A Level Physics Edexcel

6. Further Mechanics

CONTENTS

Momentum & Impulse

6.1 Impulse

Impulse

- Force is defined as the **rate of change of momentum** on a body
  - The change in momentum is defined as the final momentum minus the initial momentum
- These can be expressed as follows:

CHANGE IN MOMENTUM ($kgms^{-1}$)

$$F = \frac{\Delta p}{\Delta t}$$

FORCE (N)

CHANGE IN TIME (s)

$$\Delta p = p_{FINAL} - p_{BEFORE}$$

CHANGE IN MOMENTUM

Defining Impulse

- The force and momentum equation can be rearranged to find the **impulse of a force**
- Impulse, *I*, is equal to the **change in momentum:**

$$I = F\Delta t = \Delta p = mv - mu$$

- Where:
  - I = impulse (N s)
  - F = force (N)
  - t = time (s)
  - Δp = change in momentum (kg m s^{-1})
  - m = mass (kg)
  - v = final velocity (m s^{-1})
  - u = initial velocity (m s^{-1})

- This equation is only used when the force is **constant**
  - Since the impulse is proportional to the force, it is also a vector
  - The impulse is in the same direction as the force

- The unit of impulse is **N s**
- The impulse quantifies the effect of a force acting over a time interval
    - This means a **small force acting over a long time** has the same effect as a **large force acting over a short time**

Examples of Impulse

- An example in everyday life of impulse is when standing under an umbrella when it is raining, compared to hail (frozen water droplets)
    - When rain hits an umbrella, the water droplets tend to splatter and fall off it and there is only a very **small** change in momentum
    - However, hailstones have a **larger mass** and tend to bounce back off the umbrella, creating a **greater** change in momentum
    - Therefore, the impulse on an umbrella is **greater** in hail than in rain
    - This means that **more force** is required to hold an umbrella upright in hail compared to rain

Worked Example

A 58 g tennis ball moving horizontally to the left at a speed of 30 m s^{-1} is struck by a tennis racket which returns the ball back to the right at 20 m s^{-1}.

(i) Calculate the impulse delivered to the ball by the racket.

(ii) State which direction the impulse is in.

(i) Step 1: **Write the known quantities**

- Taking the initial direction of the ball as positive (the left)
- Initial velocity, u = 30 m s^{-1}
- Final velocity, v = –20 m s^{-1}
- Mass, m = 58 g = 58 × 10^{-3} kg

Step 2: Write down the impulse equation

$$\text{Impulse } I = \Delta p = m(v - u)$$

Step 3: Substitute in the values

$$I = (58 \times 10^{-3}) \times (-20 - 30) = -2.9 \text{ N s}$$

(ii) Direction of the impulse

- Since the impulse is **negative**, it must be **in the opposite direction** to which the tennis ball was initial travelling (since the left is taken as positive)
- Therefore, the direction of the impulse is **to the right**

Exam Tip

Remember that if an object changes direction, then this must be reflected by the change in sign of the velocity. As long as the magnitude is correct, the final sign for the impulse doesn't matter as long as it is consistent with which way you have considered positive (and negative). For example, if the left is taken as positive and therefore the right as negative, an impulse of 20 N s to the right is equal to -20 N s.

Core Practical 9: Investigating Impulse

Aims of the Experiment

- To determine the change in momentum of a trolley due to a force acting on it
  - This is known as the impulse

Variables

- Independent variable = accelerating mass, *m*
- Dependent variable = time taken to pass between two light gates, *t*
- Control variables
  - Overall mass of the system (trolley + accelerating masses)
  - Tilt angle of the ramp
  - Trolley and ramp used
  - Size of interrupter card

Equipment List

Apparatus	Purpose
Dynamics trolley	Momentum change of the trolley is being investigated
Ramp, slightly tilted	For the trolley to travel down
Bench pulley	To pull trolley using the suspended masses
String	To connect the suspended mass and the trolley over the pulley
5 slotted masses (10 g) and hanger	To create the force to accelerate the trolley
Light gates and computer or datalogger	To measure the time taken and velocity of the trolley passing through it
Balance	To measure the masses
Interrupt card	For the data logger to detect the motion of the trolley

- Resolution of measuring equipment
  - Balance = 0.01 g

Method

1. Measure the total mass, *M*, of the trolley and the five 10 g masses using the balance

2. Set up the equipment:
   - Secure the bench pulley to one end of the runway allowing one end to project over the end of the bench
   - Tilt the ramp slightly
     - This is to compensate for friction
   - Place the mass hanger (without the masses on them) on the floor and move the trolley backwards until the string becomes tight, with the mass on the floor
   - Place the light gates at either end of the ramp
     - There should be enough space on the ramp to allow the trolley to clear the light gate at the bottom before hitting the pulley

3. Set the start position for the experiment
   - Move the trolley further backwards until the mass hanger is closer to the pulley (it will fall to the floor as the trolley moves on the runway)
   - Put the five 10 g masses on the trolley so that they will not slide off

4. Record the total hanging mass, *m* in the results table

5. Release the trolley and start the timing software
   - The computer will record the velocity through each gate, and the the time taken for the trolley to travel between them
   - Record the values in the results table

6. Repeat the readings and calculate the mean time and velocity for this value of *m*

7. Move one 10 g mass from the trolley to the hanger and repeat steps 4 and 5
   - Repeat this process, moving one 10 g mass at a time
   - The last reading is when all of the masses are on the hanger

Table of Results:

Mass of system, M =

Hanging mass, m/kg	Velocity at A, V_A / m s^{-1}				Velocity at B, V_B / m s^{-1}				Time to travel between A and B, t/s			
	1	2	3	Average	1	2	3	Average	1	2	3	Average
0.01												
0.02												
0.03												
0.04												
0.05												
0.06												

Analysing the Results

- The momentum of the trolley can be represented by two equations:

$$\Delta p = M(v_B - v_A) \text{ (equation 1)}$$

- Where:
  - Δp = change in momentum (kg m s^{-1})
  - M = mass of the system (kg)
  - v_B = velocity at light-gate B
  - v_A = velocity at light-gate A

$$\Delta p = mgt \text{ (equation 2)}$$

- Where:
  - m = mass on the hanger (kg)
  - g = acceleration due to gravity, (9.81 m s^{-1})
  - t = time taken between light-gates A and B

- Combing equations 1 and 2 gives:

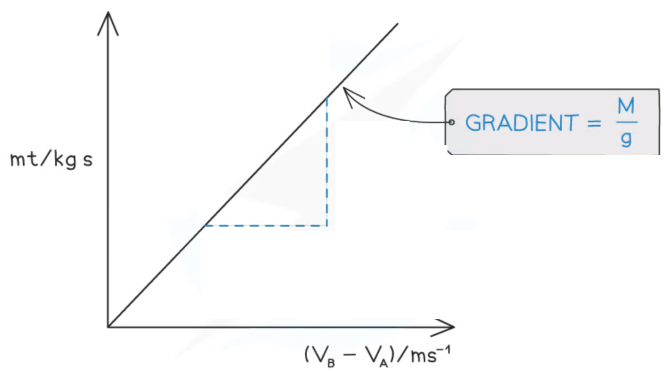
$$mgt = M(v_B - v_A)$$

- This can be rearranged to give:

$$mt = \frac{M}{g}(v_B - v_A)$$

- This is in the form of $y = mx + c$, where:
  - $y = mt$
  - $x = (v_B - v_A)$
  - $m = \frac{M}{g}$
  - c = 0

- Therefore, a graph can be plotted of mt against $(v_B - v_A)$
  - This should be a straight line graph to prove the relationship
  - The gradient should be equal to $\frac{M}{g}$



A straight line with gradient M/g confirms the relationship between the variables

Evaluating the Experiment

Systematic errors:

- The interrupt card may be a different width to that recorded in the data logger
  - Measure it three times and calculate an average value
- The interrupt card may not be of sufficient height to trigger the light gate
  - Move the light gates down, or use a taller card
- Mass of the system, *M*, may not be measured correctly
  - Measure it three times and calculate an average value
- The overall mass, *M*, of the system may not be kept constant
  - Ensure each hanging mass, *m*, which is removed is transferred to the trolley so the overall mass of the system (trolley + hanging masses) stays the same

Random errors:

- The trolley may not travel in a straight line
  - Discard this result
- The trolley may hit one of the light gates when passing through
  - Discard this result

Safety Considerations

- Stand well away from the masses in case they fall onto the floor
- Place a crash mat or any soft surface, such as a small cushion, under the masses to break their fall
- Keep liquids away from the data logger and other electronic equipment
- Make sure no other objects are obstructing the motion of the trolley throughout the experiment

Exam Tip

This practical can be completed with one light gate, where a card of known length, *L* is passed through a lightgate. The time is recorded and *v* is found using $v = \frac{L}{t}$. The overall time is taken from the trolley at rest and the stop watch is started when the trolley is released.

A graph is then plotted as above, but with *v* not (v_B – v_A) on the x-axis as initial velocity is 0.

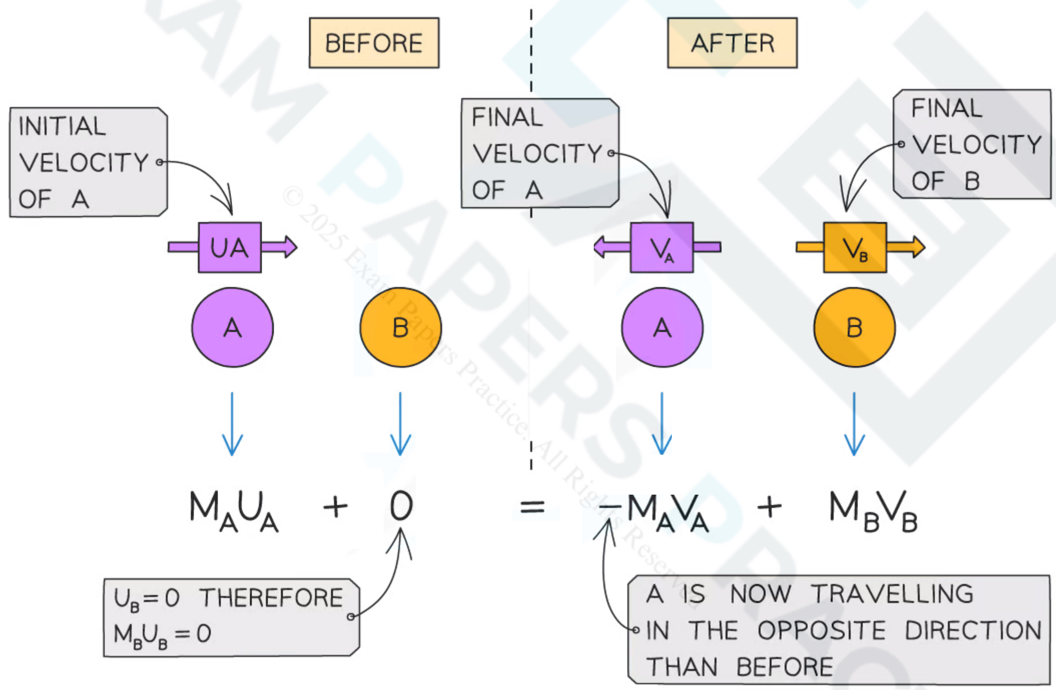
Applying Conservation of Linear Momentum

- The principle of conservation of linear momentum states:

  The total momentum before a collision = the total momentum after a collision provided no external force acts

- Linear momentum is the momentum of an object that only moves in a straight line
- Momentum is a **vector** quantity
  - This means oppositely-directed vectors can cancel each other out resulting in a net momentum of zero
  - If after a collision an object starts to move in the opposite direction to which it was initially travelling, its velocity will now be **negative**
- Momentum, just like energy, is **always conserved**

Conservation of Linear Momentum in 1D



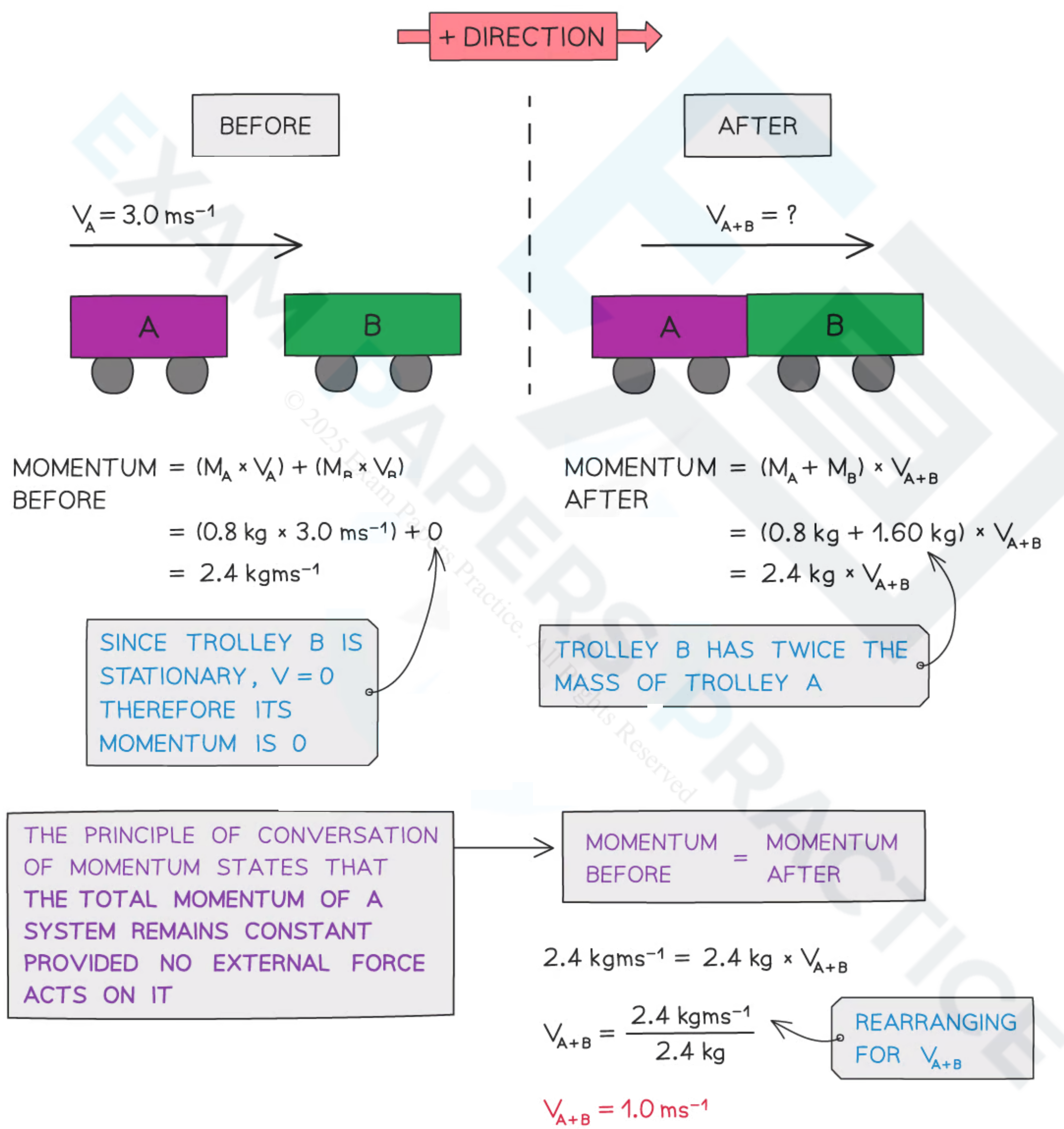
The conservation of momentum in 1D, for two objects A and B colliding then moving apart

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** whilst travelling at

3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. On impact, the trolleys stick together.

Using the conversation of momentum, calculate the common velocity of both trolleys after the collision.



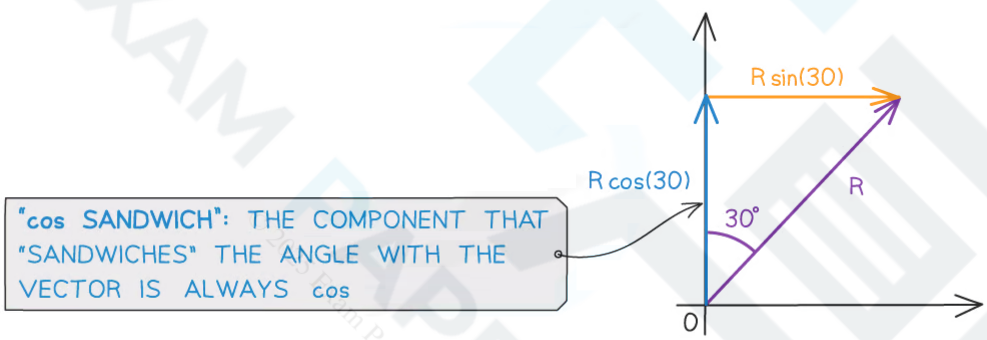
Exam Tip

Momentum is a **vector** quantity, therefore, you should always define a direction to be 'positive' when applying the principle of conservation of momentum. In this worked example, we implicitly took velocity 'to the right' as the positive direction.

Sometimes, however, you might encounter two objects moving **towards** each other before colliding. If both objects have the same mass m and speed v, then the total momentum (before collision) is **zero**, because $p_{total} = (mv) + (-mv) = 0$. Note the negative sign indicates a body travelling in the opposite direction.

Conservation of Linear Momentum in 2D

- For objects moving in 2D, there are components of momentum to consider
  - This is similar to projectile motion in 2D, in which we consider horizontal and vertical components of motion



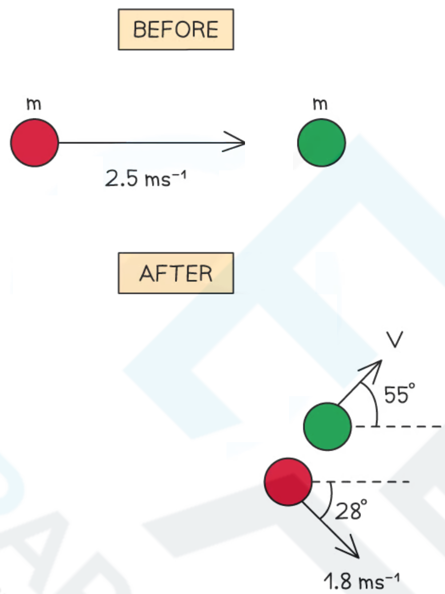
Vector R split into its vertical, R cos (30) and horizontal, R sin (30), components

- Each **component** of momentum is conserved separately
  - Since momentum is a vector, it can be **resolved** into horizontal and vertical components
  - The **sum of horizontal components** will be equal before and after a collision
  - The **sum of vertical components** will be equal before and after a collision

Worked Example

A red snooker ball, travelling at 2.5 m s−1 collides with a green snooker ball, which is at rest. Both snooker balls have the same mass *m*.

The angle of collision is such that the red ball moves off at 28° below the horizontal at 1.8 m s−1 and the green ball moves off at 55° above the horizontal, with a speed *v*, as shown.



Determine the size of *v*.

Step 1: Write the conservation of linear momentum for horizontal components

- The question is worded in terms of the horizontal direction, so write:

Horizontal momentum before = horizontal momentum after

Step 2: Resolve the velocity of each ball to find the horizontal component:

- Since momentum $p = mv$, then the horizontal component of momentum $p_{horiz} = mv_{horiz}$
- Therefore, the horizontal component of the green ball is 1.8 cos 28°
- The horizontal component of the red ball is v cos 55°

Step 3: Substitute quantities into the conservation of momentum

Horizontal momentum before = horizontal momentum after

$$mu_{red} + mu_{green} = mv_{horiz(red)} + mv_{horiz(green)}$$

$$m(2.5) + 0 = m(v\cos 55°) + m(1.8\cos 28°)$$

Step 4: Simplify and rearrange to calculate *v*

$$2.5 = v \cos 55° + 1.8 \cos 28°$$

$$2.5 = v \cos 55° + 1.6$$

$$0.9 = v \cos 55°$$

$$v = 0.9 \cos 55° = 1.6 \text{ m s}^{-1}$$

Exam Tip

Generally speaking, whenever you see any vector given at an angle to the horizontal or the vertical (e.g. velocity, or momentum), think "**resolve**"! It's extremely likely you will need to consider the separate components of motion for a projectiles question or for a conservation of momentum question.

Questions which ask you to use the principle of conservation of linear momentum in 2D are usually worth a lot of marks, so make sure you practise lots of questions involving resolving vectors!

Core Practical 10: Investigating Collisions using ICT

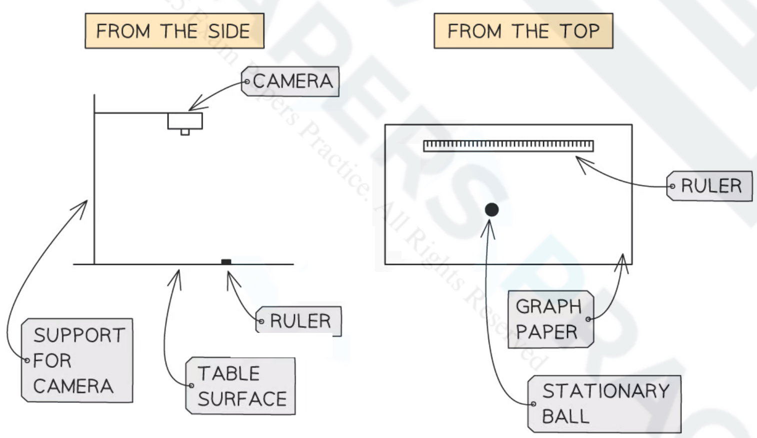
Aims of the Experiment

- To investigate conservation of momentum in two directions
  - Considering if collisions are elastic
  - Constructing a diagram of 2D collisions
- Use of ICT software is required
  - 'Tracker' is recommended by Edexcel

Equipment List

- Small spheres
  - Of two different diameters (ball bearings are ideal)
- Digital camera able to record video
  - Support to allow it to be positioned directly above the collision
- Computer with *Tracker* installed
- 30 cm ruler
- Micrometer or calipers
- Balance
- Graph paper

Method



1. Measure the mass of the spheres using the balance and record

2. Measure the diameter of the spheres using a micrometer or Vernier calipers

3. Mark an approximately central point on the graph paper

   - This will be where the stationary sphere is placed

4. Start the camera recording

5. Within the area of the graph paper, roll a sphere into the stationary one

6. Replace the stationary sphere in its initial place and repeat the experiment up to three times

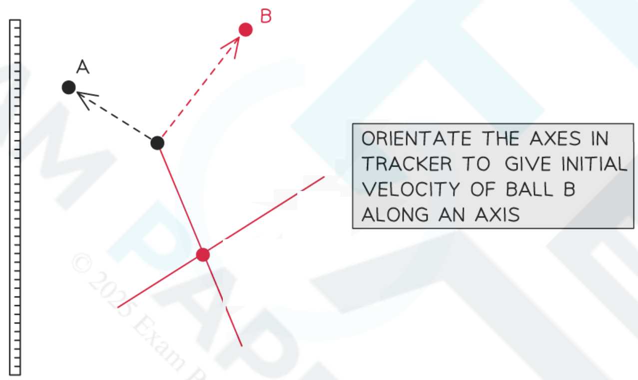
    - A slightly different angle of approach should be used for each collision

7. Download the video file from the camera to the computer that runs *Tracker*

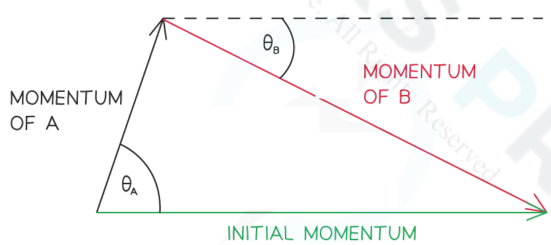
    - Load the clip into the program.

Analysing the Results

- Use *Tracker* to analyse the video clips.
    - Input the mass and diameter of each sphere when prompted
    - Use the 'velocity overlay' feature so that the software can analyse velocities
- The *Tracker* software allows for frame-by-frame analysis of the movement of the spheres
    - Orientate the axes to make the velocity of the moving ball along one of the axes
    - Record the momentum of each ball as indicated in *Tracker*



- Construct a vector diagram from the results



Evaluating the Experiment

- ICT is used in this experiment because
    - The events happen to swiftly for the unaided eye to take readings
    - ICT generally provides more precise and reliable data

Systematic errors:

- Parallax error from camera to the table
- The precision of the balance may give a wide range of possible values for mass
    - If possible use a more precise balance

- The spheres may have damage
  - Check there is no damage to the surface of each sphere before using
- The *Tracker* axes may not be correctly aligned when analysing

Random errors:

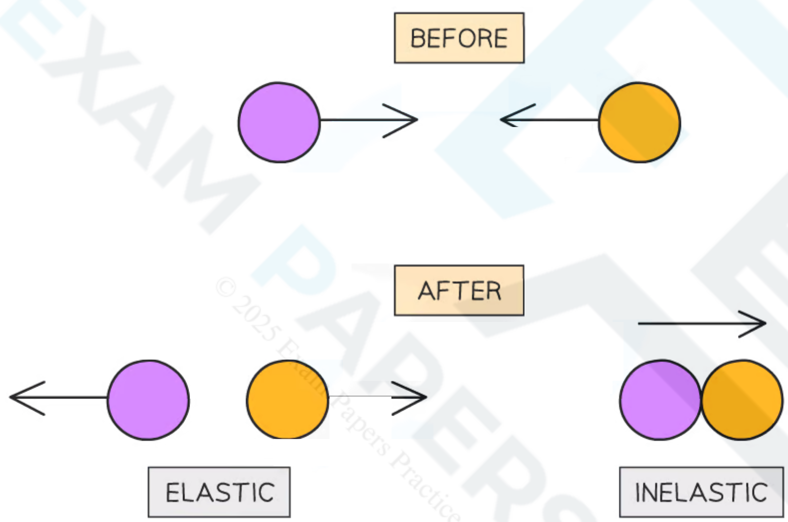
- The collision event may happen between frames
- From variations in the table surface
  - This could cause loss or gain of kinetic energy due to friction or slopes
- The sphere may not travel far enough to hit the second stationary sphere
  - Discard this result and release with greater initial velocity

> **Exam Tip**
>
> It can be helpful to practice a few collisions before making your final readings. This will help you become familiar with how fast to release the sphere.

Elastic & Inelastic Collisions

- In both collisions and explosions, momentum is **always** conserved
- However, **kinetic energy** might not always be
- A collision (or explosion) is either:
  - **Elastic** – if the kinetic energy **is** conserved
  - **Inelastic** – if the kinetic energy is **not** conserved

- Collisions happen when objects strike against each other
  - **Elastic** collisions are commonly those where objects colliding do not stick together; instead, they strike each other then **move away in opposite directions**
  - **Inelastic** collisions are commonly those where objects collide and **stick together** after the collision



Elastic collisions are those following which objects move away in opposite directions. Inelastic collisions are where two objects stick together

- An explosion is commonly to do with **recoil**
  - For example, a gun recoiling after shooting a bullet or an unstable nucleus emitting an alpha particle and a daughter nucleus

- To find out whether a collision is elastic or inelastic, **compare the kinetic energy before and after the collision**
- The equation for kinetic energy is:

MASS (kg)

$$KE = \frac{1}{2}mv^2$$

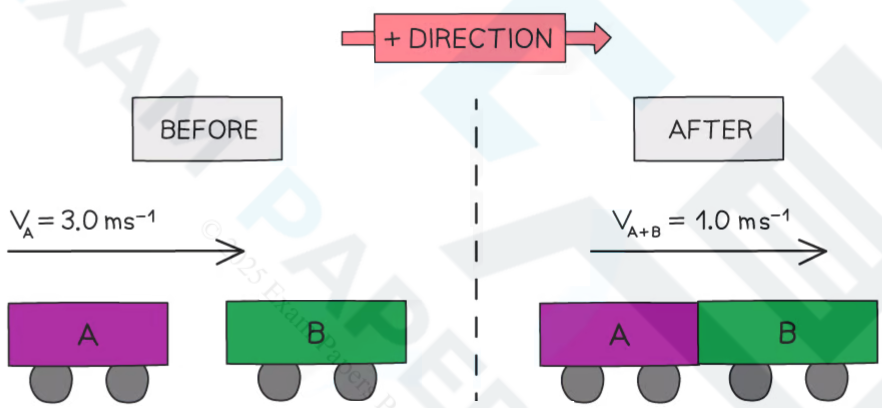
KINETIC ENERGY (J)

VELOCITY (ms^{-1})

Worked Example

Trolley A of mass 0.80 kg collides head-on with stationary trolley B at speed 3.0 m s^{-1}. Trolley B has twice the mass of trolley A.

The trolleys stick together and travel at a velocity of 1.0 m s^{-1}. Determine whether this is an elastic or inelastic collision.



KINETIC ENERGY = 1/2 mv^2

KINETIC ENERGY BEFORE $= \frac{1}{2} \times M_A \times (V_A)^2 + \frac{1}{2} \times M_B \times (V_B^2)$

$= \frac{1}{2} \times 0.8 \times (3.0)^2 + 0$ ← $V_B = 0$

$= 3.6$ J

KINETIC ENERGY AFTER $= \frac{1}{2} \times M_{A+B} \times (V_{A+B})^2$

$= \frac{1}{2} \times 2.4 \times (1.0)^2$

$= 1.2$ J

THIS IS AN INELASTIC COLLISION SINCE KINETIC ENERGY IS **NOT** CONSERVED

Worked Example

Discuss whether a head-on collision between two cars is likely to be an elastic or inelastic collision.

Step 1: Define an elastic and inelastic collision

- An elastic collision is one in which kinetic energy is conserved
- An inelastic collision is one in which kinetic energy is not conserved, but is transferred to other forms, e.g. heat and sound

Step 2: Describe the effects of head-on car collisions

- When cars collide, a large amount of kinetic energy is transferred due to work by internal forces
- This is mainly due to **crumpling** where the collision of the car causes **plastic defamation** of the car's bodywork
- Other energy transfers will include kinetic energy into **heat** and **sound**

Step 3: Link the effects to energy transfers

- Since the cars are brought to rest by the collision, the total KE before the collision does not equal the total KE after
- Therefore, the collision is inelastic

Exam Tip

If an object is stationary or at rest, its velocity equals **0**, therefore, the momentum and kinetic energy are also equal to 0.

When a collision occurs in which two objects stick together, treat the final object as a single object with a mass equal to the **sum** of the two individual objects.

Deriving the Energy-Momentum Relation

- The equation for calculating the **kinetic energy** E_k of a particle m moving at velocity v is given by:

$$E_k = \frac{1}{2}mv^2$$

- The formula for the momentum p of the same particle is:

$$p = mv$$

- Combining these gives an equation that links kinetic energy to momentum, called the **energy-momentum relation**
  - Firstly, substituting the equation for velocity $v = \frac{p}{m}$ into the equation for kinetic energy gives:

$$E_k = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$

  - Multiplying brackets out and simplifying gives:

$$E_k = \frac{1}{2}m\frac{p^2}{m^2} = \frac{1}{2}\frac{p^2}{m}$$

- Therefore the **energy-momentum** is presented as:

$$E_k = \frac{p^2}{2m}$$

- Where:
  - E_k = kinetic energy (J)
  - p = momentum (kg m s^{-1})
  - m = mass (kg)

Exam Tip

This is a common derivation, so make sure you're comfortable with deriving this from scratch! Think carefully about the algebra on each step.

Using the Energy-Momentum Relation

- The energy-momentum relation is particularly useful for:
  - Calculations involving the kinetic energy of **subatomic particles** travelling at non-relativistic speeds (i.e. much slower than the speed of light)
  - Projectiles and collisions involving large masses

Worked Example

Calculate the kinetic energy, in MeV, of an alpha particle which has a momentum of 1.1×10^{-19} kg m s^{-1}.

Use the following data:

- Mass of a proton = 1.67×10^{-27} kg
- Mass of a neutron = 1.67×10^{-27} kg

Step 1: Write the energy-momentum relation

- The energy-momentum relation is given by $E_k = \dfrac{p^2}{2m}$

Step 2: Determine the mass of an alpha particle

- An alpha particle is comprised of two protons and two neutrons
- Therefore, the mass of an alpha particle $m_\alpha = 2m_p + 2m_n$, where m_p and m_n is the mass of a proton and neutron respectively
- So $m_\alpha = 2(1.67 \times 10^{-27}) + 2(1.67 \times 10^{-27}) = 6.68 \times 10^{-27}$ kg

Step 3: Substitute the momentum and the mass of the alpha particle into the energy-momentum relation

$$E_k = \frac{p^2}{2m}$$

$$E_k = \frac{(1.1 \times 10^{-19})^2}{2 \times (6.68 \times 10^{-27})} = 9.1 \times 10^{-13} \text{ J}$$

Step 4: Convert the value of kinetic energy from J to MeV

$$1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

- Therefore:

$$9.1 \times 10^{-13} \text{ J} = \frac{9.1 \times 10^{-13}}{1.6 \times 10^{-13}} \text{ MeV} = 5.7 \text{ MeV}$$

Exam Tip

Calculations with the energy-momentum equation often require changing units, especially between eV and J due to it commonly being used for particles. Remember that 1 eV = 1.60×10^{-19} J. Therefore

$$eV \rightarrow J = \times (1.60 \times 10^{-19})$$

$$J \rightarrow eV = \div (1.60 \times 10^{-19})$$

The prefix 'mega' (M) means $\times 10^6$ therefore, 1 MeV = $(1.60 \times 10^{-19}) \times 10^6 = 1.60 \times 10^{-13}$

Circular Motion

6.7 Radians & Angular Displacement

Radians

- A **radian** (rad) is defined as:

  The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

- Radians are used whenever describing the angular displacement of objects in circular motion
- Angular displacement can be calculated using the equation:

$$\Delta\theta = \frac{\Delta S}{r}$$

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - s = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)

- Radians are commonly written in terms of π
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{\text{circumference of circle}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$

Radian Conversions

- If an angle of 360° = 2π radians, then 1 radian in degrees is equal to:

$$\frac{360}{2\pi} = \frac{180}{\pi} \approx 57.3^{\circ}$$

- Use the following equation to convert from degrees to radians:

$$\theta^{\circ} \times \frac{\pi}{180} = \theta \text{ rad}$$

Table of common degrees to radians conversions

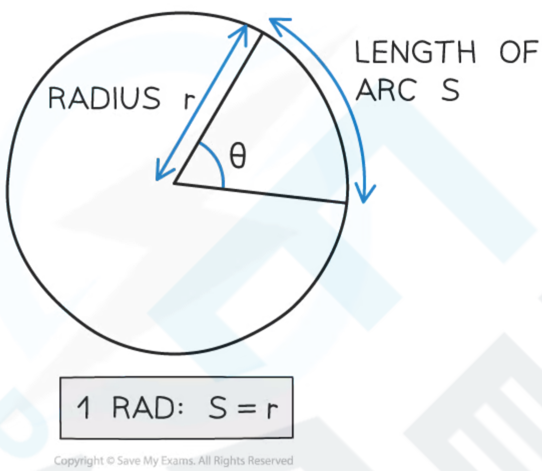
Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

Angular Displacement

- The **angular displacement** $\Delta\theta$ is the ratio of:

$$\Delta\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}}$$

- Angular displacement describes the **change in angle**, in **radians**, of a body as it moves in a circle
  - This angle is measured with respect to the centre of orbit



When the angle is equal to one radian, the length of the arc (Δs) is equal to the radius (r) of the circle

> **Exam Tip**
>
> Since the equation for angular displacement gives the angle in **radians**, make sure you're comfortable with then converting to degrees if you need to for the question!

6.8 Angular Velocity

Angular Velocity

- The **angular velocity** ω of a body in circular motion is defined as:

   The rate of change of angular displacement

- In other words, angular velocity is the angle swept out by an object in circular motion, per second
- Angular velocity is a **vector quantity** and is measured in rad s^{-1}
   - Since it is a vector, it has a magnitude (angular **speed**) and direction

- Angular velocity is calculated using:

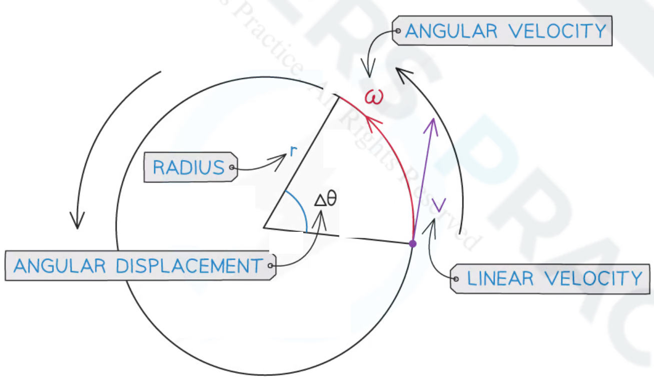
$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Where:
   - $\Delta\theta$ = change in angular displacement (radians)
   - Δt = time interval (s)

- It is related to linear speed, v by the equation

$$v = \omega r$$

- Where:
   - v = linear speed, v (m s^{-1})
   - ω = angular speed (rad s^{-1})
   - r = radius of orbit (m)



When an object is in uniform circular motion, velocity constantly changes direction, but the speed stays the same

- Taking the angular displacement of a complete orbit or revolution as 2π radians, the angular velocity ω an be calculated using the equation:

$$\omega = \frac{v}{r} = 2\pi f = \frac{2\pi}{T}$$

- Where:
  - T = the time period (s)
  - f = frequency (Hz)
- This equation shows that:
  - The greater the rotation angle θ in a given amount of time T, the greater the angular velocity ω
  - An object travelling with the same linear velocity, but further from the centre of orbit (larger r) moves with a smaller angular velocity (smaller ω)

Exam Tip

Try not to be confused by similar sounding terms like "angular velocity" and "angular speed". Just like in regular linear motion, you have linear velocity and linear speed: one is a scalar (speed) and the other is a vector (velocity).

In this worked example, the equation $v = r\omega$ is used to calculate the linear speed. This is fine, because v in this context is just the **magnitude** of the linear velocity (and similarly, ω is the magnitude of the angular velocity).

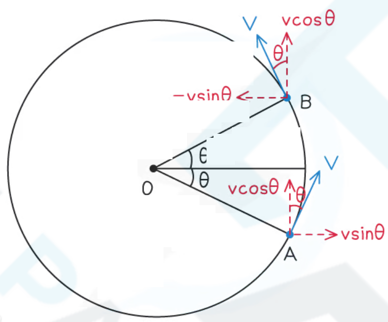
Finally, you may sometimes come across ω being labelled as 'angular frequency', because of its relationship to linear frequency f as given by the alternative equation $\omega = 2\pi f$. Remember, the units of ω are rad s^{-1}, whereas the units of f are Hz.

Deriving Equations for Centripetal Acceleration

- An object moving in **uniform circular motion** travels with a constant **angular velocity** and **angular speed**
- However, its **direction** is always changing
  - Therefore, its **linear velocity** changes, so it must be **accelerating**
  - This is called a **centripetal acceleration**

- An object in circular motion is thus always accelerating
- This acceleration is called 'centripetal' because it is directed toward the **centre of orbit**

- To derive an equation for the magnitude of centripetal acceleration, consider an object in uniform circular motion between point A and B on a circle, as shown below:



An object in uniform circular motion is accelerating toward the centre of orbit, O. Between A and B, the horizontal component of motion changes from v sinθ to –v sinθ

- At A and B, by resolving the horizontal and vertical components of linear velocity *v*, it can be seen that:
  - Initial vertical component of *v* = final vertical component of *v* which is *v* cosθ
  - Initial horizontal component of *v* is *v* sinθ
  - Final horizontal component of *v* is –*v* sinθ

- This means the acceleration of the object is only horizontal, given by:

$$a = \frac{\Delta v}{\Delta t} = \frac{(-v\sin\theta) - (v\sin\theta)}{t} = \frac{-2v\sin\theta}{t}$$

- Recalling the equations for angular velocity $\omega = \theta / t$ and $v = r\omega$, then:

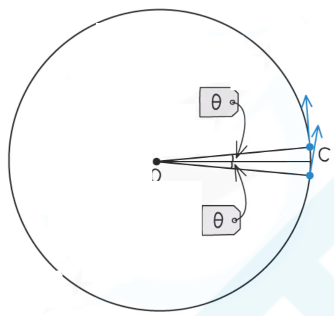
$$t = \frac{\theta}{\omega} = \frac{\theta}{\left(\frac{v}{r}\right)} = \frac{r\theta}{v}$$

- The object's angular displacement is actually 2θ, therefore, the time *t* is given by $t = \dfrac{2r\theta}{v}$

- Therefore, substituting this into the equation for acceleration gives:

$$a = \frac{-2v\sin\theta}{\left(\dfrac{2r\theta}{v}\right)} = \frac{-v^2\sin\theta}{r\theta}$$

- This equation is the acceleration of the object between points A and B
  - To find the **instantaneous acceleration** at an exact point on the circle, say point C, reduce the size of the angular displacement θ so it becomes infinitesimally small
  - This is shown in the image below:



By taking the limit of angular displacement as zero, we can derive an equation for the instantaneous centripetal acceleration of the object at point C

- In the limit $\theta \rightarrow 0$ radians
  - The value of sin θ is approximately equal to θ
  - Therefore, $\dfrac{\sin\,\theta}{\theta} \approx 1$ (for very small angles)
  - This is known as the **small angle approximation**
- Therefore, the **instantaneous acceleration** is the centripetal acceleration:

$$a = -\frac{v^2}{r} = -r\omega^2$$

- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear velocity (m s^{-1})
  - r = radius of orbit (m)
  - ω = angular velocity (rad s^{-1})
- The negative sign indicates that the centripetal acceleration is directed toward the **centre of orbit**

Exam Tip

This seems like a complicated derivation, but there is no maths in there that you haven't been introduced to already. It is important you know how to use the **vector diagrams** to reach the final equations for angular accelerations, understanding every step along the way. Try and do it without the notes to help memorise and see how far you get!

Using Equations for Centripetal Acceleration

- Centripetal acceleration is defined as:

  The acceleration of an object towards the centre of a circle when an object is in motion (rotating) around a circle at a constant speed

- Its magnitude is calculated using the radius *r* and linear speed *v*:

$$a = \frac{v^2}{r}$$

- Using the equation relating angular speed ω and linear speed *v*:

$$v = r\omega$$

- These equations can be combined to give another form of the centripetal acceleration equation:

$$a = \frac{(r\omega)^2}{r}$$

$$a = r\omega^2$$

- This equation shows that centripetal acceleration is equal to the radius times the square of the angular speed
- Alternatively, rearrange for *r*:

$$r = \frac{v}{\omega}$$

- This equation can be combined with the first one to give us another form of the centripetal acceleration equation:

$$a = \frac{v^2}{\left(\frac{v}{\omega}\right)}$$

$$a = v\omega$$

- This equation shows how the centripetal acceleration relates to the linear speed and the angular speed

- Where:
  - a = centripetal acceleration ($m\ s^{-2}$)
  - v = linear speed ($m\ s^{-1}$)
  - ω = angular speed ($rad\ s^{-1}$)
  - r = radius of the orbit (m)

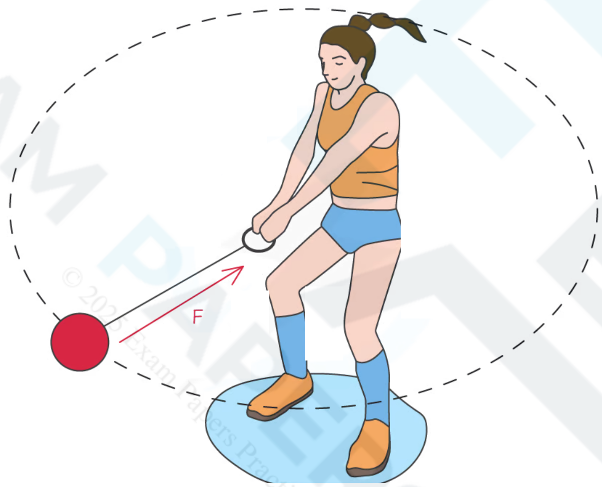
Exam Tip

Make sure you understand both the derivation and how to use the equation for centripetal acceleration. The most crucial step is to remember the **small angle approximation,** that sin θ is approximately equal to θ when the angle is very very small. Try this in your calculator (in radians!) and see for yourself!

Maintaining Circular Motion

- An object moving in a circle is not in equilibrium, it is constantly changing direction
  - Therefore, in order to produce circular motion, an object requires a **resultant force** to act on it
  - This resultant force is known as the **centripetal force** and is what keeps an object moving in a circle
- The centripetal force *F* is defined as:

  The resultant force towards the centre of the circle required to keep a body in uniform circular motion. It is always directed towards the centre of the body's rotation.



The tension in the string provides the centripetal force F to keep the hammer in circular orbit

- **Note:** centripetal force and centripetal acceleration act in the **same direction**
  - This is due to Newton's Second Law
- The centripetal force is **not** a separate force of its own
  - It can be any type of force, depending on the situation, which keeps an object moving in a circular path

Examples of centripetal force

Situation	Centripetal Force
Car travelling around a roundabout	Friction between car tires and the road
Ball attached to a rope moving in a circle	Tension in the Rope
Earth orbiting the Sun	Gravitational Force

Exam Tip

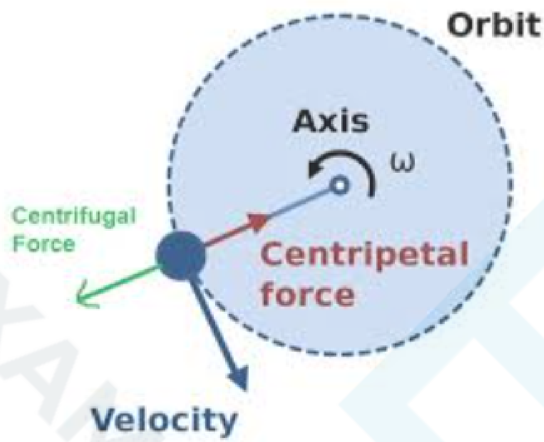
Make sure you are able to give examples of centripetal forces, understanding that many types of familiar forces (e.g., gravity, electric) can act as centripetal forces.

A classic example that often comes up in your magnetic fields topic is the magnetic force on a charged particle, which is always centripetal. This is because the force acts at 90° to the charged particle's velocity, causing it to move in a circle.

Centripetal Force

- Centripetal force can be calculated using any of the following equations:

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$



Centripetal force is always perpendicular to the direction of travel

- Where:
  - F = centripetal force (N)
  - v = linear velocity (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)
- The centripetal force is the **resultant** force on the object moving in a circle
  - This is particular important if there are multiple forces on the object, such as weight