A Level Physics Edexcel

7. Electric & Magnetic Fields

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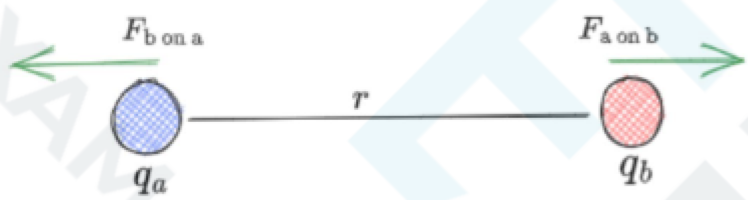
Electric Fields

7.1 Defining an Electric Field

Defining an Electric Field

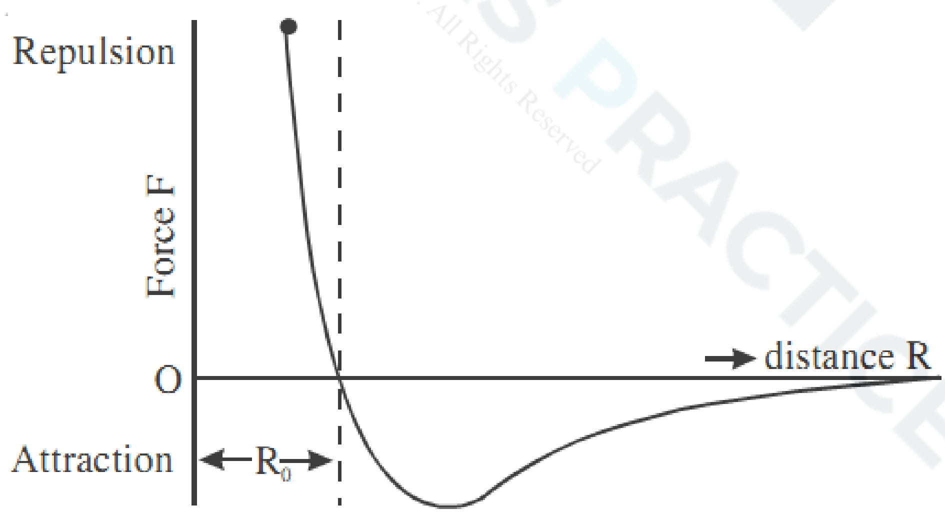
- An electric field is defined as a region of space in which a **charged** particle experiences a **force**
  - Hence, electric fields are a type of **force field**
- The charged particle could be stationary or moving, and will experience an electric force in that field

- All charged particles create their own electric fields
  - These fields exert an electrostatic force, F_E on other charged particles



The electrostatic force between two charges

- Like charges (positive and positive, or negative and negative) **repel** each other
  - This means the force on each charge are away from the other charge
- Opposite charged (positive and negative) **attract** each other
  - This means the force on each charge is towards the other charge
- The size of the force changes with distance



A repulsive force decreases with distance

Exam Tip

Electric fields are slightly different in that a charged particle will experience a force in this field whether it's stationary or moving. Don't get this mixed up with a **magnetic** field, where a charged particle only experiences a force if it's **moving**.

Electric Field Strength

- The **electric field strength** at a point is defined as:

  The force per unit charge acting on a positive test charge at that point

- The electric field strength can be calculated using the equation:

$$E = \frac{F}{Q}$$

- Where:
  - E = electric field strength ($N\ C^{-1}$)
  - F = electrostatic force on the charge (N)
  - Q = charge (C)

- It is important to use a positive test charge in this definition, as this determines the direction of the electric field
- Recall, the electric field strength is a **vector** quantity, it is always directed:
  - **Away** from a positive charge
  - **Towards** a negative charge
- This direction is also denoted by the direction of the electric field

Worked Example

A charged particle is in an electric field with electric field strength $3.5 \times 10^4\ N\ C^{-1}$ where it experiences a force of 0.3 N.

Calculate the charge of the particle.

Step 1: Write down the equation for electric field strength

$$E = \frac{F}{Q}$$

Step 2: Rearrange for charge Q

$$Q = \frac{F}{E}$$

Step 3: Substitute in values and calculate

$$Q = \frac{0.3}{3.5 \times 10^4} = 8.571 \times 10^{-6} = \mathbf{8.6 \times 10^{-6}\ C\ (2\ s.f)}$$

Exam Tip

While the defining equation for electric field strength, $E = F / Q$ is defined for a positive test charge, it is still useable for negative charges in an electric field. You will find that, if you substitute a negative charge in for Q, the electric field strength *E* is also negative. This simply means that the vector representing the field points in the **opposite direction** than it would for a positive charge, as you should expect. Make sure you can interpret the direction of electric field lines for your exam!

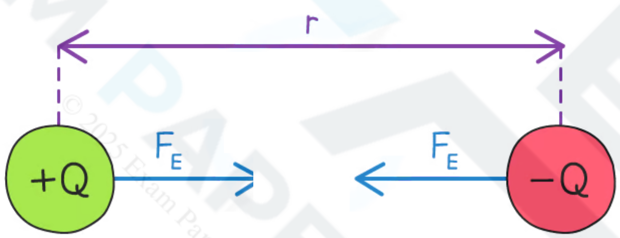
Electric Force between Two Charges

- All charged particles produce an electric field around them
  - This field exerts a force on any other charged particle within range
- The electrostatic force between two charges is defined by **Coulomb's Law**
  - Recall that the charge of a uniform spherical conductor can be considered as a point charge at its centre
- Coulomb's Law states that:

  The electrostatic force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation

- The force F_E between two charges as expressed by Coulomb's Law is given by the equation:

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$



The electrostatic force between two charges is defined by Coulomb's Law

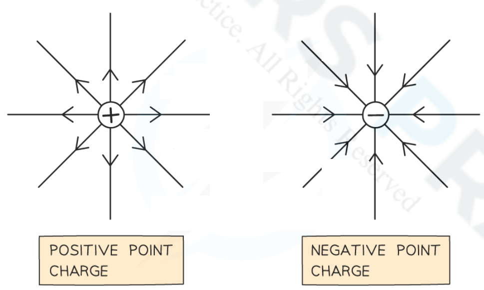
- Where:
  - F_E = electrostatic force between two charges (N)
  - Q_1 and Q_2 = two point charges (C)
  - ε_0 = permittivity of free space
  - r = distance between the centre of the charges (m)
- The $1/r^2$ relation is called the inverse square law
- This means that when the separation of two charges doubles, the electrostatic force between them reduces to $(½)^2 = ¼$ of its original size
- ε_0 is a physical constant used to show the capability of a vacuum to permit electric fields
- If Q_1 and Q_2 are oppositely charged, then the electrostatic force F_E is negative
  - This can be interpreted as an **attractive force** between Q_1 and Q_2
- If Q_1 and Q_2 are the same charge, then the electrostatic force F_E is positive
  - This can be interpreted as a **repulsive force** between Q_1 and Q_2

Electric Field due to a Point Charge

- The electric field strength describes how strong or weak an electric field is at that point
- A point charge produces a **radial** field
  - A charge sphere also acts like a point charge
- The electric field strength *E* at a distance *r* due to a point charge Q in free space is defined by:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

- Where:
  - Q = the point charge producing the radial electric field (C)
  - *r* = distance from the centre of the charge (m)
  - ε_0 = permittivity of free space ($F\ m^{-1}$)
- This equation shows:
  - Electric field strength in a radial field is **not constant**
  - As the distance from the charge *r* increases, *E* decreases by a factor of $1/r^2$
- This is an inverse square law relationship with distance
  - This means the field strength *E* decreases by a factor of **four** when the distance *r* is **doubled**
- **Note:** this equation is only for the field strength around a **point charge** since it produces a radial field

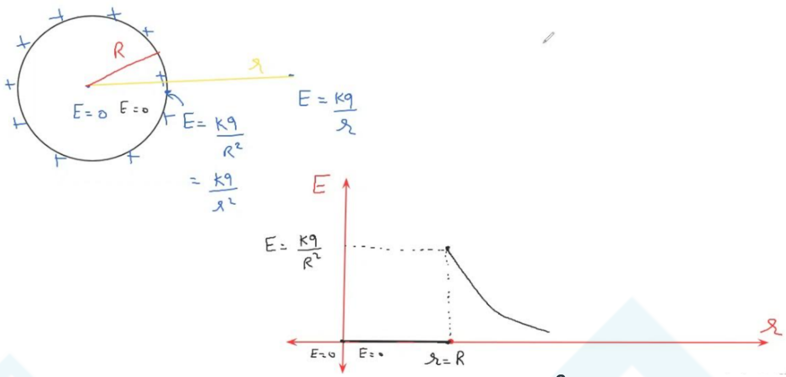


Positive and negative point charges and the direction of the electric field lines

- The electric field strength is a **vector** Its direction is the same as the electric field lines
  - If the charge is negative, the E field strength is negative and points **towards** the centre of the charge
  - If the charge is positive, the E field strength is positive and points **away** from the centre of the charge
- This equation is analogous to the gravitational field strength around a point mass

- The only difference is, gravitational field lines are always **towards** the mass, whilst electric field lines can be towards **or** away from the point charge

- The graph of *E* against *r* for a charge is:



The electric field strength E has a 1/r² relationship

- **The key features of this graph are**:
  - The values for *E* are all positive
  - As *r* increases, *E* against *r* follows a $1/r^2$ relation (inverse square law)
  - The **area** under this graph is the change in electric potential ΔV
  - The graph has a steep decline as *r* increases

Worked Example

Calculate the strength of the electric field at a distance of 2 m away from an electron, and state its direction.

Step 1: Write out the equation for electric field strength

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Step 2: Substitute quantities for charge, distance and permittivity of free space

- The charge on an electron $Q = -1.6 \times 10^{-19}$ C
- The distance $r = 2$ m
- Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}$
- Therefore:

$$E = \frac{-1.6 \times 10^{-19}}{4\pi \times (8.85 \times 10^{-12}) \times 2^2} = -3.6 \times 10^{-10} \text{ N C}^{-1}$$

Step 3: State the direction of the field

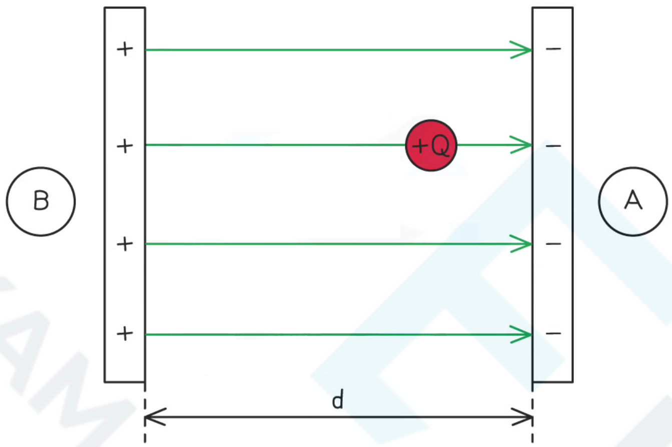
- The negative sign indicates the electric field is directed **towards the electron**

Exam Tip

Remember to **square** the distance in the electric field strength equation! Don't get this mixed up with the electric force between two charges equation, which has **two** charges (Q) in the equation, whilst the equation for *E* only has 1 Q, which is the one producing the electric field.

Electric Field & Potential

- A positive test charge has electric **potential energy** due to its position in an electric field
- The amount of electric potential energy depends on:
  - The magnitude of charge
  - The value of the **electric potential** in the field



Work is done on a positive test charge Q to move it from the negatively charged plate A to the positively charged plate B. This means its electric potential energy increases

- Electric potential is defined as the amount of work done per unit of charge at that point
- A stronger electric field means the electric potential changes **more rapidly** with distance as the test charge moves through it
- Hence, the relationship between the electric field strength and the electric potential is summarised as:

The electric field strength is proportional to the gradient of the electric potential

- This means:
  - If the electric potential changes very rapidly with distance, the electric field strength is large
  - If the electric potential changes very gradually with distance, the electric field strength is small

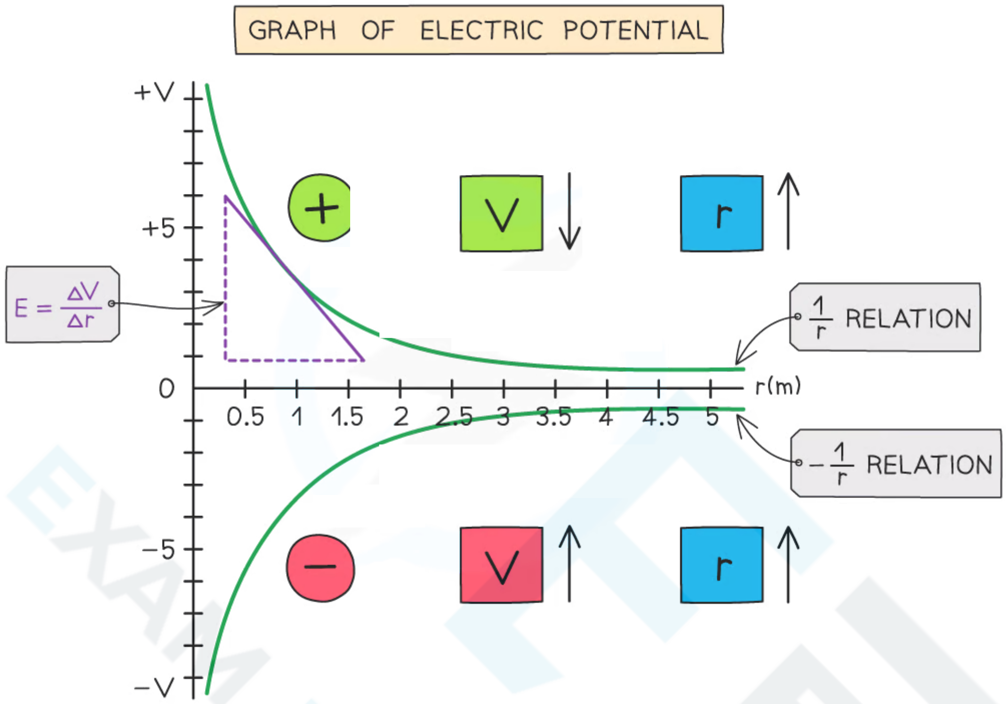
- An electric field can be defined in terms of the variation of **electric potential** at different points in the field**:**

The electric field at a particular point is equal to the gradient of a potential–distance graph at that point

- The potential gradient in an electric field is defined as:

The rate of change of electric potential with respect to displacement in the direction of the field

- The graph of potential *V* against distance *r* for a negative or positive charge is:



The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- **The key features of this graph are:**
  - The values for *V* are all negative for a negative charge
  - The values for *V* are all positive for a positive charge
  - As *r* increases, *V* against *r* follows a 1/*r* relation for a positive charge and -1/*r* relation for a negative charge
  - The **gradient** of the graph at any particular point is the value of *E* at that point
  - The graph has a shallow increase (or decrease) as *r* increases

- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance

7.6 Electric Field between Parallel Plates

Electric Field between Parallel Plates

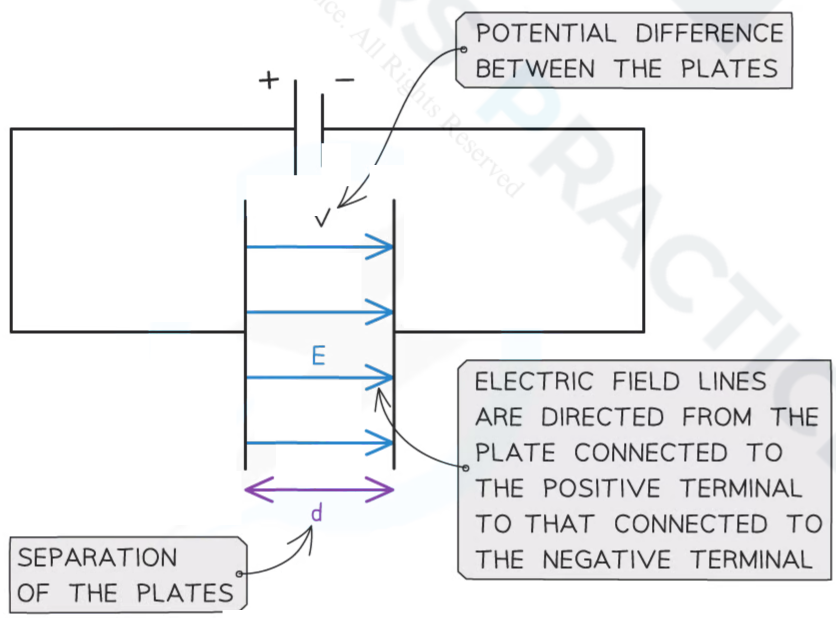
- The magnitude of the electric field strength in a **uniform** field between two charged parallel plates is defined as:

$$E = \frac{V}{d}$$

- Where:
  - E = electric field strength ($V\ m^{-1}$)
  - V = potential difference between the plates (V)
  - d = separation between the plates (m)

- The electric field strength is now defined by the units **$V\ m^{-1}$**
  - Therefore, the units $V\ m^{-1}$ are equivalent to the units **$N\ C^{-1}$**
- The equation shows:
  - The greater the **voltage** (potential difference) between the plates, the **stronger** the field
  - The greater the **separation** between the plates, the **weaker** the field

- Remember this equation **cannot** be used to find the electric field strength around a point charge (since this would be a radial field)
- The direction of the electric field is from the plate connected to the **positive** terminal of the cell to the plate connected to the **negative** terminal



The E field strength between two charged parallel plates is the ratio of the potential difference and separation of the plates

- **Note:** if one of the parallel plates is **earthed**, it has a voltage of 0 V

Worked Example

Two parallel metal plates are separated by 3.5 cm and have a potential difference of 7.9 kV.

Calculate the electric force acting on a stationary charged particle between the plates that has a charge of 2.6×10^{-15} C.

Step 1: Write down the known values

- Potential difference, $V = 7.9$ kV $= 7.9 \times 10^3$ V
- Distance between plates, $d = 3.5$ cm $= 3.5 \times 10^{-2}$ m
- Charge, $Q = 2.6 \times 10^{-15}$ C

Step 2: Calculate the electric field strength between the parallel plates

$$E = \frac{V}{d}$$

$$E = \frac{7.9 \times 10^3}{3.5 \times 10^{-2}} = 2.257 \times 10^5 \text{ V m}^{-1}$$

Step 3: Write out the equation for electric force on a charged particle

$$F = QE$$

Step 4: Substitute electric field strength and charge into electric force equation

$$F = QE = (2.6 \times 10^{-15}) \times (2.257 \times 10^5) = 5.87 \times 10^{-10} \text{ N} = 5.9 \times 10^{-10} \text{ N (2 s.f.)}$$

Exam Tip

Remember the equation for electric field strength with V and d is only used for **parallel plates**, and not for point charges (where you would use $E = F/Q$)

Electric Potential for a Radial Field

Electric Potential Energy

- In order to move a positive charge closer to another positive charge, work must be done to overcome the force of repulsion between them
    - Similarly, to move a positive charge away from a negative charge, work must be done to overcome the force of attraction between them
- Energy is therefore transferred to the charge that is being pushed upon
    - This means its **potential energy** increases
- If the positive charge is free to move, it will start to move away from the repelling charge
    - As a result, its potential energy decreases back to 0
- This is analogous to the gravitational potential energy of a mass increasing as it is being lifted upwards and decreasing as it falls
- The electric potential at a point is defined as:

The work done per unit charge in bringing a positive test charge from infinity to that point

- Electric potential is a **scalar** quantity
    - This means it doesn't have a direction
- However, you will still see the electric potential with a positive or negative sign. This is because the electric potential is:
    - **Positive** around an isolated positive charge
    - **Negative** around an isolated negative charge
    - **Zero** at infinity
- Positive work is done to move a positive test charge from infinity to a point around a positive charge and negative work is done to move it to a point around a negative charge. This means:
    - When a positive **test charge** moves closer to a **negative** charge, its electric potential **decreases**
    - When a positive test charge moves closer to a **positive** charge, its electric potential **increases**

Electric Potential due to a Point Charge

- The **electric potential** in the radial field due to a **point charge** is defined as:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Where:
  - V = the electric potential (V)
  - Q = the point charge producing the potential (C)
  - ε_0 = permittivity of free space (F m^{-1})
  - r = distance from the centre of the point charge (m)
- This equation shows that for a positive test charge:
  - As the distance *r* from the charge Q **decreases**, the potential *V* **increases** (becomes more positive)
  - This is because more work has to be done on the positive test charge to overcome the repulsive force of Q
- For a negative test charge:

- As the distance from the charge *r* **decreases**, the potential *V* **decreases** (becomes more negative)
- This is because less work has to be done on the negative test charge since the attractive force becomes stronger the nearer it gets to Q

- Unlike the **gravitational potential** equation, the electric potential can be positive or negative, because Q can be positive or negative
- The electric **potential** varies according to 1 / r
  - Note, this is different to electric **field strength**, which varies according to 1 / r^2

Worked Example

The electric potential at a distance *r* from a proton is *V*.

What is the value of the electric potential at a distance three-times farther?

Step 1: Write the equation for electric potential

- The electric potential is given by the equation:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 2: Write the transformed equation for a distance three times as large

- The charge Q remains constant (due to the proton)
- The potential *V* becomes *V'*
- The distance *r* becomes 3*r*
- Hence the transformed equation becomes:

$$V' = \frac{Q}{4\pi\varepsilon_0 (3r)} = \frac{1}{3}\frac{Q}{4\pi\varepsilon_0 r} = \frac{1}{3}V$$

Step 3: Write a conclusion

- Therefore, when the distance from a charge Q gets three times larger, the value of the electric potential decreases by a factor 1/3, because the potential is inversely proportional to distance *r*

Exam Tip

- Electric **potential** *V* is inversely proportional to radial distance, $V \propto \frac{1}{r}$
- Electric **field strength** *E* is inversely proportional to radial distance squared, $E \propto \frac{1}{r^2}$
- Make sure you remember these variations and that you can describe them in words!

One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines. The potential always **decreases** in the **same** direction as the field lines and vice versa.

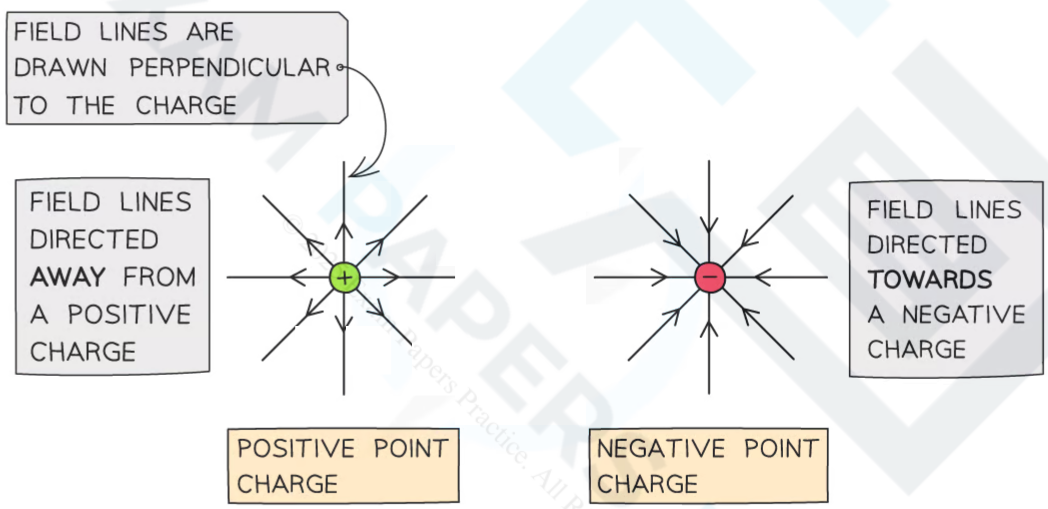
Using Field Lines & Equipotential Diagrams

- The direction of electric fields is represented by **electric field lines**
- Electric field lines are directed from positive to negative
    - Therefore, the field lines must be pointed **away** from the **positive** charge and **towards** the **negative** charge
    - Hence, field lines show the direction of force on a **positive test charge**

Representing Radial Fields

- A radial field spreads out from a spherical charge in all directions
    - e.g. the field around a point charge

- Around a **point charge**, the electric field lines are directly radially inwards or outwards:
    - If the charge is **positive** (+), the field lines are radially **outwards**
    - If the charge is **negative** (-), the field lines are radially **inwards**



Radial electric field lines point away from a positive charge and point towards a negative charge

- This shares many similarities to radial gravitational field lines around a point mass
    - Since gravity is only **attractive**, the field lines will look similar to the negative point charge, directed inward
    - However, electric field lines can be in **either direction**

- The electric field strength in a radial field follows an inverse square law
    - This means the field strength varies with distance *r* by $1 / r^2$

Representing Uniform Electric Fields

- A uniform electric field has the same electric field strength throughout the field
    - For example, the field between oppositely charged parallel plates

- This is represented by **equally spaced** field lines

- This shares many similarities to uniform gravitational field lines on the surface of a planet

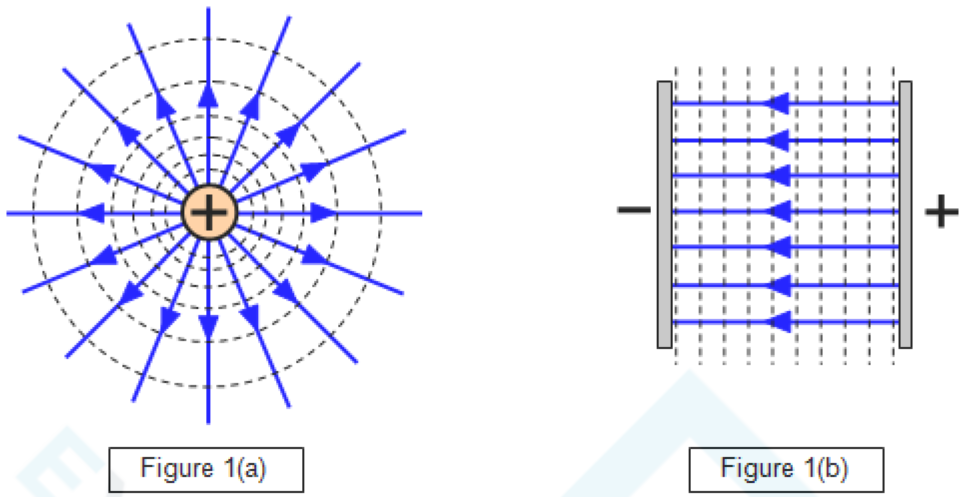
- A **non-uniform** electric field has varying electric field strength throughout
- The strength of an electric field is determined by the spacing of the field lines:
  - A **stronger** field is represented by the field lines **closer** together
  - A **weaker** field is represented by the field lines **further** apart

- The electric field lines are directed from the **positive** to the **negative** plate
- The electric field strength in a **uniform** field is given by the equation $E = V / d$
  - Hence, E proportional to the potential difference V between the plates
  - E is inversely proportional to the distance d between the plates

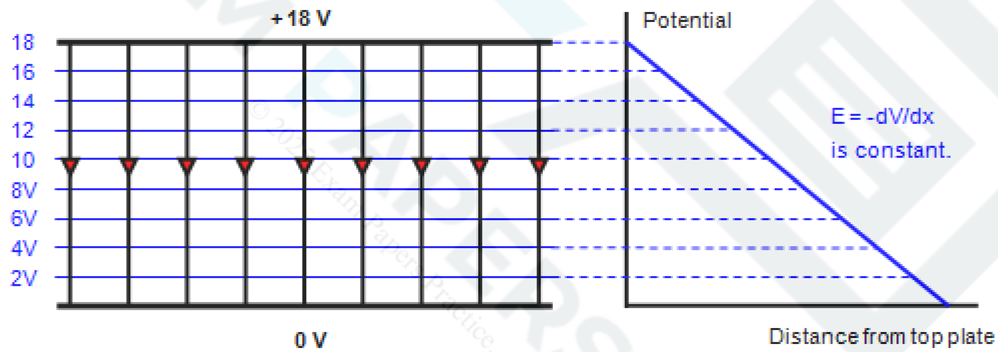
Equipotential Diagrams

- Equipotential lines (2D) and surfaces (3D) join together points that have the **same electric potential**
- These are always:
  - **Perpendicular** to the electric field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)

- The potential gradient is defined by the **equipotential lines**



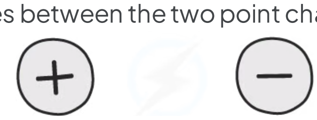
Equipotential lines in a radial field are circles, showing lines of equal potential around a charge. They intersect radial field lines at 90°



Equipotential lines in a uniform field are straight lines. They too intersect uniform field lines at 90°

Worked Example

Sketch the electric field lines between the two point charges in the diagram below.

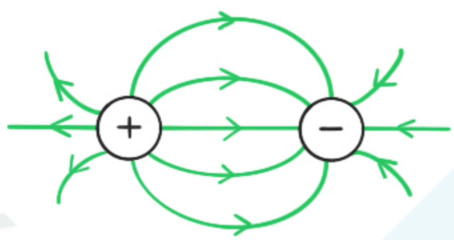


- Electric field lines around point charges are radially outwards for positive charges and radially inwards for negative charges
- The field lines must be drawn with arrows **from the positive charge to the negative charge**

- In a **radial field** (eg. a point charge), the equipotential lines:
  - Are concentric circles around the charge
  - Become further apart further away from the charge

- In a **uniform field** (eg. between charged parallel plates), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced

- **No work is done** when moving along an equipotential line or surface
- Work is only done when moving **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$



Exam Tip

Always label the arrows on the field lines! The lines must also touch the surface of the source charge or plates.

Capacitance

7.9 Capacitance

Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- Capacitors do this by storing electric **charge**, which creates a build up of electric **potential energy**
- They are made in the form of two conductive **metal plates** connected to a voltage supply (parallel plate capacitor)
  - There is commonly a **dielectric** in between the plates, to ensure charge does not flow across them
- The capacitor circuit symbol is:

The capacitor circuit symbol is two parallel lines

- Capacitors are marked with a value of their **capacitance**
- Capacitance is defined as:

The charge stored per unit potential difference (between the plates)

- The greater the **capacitance**, the greater the **charge stored** in the capacitor
- The capacitance of a capacitor is defined by the equation:

$$C = \frac{Q}{V}$$

- Where:
  - C = capacitance (F)
  - Q = charge stored (C)
  - *V* = potential difference across the capacitor plates (V)

A capacitor used in small circuits

- Capacitance is measured in the unit **Farad (F)**
  - In practice, 1 F is a very large unit
  - Often it will be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)
- If the capacitor is made of parallel plates, Q is the charge on the plates and *V* is the potential difference across the capacitor
  - The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates
- This capacitance equation shows that an object's capacitance is the **ratio of the charge stored by the capacitor to the potential difference between the plates**

Worked Example

A parallel plate capacitor has a capacitance of 1 nF and is connected to a voltage supply of 0.3 kV.

Calculate the charge on the plates.

Step 1: Write down the known quantities

- Capacitance, $C = 1\text{ nF} = 1 \times 10^{-9}\text{ F}$
- Potential difference, $V = 0.3\text{ kV} = 0.3 \times 10^{3}\text{ V}$

Step 2: Write out the equation for capacitance

$$C = \frac{Q}{V}$$

Step 3: Rearrange for charge Q

$$Q = CV$$

Step 4: Substitute in values

$$Q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\text{ C} = 300\text{ nC}$$

Exam Tip

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The letter 'C' is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

Energy Stored by a Capacitor

- When charging a capacitor, the power supply 'pushes' electrons to one of the metal plates
  - It therefore does **work** on the electrons and **electrical energy** becomes stored on the plates
- The power supply 'pulls' electrons off of the other metal plate, attracting them to the positive terminal
  - This leaves one side positively charged, while the other side becomes negatively charged
  - Hence, in this way, charge is 'stored' by the capacitor
- Gradually, this stored charge builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion
- As the charge of the negative plate increases, i.e., becomes more negatively charged, the force of repulsion between the electrons on the plate and the new electrons being pushed onto it increases
- This means a greater amount of work must be done to increase the charge on the negative plate or in other words:

The potential difference across the capacitor increases as the amount of charge increases

Alternative Equations for Energy Stored

- The energy stored by a capacitor is given by:

$$W = \frac{1}{2}QV$$

- Substituting the charge Q with the **capacitance** equation $Q = CV$, the energy stored can also be calculated by the following equation:

$$W = \frac{1}{2}CV^2$$

- By substituting the potential difference V, the energy stored can also be defined in terms of just the charge stored Q and the capacitance, C:

$$W = \frac{Q^2}{2C}$$

Worked Example

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 10 V to 30 V.

Step 1: Write down the equation for energy stored, in terms of C and V and list the known values

$$E = \frac{1}{2}CV^2$$

Capacitance, C = 1500μF

Final p.d, V_2 = 30 V

Initial p.d V_1=10 V

Step 2: The change in energy stored in proportional to the change in p.d

$$\Delta E = \frac{1}{2}C(\Delta V^2) = \frac{1}{2}C(V_2^{\,2} - V_1^{\,2})$$

Step 3: Substitute in the values

$$\Delta E = \frac{1}{2}(1500 \times 10^{-6})(30^2 - 20^2) = 0.4 \text{ J}$$

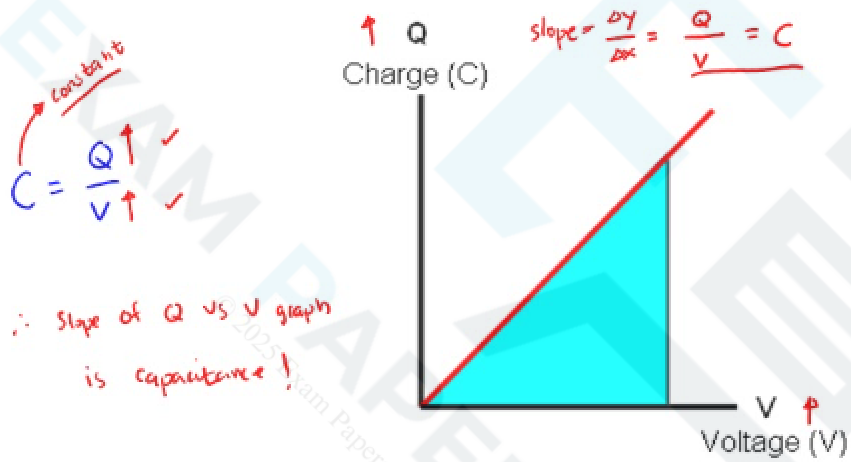
Exam Tip

Energy stored or work done are used interchangeably (and sometimes written as E or W as shown above). You should be comfortable linking the two equivalent ideas - the energy stored *in* the capacitor is equal to the work done *on* it, by the power supply which charges it. Make sure you can apply each of the three equations given above!

Area Under a Potential Difference-Charge Graph

- The charge stored Q on the capacitor is given by the equation $Q = CV$
  - Therefore, the charge stored Q is **directly proportional** to the potential difference across the plates V
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The gradient of the graph represents the capacitance C, which is a constant
- The electrical (potential) energy stored in the capacitor can be determined from the **area under the potential-charge graph** which is equal to the **area** of a right-angled triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$



The area under a potential difference-charge graph represents the energy stored by a capacitor

- Therefore the work done, or **energy stored *W*** in a capacitor is defined by the equation:

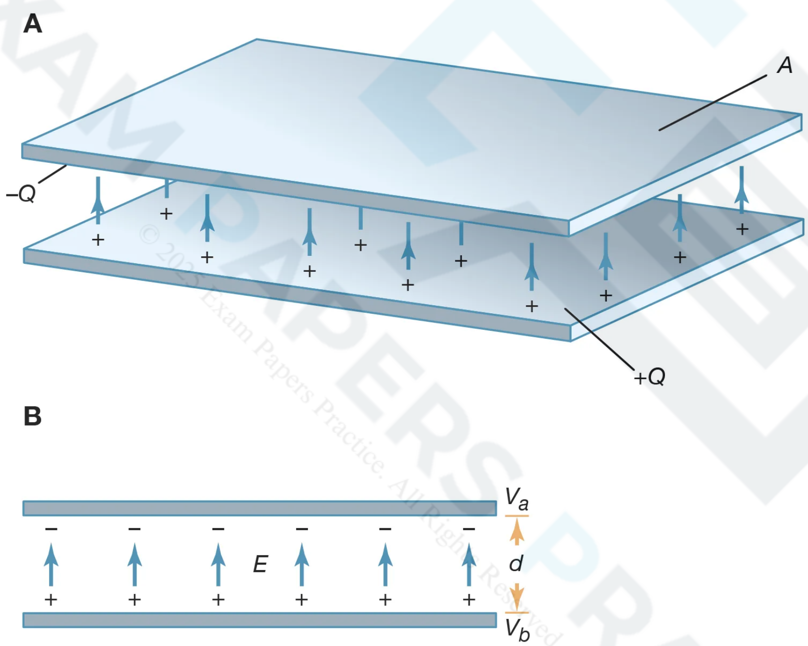
$$W = \frac{1}{2} QV$$

- Where:
  - W = energy stored (J)
  - Q = charge stored (C)
  - V = potential difference across the plates (V)

Charge & Discharge Curves

Charging Curves

- Capacitors are charged by a **power supply** (e.g. a battery)
- When charging, electrons are 'pulled' from the plate connected to the positive terminal of the power supply
  - Hence the plate nearest the positive terminal is **positively charged**
- Oppositely, electrons are 'pushed' onto the plate connected to the negative terminal
  - Hence the plate nearest the negative terminal is **negatively charged**
- As the negative charge builds up, fewer electrons are pushed onto the plate due to electrostatic repulsion from the electrons already on the plate
- When no more electrons can be pushed onto the negative plate, the charging stops



A parallel plate capacitor is made up of two conductive plates with opposite charges building up on each plate

- At the start of charging, the current is large and gradually falls to zero as the electrons stop flowing through the circuit
  - The current decreases **exponentially**
  - This means the rate at which the charge decreases is proportional to the amount of charge it has left
- Since an equal but opposite charge builds up on each plate, the potential difference between the plates slowly increases until it is the same as that of the power supply

YOUR NOTES
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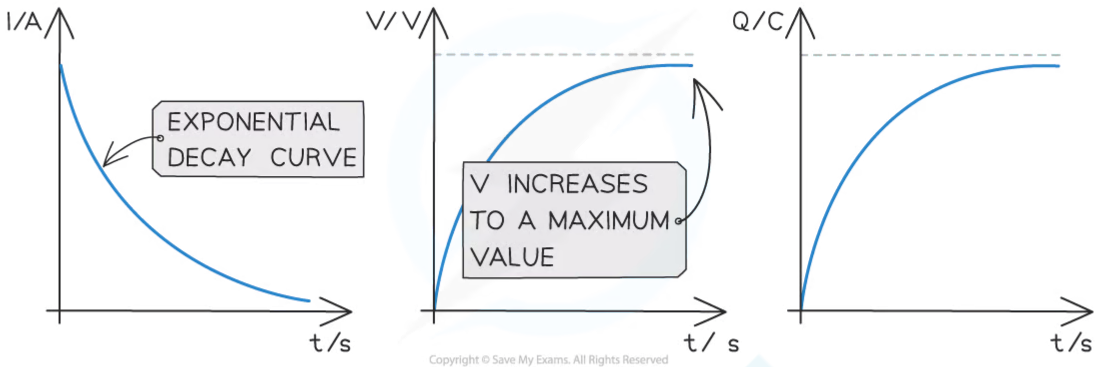
- Therefore, the charge stored on the capacitor plates increases until the potential difference across the plates matches that of the power supply

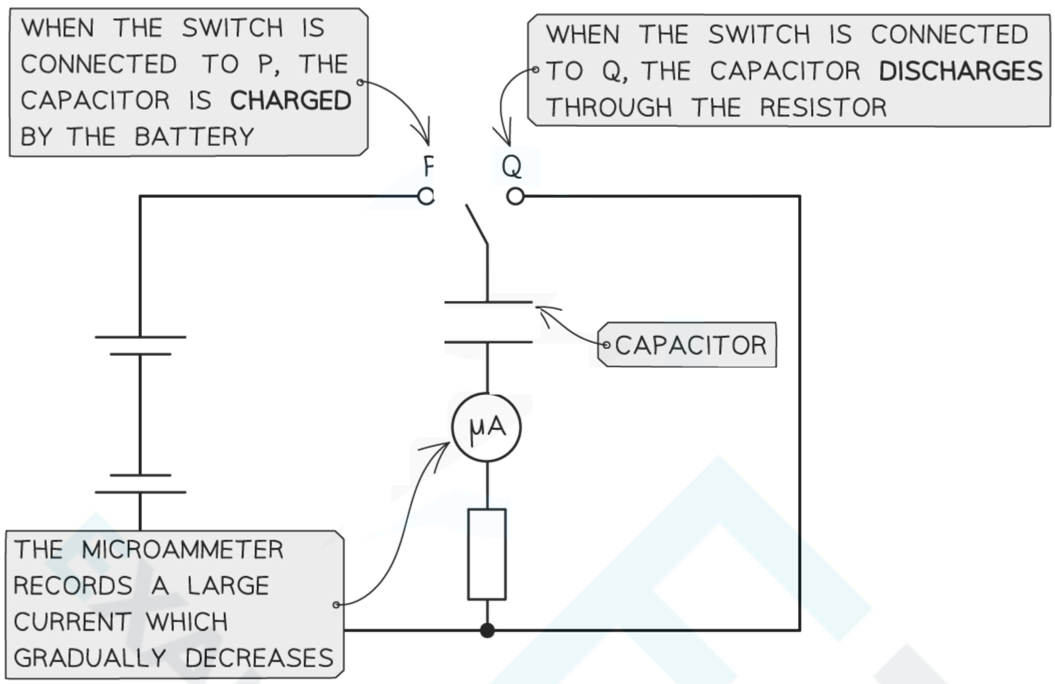


Graphs of variation of current, p.d and charge with time for a capacitor charging through a battery

- **The key features of the charging graphs are:**
  - The shapes of the p.d. and charge against time graphs are identical
  - The current against time graph is an **exponential decay** curve
  - The initial value of the current starts on the y axis and decreases exponentially
  - The initial value of the p.d and charge starts at 0 up to a maximum value

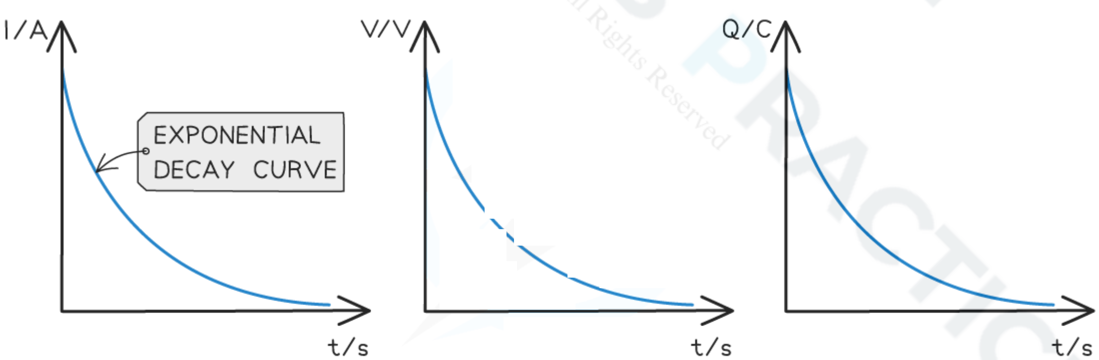
Discharging Curves

- Capacitors are **discharged** through a resistor with **no** power supply present
- The electrons now flow back from the negative plate to the positive terminal of the power supply until there is potential difference across the capacitor plates
- Charging and discharging is commonly achieved by moving a switch that connects the capacitor between a power supply and a resistor



The capacitor charges when connected to terminal P and discharges when connected to terminal Q

- At the start of discharge, the current is **large** (but in the opposite direction to when it was charging) and gradually falls to zero
- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
  - This means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left
- The graphs of the variation with time of current, p.d and charge are all identical and follow a pattern of **exponential decay**



Graphs of variation of current, p.d and charge with time for a capacitor discharging through a resistor

- **The key features of the discharge graphs are:**
  - The shape of the current, p.d. and charge against time graphs are identical
  - Each graph shows exponential decay curves with decreasing gradient

- The initial values (typically called I_0, V_0 and Q_0 respectively) start on the y axis and decrease exponentially

- The rate at which a capacitor discharges depends on the **resistance** of the circuit
  - If the resistance is **high**, the current will decrease **more slowly** and charge will flow from the capacitor plates more slowly, meaning the capacitor will take **longer** to discharge
  - If the resistance is **low**, the current will decrease **quickly** and charge will flow from the capacitor plates quickly, meaning the capacitor will discharge **faster**

Exam Tip

Make sure you're comfortable with sketching and interpreting charging and discharging graphs, as these are common exam questions. A quick summary to help you remember:

- Discharging curves are all identical
- **C**urrent decreases for the **C**harging curve (but increases for potential difference and charge stored!)

The Time Constant

- The time constant of a capacitor discharging through a **resistor** is a measure of how long it takes for the capacitor to discharge
- The definition of the time constant for a **discharging** capacitor is:

> **The time taken for the charge, current or potential difference of a discharging capacitor to decrease to 37% of its original value**

- Alternatively, for a **charging** capacitor:

> **The time taken for the charge or potential difference of a charging capacitor to rise to 63% of its maximum value**

- 37% is 0.37 or 1 / e (where e is the exponential function) multiplied by the original value (I_0, Q_0 or V_0)
- This is represented by the Greek letter tau, τ, and measured in units of **seconds** (s)
- The time constant provides an easy way to compare the rate of change of similar quantities eg. charge, current and p.d.
- It is defined by the equation:

$$\tau = RC$$

- Where:
  - τ = time constant (s)
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)

- For example, to find the time constant for a **discharging** capacitor:
  - Calculate $0.37V_0$, where V_0 is the **initial** potential difference across it
  - Determine the corresponding time taken for the potential difference to decrease to that value

- To find the time constant for a **charging** capacitor:
  - Calculate $0.63V_0$, where V_0 is the **maximum** potential difference across it
  - Determine the corresponding time taken for the potential difference to rise to that value

Required Practical: Charging & Discharging Capacitors

Aim of the Experiment

- The overall aim of this experiment is to calculate the capacitance of a capacitor. This is just one example of how this required practical might be carried out

Variables

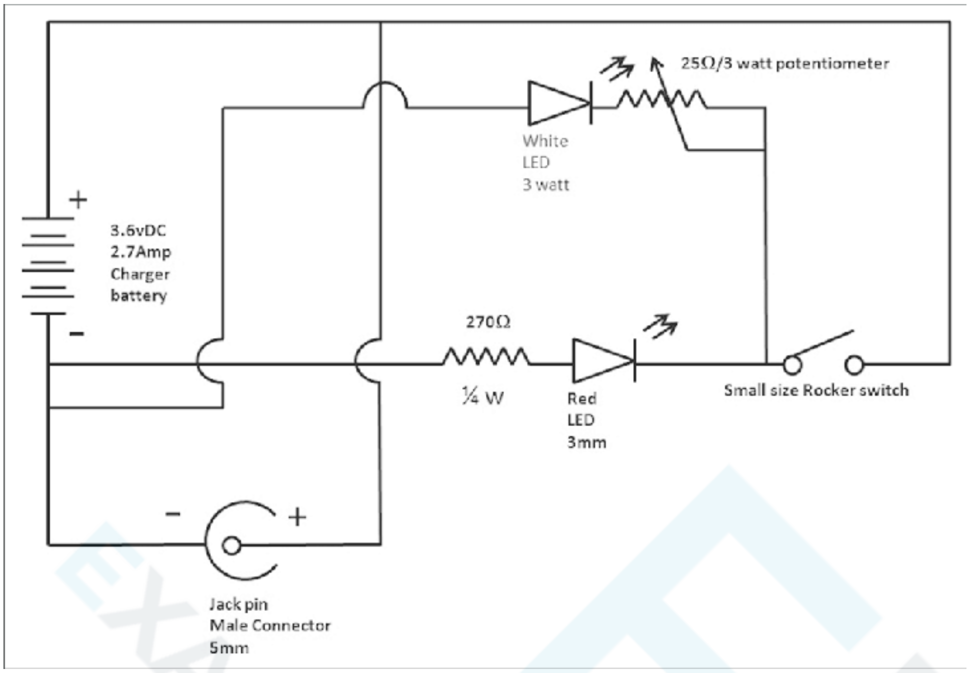
- Independent variable = time, *t*
- Dependent variable= potential difference, *V*
- Control variables:
  - Resistance of the resistor
  - Current in the circuit

Equipment List

Apparatus	Purpose
Switch	To switch between the charging and discharging circuit
Capacitor	To measure the capacitance
10 kΩ Resistor	To discharge the capacitor
Battery pack (power supply)	To provide the potential difference across the capacitor
Voltmeter	To measure the potential difference across the capacitor
Stopwatch	To measure the time taken for the capacitor to discharge

- **Resolution** of measuring equipment:
  - Voltmeter = 0.1 V
  - Stopwatch = 0.01 s

Method



1. Set up the apparatus like the circuit above, making sure the switch is not connected to **X** or **Y** (no current should be flowing through)
2. Set the battery pack to a potential difference of 10 V and use a 10 kΩ resistor. The capacitor should initially be fully discharged
3. Charge the capacitor fully by placing the switch at point **X**. The voltmeter reading should read the same voltage as the battery (10 V)
4. Move the switch to point **Y**
5. Record the voltage reading every 10 s down to a value of 0 V. A total of 8–10 readings should be taken

- An example table might look like this:

FROM STOPWATCH

FROM VOLTMETER

TIME t/s	POTENTIAL DIFFERENCE / V
0.00	
10.00	
20.00	
30.00	
40.00	
50.00	
60.00	
70.00	
80.00	

Analysing the Results

- The potential difference (p.d) across the capacitance is defined by the equation:

$$V = V_0 e^{-\frac{t}{RC}}$$

- Where:
  - V = p.d across the capacitor (V)
  - V_0 = initial p.d across the capacitor (V)
  - t = time (s)
  - e = exponential function
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)

- Rearranging this equation for ln(V) by taking the natural log (ln) of both sides:

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}$$

$$\ln(V) - \ln(V_0) = -\frac{t}{RC}$$

$$\ln(V) = -\frac{t}{RC} + \ln(V_0)$$

- Comparing this to the equation of a straight line: y = mx + c
  - y = ln(V)
  - x = t
  - gradient = -1/RC
  - c = $\ln(V_0)$

1. Plot a graph of ln(V) against t and draw a line of best fit
2. Calculate the gradient (this should be negative)
3. The capacitance of the capacitor is equal to:

$$C = -\frac{1}{R \times \text{gradient}}$$

Evaluating the Experiment

Systematic Errors:

- If a digital voltmeter is used, wait until the reading is settled on a value if it is switching between two
- If an analogue voltmeter is used, reduce parallax error by reading the p.d at eye level to the meter
- Make sure the voltmeter starts at zero to avoid a zero error

Random Errors:

- Use a resistor with a large resistance so the capacitor discharges slowly enough for the time to be taken accurately at p.d intervals
- Using a datalogger will provide more accurate results for the p.d at a certain time. This will reduce the error in the speed of the reflex needed to stop the stopwatch at a certain p.d
- The experiment could be repeated by measuring the time for the capacitor to charge instead

Safety Considerations

- Keep water or any fluids away from the electrical equipment
- Make sure no wires or connections are damaged and contain appropriate fuses to avoid a short circuit or a fire
- Using a resistor with too low a resistance will not only mean the capacitor discharges too quickly but also that the wires will become very hot due to the high current
- Capacitors can still retain charge after power is removed which could cause an electric shock. These should be fully discharged and removed after a few minutes

Exponential Discharge in a Capacitor

The Discharge Equation

- When a capacitor discharges through a resistor, the charge stored on it decreases **exponentially**
- The amount of charge remaining on the capacitor Q after some elapsed time *t* is governed by the **exponential decay equation**:

$$Q = Q_0 e^{-(t/RC)}$$

- Where:
  - Q = charge remaining (C)
  - Q_0 = initial charge stored (C)
  - e = exponential function
  - *t* = elapsed time (s)
  - *R* = circuit resistance (Ω)
  - C = capacitance (F)

Discharge Equation for Potential Difference

- The exponential decay equation for **charge** can be used to derive a decay equation for potential difference
- Recall the equation for charge Q = *CV*
  - It also follows that the initial charge $Q_0 = CV_0$ (where V_0 is the initial potential difference)
- Therefore, substituting *CV* for Q into the original exponential decay equation gives:

$$CV = CV_0 e^{-(t/RC)}$$

- Cancelling C from both sides gives the exponential decay equation for **potential difference *V*:**

$$V = V_0 e^{-(t/RC)}$$

- Where:
  - *V* = potential difference after some time *t* (V)
  - V_0 = initial potential difference (V)
  - *t* = elapsed time (s)
  - *R* = resistance (Ω)
  - C = capacitance (F)
- This equation shows that the potential difference also decreases **exponentially,** from some initial value V_0

Discharge Equation for Current

- The exponential decay equation for **potential difference** can be used to derive a decay equation for current
- Recall Ohm's law ***V* = *IR***
  - It follows that the initial potential difference $V_0 = I_0R$ (where I_0 is the initial current)
- Therefore, substituting *IR* for *V* into the decay equation for potential difference gives:

$$IR = I_0 R e^{-(t/RC)}$$

- Cancelling *R* from both sides gives the exponential decay equation for **current *I*:**

$$I = I_0 e^{-(t/RC)}$$

- Where:
  - *I* = current after some time *t* (A)
  - I_0 = initial current (A)
  - *t* = elapsed time (s)
  - *R* = resistance (Ω)
  - *C* = capacitance (F)
- This equation shows that the current also decreases **exponentially,** from some initial value I_0

Worked Example

A 10 mF capacitor is fully charged by a 12 V power supply and then discharged through a 1 kΩ resistor.

What is the discharge current after 15 s?

Step 1: Write the known quantities

- Initial potential difference V_0 = 12 V
- Resistance *R* = 1 kΩ = 1000 Ω
- Capacitance C = 10 mF = 0.01 F
- Time elapsed = 15 s

Step 2: Determine the initial current I_0

- Since the initial potential difference is 12 V and the resistance is 1000 Ω, then:

$$I_0 = \frac{V_0}{R} = \frac{12}{1000} = 0.012 \text{ A}$$

Step 3: Write the decay equation for current

- The decay equation for current is:

$$I = I_0 e^{-(t/RC)}$$

Step 4: Substitute quantities and calculate the current after 15 s

- Substituting quantities gives the following:

$$I = (0.012) \times (e^{-(15/(1000 \times 0.01)})$$

$$I = (0.012) \times (e^{-1.5})$$

$$I = (0.012) \times (0.223...)$$

$$I = 2.7 \times 10^{-3} \text{ A} = 2.7 \text{ mA}$$

Exam Tip

Remember you can work out initial quantities like current or potential difference or charge using the equations:

- $V_0 = I_0 R$
- $Q_0 = CV_0$

You will then usually have enough information to substitute all necessary values into the decay equations!

Natural Logarithms & Discharge Equations

- The exponential decay equations are not **linear**
- They can be turned into linear equations by using the **natural logarithm** function
- Recall the exponential decay equation for charge:

$$Q = Q_0 e^{-(t/RC)}$$

- Dividing both sides by Q_0 gives:

$$\frac{Q}{Q_0} = e^{-(t/RC)}$$

- Taking the **natural logarithm** of both sides 'cancels' the exponential function e, giving:

$$\ln\left(\frac{Q}{Q_0}\right) = \ln\left(e^{-(t/RC)}\right) = -\frac{t}{RC}$$

- This simplifies to:

$$\ln Q - \ln Q_0 = -\frac{t}{RC}$$

- Leaving an equation for the natural logarithm of charge Q as:

$$\ln Q = -\frac{1}{RC}t + \ln Q_0$$

- This is the equation of a **straight line graph**, where:
  - ln Q is plotted on the *y*-axis

- t is plotted on the x-axis
- The gradient of the line is therefore equal to $-1/RC$

The natural logarithm of the exponential decay curve line arises it to a straight-line graph with a gradient equal to −1/RC

- Following similar steps, the linearised versions of the decay equations for **potential difference** V is:

$$\ln V = -\frac{1}{RC}t + \ln V_0$$

- And for current I is:

$$\ln I = -\frac{1}{RC}t + \ln I_0$$

Magnetic Fields

7.14 Magnetic Flux Density, Flux & Flux Linkage

Magnetic Flux, Flux Density & Flux Linkage

Magnetic Flux Density

- The strength of a magnetic field is defined by the density of the magnetic field lines, or **magnetic flux density**, at that point
  - Magnetic flux density is defined by the symbol *B*
  - It is measured in **Tesla** (T)

- Rearranging the equation for magnetic force on a wire, the magnetic flux density is defined by the equation:

$$B = \frac{F}{IL}$$

- Where:
  - *B* = magnetic flux density (T)
  - *F* = magnetic force on a current-carrying wire (N)
  - *I* = current (A)
  - *L* = length of the wire (m)

- For reference, the Earth's magnetic flux density is around 0.032 mT and an ordinary fridge magnet is around 5 mT
- The magnetic flux density is sometimes referred to as the **magnetic field strength**

Magnetic Flux

- Magnetic flux is a quantity which signifies how much of a magnetic field passes **perpendicularly** through some area
- For example, the amount of magnetic flux through a rotating coil will vary as the coil rotates in the magnetic field
  - It is a maximum when the magnetic field lines are **perpendicular** to the coil area
  - It is at a minimum when the magnetic field lines are **parallel** to the coil area

- The **magnetic flux** is defined as:

  The product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density

- Magnetic flux is defined by the symbol Φ (greek letter 'phi')
- It is measured in units of **Webers (Wb)**
- Magnetic flux can be calculated using the equation:

$$\Phi = BA$$

- This means the magnetic flux is:
  - **Maximum** = BA when $\cos(\theta)$ =1 therefore **θ = 0°**. The magnetic field lines are perpendicular to the plane of the area
  - **Minimum** = 0 when $\cos(\theta)$ = 0 therefore **θ = 90°**. The magnetic fields lines are parallel to the plane of the area

- An e.m.f is induced in a circuit when the magnetic flux linkage changes with respect to time
- This means an e.m.f is induced when there is:
  - A changing magnetic flux density B
  - A changing cross-sectional area A
  - A change in angle θ

Flux Linkage

- The magnetic **flux linkage** is a quantity commonly used for solenoids which are made of N turns of wire
- The flux linkage is defined as:

The product of the magnetic flux and the number of turns of the coil

- It is calculated using the equation:

$$\textbf{Flux linkage} = \Phi N = BAN$$

- Where:
  - Φ = magnetic flux (Wb)
  - N = number of turns of the coil
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)

- The flux linkage ΦN has the units of **Weber turns (Wb turns)**

Magnetic Force on a Charged Particle

- The magnetic force on an isolated moving charged particle, such as a proton, is given by the equation:

$$F = BQv$$

- Where:
  - F = magnetic force on the particle (N)
  - B = magnetic flux density (T)
  - Q = charge of the particle (C)
  - v = speed of the particle (m s^{-1})
- This is the maximum force on the charged particle, when F, B and v are mutually perpendicular
  - Therefore if a particle travels **parallel** to a magnetic field, it will not experience a magnetic force

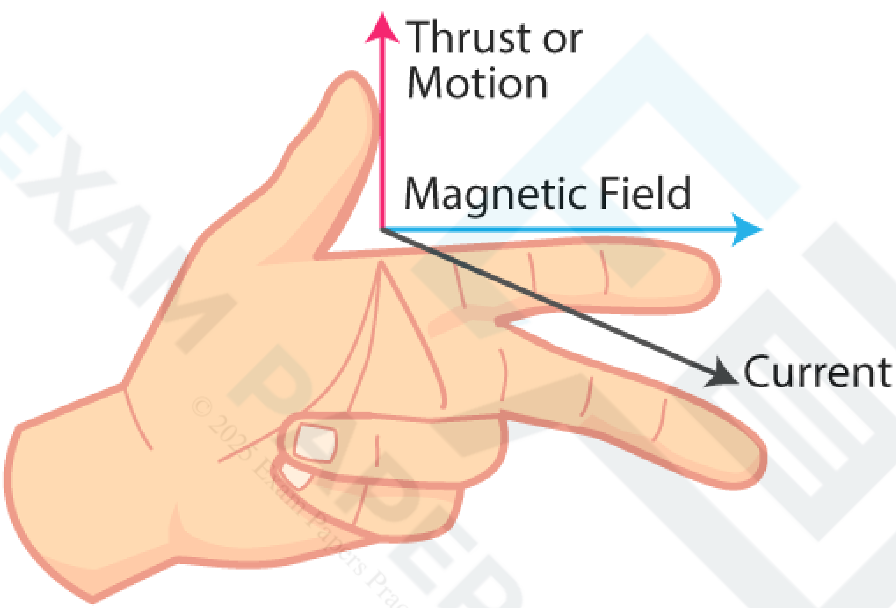
- Current is the rate of flow of **positive** charge
  - This means that the direction of the 'current' for a flow of negative charge (e.g. an electron beam) is in the opposite direction to its motion
- If the charged particle is moving at an angle θ to the magnetic field lines, then the size of the magnetic force F is given by the equation:

$$F = BQv \sin \theta$$

- This equation shows that:
  - The size of the magnetic force is **zero** if the angle θ is **zero** (i.e. the particle moves **parallel** to the field lines)
  - The size of the magnetic force is **maximum** if the angle θ is **90°** (i.e. the particle moves **perpendicular** to field lines)

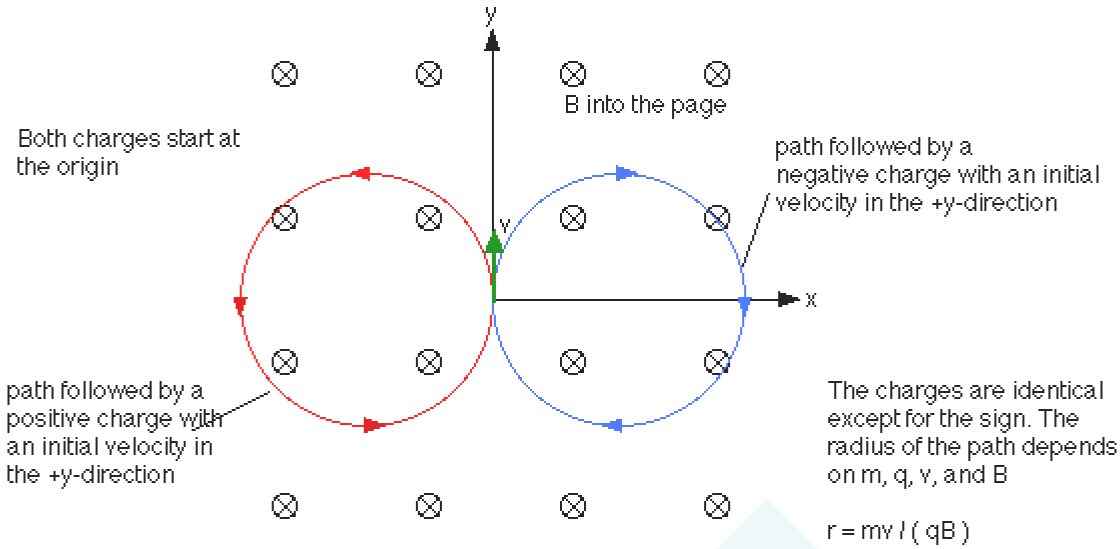
Fleming's Left Hand Rule for a Charged Particle

- **Fleming's left hand rule** can be used to determine the direction of the magnetic force on a moving charged particle in a magnetic field
  - The **First Finger** = direction of the magnetic field
  - The **Second Finger** = direction of conventional current (i.e. the velocity of a moving **positive charge**)
  - The **Thumb** = direction of the magnetic force
- Fleming's Left Hand Rule is illustrated in the image below:



Fleming's Left Hand Rule shows the magnetic force, magnetic field and conventional current (flow of positive charge) are all perpendicular to each other

- Since this is represented in 3D space, sometimes the flow of charge, magnetic force or magnetic field could be directed into or out of the page, not just left, right, up and down
- The direction of the magnetic field **into** or **out of** the page in 3D is represented by the following symbols:
  - **Dots** (sometimes with a circle around them) represent the magnetic field directed **out** of the plane of the page
  - **Crosses** represent the magnetic field directed **into** the plane of the page

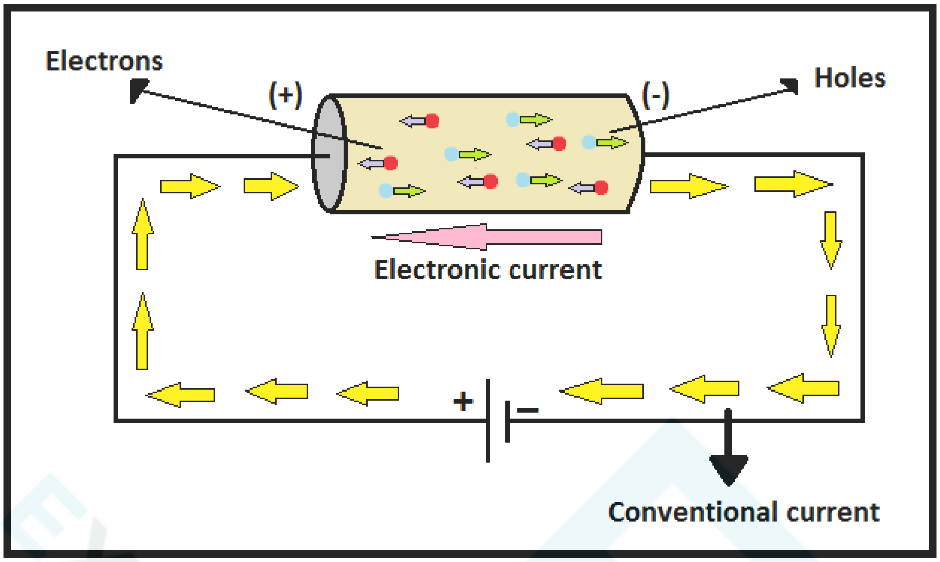


The magnetic field into or out of the page is represented by circles with dots or crosses

- The way to remember this is by imagining an arrow used in archery or darts:
  - If the arrow is approaching **head-on**, such as out of a page, only the very tip of the arrow can be seen (a dot)
  - When the arrow is **moving away**, such as into a page, only the cross of the feathers at the back can be seen (a cross)

An Electron Moving in a Magnetic Field

- The maximum magnetic force on a moving charged particle is always perpendicular to its velocity
  - This means magnetic forces cause charged particles to move in a **circle**
- The **direction** of magnetic force on the charged particle can be determined using Fleming's Left Hand Rule
  - The image below shows an electron incident on a uniform magnetic field *B* directed into the page:



An electron moving to the left as shown is equivalent to a conventional positive charge or current moving to the right. Using Fleming's Left Hand Rule, the direction of the force can be determined

- According to Fleming's Left Hand Rule:
  - *B* is directed into the page, therefore the first finger should point **into** the page
  - The conventional current (or velocity of a **positive charge**) is directed to the **right** (because an electron is moving to the left), therefore the second finger should point to the **right**
  - Therefore, the force on the electron as shown by the thumb is initially **upwards** as it enters the magnetic field
- The force due to the magnetic field **is always perpendicular to the velocity** of the electron
  - **Note:** this is equivalent to circular motion
  - Therefore, the magnetic force on a moving charge is a **centripetal force**
- The centripetal force is what keeps moving charges following a **circular** trajectory

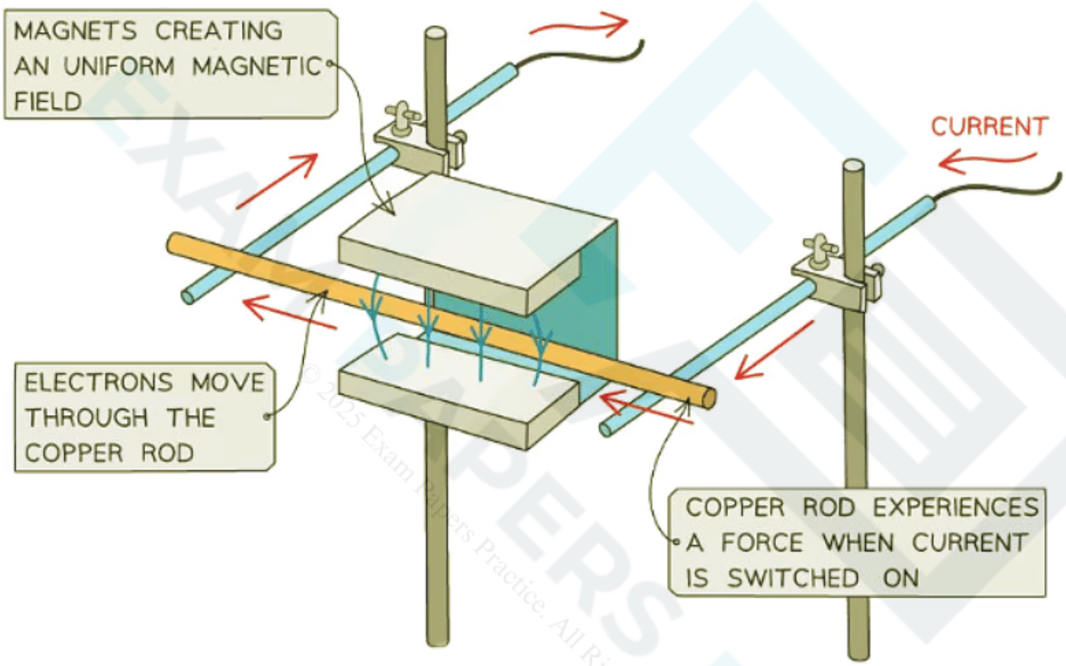
- Fleming's Left Hand Rule can be used again to find the direction of the force, magnetic field and velocity
  - The key difference is that the second finger, representing current *I* (direction of positive charge), can now be used as the **direction of velocity *v*** of a **positive** charge

Exam Tip

The most important point when using Fleming's left hand rule is the direction of the **charge** (or **current** flow). This is always the direction of **positive** charge. Therefore, for electrons, or negatively charged ions, you should point your second finger for the current in the **opposite** direction to its motion.

Magnetic Force on a Current-Carrying Conductor

- A current-carrying conductor produces its own **magnetic field**
  - An **external magnetic field** will therefore exert a **magnetic force** on it
- A current-carrying conductor (eg. a wire) will experience the **maximum** magnetic force if the current through it is **perpendicular** to the direction of the magnetic flux lines
  - A simple situation would be a copper rod placed within a uniform magnetic field
  - When current is passed through the copper rod, it experiences a **force** which makes it accelerate



A copper rod moves within a magnetic field when current is passed through it

- The force *F* on a conductor carrying current *I* in a magnetic field with flux density *B* is defined by the equation

$$F = BIL \sin \theta$$

- Where:
  - F = magnetic force on the current-carrying conductor (N)
  - B = magnetic flux density of external magnetic field (T)
  - I = current in the conductor (A)
  - L = length of the conductor in the field (m)
  - θ = angle between the conductor and external flux lines (degrees)
- This equation shows that the magnitude of the magnetic force *F* is **proportional** to:
  - Current *I*
  - Magnetic flux density *B*

- Length of conductor in the field L
- The sine of the angle θ between the conductor and the magnetic flux lines

- The **maximum** force occurs when $\sin\theta = 1$
  - This means $\theta = 90^\circ$ and the conductor is **perpendicular** to the B field
  - This equation for the magnetic force now becomes:

$$F = BIL$$

- The **minimum** force (0) is when $\sin\theta = 0$
  - This means $\theta = 0^\circ$ and the conductor is **parallel** to the B field

- It is important to note that a current-carrying conductor will experience **no** force if the current in the conductor is parallel to the field

Worked Example

A current of 0.87 A flows in a wire of length 1.4 m placed at 30° to a magnetic field of flux density 80 mT.

Calculate the force on the wire.

Step 1: Write down the known quantities

- Magnetic flux density, $B = 80\text{ mT} = 80 \times 10^{-3}\text{ T}$
- Current, $I = 0.87\text{ A}$
- Length of wire, $L = 1.4\text{ m}$
- Angle between the wire and the magnetic flux lines, $\theta = 30^\circ$

Step 2: Write down the equation for the magnetic force on a current-carrying conductor

$$F = BIL\sin\theta$$

Step 3: Substitute in values and calculate

$$F = (80 \times 10^{-3}) \times (0.87) \times (1.4) \times \sin(30) = 0.04872 = 0.049\text{ N (2 s.f)}$$

Exam Tip

Remember that the direction of current is the flow of **positive** charge (i.e. conventional current) and this is in the **opposite direction** to the flow of electrons (i.e. electron flow)!

Fleming's Left Hand Rule for a Current-Carrying Conductor

- Fleming's Left Hand Rule was previously used to determine the direction of the magnetic force on a moving **charged particle** in a magnetic field
- It can also be used to determine the direction of the magnetic force on a **current-carrying conductor** in a magnetic field
  - This is because inside a conductor (e.g. a wire) there are **many** charged particles flowing as a current

- Using the conventional symbols representing vectors like magnetic flux density ***B*** and force ***F*** that go into the page (arrows) or out of the page (dots) we can apply Fleming's Left Hand Rule to problems in 3D

YOUR NOTES
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Induced E.M.F in a Moving Coil

- Electromagnetic induction is a phenomenon which occurs when an e.m.f is induced when a conductor moves through a magnetic field
- If there is a change in **magnetic flux** Φ or magnetic flux linkage $N\Phi$
  - **Mechanical work** (from moving the conductor in the field) is transformed into **electrical energy**
- Therefore, if attached to a complete circuit, a current will be induced in the conductor
- This is known as **electromagnetic induction** and is defined as:

The process in which an e.m.f is induced in a closed circuit due to changes in magnetic flux (linkage)

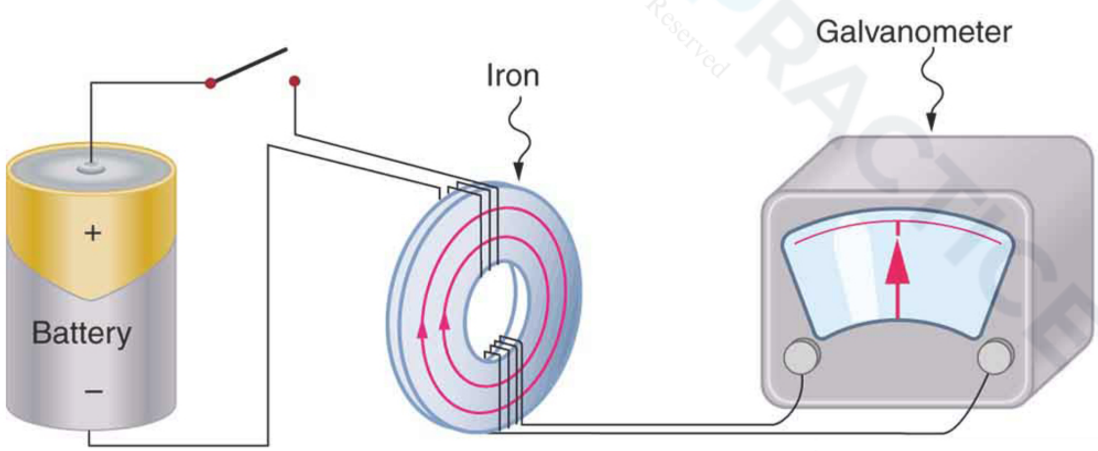
- This can occur either when:
  - A conductor cuts through a **magnetic field**
  - The magnetic flux (linkage) through a coil changes, e.g. becomes more or less dense, or changes direction

- Electromagnetic induction is used in:
  - Electrical **generators** which convert mechanical energy to electrical energy
  - **Transformers** which are used in electrical power transmission

- This phenomenon can easily be demonstrated with a magnet and a coil, or a wire and two magnets

Relative Motion between a Coil and a Magnet

- When a coil is connected to a sensitive voltmeter, a bar magnet can be moved in and out of the coil to induce an e.m.f in the coil



A bar magnet is moved through a coil connected to a voltmeter to induce an e.m.f

The observations are:

- When the bar magnet is **not moving**, the voltmeter shows a **zero reading**
  - When the bar magnet is held still inside, or outside, the coil, the rate of change of flux is zero, so, there is **no e.m.f induced**
- When the bar magnet begins to move inside the coil, there is a reading on the voltmeter
  - As the bar magnet moves, its magnetic field lines 'cut through' the coil, generating a **change in magnetic flux** ($\Delta\Phi$)
  - This induces an **e.m.f** within the coil, shown momentarily by the reading on the voltmeter
- When the bar magnet is taken back out of the coil, an e.m.f is induced in the **opposite direction**
  - As the magnet changes direction, the direction of the current changes
  - The voltmeter will momentarily show a reading with the opposite sign
- Increasing the **speed** of the magnet induces an e.m.f with a **higher magnitude**
  - As the speed of the magnet increases, the rate of change of flux increases
- The direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it

- Factors that will increase the induced e.m.f are:
  - Moving the magnet **faster** through the coil
  - Adding more **turns** to the coil
  - Increasing the **strength** of the bar magnet

Rotating Coils

- When a coil rotates in a uniform magnetic field, the **magnetic flux** through the coil will vary as it rotates
- Therefore, since the flux linkage through the coil also varies, this will induce an e.m.f that also varies
  - The maximum e.m.f is when the coil **cuts through** the most field lines
  - The varying e.m.f induced is called an **alternating voltage**

Even though the flux linkage through the coil is maximum when θ = 0°, the change in flux linkage is minimal as the coil rotates, so the induced e.m.f is a minimum. The opposite is true when θ = 90°

- Increasing the coil's frequency of rotation increases:
  - The **frequency** of the alternating voltage
  - The **amplitude** of the alternating voltage

Induced E.M.F between Linked Coils

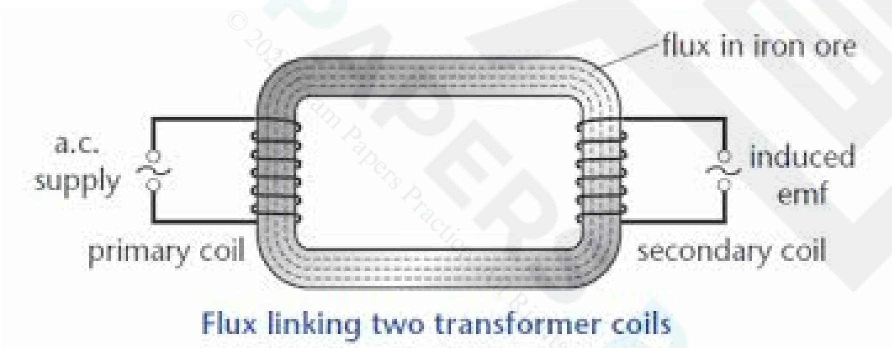
- An e.m.f can be induced in a coil when there is a change of current in another coil linked with this coil
  - This is what happens in a **transformer**

Transformers

- A transformer is a device that works by the principles of electromagnetic induction
- It changes high alternating voltages at low current to low alternating voltage at high current, and vice versa

- A transformer is made up of:
  - A primary coil
  - A secondary coil
  - An iron core

- The primary and secondary coils are wound around the soft iron core
  - The soft iron core is necessary because it creates **flux linkage** between the primary and secondary coils
  - Soft iron is used because it can easily be magnetised and demagnetised



Coils are magnetically linked, through their combined magnetic flux linkage, using a soft iron core

- In the primary coil, an alternating current producing an alternating voltage is applied
  - This creates an **alternating magnetic field** inside the iron core and therefore a changing magnetic flux linkage

- A changing magnetic field passes through to the secondary coil through the iron core
  - This results in a changing magnetic flux linkage in the secondary coil and from Faraday's Law, an **e.m.f is induced**

- An e.m.f produces an alternating output voltage from the secondary coil
- The output alternating voltage is at the **same** frequency as the input voltage

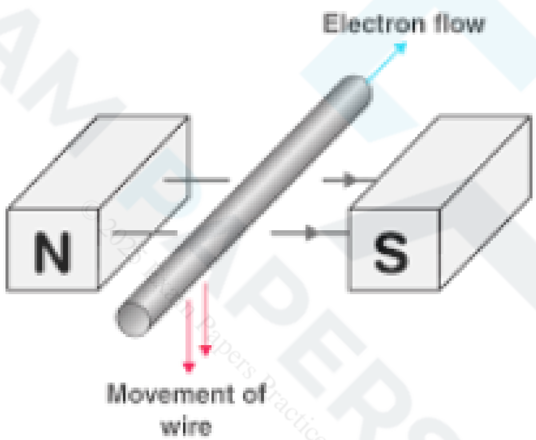
Lenz's Law

- Lenz's Law is used to predict the **direction** of an induced e.m.f in a coil or wire
- Lenz's Law is summarised below:

The induced e.m.f is set up in a direction to produce effects that oppose the change causing it

Experimental Evidence for Lenz's Law

- To verify Lenz's Law, the only apparatus needed is:
  - A bar magnet
  - A coil of wire
  - A sensitive ammeter
- Note, a cell is **not** required



Lenz's law can be verified using a coil connected in series with a sensitive ammeter and a bar magnet

- A known pole (either north or south) of a bar magnet is pushed into the coil
  - This induces an e.m.f in the coil
  - The induced e.m.f drives a current (because it is a complete circuit)
- Lenz's Law dictates:
  - The direction of the e.m.f, and hence the current, must be set up to **oppose** the incoming magnet
  - Since a **north pole** approaches the coil face, the e.m.f must be set up to create an induced **north pole**
  - This is because two north poles will **repel** each other
- The direction of the current is therefore as shown in the image above
  - The direction of current can be verified using the right hand grip rule

- Fingers curl around the coil in the direction of current and the thumb points along the direction of the flux lines, from north to south
- Therefore, the current flows in an anti-clockwise direction in the image shown, in order to induce a north pole opposing the incoming magnet

- Reversing the magnet direction would give an opposite deflection on the voltmeter
  - Lenz's Law now predicts a south pole induced at the coil entrance
  - This would **attract** the north pole attempting to leave
  - Therefore, the induced e.m.f **always** produces effects to oppose the changes causing it

- Lenz's Law is a direct consequence of the **principle of conservation of energy**
  - Electromagnetic effects will not create electrical energy out of nothing
  - In order to induce and sustain an e.m.f, for instance, **work** must be done in order to overcome the repulsive effect due to Lenz's Law

Exam Tip

A typical exam question may ask you to explain the presence of the negative sign in Faraday's Law, which is the equation that tells you the size of the induced e.m.f ε as:

$$\varepsilon = -\frac{d(N\Phi)}{dt}$$

You should remember that the negative sign is representative of **Lenz's Law,** which says that the induced e.m.f ε is set up **to oppose the change causing it.** The 'change' causing an induced e.m.f, in this case, is the changing flux linkage (represented by the quantity $\frac{d(N\Phi)}{dt}$).

Faraday's Law

- Faraday's Law connects the **rate** of change of flux linkage with induced e.m.f
- It is defined in words as:

The magnitude of the induced e.m.f is directly proportional to the rate of change of magnetic flux linkage

- Faraday's Law as an equation is defined as:

$$\varepsilon = \frac{\Delta(N\Phi)}{\Delta t}$$

- Where:
  - ε = induced e.m.f (V)
  - $\Delta(N\phi)$ = change in flux linkage (Wb turns)
  - Δt = time interval (s)
- If the interval of time becomes very small (i.e., in the limit of $\Delta t \to 0$) the equation for Faraday's Law can be written as:

$$\varepsilon = \frac{d(N\Phi)}{dt}$$

Combining Lenz's Law and Faraday's Law

- Combining Lenz's Law into the equation for Faraday's Law is written as:

$$\varepsilon = -\frac{d(N\Phi)}{dt}$$

- The **negative sign** represents Lenz's Law
  - This is because it shows the induced e.m.f ε is set up in an '**opposite direction**' to oppose the changing flux linkage
- This equation also shows that the **gradient** of the graph of magnetic flux (linkage) against time, $\frac{\Delta(N\Phi)}{\Delta t}$ represents the **magnitude** of the induced e.m.f
  - Note: the negative sign means if the gradient is **positive**, the induced e.m.f is **negative**
  - This is again due to Lenz's Law, which says the e.m.f is set up to **oppose** the effects of the changing flux linkage

Alternating Currents & Potential Differences

- An alternating current (a.c) is defined as:

A current which periodically varies between a positive and negative value

- This means the direction of an alternating current switches every **half cycle**
- The variation of current, or p.d., with time can be described as a sine curve ie. **sinusoidal**
  - Therefore, the electrons in a wire carrying a.c. move back and forth with simple harmonic motion

- As with SHM, the relationship between time period *T* and frequency *f* for a.c is related by the equation:

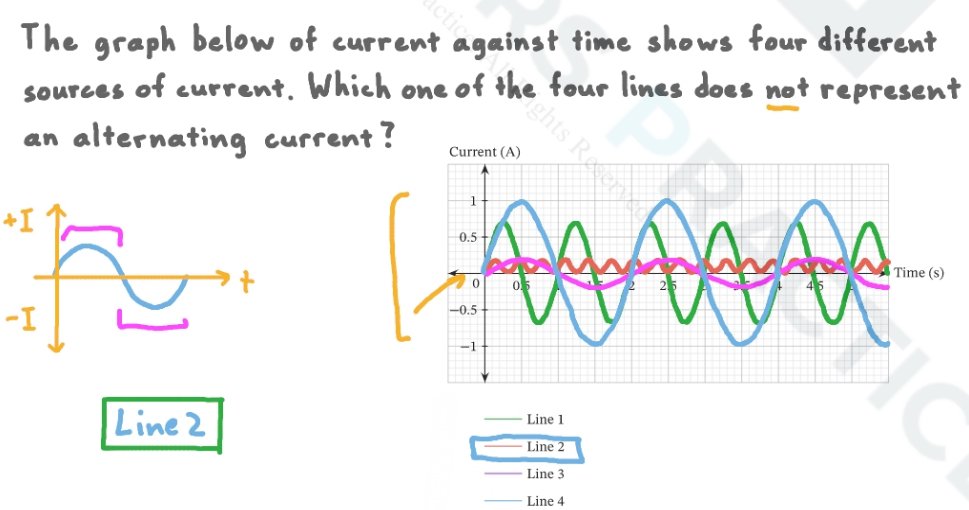
$$T = \frac{1}{f}$$

- Where:
  - T = time period (s)
  - f = frequency (Hz)

- Peak current (I_0), or peak voltage (V_0), is defined as:

The maximum value of the alternating current or voltage

- Peak current, or voltage, can be determined from the **amplitude** of a current-time or voltage-time graph
- The **peak-to-peak** current or voltage is the distance between a positive and consecutive negative peak. This means:



Graph of alternating current against time showing the time period, peak current and peak-to-peak current

Root-Mean-Square Current & Voltage

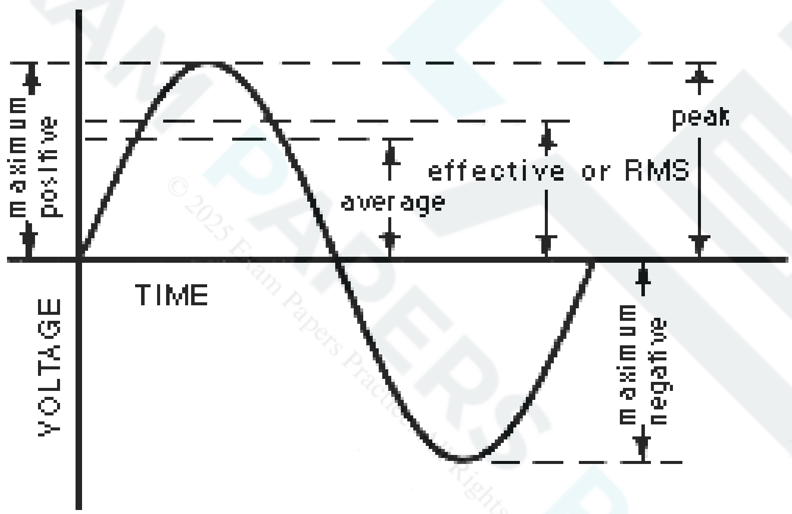
- Root-mean-square (rms) values of current, or voltage, are a useful way of **comparing** a.c current, or voltage, to its equivalent direct current (d.c), or voltage
- The rms values represent the direct current, or voltage, values that will produce the same **heating effect**, or power dissipation, as the alternating current, or voltage
- The **rms** value of an **alternating current** is defined as:

  The equivalent direct current that produces the same power

- In other words, an rms current is 'equivalent', in a sense, to a DC current, because they produce the same overall effect in a circuit
- The rms value of an alternating voltage is similarly defined as:

  The equivalent dc voltage that produces the same power

- Rms current is equal to 0.707 × I_0, which is about 70% of the peak current I_0
  - This is also the case for rms voltage



V_{rms} and peak voltage. The rms voltage is about 70% of the peak voltage

Root-Mean-Square Current & Potential Difference

- The root-mean-square (rms) current I_{rms} is defined by the equation:

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

- Where:
  - I_0 = peak current (A)
- The rms voltage V_{rms} is defined by the equation:

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

- Where:
  - V_0 = peak voltage (V)

- Rms current is equal to $\mathbf{0.707 I_0}$, which is about **70%** of the peak current I_0
  - This is also the case for rms voltage

Worked Example

An electric oven is connected to a 230 V root mean square (rms) mains supply using a cable of negligible resistance.

Calculate the peak-to-peak voltage of the mains supply.

Step 1: Write down the V_{rms} equation

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Step 2: Rearrange for the peak voltage, V_0

$$V_0 = \sqrt{2} \times V_{rms}$$

Step 3: Substitute in the values

$$V_0 = \sqrt{2} \times 230$$

Step 4: Calculate the peak-to-peak voltage

- The peak-to-peak voltage is the peak voltage (V_0) × 2

Peak-to-peak voltage = ($\sqrt{2}$ × 230) × 2 = 650.538 = **651 V (3 s.f)**

Exam Tip

You are expected to know how to apply these equations, which simply relate the peak current or voltage of an AC circuit to its rms value.

Remember, the rms value in an AC circuit is **equivalent to values of current and voltage that would produce the same heating effect in DC circuits**. This means, you can use the rms values in AC circuits for equivalent calculations involving DC circuits.