A Level Physics Edexcel

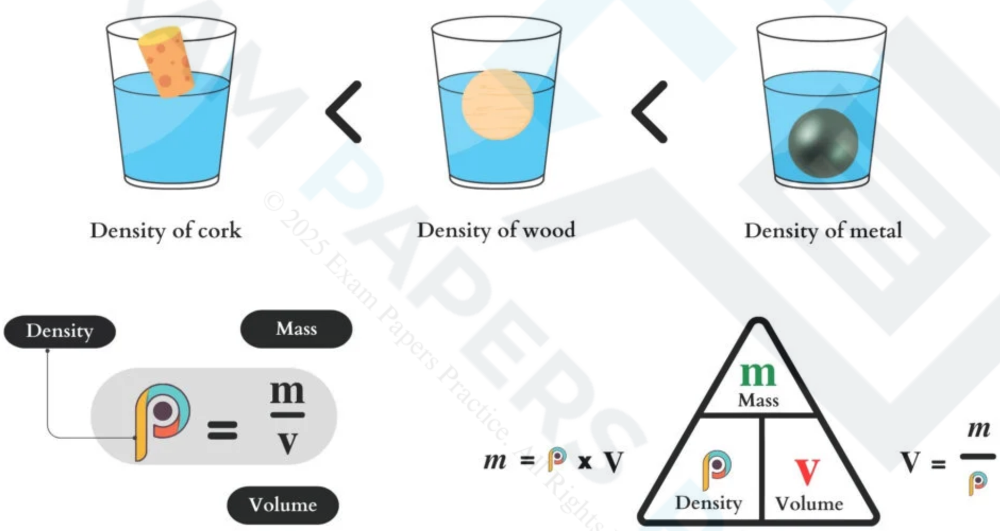
4. Materials

CONTENTS

Density

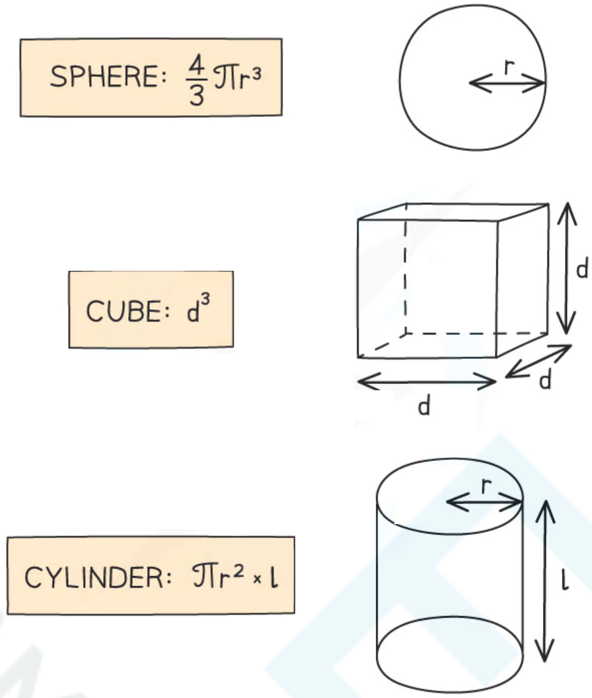
- Density is the **mass per unit volume** of an object
  - Objects made from low-density materials typically have a lower mass
  - For example, a balloon is less dense than a small bar of lead despite occupying a larger volume

- The units of density depend on the units used for mass and volume:
  - If the mass is measured in *g* and volume in **cm^3**, then the density will be in **g / cm^3**
  - If the mass is measured in *kg* and volume in **m^3**, then the density will be in **kg / m^3**



Gases are less dense than a solid

- The volume of an object may not always be given directly, but can be calculated with the appropriate equation depending on the object's shape



Volumes of common 3D shapes

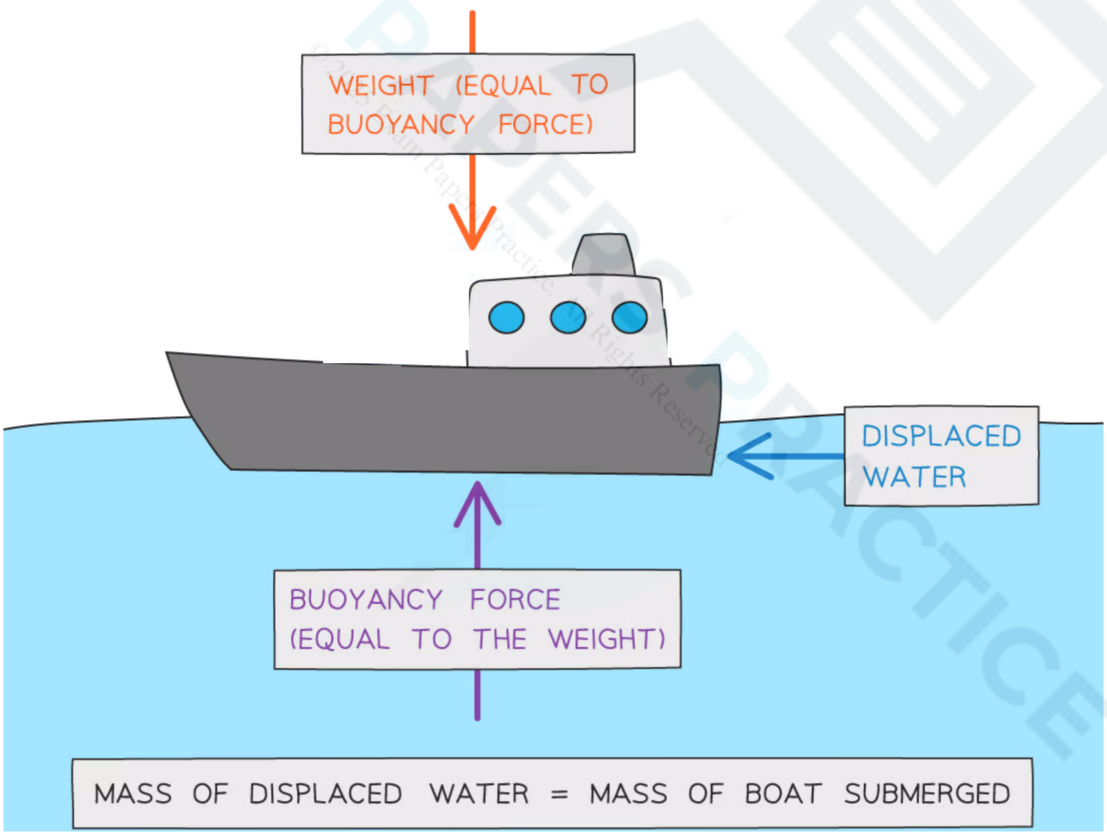
Upthrust

Archimedes' Principle

- Archimedes' principle states:

  An object submerged in a fluid at rest has an upward buoyancy force (upthrust) equal to the weight of the fluid displaced by the object

- The object sinks until the weight of the fluid displaced is equal to its own weight
  - Therefore the object floats when the magnitude of the upthrust equals the weight of the object
- The magnitude of upthrust can be calculated in steps by:
  - Find the volume of the submerged object, which is also the volume of the displaced fluid
  - Find the weight of the displaced fluid
  - Since $m = \rho V$ (density × volume), upthrust is equal to $F = mg$ which is the weight of the fluid displaced by the object
- Archimedes' Principle explains how ships float:



Boats float because they displace an amount of water that is equal to their weight

Exam Tip

Don't get confused by the two step process to find upthrust.

- Step 1: You need the volume of the submerged object, but **only** because you want to know **how much fluid was displaced**
- Step 2: What you **really** want to know is the **weight of the displaced fluid**.

A couple of familiar equations will help;

- $m = \rho V$ to get mass (and that's the V from step 1 out of the way),

then

- $W = mg$ to get weight

If you are feeling particularly mathematical, you can combine your equations, so that $W = \rho V g$

Stoke's Law

Viscous Drag

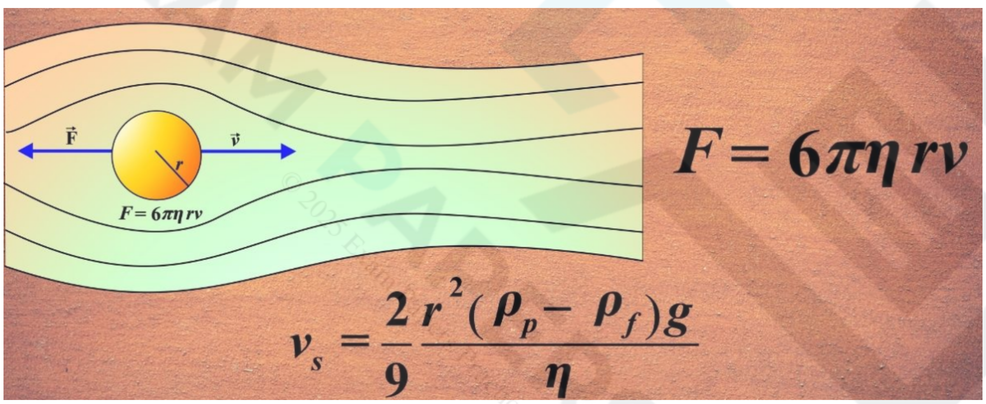
- Viscous drag is defined as

the frictional force between an object and a fluid which opposes the the motion between the object and the fluid

- Viscous drag is calculated using Stoke's Law;

$$F = 6\pi\eta rv$$

- Where
  - F = viscous drag (N)
  - η = coefficient of viscosity of the fluid (N s m^{-2} or Pa s)
  - r = radius of the object (m)
  - v = velocity of the object (ms^{-1})



- The viscosity of a fluid can be thought of as its thickness, or how much it resists flowing
  - Fluids with **low viscosity** are **easy** to pour, while those with **high viscosity** are **difficult** to pour

- The **coefficient of viscosity** is a property of the fluid (at a given temperature) that indicates how much it will resist flow
  - **The rate of flow** of a fluid is inversely proportional to the coefficient of viscosity

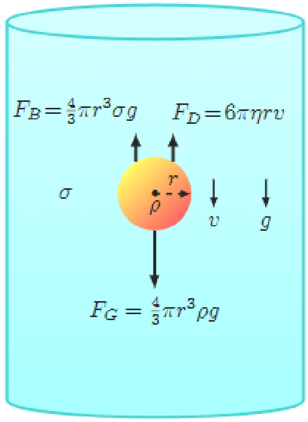
Drag Force at Terminal Velocity

- Terminal velocity is useful when working with Stoke's Law since at terminal velocity the forces in each direction are balanced

$$W_s = F_d + U \text{ (equation 1)}$$

- Where;

- W_s = weight of the sphere
- F_d = the drag force (N)
- U = upthrust (N)



At terminal velocity forces are balanced: W (downwards) = Fd + U (upwards)

- The weight of the sphere is found using volume, density and gravitational force

$$W_s = v_s \rho_s g$$

$$W_s = \frac{4}{3}\pi r^3 \rho_s g \text{ (equation 2)}$$

- Where
  - v_s = volume of the sphere (m^3)
  - ρ_s = density of the sphere ($kg\ m^{-3}$)
  - g = gravitational force ($N\ kg^{-1}$)
- Recall Stoke's Law

$$F_d = 6\pi\eta r v_{term} \text{ (equation 3)}$$

- Upthrust equals weight of the displaced fluid
  - The **volume of displaced fluid is the same as the volume of the sphere**
  - The weight of the fluid is found from volume, density and gravitational force as above

$$U = \frac{4}{3}\pi r^3 \rho_f g \text{ (equation 4)}$$

- Substitute equations 2, 3 and 4 into equation 1

$$\frac{4}{3}\pi r^3 \rho_s g = 6\pi\eta r v_{term} + \frac{4}{3}\pi r^3 \rho_f g$$

- Rearrange to make terminal velocity the subject of the equation

$$v_{term} = \frac{\frac{4}{3}\pi r^3 g(\rho_s - \rho_f)}{6\pi\eta r} = \frac{4\pi r^3 g(\rho_s - \rho_f)}{18\pi\eta r}$$

- Finally, cancel out *r* from the top and bottom to find an expression for **terminal velocity** in terms of the **radius of the sphere** and the **coefficient of viscosity**

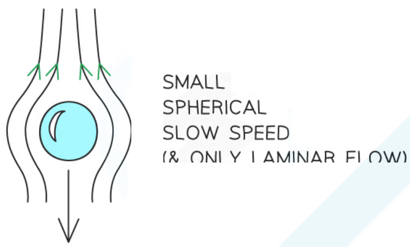
$$v_{term} = \frac{2\pi r^2 g(\rho_s - \rho_f)}{9\pi\eta}$$

- This final equation shows that terminal velocity is;
  - directly proportional to the square of the radius of the sphere
  - inversely proportional to the viscosity of the fluid

Understanding Viscosity & Stoke's Law

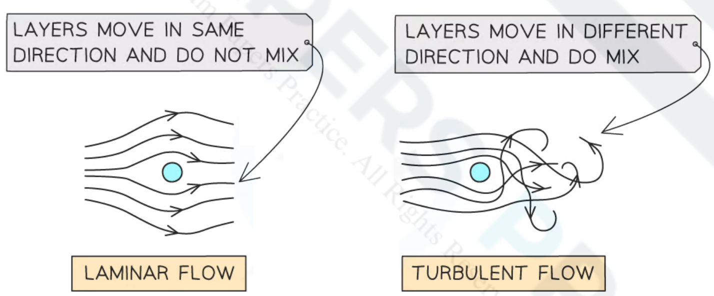
Conditions for Stoke's Law Equation

- The equation can only be used when certain conditions are met;
  - The flow is laminar
  - The object is small
  - The object is spherical
  - Motion between the sphere and the fluid is at a slow speed



Laminar and Turbulent Flow

- As an object moves through a fluid, or a fluid moves around an object, layers in the fluid are created
- In laminar flow all the layers are moving in the same direction and they do not mix
- This tends to happen for slow moving objects, or slow flowing liquids
- The equation above only applies for laminar flow
- In turbulent flow the layers move in different directions and the layers do mix



Changing Viscosity

- Viscosity is temperature-dependent
  - **Liquids** are **less** viscous as temperature increases
  - **Gases** get **more** viscous as temperature increases

Worked Example

A ball bearing of radius 5.0 mm falls at a constant speed of 0.030 ms^{-1} through a oil which has viscosity 0.3 Pa s and density 900 kg m^{-3}.

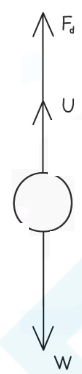
Determine the viscous drag acting on the ball bearing.

Step 1: List the known quantities in SI units

- Radius of the sphere, $r_s = 5.0 \text{ mm} = 5.0 \times 10^{-3}$ m
- Terminal velocity of the sphere, $v = 0.03$ m s^{-1}
- Viscosity of oil, $\eta = 0.3$ Pa s
- Density of oil, $\rho_f = 900$ kg m^{-3}

Step 2: Sketch a free-body diagram to resolve the forces at constant speed

$$W_s = F_d + U$$



Step 3: Calculate the value for viscous drag, F_d

$$F_d = 6\pi\eta rv = 6 \times \pi \times 0.3 \times 5.0 \times 10^{-3} \times 0.03 = 0.008482$$

Step 4: Write the complete answer to the correct significant figures and include units

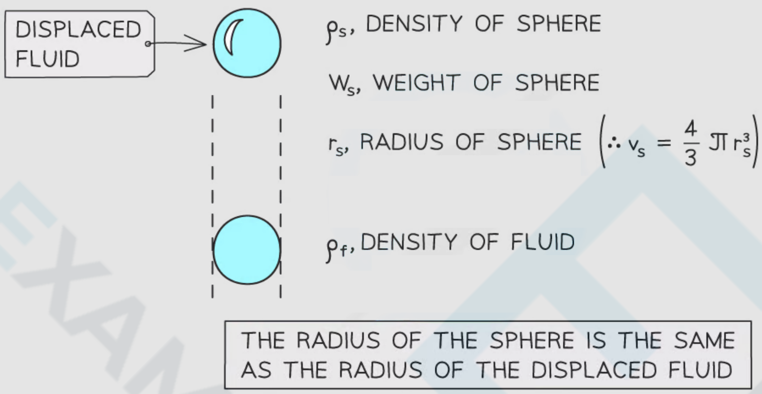
- The viscous drag, $F_d = 8.5 \times 10^{-4}$ N

Exam Tip

You may need to write out some or all of the derivation given in the first part above.

It is really important to keep clear whether you are talking about density of the sphere or the fluid, and mass of the sphere or the fluid.

Practice using subscripts and do try this at home. It isn't one to do for the first time in an exam!



Core Practical 4: Investigating Viscosity of a Liquid

Aim of the Experiment

- By allowing small spherical objects of known weight to fall through a fluid until they reach terminal velocity, the **viscosity of the fluid** can be calculated

Variables

- Independent variable: weight of ball bearing, W_s
- Dependent variable: terminal velocity, v_{term}
- Control variables:
  - fluid being tested,
  - temperature

Equipment List

- Long measuring cylinder
- Viscous liquid to be tested (thin oil of known density or washing up liquid)
- Stand and clamp
- Metre rule
- Rubber bands
- Steel ball bearings of different weights
- Digital scales
- Vernier calipers
- Digital stopwatch
- Magnet

Method

1. Weigh the balls, measure their radius using Vernier callipers and calculate their density
2. Place three rubber bands around the tube. The highest should be far enough below the surface of the liquid to ensure the ball is travelling at terminal velocity when it reaches this band. The remaining two bands should be 10 – 15 cm apart so that time can be measured accurately
3. Release the ball and wait until it reaches the first rubber band. Start the timer at the first band, then use the lap timer to find the time to fall d_1 and also d_2
   i. If lap timing is not available, two stopwatches operated by different people should be used
   ii. If the ball is still accelerating as it passes the markers, they need to be moved downwards until the ball has reached terminal velocity before passing the first mark

4. Measure and record the distances d_1 (between the highest and middle rubber band) and d_2 between the highest and lowest bands.
5. Repeat at least three times for balls of this diameter and three times for each different diameter
6. Ball bearings are removed from the bottom of the tube using the magnet against the outside wall of the measuring cylinder

Table of Results:

MASS OF BALL BEARING, m_s = ________kg

∴ WEIGHT OF BALL BEARING, $W_s = m_s g$ = ________N

DENSITY FLUID, ρ_f = ________kgm^{-3}

d_1/m	d_2/m	t_1/s	t_2/s	t_3/s	t_{av}/s	V_{term}/ms^{-1}

Analysis

- Terminal velocity is used in this investigation since at terminal velocity the forces in each direction are balanced

$$W_s = F_d + U \text{ (equation 1)}$$

- Where;
  - W_s = weight of the sphere
  - F_d = the drag force (N)
  - U = upthrust (N)
- The weight of the sphere is found using volume, density and gravitational force

$$W_s = v_s \rho_s g$$

$$W_s = \frac{4}{3}\pi r^3 \rho_s g \text{ (equation 2)}$$

- Where
  - v_S = volume of the sphere (m^3)
  - ρ_S = density of the sphere ($kg\ m^{-3}$)
  - g = gravitational force ($N\ kg^{-1}$)
- Recall Stoke's Law

$$F_d = 6\pi\eta r v_{term} \text{ (equation 3)}$$

- Upthrust equals the weight of the displaced fluid
  - The **volume of displaced fluid is the same as the volume of the sphere**
  - The weight of the fluid is found from volume, density and gravitational force as above

$$U = \frac{4}{3}\pi r^3 \rho_f g \text{ (equation 4)}$$

- Substitute equations 2, 3 and 4 into equation 1

$$\frac{4}{3}\pi r^3 \rho_s g = 6\pi\eta r v_{term} + \frac{4}{3}\pi r^3 \rho_f g$$

- Rearrange to make viscosity the subject of the equation

$$\frac{4}{3}\pi r^3 \rho_s g - \frac{4}{3}\pi r^3 \rho_f g = 6\pi\eta r v_{term}$$

$$\frac{4\pi r^3 g(\rho_s - \rho_f)}{3 \times (6\pi r v_{term})} = \eta$$

$$\eta = \frac{2r^2 g(\rho_s - \rho_f)}{9v_{term}}$$

Evaluating the Experiment

Systematic Errors:

- Ruler must be clamped vertically and close to the tube to avoid parallax errors in measurement
- Ball bearing must reach terminal velocity before the first marker

Random errors:

- Cylinder must have a large diameter compared to the ball bearing to avoid the possibility of turbulent flow
- Ball must fall in the centre of the tube to avoid pressure differences caused by being too close to the wall which will affect the velocity

Safety Considerations

- Measuring cylinders are not stable and should be clamped into position at the top and bottom
- Spillages will be slippery and must be cleaned up immediately
- Avoid getting fluids in the eyes

Stretching Materials

4.5 Hooke's Law

Hooke's Law

- When a force F is added to the bottom of a vertical metal wire of length L, the wire **stretches**
- A material obeys Hooke's Law if:

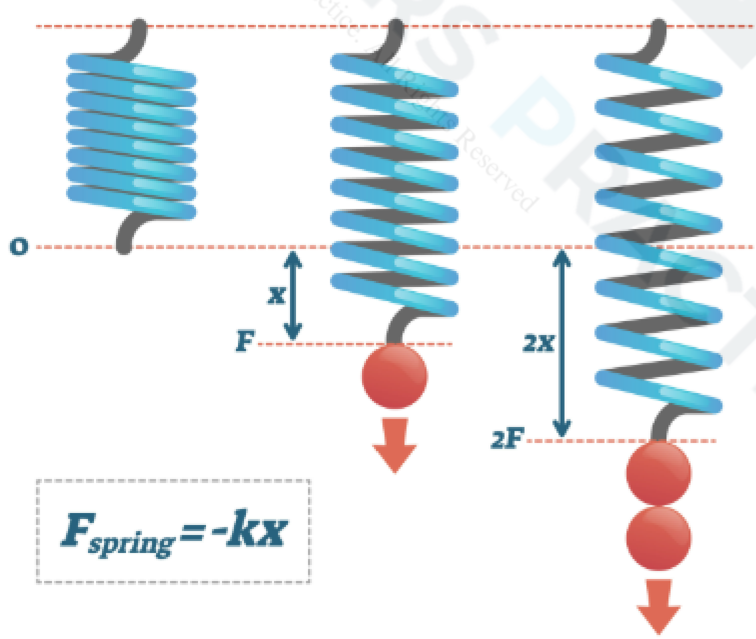
> **The extension of the material is directly proportional to the applied force (load) up to the limit of proportionality**

- This linear relationship is represented by the Hooke's law equation:

$$\Delta F = k\Delta x$$

- Where:
  - F = applied force (N)
  - k = spring constant (N m^{-1})
  - Δx = extension (m)

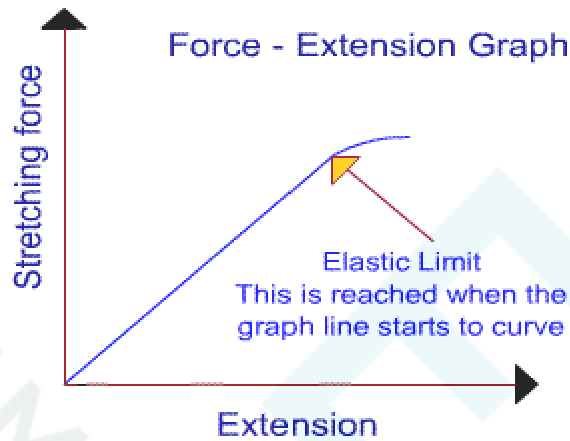
- The spring constant is a property of the material being stretched and measures the **stiffness** of a material
  - The larger the spring constant, the stiffer the material
- Hooke's Law applies to both **extensions** and **compressions**:
  - The extension of an object is determined by how much it has **increased** in length
  - The compression of an object is determined by how much it has **decreased** in length



Stretching a spring with a load produces a force that leads to an extension

Force-Extension Graphs

- The way a material responds to a given force can be shown on a force-extension graph
- A material may obey Hooke's Law up to a point
  - This is shown on its force-extension graph by a **straight line through the origin**
- As more force is added, the graph may start to curve slightly



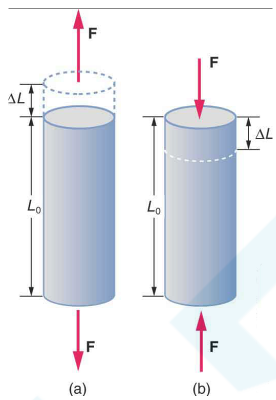
The Hooke's Law region of a force-extension graph is a straight line. The spring constant is the gradient of that region

- The key features of the graph are:
  - **The limit of proportionality:** The point beyond which Hooke's law is no longer true when stretching a material i.e. the extension is no longer proportional to the applied force
    - The point is identified on the graph where the line starts to curve (flattens out)
  - **Elastic limit:** The maximum amount a material can be stretched and still return to its original length (above which the material will no longer be elastic). This point is always **after** the limit of proportionality
    - The **gradient** of this graph is equal to the **spring constant** ***k***

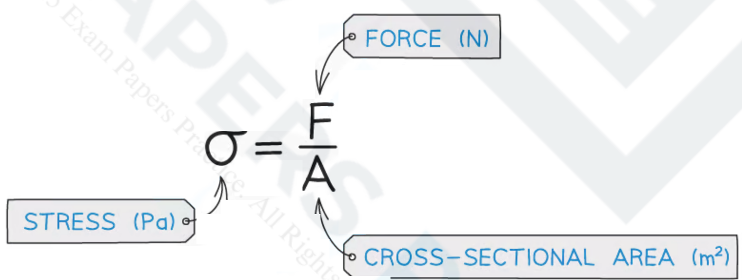
Stress & Strain

Stress

- Stress is the **applied force per unit cross sectional area** of a material
- Forces can be;
  - Tensile forces, which pull on an object and extend it
  - Compressive forces, which push onto an object and compress (or squash) it



- The equation for stress is the force per unit area, and so the units are N m^{-2}, or Pascals, the same unit as pressure

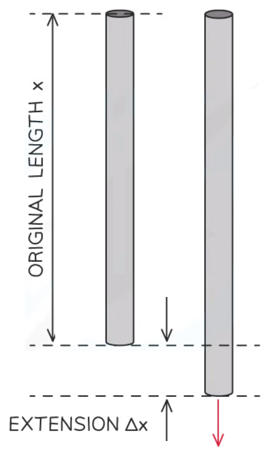


Stress equation

- The **ultimate tensile stress** is the **maximum** force per original cross-sectional area a wire is able to support until it breaks

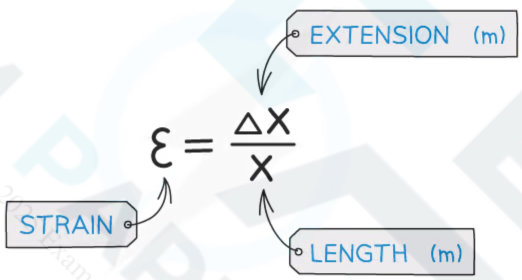
Strain

- Strain is the **extension per unit length**



Strain is the ratio of the extension (or compression) and the original length

- This is a deformation of a solid due to stress in the form of elongation or contraction
- Note that strain is a **dimensionless** unit because it's the ratio of lengths



Strain equation

The Young Modulus

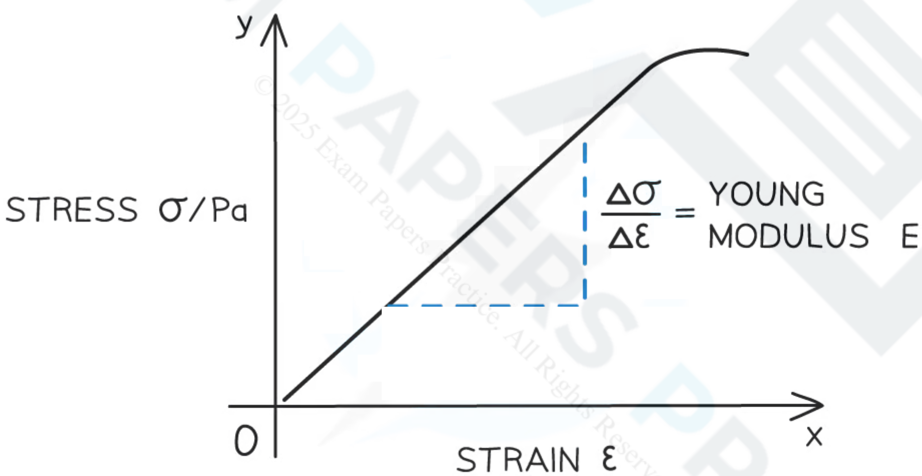
Young Modulus

- The Young modulus (sometimes called Young's Modulus) is the measure of the ability of a material to withstand changes in length with an added load ie. how stiff a material is
- This gives information about the elasticity of a material
- The Young Modulus is defined as the **ratio of stress and strain**

$$\text{YOUNG MODULUS } E \text{ (Pa)} = \frac{\text{STRESS } \sigma}{\text{STRAIN } \varepsilon} = \frac{Fx}{A\Delta x}$$

Young Modulus equation

- Its unit is the same as stress: **Pa** (since strain is unitless)
- Just like the Force-Extension graph, stress and strain are directly proportional to one another for a material exhibiting elastic behaviour



A stress-strain graph is a straight line with its gradient equal to Young modulus

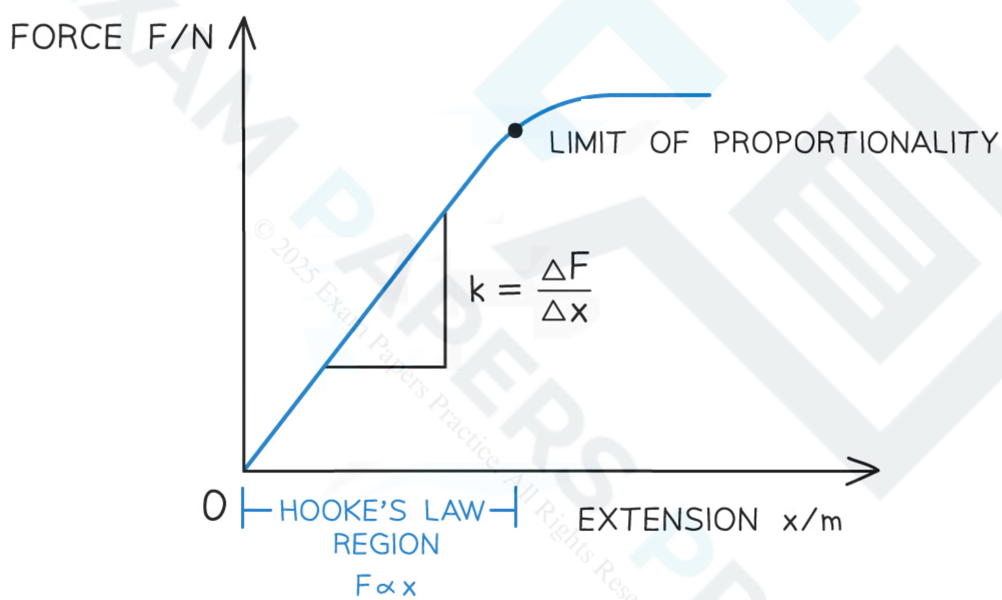
- The **gradient** of a stress-stress graph when it is linear is the **Young Modulus**

Force-Extension Graphs

- The way a material responds to a tensile or compressive force can be shown on a force-extension, or a force-compression graph
  - Although compression can be put into **equations** as a **negative** value, the graphs have the same shaped curves
  - Compression is plotted on the **graph** as a **positive**, increasing value

- Every material will have a unique force-extension graph depending on how **brittle** or **ductile** it is
- In the same way, materials have unique force-compression graphs, which will not be the same as their force-extension graph
  - This is because materials behave differently under tensile and compressive strain

Simple Force-Extension Graphs



Simple force-extension graph showing the Hooke's Law region, and the calculation to find k, the spring constant

- A material may obey Hooke's Law up to a point
  - This is shown on its force-extension graph by a **straight line through the origin**
- As more force is added, the graph may start to curve slightly

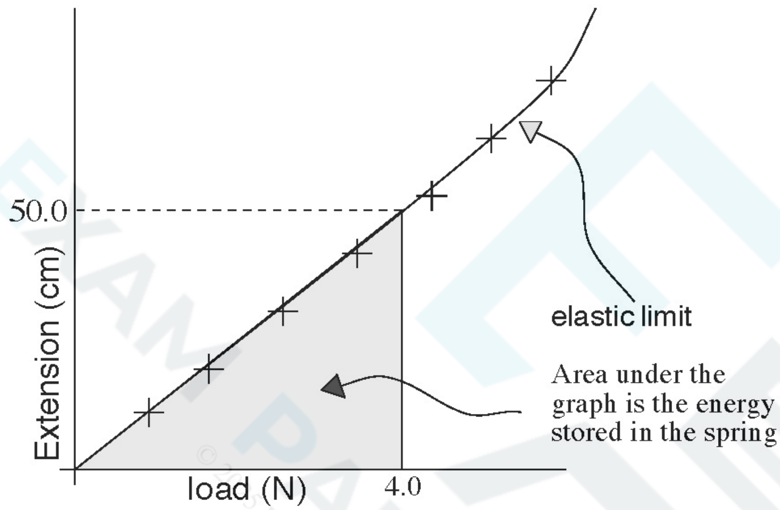
The key features of the graph are:

- **The limit of proportionality**
  - The point **beyond** which Hooke's law is no longer true when stretching or compressing a material i.e. the extension/ compression is no longer proportional to the applied force
  - The point is identified on the graph where the line starts to curve

- **The elastic limit**
  - The point **before** which a material will **return to its original length or shape** when the deforming force is removed
  - This point is always **after** the limit of proportionality
- **The spring constant *k*** is found from the **gradient** of the straight part of the graph

More Detailed Force-Extension Graphs

- Graphs of applied load-extension can give more detailed information about materials
  - This will apply when loads were continued well past the elastic limit



Detailed force-extension graph showing a material under loads which exceed the elastic limit

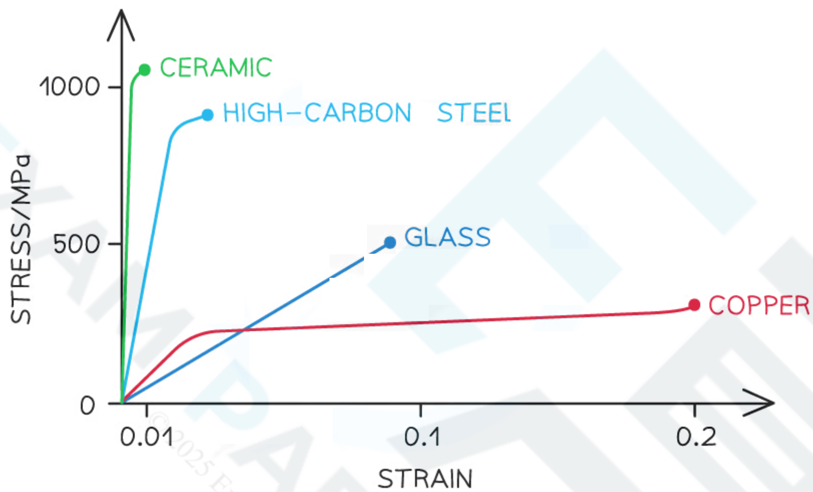
- **The yield point** is where the material continues to stretch even though no extra force is being applied to it
- **Elastic deformation** is a change of shape where the material **will return to its original shape** when the load is removed
- **Plastic deformation** occurs after the yield point
  - It is a change of shape where the material will **not return to its original shape** when the load is removed

YOUR NOTES
↓

Stress-Strain Graphs

- Stress-strain curves give an indication of the properties of materials such as
  - Up to what stress and strain they obey Hooke's Law
  - Whether they exhibit elastic and/or plastic behaviour
  - The value of their Young Modulus
  - The value of their **breaking stress**

- Each material has a unique stress-strain curve

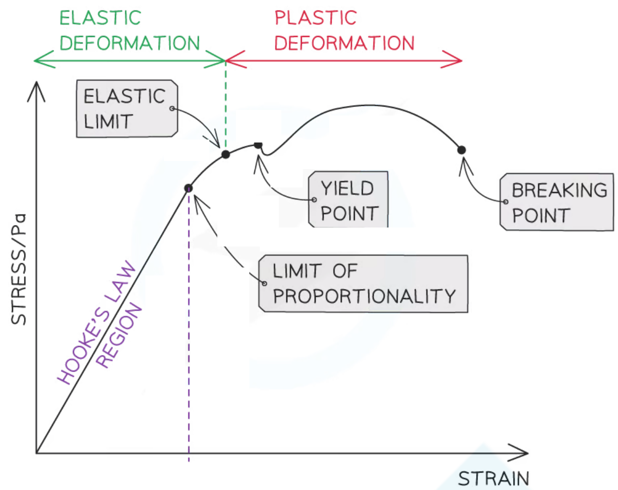


Stress-strain graph for different materials up to their breaking stress

Comparing Force-Extension to Stress-Strain Graphs

The key features of the graph which are also on the force-extension graph are:

- **Limit of proportionality**, beyond which Hooke's law no longer applies
- **The elastic limit**, before which a material returns to its original length or shape when the deforming force is removed
- The **yield point** beyond which the material continues to stretch (more strain is seen) even though no extra force is being applied to it (without additional stress)
- **Elastic deformation** where the material **will return to its original shape** when the load is removed
- **Plastic deformation** where the material will **not return to its original shape** when the load is removed



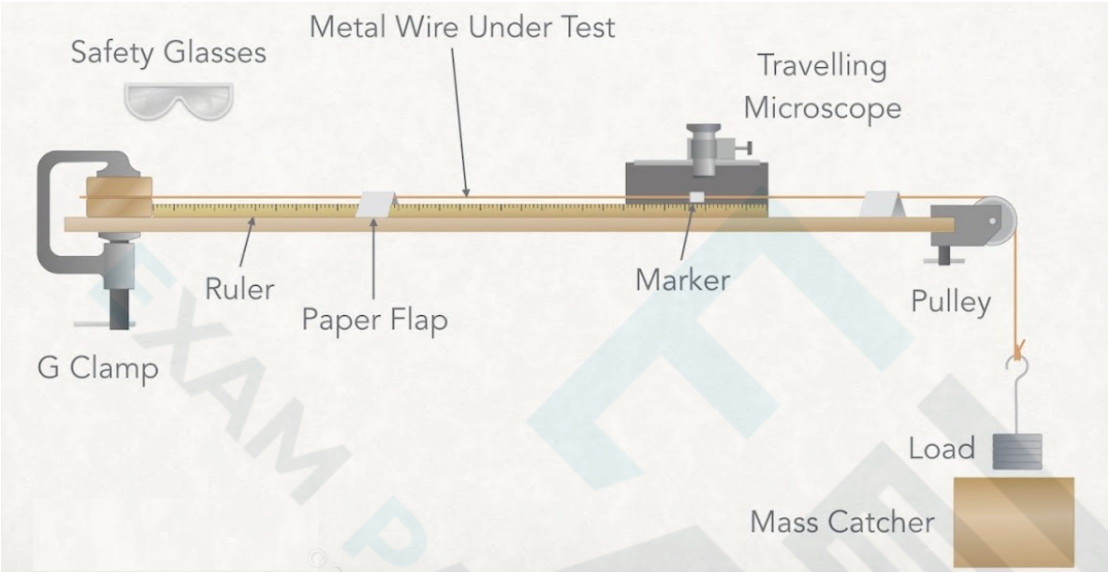
The important points shown on a stress-strain graph

The stress-strain graph is also used to find;

- The **Young Modulus** is found from the gradient of the straight part of the graph
- **Breaking stress** (also called fracture stress) is the stress at the point where the material breaks
    - At the yield point the atoms in the material had started to move relative to each other, at the breaking stress they separate completely
    - Breaking stress is not the same as ultimate tensile stress which is marked on many graphs

Core Practical 5: Investigating Young Modulus of a Material

- To measure the Young Modulus of a metal in the form of a wire requires a clamped horizontal wire over a pulley (or vertical wire attached to the ceiling with a mass attached) as shown in the diagram below



- A reference marker is needed on the wire. This is used to accurately measure the extension with the applied load
- The independent variable is the **load**
- The dependent variable is the **extension**

Method

1. Measure the original length of the wire using a metre ruler and mark this reference point with tape
2. Measure the diameter of the wire with micrometer screw gauge or digital calipers
3. Measure or record the mass or weight used for the extension e.g. 300 g
4. Record initial reading on the ruler where the reference point is
5. Add mass and record the new scale reading from the metre ruler
6. Record final reading from the new position of the reference point on the ruler
7. Add another mass and repeat method

Reducing Uncertainty

- To reduce the uncertainty in the final answer, take the following precautions when measuring
  - Take pairs of readings of the diameter right angles to each other, to ensure the wire is circular

- Six to ten readings altogether is enough to get an average value
- Remove the load and check the wire returns to the original limit after each reading. A little 'creep' is acceptable but a large amount indicates that the elastic limit has been exceeded
- Take several readings with different loads and find average
- Use a Vernier scale to measure the extension of the wire

Measurements to Determine the Young Modulus

1. Determine extension *x* from final and initial readings

Example table of results:

Mass m / g	Load F / N	Initial Length / mm	Final Length / mm	Extension x (x10 -3) / m
200	2.0	500	500.1	0.1
300	2.9	500.1	500.4	0.3
400	3.9	500.4	501.0	0.6
500	4.9	501.0	501.9	0.9
600	5.9	501.9	503.2	1.3
700	6.9	503.2	504.9	1.7
800	7.8	504.9	507.0	2.1

Table with additional data

Length L / m	1.382
Diameter 1 / mm	0.277
Diameter 2 / mm	0.280
Diameter 3 / mm	0.275
Average Diameter d / mm	0.277
Cross Sectional area A / m2	6.03 x 10 -8

2. Plot a graph of force against extension and draw line of best fit

3. Determine gradient of the force v extension graph

4. Calculate cross-sectional area from:

DIAMETER OF THE WIRE (m)

$$\text{CROSS-SECTIONAL AREA } A = \frac{\pi d^2}{4}$$

5. Calculate the Young modulus from

FORCE / LOAD (N)

LENGTH OF WIRE (m)

$$\text{YOUNG'S MODULUS } E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{Fl}{Ax} = \text{GRADIENT} \times \frac{l}{A}$$

CROSS-SECTIONAL AREA (m^2)

EXTENSION (m)

Safety Considerations

- Safety glasses should be worn in case of the wire snapping
- Protect feet and the floor from falling weights by cushioning the area underneath the weights

Exam Tip

Although every care should be taken to make the experiment as reliable as possible, you will be expected to suggest improvements in producing more accurate and reliable results

Good examples of improvements in any experiment are:

- Take repeat readings and take an average to improve accuracy
- Measure longer distances, such as using a longer length of wire, to reduce percentage error

4.10 Elastic Strain Energy

Area Under a Force-Extension Graph

- For a material which obeys Hooke's law, the elastic strain energy, E_{el} can be determined by finding the area under the force-extension graph
  - Since this area will be a triangle with sides *F* (force) and *x* (extension) the equation is:

$$\Delta E_{el} = \frac{1}{2} F \Delta x$$

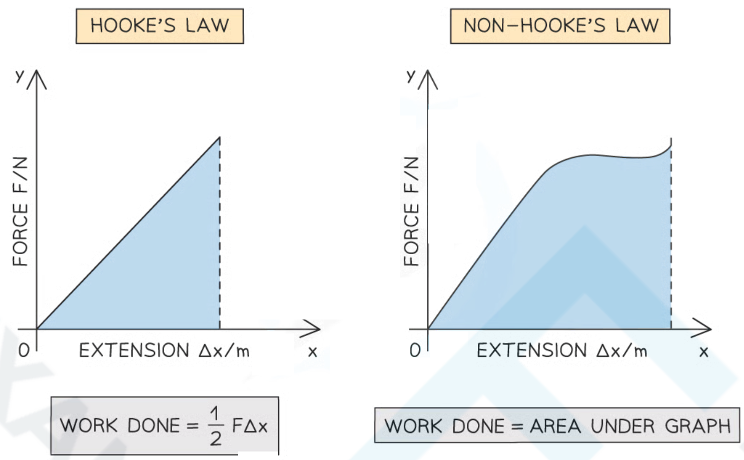
- Where:
  - E_{el} = elastic strain energy (or work done) (J)
  - *F* = average force (N)
  - Δx = extension (m)
- Since Hooke's Law states that ***F* = *k*Δ*x***, the elastic strain energy can also be written as:

$$\Delta E_{el} = \frac{1}{2} k (\Delta x)^2$$

- Where:
  - *k* = spring constant (N m^{-1})

Elastic Strain Energy

- Work has to be done to stretch a material
- Before a material reaches its elastic limit (whilst it obeys Hooke's Law), all the work is done is stored as **elastic strain energy**
- The work done, or the elastic strain energy is the **area under the force-extension graph**



Work done is the area under the force-extension graph

- This is true for whether the material obeys Hooke's law or not

Linear Graphs

- For the region where the material **obeys** Hooke's law, the work done is the area of a **right-angled triangle** under the graph

Non-linear Graphs

- For the region where the material **doesn't obey Hooke's law, the area is the full region** under the graph.
- To calculate this area, split the graph into separate segments and add up the individual areas of each
- For the remaining part, **count the squares** left over
  - Before adding squares to the total they must be converted using the values on the axes
  - For example, if each division on the y-axis = 0.1 N and each division on the x-axis = 0.2 m, then each square = 0.1 × 0.2 N m = 0.02 N m