 mm

Method



Apparatus setup for Boyle's Law

1. With the plunger removed from the syringe, measure the inside diameter, d of the syringe using a Vernier caliper.
 - Take 3 readings and find an average
 2. The plunger should be replaced and the rubber tubing should be fit over the nozzle and clamped with a pinch clip as close to the nozzle as possible
 3. Set up the apparatus as shown in the diagram
 4. Push the syringe upwards until it reads the lowest volume of air visible. Record this volume
 5. Add the 100 g mass holder to the loop of string at the bottom of the plunger. Wait a few seconds before taking the reading (this allows temperature to equilibrate after work is done against the plunger when the volume increases)
 6. Record the value of the new volume from the syringe scale
 7. Repeat the experiment by adding 100 g masses and recording the readings up to 10 readings.
- An example table of results might look like this:

		MASS / kg	VOLUME / cm ³
NO MASS HOLDER	→	0.0	
	→	0.2	
100g MASS HOLDER AND A 100g MASS	→	0.4	
		0.6	
		0.8	
		1.0	
		1.2	

DIAMETER OF THE SYRINGE	DIAMETER 1/mm	DIAMETER 2/mm	DIAMETER 3/mm	AVERAGE DIAMETER/mm

Analysing the Results

- Boyle's Law can be represented by the equation:

$$pV = \text{constant}$$

- This means the pressure must be calculated from the experiment
- The exerted pressure of the masses is calculated by:

$$p = \frac{F}{A}$$

- Where:
 - F = weight of the masses, mg (N)
 - A = cross-sectional area of the syringe (m²)
- The cross-sectional area is found from the equation for the area of a circle:

$$A = \frac{\pi d^2}{4}$$

- To calculate the pressure of the gas:

$$\text{Pressure of the gas} = \text{Atmospheric pressure} - \text{Exerted pressure from the masses}$$

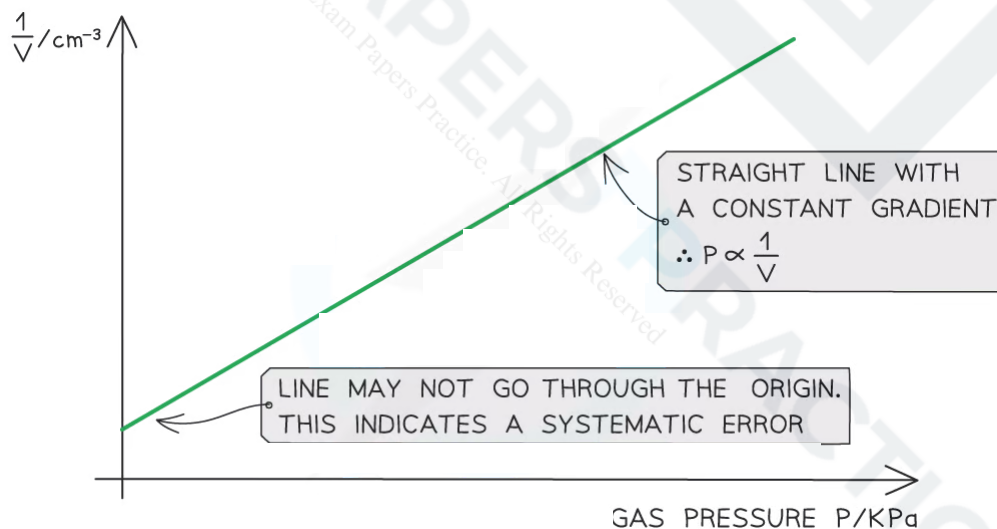
- Where:
 - Atmospheric pressure = 101 kPa

- The table of results may need to be modified to fit these extra calculations. Here is an example of how this might look:

	$F = mg$	$P = \frac{F}{A}$	ATMOSPHERIC PRESSURE	EXERTED PRESSURE	
MASS / kg	FORCE / N	EXERTED PRESSURE / Pa	GAS PRESSURE P/KPa	VOLUME V/cm ³	$\frac{1}{V}$ /cm ⁻³
0.0					
0.2					
0.4					
0.6					
0.8					
1.0					
1.2					

- Once these values are calculated:

- Plot a graph of p against $1/V$ and draw a line of best fit
- If this plot is a straight line graph, this means that the pressure is proportional to the inverse of the volume, hence confirming Boyle's Law ($pV = \text{constant}$)



Evaluating the Experiment

Systematic Errors:

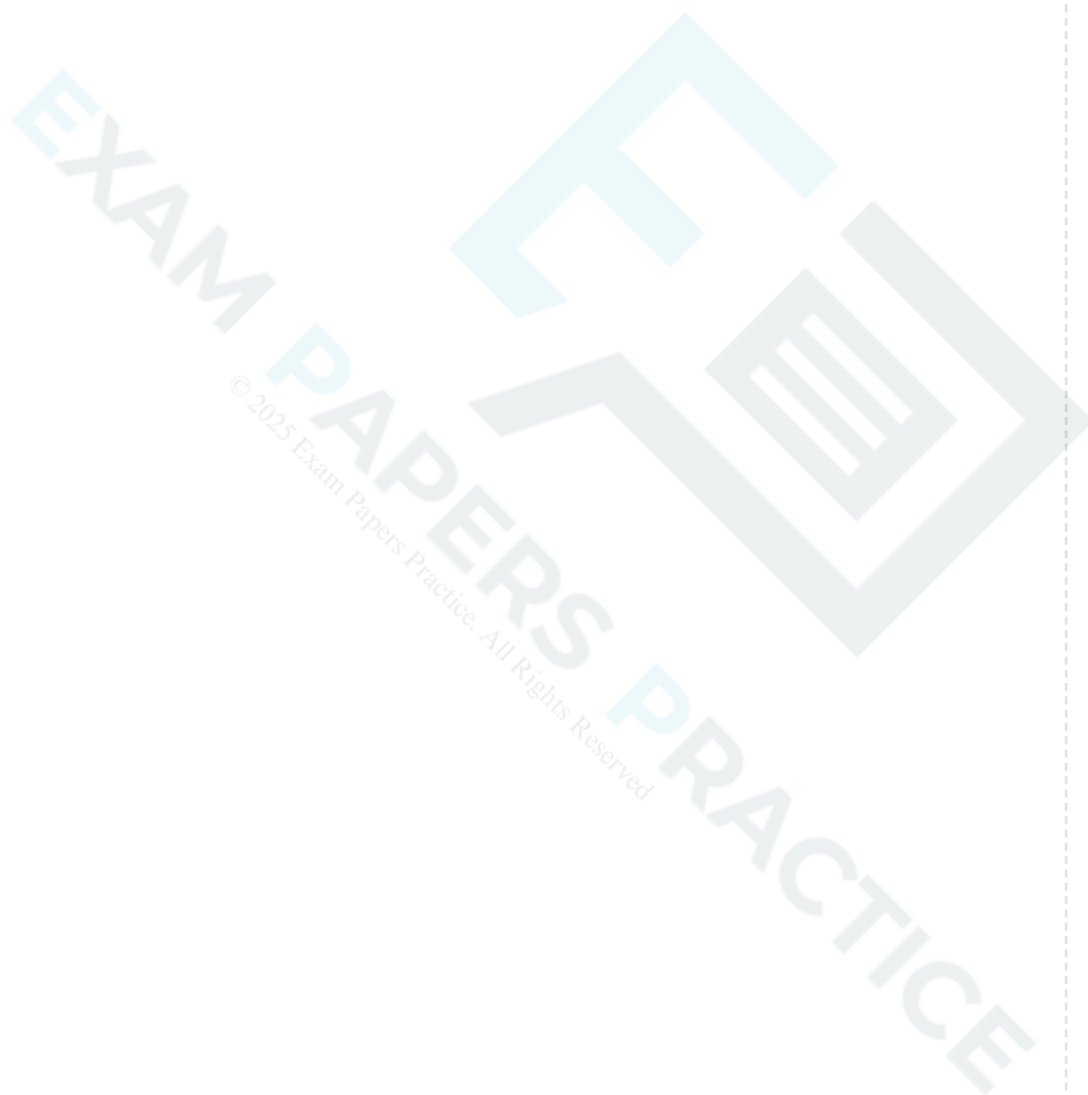
- There may be friction in the syringe which causes a systematic error
 - Use a syringe that has very little friction or lubricated it, so the only force is from the weights pulling the syringe downwards

Random Errors:

- The reading of the volume should be taken a few seconds after the mass has been added to the holder to allow temperature changes to equilibrate
- Room temperature must be kept constant

Safety Considerations

- A counterweight or G-clamp must be used to avoid the stand toppling over and causing injury, especially if the surface is not completely flat



9.9 Average Molecular Kinetic Energy

Average Molecular Kinetic Energy

- An important property of molecules in a gas is their **average kinetic energy**
- This can be deduced from the ideal gas equations relating pressure, volume, temperature and speed
- Recall the ideal gas equation in terms of number of molecules:

$$pV = NkT$$

- Also, recall the equation linking pressure and mean square speed of the molecules:

$$pV = \frac{1}{3}Nm(c_{rms})^2$$

- The left-hand side of both equations are equal to pV
- This means the right-hand sides of both equations are also equal:

$$\frac{1}{3}Nm(c_{rms})^2 = NkT$$

- N will cancel out on both sides and multiplying by 3 on both sides too obtains the equation:

$$m(c_{rms})^2 = 3kT$$

- Recall the familiar kinetic energy equation from mechanics:

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

- Instead of v^2 for the velocity of one particle, $(c_{rms})^2$ is the average speed of all molecules
- Multiplying both sides of the equation by $\frac{1}{2}$ obtains the **average molecular kinetic energy** of the molecules of an ideal gas:

$$E_k = \frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT$$

- Where:

- E_k = kinetic energy of a molecule (J)
- m = mass of one molecule (kg)
- $(c_{rms})^2$ = mean square speed of a molecule (m^2s^{-2})
- k = Boltzmann constant
- T = temperature of the gas (K)

- **Note:** this is the average kinetic energy for only **one** molecule of the gas
- To find the average kinetic energy for many molecules of the gas, multiply both sides of the equation by the number of molecules N to obtain:

$$E_k = \frac{1}{2} Nm(c)^2 = \frac{3}{2} NkT$$

- A key feature of this equation is that the mean kinetic energy of an ideal gas molecule is proportional to its thermodynamic temperature

$$E_k \propto T$$

- The Boltzmann constant k can be replaced with

$$k = \frac{R}{N_A}$$

- Substituting this into the average molecular kinetic energy equation means it can also be written as:

$$E_k = \frac{1}{2} m(c_{rms})^2 = \frac{3}{2} kT = \frac{3RT}{2N_A}$$



Worked Example

Helium can be treated as an ideal gas. Helium molecules have a root-mean-square (r.m.s.) speed of 720 m s^{-1} at a temperature of 45°C . Calculate the r.m.s. speed of the molecules at a temperature of 80°C .

Step 1: Write down the equation for the average kinetic energy

$$E_k = \frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT$$

Step 2: Determine the relation between c_{rms} and the temperature T

Since m and k are constant, $(c_{rms})^2$ is directly proportional to T

$$(c_{rms})^2 \propto T$$

Therefore

$$c_{rms} \propto \sqrt{T}$$

Step 3: Change the proportionality into an equation

$$c_{rms} = a\sqrt{T}$$

where a is the constant of proportionality

Step 4: Calculate the constant of proportionality

Substitute the values given already for a temperature and corresponding c_{rms} for helium

$c_{rms} = 720 \text{ m s}^{-1}$ at a temperature of 45°C

$$T = 45^\circ\text{C} + 273.15 = 318.15 \text{ K}$$

$$a = \frac{c_{rms}}{\sqrt{T}} = \frac{720}{\sqrt{318.15}}$$

Step 5: Calculate c_{rms} at $T = 80^\circ\text{C}$ by substituting the value of a

$$T = 80^\circ\text{C} + 273.15 = 353.15 \text{ K}$$

$$c_{rms} = \frac{720}{\sqrt{318.15}} \times \sqrt{353.15} = 758.57 = 760 \text{ m s}^{-1} \text{ (2 s.f.)}$$



Exam Tip

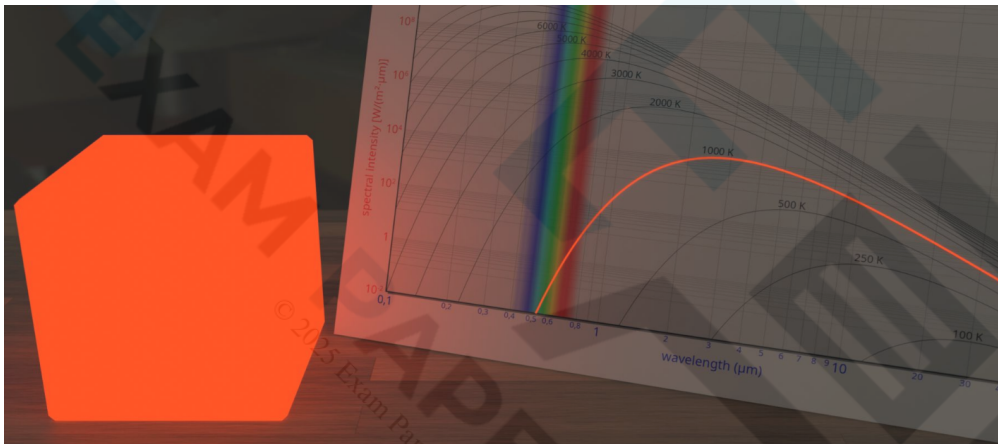
You can remember the equation through the rhyme 'Average K.E is three-halves kT'.

Black Body Radiation

9.10 Black Body Radiation

Black Body Radiation

- Black body radiation is the name given to the **thermal radiation** emitted by all bodies (objects)
- All objects, no matter what temperature, emit black body radiation in the form of electromagnetic waves
- These electromagnetic waves usually lie in the **infrared** region of the spectrum but could be emitted in the form of visible light or other wavelengths, depending on the temperature
- The **hotter** object, the **more** infrared radiation it radiates in a given time



The infrared emitted from a hot object can be detected using a special camera

- A perfect black body is defined as:

An object that absorbs all of the radiation incident on it and does not reflect or transmit any radiation

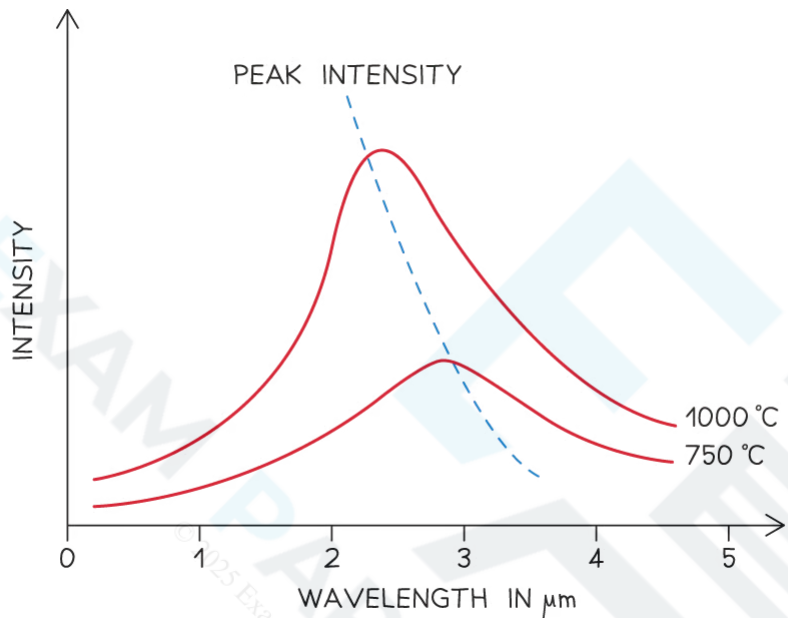
- Since a good absorber is also a good emitter, a perfect black body would be the best possible emitter too
- As a result, an object which perfectly absorbs all radiation will be black
 - This is because the colour black is what is seen when **all** colours from the visible light spectrum are absorbed

Absorption and Emission For Different Colours Table

Colour	Absorbing	Emitting
Black	Good absorber	Good emitter
Dull/Dark	Reasonable absorber	Reasonable emitter
White	Poor absorber	Poor emitter
Shiny	Very poor absorber <i>(Therefore a good reflector)</i>	Very poor emitter

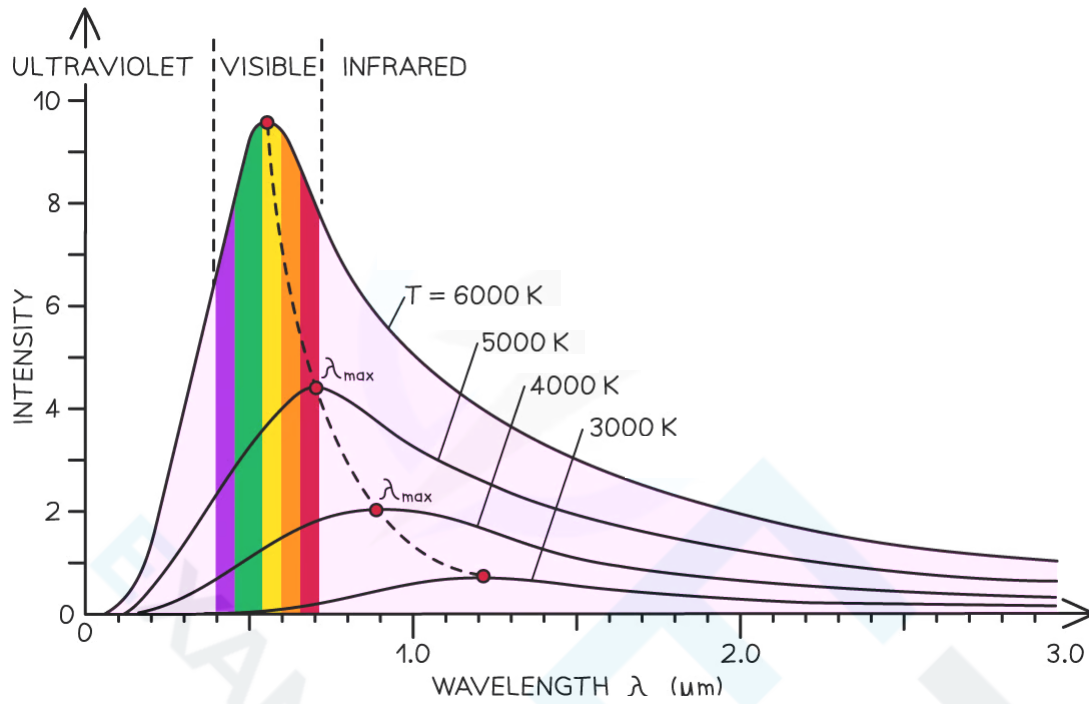
Black Body Radiation Curves

- All bodies (objects) emit a spectrum of thermal radiation in the form of electromagnetic waves
- The **intensity** and **wavelength** distribution of any emitted waves depends on the **temperature** of the body
- This is represented on a black body radiation curve
 - As the temperature increases, the peak of the curve moves



Black body spectrum for objects of different temperatures

- From the electromagnetic spectrum, waves with a **smaller** wavelength have **higher** energy (e.g. UV rays, X-rays)
- When an object gets hotter, the amount of **thermal radiation** it emits increases
 - This increases the thermal energy emitted and therefore the **wavelength** of the emitted radiation **decreases**
- An ideal black body radiator is one that **absorbs and emits all wavelengths**
 - This idealised black body is a theoretical object
 - However, **stars** are the best approximation there is
- The radiation emitted from a black body has a **characteristic spectrum** that is determined by the temperature alone



The intensity-wavelength graph shows how thermodynamic temperature links to the peak wavelength for four different bodies

9.11 Stefan-Boltzmann Law

Stefan-Boltzmann Law

- An object's luminosity depends on two factors:
 - Its surface temperature
 - Its surface area
- The relationship between these is known as the **Stefan-Boltzmann Law**, which states:

The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body

- So, $L \propto AT^4$
- The Stefan-Boltzmann Law equation is:

$$L = \sigma AT^4$$

- Where:
 - L = luminosity of the star (W)
 - A = surface area of the star
 - σ = the Stefan-Boltzmann constant
 - T = surface temperature of the star (K)
- The surface area of a star (or other spherical object) can be calculated using:

$$A = 4\pi r^2$$

- Where:
 - r = radius of the star



Worked Example

A camel has a body temperature of 40°C and a surface area of 16 m^2 . The peak wavelength of the emitted spectrum from the camel is $\lambda_{\text{max}} = 8.6 \times 10^{-6}\text{ m}$. Calculate the total power radiated by the camel.

Step 1: Write down the equation

$$L = \sigma AT^4$$

Step 2: Convert temperature from $^\circ\text{C}$ to K

$$40 + 273 = 313\text{ K}$$

Step 3: Substitute in the values

$$L = (5.67 \times 10^{-8}) \times 16 \times 313^4 = 29387\text{ W} = 8\,707$$

Step 4: Write answer to correct significant figures and include units

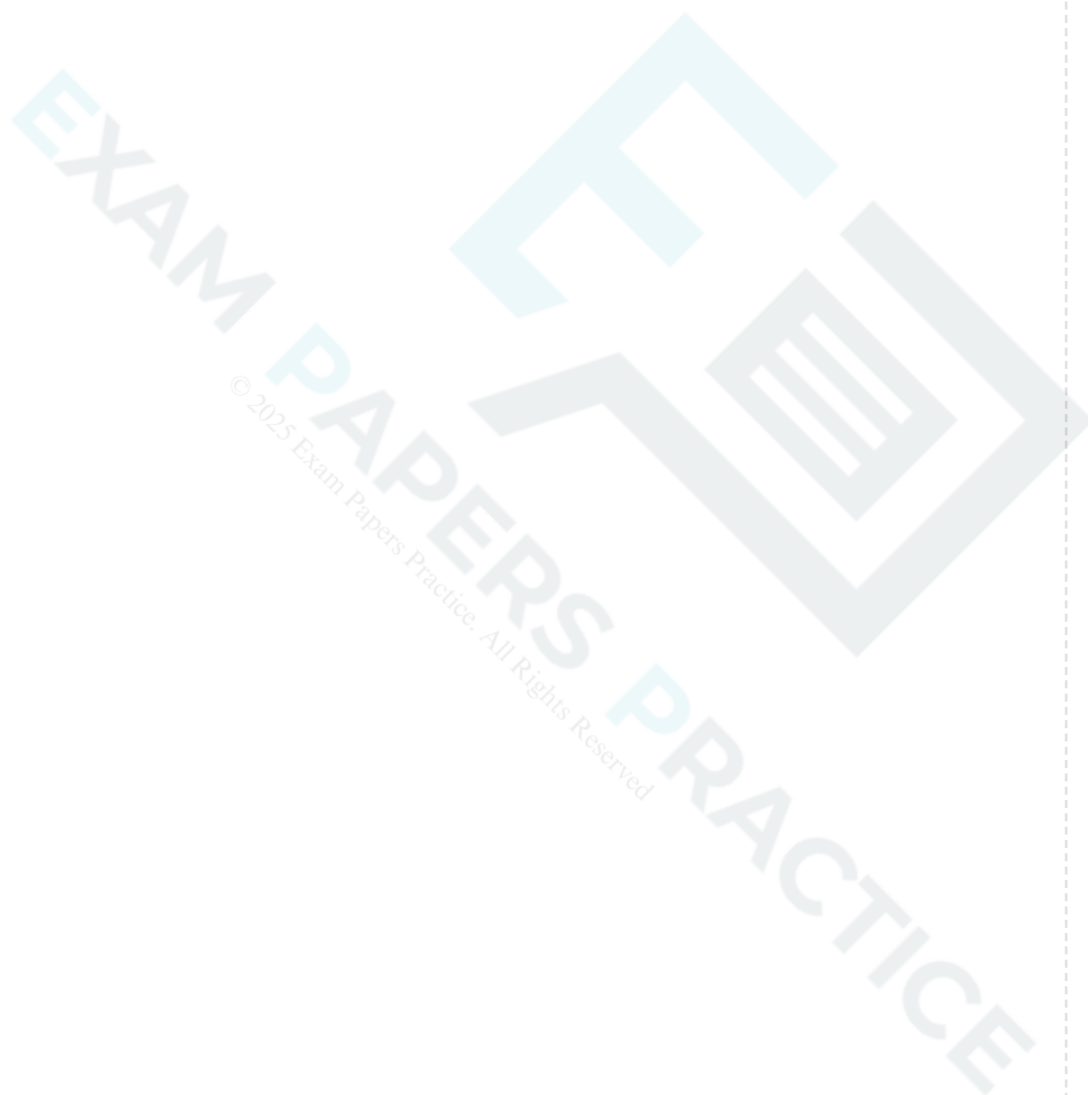
Luminosity (power emitted) of the camel = $8\,700\text{ W}$ (2 sig figs)



Exam Tip

Remember to convert temperatures into Kelvin.

If you are given the radius of a spherical object then its surface area A can be calculated using $A = 4\pi r^2$ for the radius of the object r .



9.12 Wien's Law

Wien's Law

- Wien's displacement law relates the observed wavelength of light from an object to its surface temperature, it states:

The black body radiation curve for different temperatures peaks at a wavelength that is inversely proportional to the temperature

- This relation can be written as:

$$\lambda_{\max} \propto \frac{1}{T}$$

- λ_{\max} is the maximum wavelength (m) emitted by an object at the peak intensity
- T is the surface temperature (K) of an object
- Wien's Law equation is given by:

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m K}$$

- This equation tells us the higher the temperature of a body:
 - The shorter the wavelength at the peak intensity, so **hotter** objects tend to be **white or blue**, and **cooler** objects tend to be **red or yellow**
 - The greater the intensity of the radiation at each wavelength

Table to compare surface temperature and star colour

Colour of Star	Temperature / K
Blue	> 33,000
Blue-White	10,000 – 30,000
White	7,500 – 10,000
Yellow-White	6,000 – 7,500
Yellow	5,000 – 6,000
Orange	3,500 – 5,000
Red	< 3,500



Worked Example

The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm. Which of these two stars is cooler, Betelgeuse or Rigel?

Step 1: Write down Wien's displacement law

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m K}$$

Step 2: Rearrange for temperature T

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{\max}}$$

Step 3: Calculate the surface temperature of each star

$$\text{Rigel: } T = \frac{2.9 \times 10^{-3}}{\lambda_{\max}} = \frac{2.9 \times 10^{-3}}{263 \times 10^{-9}} = 11026 = \mathbf{11\ 000\ K}$$

$$\text{Betelgeuse: } T = \frac{2.9 \times 10^{-3}}{\lambda_{\max}} = \frac{2.9 \times 10^{-3}}{828 \times 10^{-9}} = 3502 = \mathbf{3\ 500\ K}$$

Step 4: Write a concluding sentence

Betelgeuse has a surface temperature of 3500 K, therefore, it is much cooler than Rigel



The Orion Constellation; cooler stars, such as Betelgeuse, appear red or yellow, while hotter stars, such as Rigel, appear white or blue



Exam Tip

Note that the temperature used in Wien's Law is in **Kelvin (K)**. Remember to convert from °C if the temperature is given in degrees in the question before using the Wien's Law equation.