A Level Physics Edexcel

2. Mechanics

CONTENTS

Equations of Motion

- The kinematic equations of motion are a set of four equations which can describe any object moving with **constant** acceleration
- They are often referred to as the 'suvat' equations due to the variables they include
- These are:
  - s = **displacement** (m)
  - u = **initial velocity** (m s^{-1})
  - v = **final velocity** (m s^{-1})
  - a = **acceleration** (m s^{-2})
  - t = **time interval** (s)
- The four SUVAT equations are:

SUMMARY

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{(v + u)}{2}t \qquad v^2 = u^2 + 2as$$

- These are all given on the data sheet
- All the variables, apart from **time t**, are vector quantities
  - This means they can either be positive or negative depending on their direction

Mastering the SUVAT Equations

- The SUVAT equations are **always** used when acceleration is constant, but not zero
- This is indicated in questions by
  - An explicit statement that an object is **'constantly accelerating'** or **'constantly decelerating'**, or words to that effect
  - A description of an object in freefall where either **'air resistance can be ignored'** or **'air resistance is negligible'**
  - A statement that an accelerating or decelerating force is constant, since $F = ma$, constant force → constant acceleration
- Questions may appear to be missing some information, this should be checked before starting to calculate
  - Objects in motion often start or end at rest, making either u or v = zero
  - Objects in freefall have a **constant** acceleration of 9.81 m s^{-2}
  - When an object is slowing down it has a **negative** value for acceleration, this must be included in the calculation
  - Objects thrown or shot upwards also have negative acceleration (since they are **slowing down** as they ascend) and reach a **final velocity of zero** at the top of their

path

Worked Example

A pilot is landing a small aircraft. The airstrip is only 450 m long and the aircraft decelerates from 40 m s^{-1} at a constant rate of 2 m s^{-2}.

Determine if the plane will stop before it reaches the end of the runway.

Step 1: List the known values

- Length of the runway, s_{max} = 450 m (this is the maximum value of displacement)
- Initial velocity, u = 40 m s^{-1}
- Final velocity, v = 0 m s^{-1} (since the plane will stop)
- Acceleration, a = – 2 m s^{-2} (the negative sign indicates slowing down)

Step 2: Identify the missing values (the unknown answer and the one not given)

- Known quantities: u, v and a
- Unknown quantity: displacement, s
- Time, t is not required in this question

Step 3: Choose the correct SUVAT equation and rearrange it

- The SUVAT equation which contains u, v, a and s and omits t is:

$$v^2 = u^2 + 2as$$

- Rearrange to make s the subject:

$$s = \frac{v^2 - u^2}{2a}$$

Step 4: Substitute the values into the equation and calculate s

$$s = \frac{0^2 - 40^2}{2 \times (-2)} = 400 \text{ m}$$

Step 5: Write the full answer to the question

- The plane stops after 400 m, which is 50 m before the end of the runway
- **Note:** In this solution, the negative value produced by $v^2 - u^2$ is cancelled out by the negative value for acceleration, giving a positive value for displacement

Worked Example

A rocket is launched vertically, using motors which give it a constant vertical acceleration of 6.5 m s^{-2} for 30 s before switching off.

At the point when the motors switch off, calculate:

a) The velocity of the rocket

b) The height of the rocket above the launchpad

Part (a)

Step 1: List the known values

- Initial velocity, u = 0 m s^{-1}
- Acceleration, a = 6.5 m s^{-2} (the positive sign indicates the speed is increasing due to the motor)
- Time, t = 30 s

Step 2: Identify the missing values (the unknown answer and the one not given)

- Known quantities: u, a, t
- Unknown quantity: final velocity, v
- Displacement, s, is not required in this question

Step 3: Choose the correct SUVAT equation

- The SUVAT equation which omits displacement, s, is:

$$v = u + at$$

Step 4: Substitute the values into the equation and calculate

$$v = 0 + (6.5 \times 30) = 195 \text{ m s}^{-1}$$

Step 5: Write the full answer to the question, taking care to check significant figures

- The final velocity, v, at 30 s is 200 m s^{-1} (2 s.f.)

Part (b)

Step 1: List the known values

- Initial velocity, u = 0 m s^{-1}
- Acceleration, a = 6.5 m s^{-2}
- Time, t = 30 s
- Final velocity, v = 195 m s^{-1}

Step 2: Identify the missing values (the unknown answer and the one not given)

- Known quantities: u, a, t, and v (from part a)
- Unknown quantity: final displacement, s

- All the SUVAT values are available in this case, but since *v* was calculated, first look for an equation that doesn't use it (in case of mistakes)

Step 3: Choose the correct SUVAT equation

- The SUVAT equation which omits velocity is:

$$s = ut + \frac{1}{2}at^2$$

Step 4: Substitute the values into the equation and calculate *s*

$$s = (0 \times 30) + \frac{1}{2}(6.5 \times 30^2) = 2925 \text{ m}$$

Step 5: Write the full answer to the question, taking care to check significant figures

- The height, *s*, above the launchpad is 2900 m (2 s.f)

Exam Tip

SUVAT questions are popular with examiners, who will find all sorts of ways to set these problems. The trick is practice.

Learn the four equations (even though you are given them) so that you can easily pick the one you need. With lots of practice you will see that often the problem can be simplified, especially when the initial or final velocity is zero.

Finally, never cut corners with these solutions! It's common for questions to have 5–6 marks available for a long calculation. Your job is to write down every step so that the examiner can award marks for as many elements of your answer as possible.

Motion Graphs

- Three types of graph that are used to represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- Graphs are named 'y-axis - x-axis', so 'displacement-time' simply means 'displacement on the y-axis and time on the x-axis'

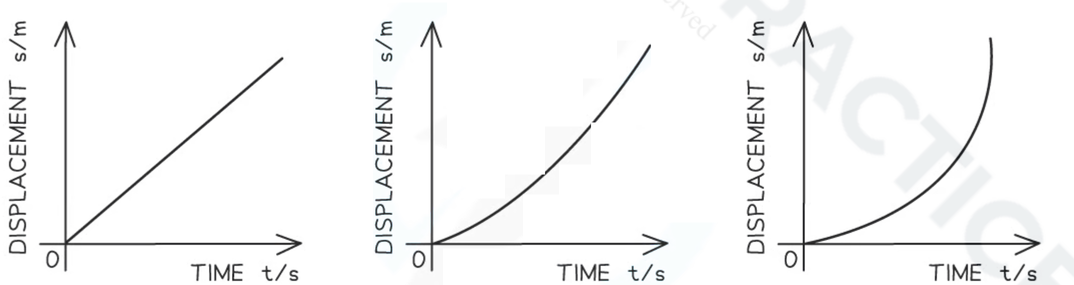
- Graphs of motion can be thought of as telling a story of something which has happened in the real world, such as
  - the time, direction and average speed of a journey to school
  - initial and final velocity and acceleration as a rocket takes off from the launchpad
  - movement and speed in various directions as an athlete takes part in an event

- **Interpreting graphs** means using the slope of the line, or the area under the line to find information about that 'story'
- **Drawing graphs** is a skill where careful planning, plotting and line drawing from some data yields further information from an investigation

Displacement-Time

- On a **displacement-time graph;**
  - **Slope** equals **velocity**
    - A **straight** (diagonal) slope represents a **constant** velocity
    - A **curved** slope represents an **acceleration**
  - A **positive slope** represents motion in the **positive direction**
  - A **negative slope** represents motion in the **negative direction**
  - A **zero** slope (horizontal line) represents a state of **rest**
  - The area under the curve is meaningless

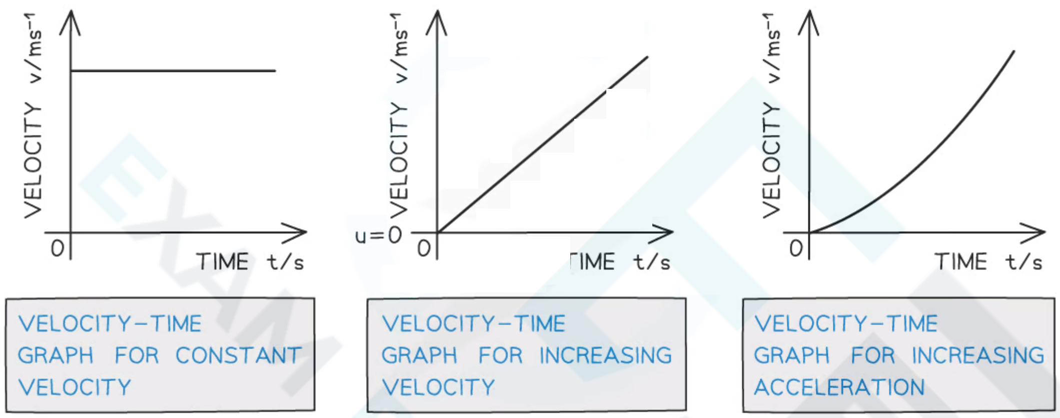
CONSTANT VELOCITY | VELOCITY INCREASING AT A CONSTANT RATE | VELOCITY INCREASING, ACCELERATION INCREASING AT A CONSTANT RATE



DISPLACEMENT-TIME GRAPH FOR CONSTANT VELOCITY | DISPLACEMENT-TIME GRAPH FOR INCREASING VELOCITY | DISPLACEMENT-TIME GRAPH FOR INCREASING ACCELERATION

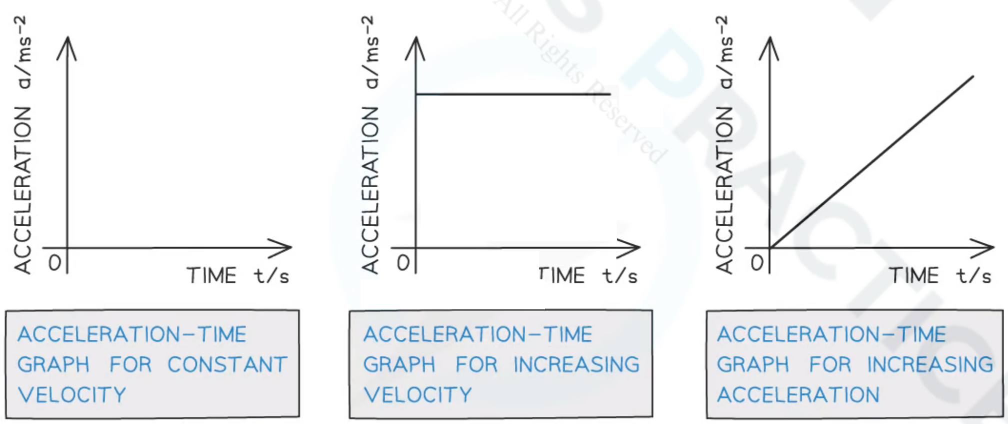
Velocity-Time

- On a **velocity-time graph…**
  - **Slope** equals **acceleration**
    - A **straight** line represents **uniform** acceleration
    - A **curved** line represents **non-uniform acceleration**
  - A **positive** slope represents an **increase** in **velocity** in the **positive direction**
  - A **negative** slope represents an **increase** in **velocity** in the **negative direction**
  - a **zero** slope (horizontal line) represents motion with **constant velocity**
  - The **area** under the curve equals the **displacement** or **distance** travelled



Acceleration-Time

- On an **acceleration-time graph…**
- A zero slope (horizontal line) represents an object undergoing constant acceleration
- The area under the curve equals the change in velocity
- The steepness of the slope is meaningless

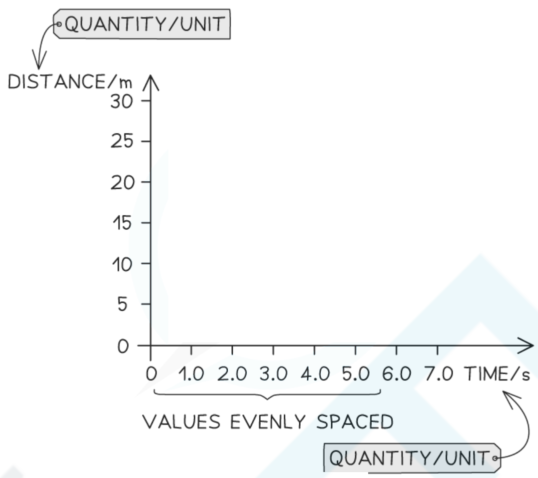


How displacement, velocity and acceleration graphs relate to each other

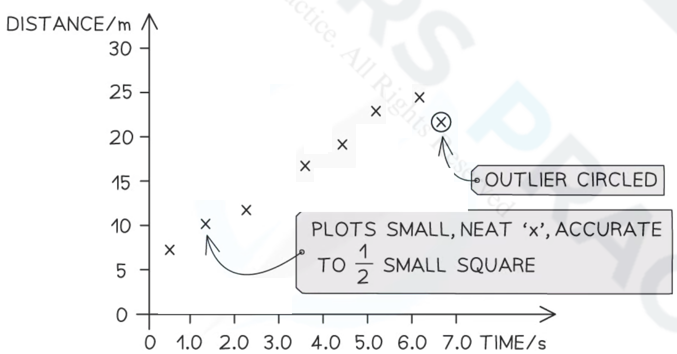
Drawing Graphs

- Drawing a good graph is an important skill

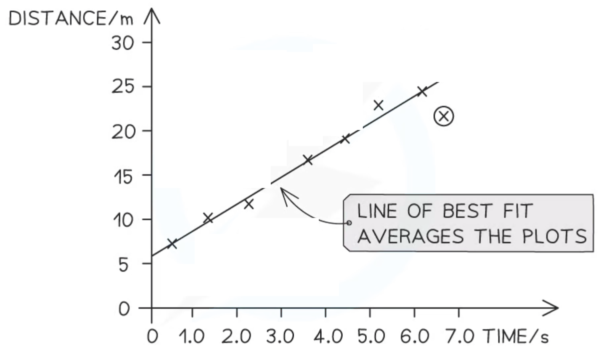
- Good graphs have the following:
  - Each axis has two labels; the name of the variable and the unit (and may include a multiplier, for example, m × 10^{-9})
  - Values on the axes are evenly spaced, with the same number of divisions for each amount



  - The axes start at the origin **only** when this fits the data or the relationship being graphed is directly proportional **or** when specified in the question
  - Increments on the scale are in multiples of 2, 5 and 10 but **never** multiples of 3.
  - The range of values being plotted takes up the length of the axis, so that the plotted line takes up most of the space on the graph
  - Plots are accurate to within half a small square
  - Plots and curves are drawn with sharp pencil so that they are narrow (don't take up more than half a small square)

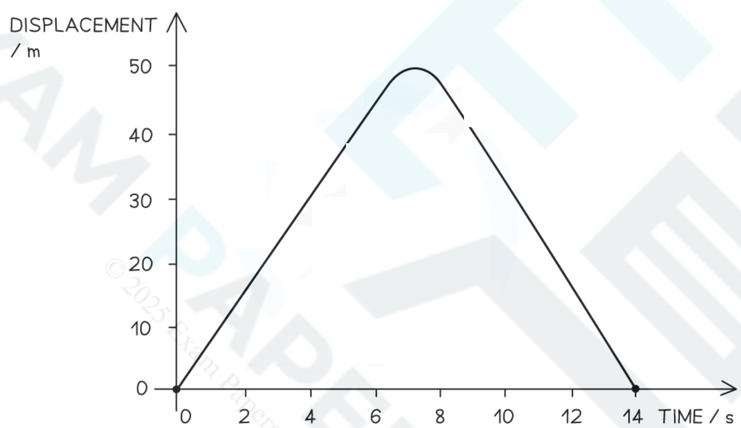


  - Lines of best fit match the data, so that lines go through as many of the plots as possible (ignoring outliers) with the remainder evenly spaced above and below the line



Worked Example

A runner is practicing short sprints between two markers. A displacement-time graph showing part of their training is shown.



(a) Find the distance between the markers

(b) Determine the speed of the runner at the fastest part of their sprint

Part (a)

Step 1: Determine the direction of motion from the graph

- Initially displacement increases from 0 - 50 m
- This represents the sprinter running away from the starting position
- Secondly displacement decreases from 50 - 0 m
- This represents the sprinter returning towards the starting position

Step 2: Identify the sprinter's turning point on the graph

- The distance between the markers is from 0 → turning around point
- This is 50 m from the start point

Step 3: Write the correct answer

Distance between the markers = 50 m

Part (b)

Step 1: Identify the part of the graph that represents the velocity

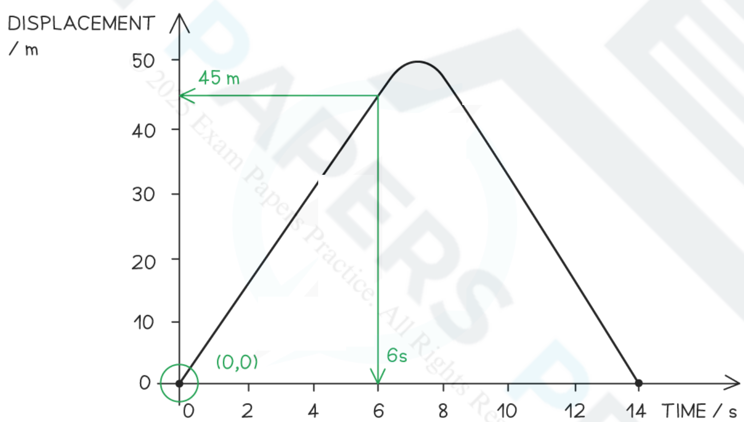
- The graph shows displacement against time
- Velocity is equal to:

$$Velocity = \frac{displacement}{time}$$

- Therefore, the slope of the displacement-time graph gives velocity
- Displacement × time (area under the graph) is not a physical quantity

Step 2: Identify the steepest section of the graph

- Select the section on the graph where the slope is the steepest
  - This represents the **fastest velocity**
- Clearly mark two points on this line which are as far apart as possible



Step 3: Use the coordinates from the graph to calculate the slope

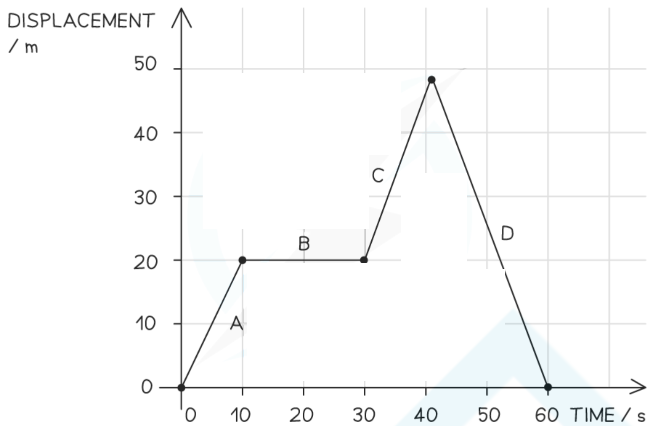
$$\text{slope} = \frac{increase\ in\ y}{increase\ in\ x} = \frac{45}{6} = 7.5\ \text{m s}^{-1}$$

Step 4: Write down the answer to the correct number of significant figures

The sprinter's fastest speed = 7.5 m s^{-1}

Worked Example

Motion has been plotted on a displacement-time graph.



Sketch a velocity-time graph for this motion. Include values and show your calculations.

Step 1: Calculate each stage:

Part A

Step 1: Determine the time interval

Time from 0 - 10 s

Step 2: Calculate the velocity by finding the gradient

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{20}{10} = 2\text{ m s}^{-1}$$

Part B

Step 1: Determine the time interval

Time from 10 - 30 s

Step 2: Calculate the velocity by finding the gradient

Gradient = 0 (horizontal line), velocity = 0 m s^{-1}

Part C

Step 1: Determine the time interval

Time from 30 - 40 s

Step 2: Calculate the velocity by finding the gradient

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{50 - 20}{40 - 30} = 3\ \text{m s}^{-1}$$

Part D

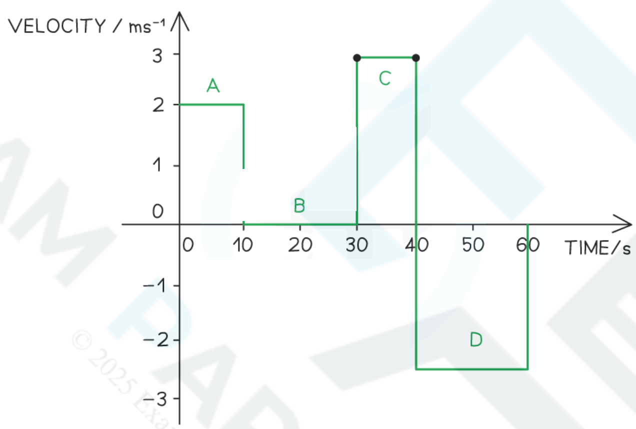
Step 1: Determine the time interval

Time from 40 - 60 s

Step 2: Calculate the velocity by finding the gradient

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{0 - 50}{60 - 40} = -2.5\ \text{m s}^{-1}$$

Step 2: Use the calculated values to sketch a graph



Exam Tip

Interpreting Graphs

It's common for a question to ask you to solve something **'using a graphical method'**. This just means 'by finding your answer on a graph'.

In AS Physics most (but not all) graphs will be a straight line, meaning they follow the equation **y = mx + c**

This means that the 'graphical method' question is asking you to look at the **gradient**, or the **area under the line**. Once you are secure in finding these then graph questions will become much easier!

Drawing Graphs

A well-drawn graph will net you four or five quick and easy marks IF you have practiced the skill.

Common mistakes are starting at the origin when there is no need (so the line takes up less than half of the page) and trying to work with a scale that goes in multiples of three (making it hard to get the plotting right).

Slope & Area of Motion Graphs

Slope & Area of Motion Graphs

- Three types of graph that are used to represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- To interpret these find either the slope or the area
- **Slope** is found using $slope = \frac{change\ in\ y}{change\ in\ x}$, which can be related to any similar algebraic relationship, for example, $v = \frac{d}{t}$
  - The slope of a distance-time graph will equal velocity
- **Area** is found using area = change in y × change in x, which can be related to any similar algebraic relationship, for example, velocity × time = distance
  - The area under a velocity-time graph will equal distance

Finding Slope of a Straight Line

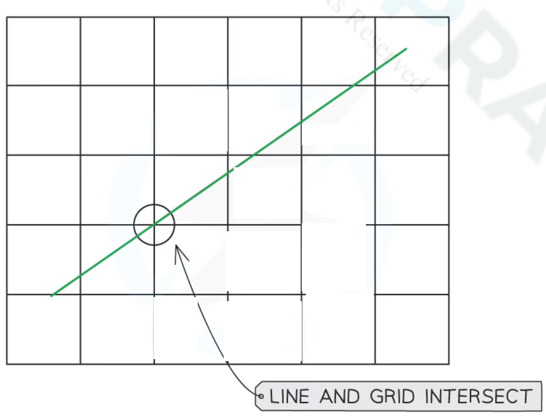
- The slope of a line is found using the formula:

$$\text{slope} = \frac{rise}{run}$$

- This is also written:

$$\text{slope} = \frac{increase\ in\ y}{increase\ in\ x}$$

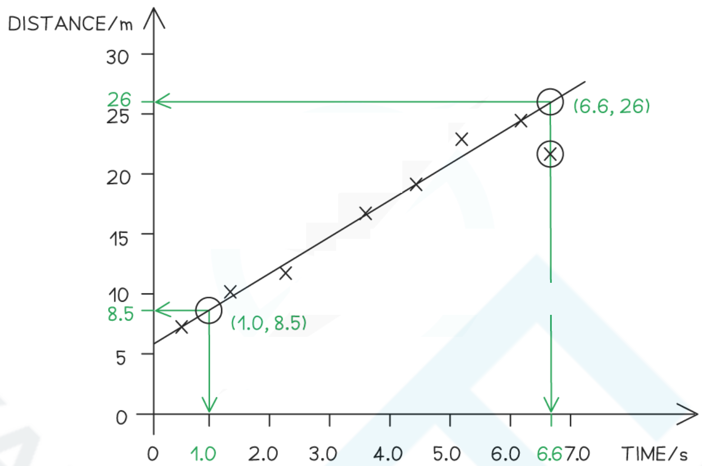
- To find the slope;
  - Identify two points **as far apart as possible** which intercept with the grid of the graph paper



  - The points must be on the line itself, **not** the furthest apart plots
  - Clearly mark the points chosen

- Find the co-ordinates of the two points
- Calculate to find the slope

$$\frac{increase\ in\ y}{increase\ in\ x} = \frac{(26 - 8.5)}{(6.6 - 1.0)} = 3.125$$



- Use the values on the axes of the graph (or in the data) to identify the correct significant figures
- Use the units to find the correct units, which also equal

$$\text{units of slope} = \frac{units\ of\ y}{units\ of\ x}$$

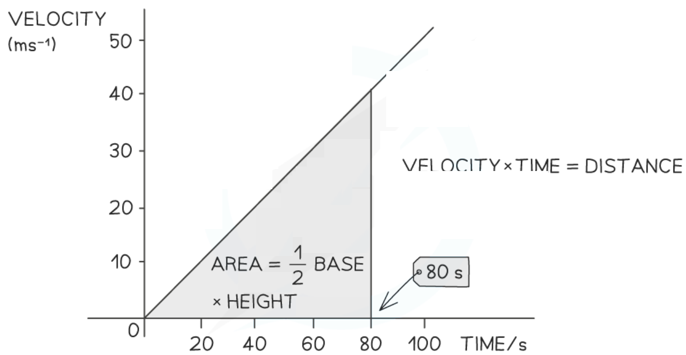
- On this graph all values are to 2 significant figures,
- Therefore slope = 3.1 m s^{-1}

Finding the Area Under the Line

- The area underneath a line represents the x axis multiplied by the y axis.
- For example, if velocity is plotted on the y-axis and time is on the x-axis, then
  - area = velocity × time = distance
- When the area is a rectangle or a triangle it is easy to find by calculating

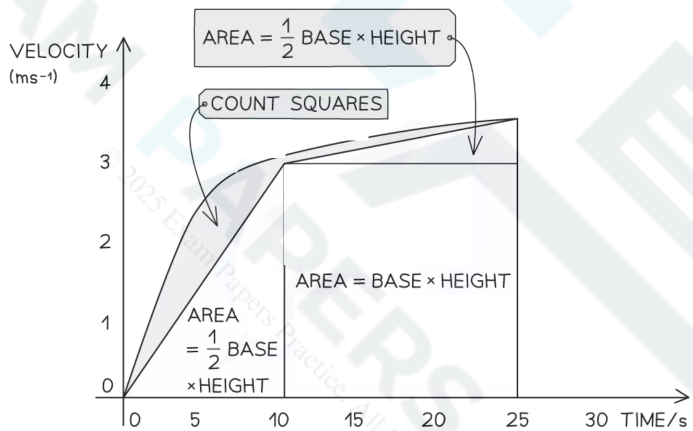
area of rectangle = base × height

area of a triangle = ½ base × height

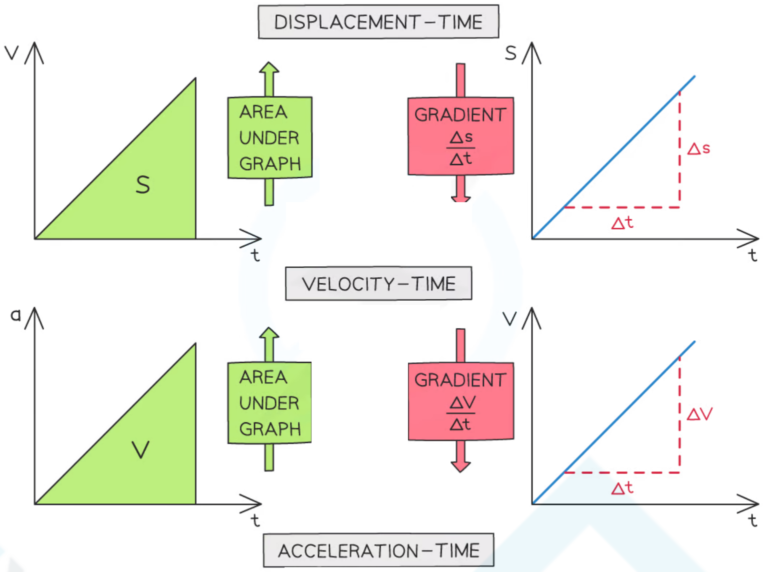


To find the area under a straight-line graph treat is as any rectangle or triangle

- When the line is curved, area is found though the following steps
  - Divide the shape into rectangles and triangles as shown
  - Find the area for each by calculating
  - Count the remaining squares
  - Add the totals together

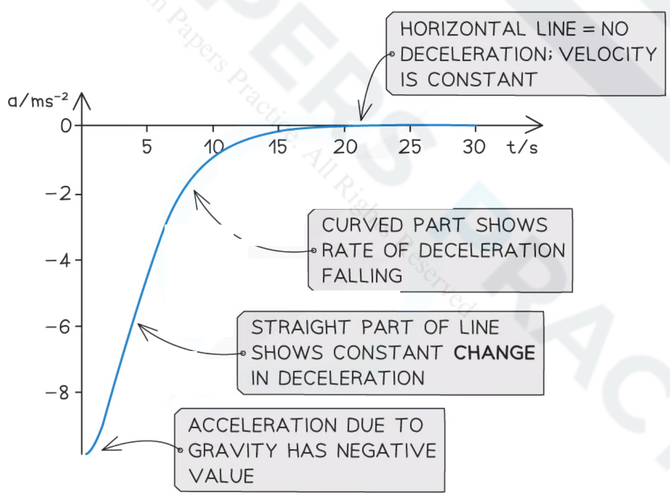


Find area under a curve by dividing it into sections which reduce the number of squares to be counted to the minimum



Non-uniform Acceleration

- In certain cases, acceleration changes throughout a period of motion
- An example of this is a skydiver falling due to their weight, against air resistance
  - As speed increases, air resistance increases
  - The increasing resultant force against the direction of motion acts to reduce acceleration
  - Eventually the weight and air resistance balance and acceleration become zero
  - Acceleration for a falling object is conventionally shown as a negative value

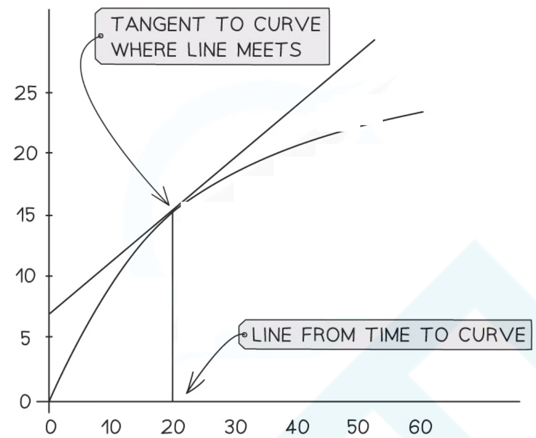


- Non-uniform acceleration will produce a curved velocity-time graph

Finding the Slope of a Curved Line

- Some questions ask for velocity or acceleration at a particular point or time
- This is called 'instantaneous' velocity or acceleration
- It is found by taking the slope of the curved line

- Identify the point on the x-axis which corresponds with the question
  - Draw a line vertically to meet the curve
- Take a tangent to the curve at this point
  - Do this carefully, trying out more than one attempt before choosing the correct tangent



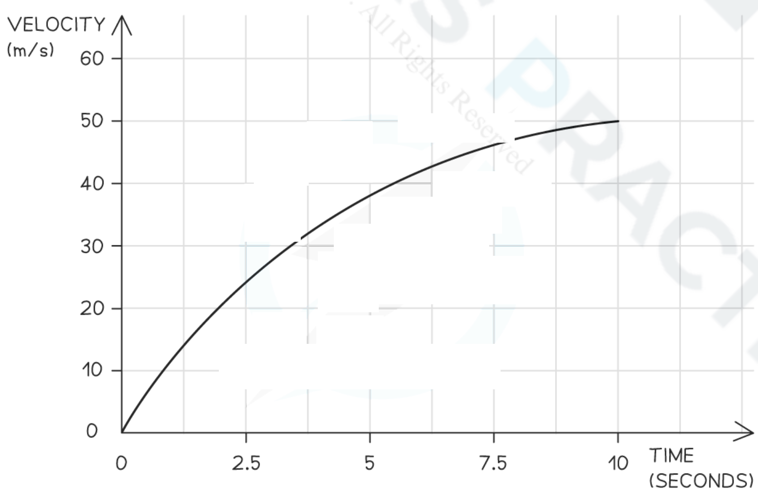
Tangents are a useful way to find the slope of a curved line

- Draw the tangent in place, making it as long as will fit onto the graph
  - Follow the steps above to find the slope of the tangent

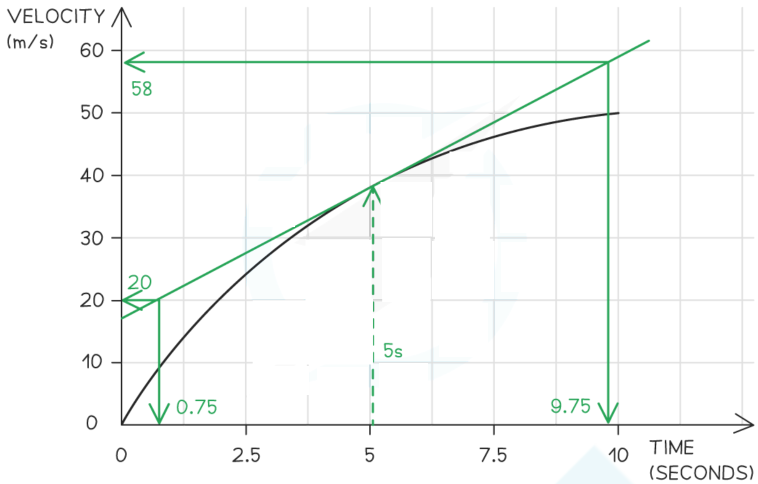
Worked Example

A skydiver jumps from a plane and reaches terminal velocity after 15 seconds. A graph of her motion is shown.

Use the graph to find the acceleration at 5 seconds.



Step 1: Draw a tangent to the curve at the point where t = 5 s



Step 2: Select points on the graph which are very far apart, and determine the corresponding values of x and y, write these down

Increase in y = (58 -20) = 38 m s^{-1}

Increase in x = (9.75 - 0.75) = 9.0 s

Step 3: Calculate the slope

$$\text{slope} = \frac{\textit{increase in y}}{\textit{inrease in x}} = \frac{38}{9} = 4.222 \text{ m s}^{-2}$$

Step 4: Write the answer to the correct significant figures

The acceleration at 5.0 s = 4.2 m s^{-2} (2 sf)

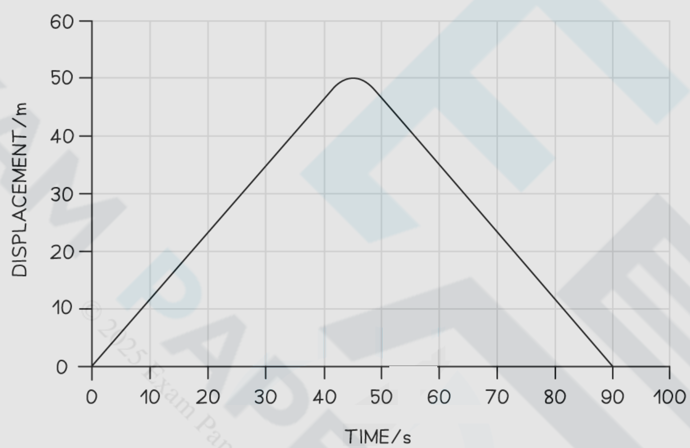
Exam Tip

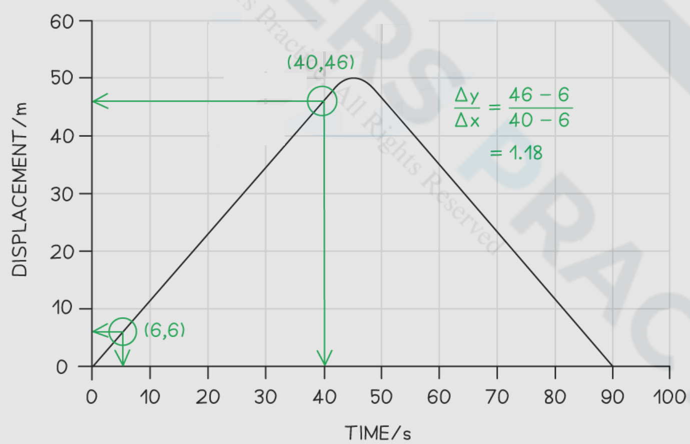
When working with graphs follow the advice, 'show don't tell'. The examiner will want to see that you used the graph to find your answer, as this skill is part of the toolkit of a good scientist.

Mark very clearly on the graph any points you use to calculate and indicate with clear lines or large triangles where you chose your data from.

And always remember to use the largest section of the graph that you can to find the slope.

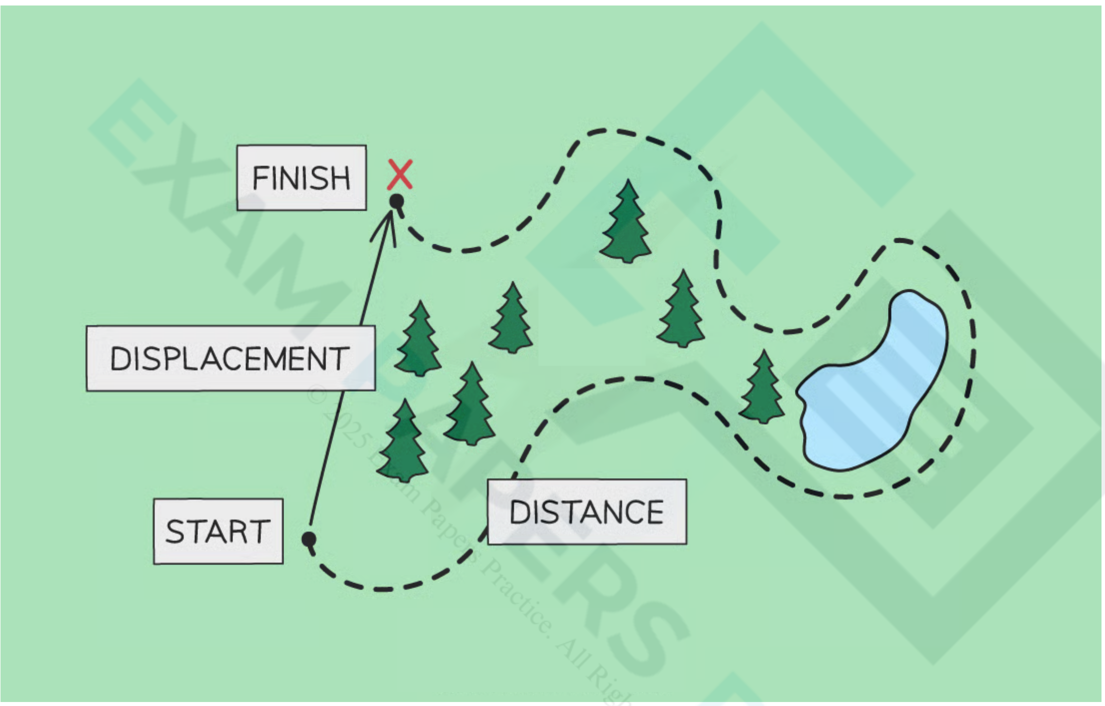
You will be given an exam paper with pristine graphs. Don't leave them that way! By the time you finish with the paper, it should be clearly annotated with your method, so the examiner can see exactly where you have earned marks.





Scalars & Vectors

- A **scalar** is a quantity which **only** has a magnitude (size)
- A **vector** is a quantity which has **both** a magnitude and a direction
- For example, if a person goes on a hike in the woods to a location which is a couple of miles from their starting point
  - As the crow flies, their **displacement** will only be a few miles but the **distance** they walked will be much longer



Displacement is a vector while distance is a scalar quantity

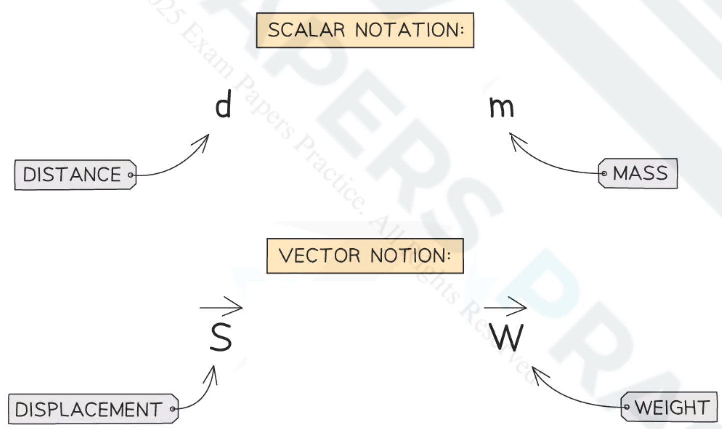
- **Distance** is a scalar quantity because it describes how an object has travelled overall, but not the direction it has travelled in
- **Displacement** is a vector quantity because it describes how far an object is from where it started and in what direction

- Some common scalar and vector quantities are shown in the table below:

Scalars and Vectors Table

SCALARS	VECTORS
DISTANCE	DISPLACEMENT
SPEED	VELOCITY
MASS	ACCELERATION
TIME	FORCE
ENERGY	MOMENTUM
VOLUME	
DENSITY	
PRESSURE	
ELECTRIC CHARGE	
TEMPERATURE	

Vector Notation



The arrow on vector notation does not indicate an actual direction, just that the quantity has a direction

- This means writing the quantity to make it clear that it is a vector
- In text books vectors are often shown as bold and italic, for example ***F*** or ***s***
- Another form of notation, and easier to do in handwriting, is putting an arrow over the top of the quantity, for example $\vec{F}$ or $\vec{s}$
  - The arrow does **not** indicate the actual direction of the vector, only that is **has** a direction

Exam Tip

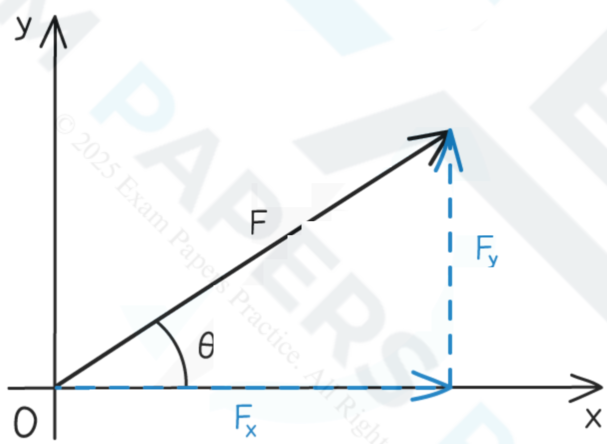
Do you have trouble figuring out if a quantity is a vector or a scalar? Just think - can this quantity have a minus sign? For example - can you have negative energy? No. Can you have negative displacement? Yes!

Resolving Vectors

- **Vectors** are represented by an arrow
  - The **arrowhead** indicates the **direction** of the vector
  - The **length** of the arrow represents the **magnitude**
  - **Component** vectors are sometimes drawn with a dotted line and a **subscript** indicating horizontal or vertical for example F_v is the vertical component of the force F

Resolving Vectors

- A single resultant vector can be resolved
  - This means it can be represented by **two** vectors, which in combination have the same effect as the original one
  - Resolving vectors makes calculating much easier when working with projectiles or any motion or force at an angle
- When a single resultant vector is broken down into its **parts**, those parts are called **components**
- A vector of **magnitude *F*** at **angle theta** to the horizontal is shown



Resolving Vectors by Scale Drawing

- To resolve vectors by scale drawing means carefully producing a scale drawing with all lengths and angles correct
  - This should be done using a sharp **pencil**, **ruler** and **protractor**

- Follow these steps
  - Choose a scale which fits to the page. For example, if resolving a resultant displacement of 100 m, use 1 cm = 10 m so that the resultant drawing is around 10 cm high
  - Clearly mark the starting point and a line to represent either the vertical or horizontal direction. This will depend on the angle given in the question, for example if the direction is '30° east of north' then start with a vertical line representing north
  - Carefully measure the angle and start by drawing the resultant vector
  - Draw a triangle using two of the sides of this rectangle

Worked Example

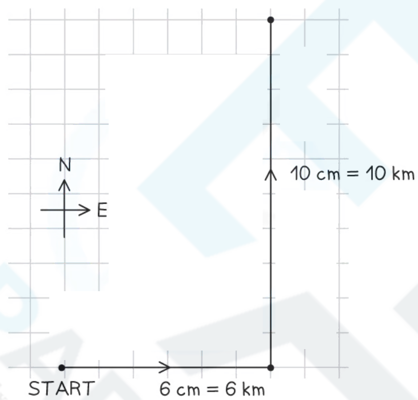
A hiker walks a distance of 6 km due east and 10 km due north.

By making a scale drawing of their route, find the magnitude of their displacement and its direction from the horizontal.

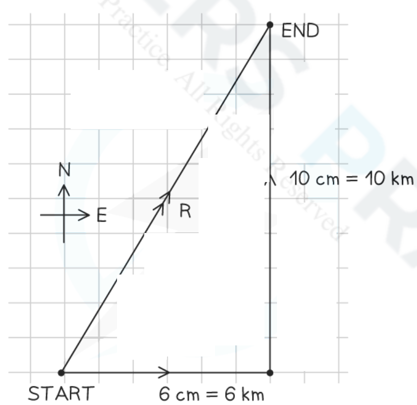
Step 1: Choose a sensible scale

The distances are 6 and 10 km, so a scale of 1 cm = 1 km will fit easily on the page, but be large enough for an accurate scale drawing

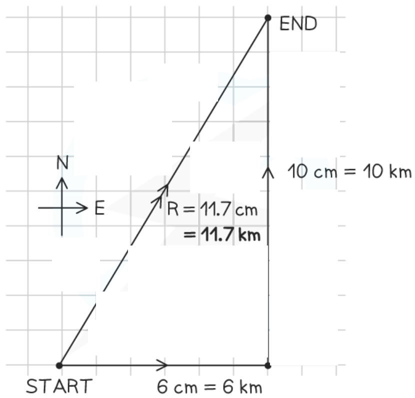
Step 2: Draw the two components using a ruler and making the measurements accurate to 1 mm



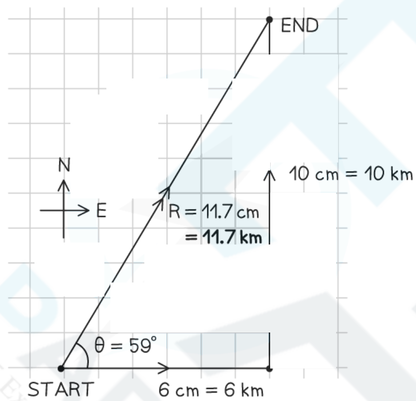
Step 3: Add the resultant vector, remembering the start and finish points of the journey



Step 4: Carefully measure the length of the resultant and convert using the scale



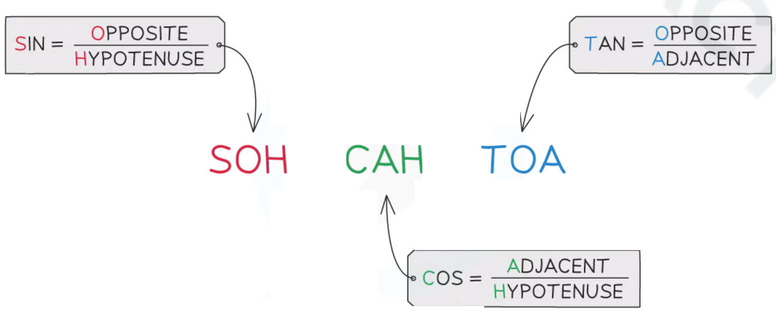
Step 5: Measure the angle between the vector and the horizontal line



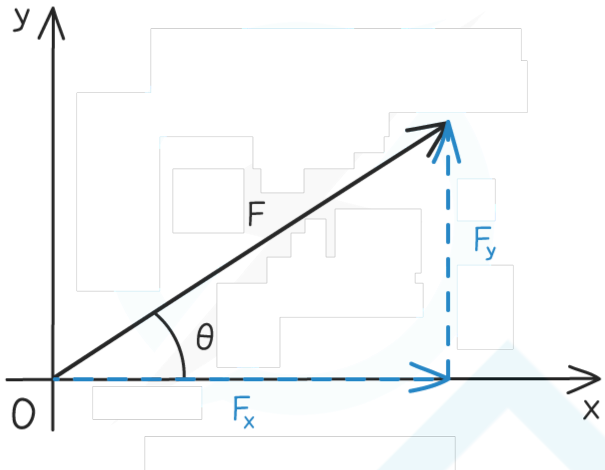
Step 6: Write the complete answer, giving both magnitude and direction

Resolving Vectors by Calculation

- In this method a diagram is still essential but it does **not** need to be exactly to scale
- The diagram can take the form of a sketch, as long as the resultant, component and sides are clearly labelled
- Use **trigonometry** to resolve the two sides
- The mnemonic '**soh-cah-toa**' is used to remember how to apply sines and cosines to resolving sides of a triangle



- It is possible to **resolve** this vector into its **horizontal** and **vertical** components using trigonometry



The resultant force F can be split into its horizontal and vertical components

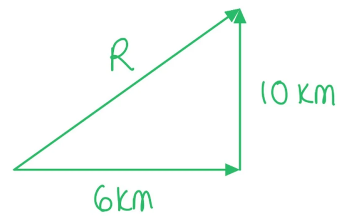
- For the **horizontal** component, $\boldsymbol{F_x = F \cos \theta}$
- For the **vertical** component, $\boldsymbol{F_y = F \sin \theta}$

Worked Example

A hiker walks a distance of 6 km due east and 10 km due north.

Calculate the magnitude of their displacement and its direction from the horizontal.

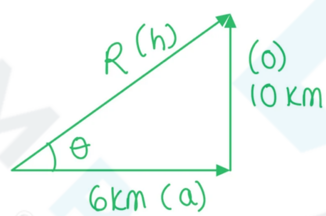
Step 1: Draw a vector diagram



Step 2: Calculate the magnitude of the resultant vector using Pythagoras' Theorem

$$R = \sqrt{6^2 + 10^2} = 2\sqrt{34}$$

Step 3: Calculate the direction of the resultant vector using trigonometry



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{6}$$

$$\theta = \tan^{-1}\left(\frac{10}{6}\right) = 59°$$

Step 4: State the final answer complete with direction

$$R = 2\sqrt{34} = 11.66 = \mathbf{12\ km}$$

θ = **59°** east and upwards from the horizontal

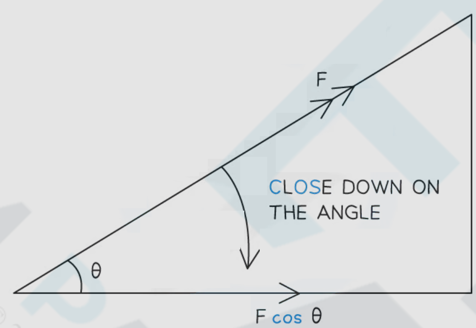
Exam Tip

Did you notice that the two worked examples above were the same question, being solved in two different ways?

If the question specifies calculation or scale drawing you must solve the problem as asked. However, if the choice is left up to you then any correct method will lead to the correct answer.

Scale drawing sometimes feels easier than calculating, but once you are confident with trigonometry and Pythagoras you will find calculating quicker and more accurate.

A quick tip to remember whether to use sin or cos is that when the resultant is closing down onto the angle, use cos.



Adding Vectors

- Vectors can be combined by **adding** or **subtracting** them to produce the resultant vector
  - The resultant vector is sometimes known as the 'net' vector (eg. the net force)

- There are two methods that can be used to add vectors
  - **Calculation** – if the vectors are perpendicular
  - **Scale drawing** – if the vectors are not perpendicular

Combining Vectors Using a Scale Diagram

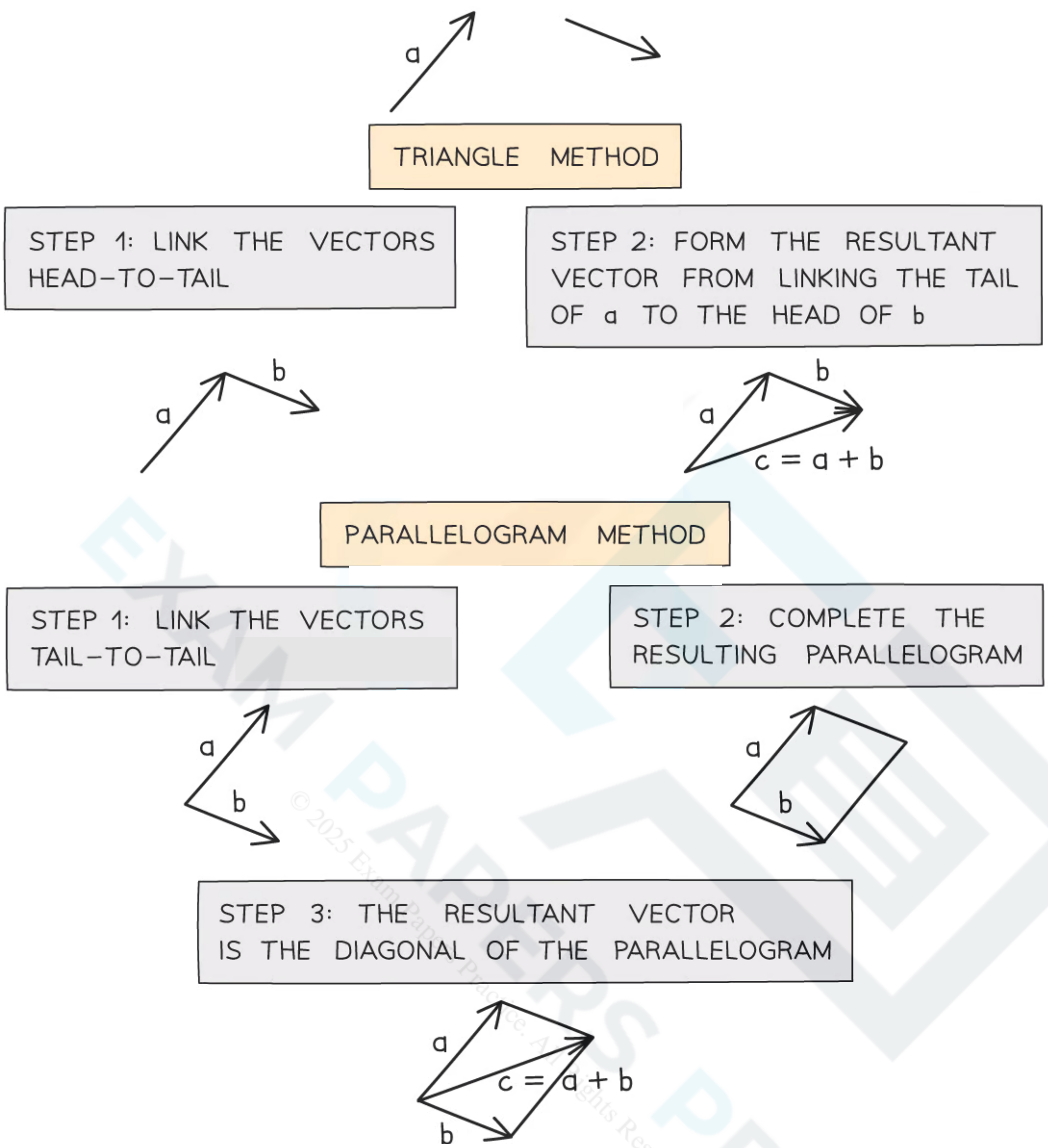
- There are two methods that can be used to combine vectors using a scale diagram: the **triangle method** and the **parallelogram method**

- To combine vectors using the triangle method:
  - **Step 1:** link the vectors head-to-tail
  - **Step 2:** the resultant vector is formed by connecting the tail of the first vector to the head of the second vector

- To combine vectors using the parallelogram method:
  - **Step 1:** link the vectors tail-to-tail
  - **Step 2:** complete the resulting parallelogram
  - **Step 3:** the resultant vector is the diagonal of the parallelogram

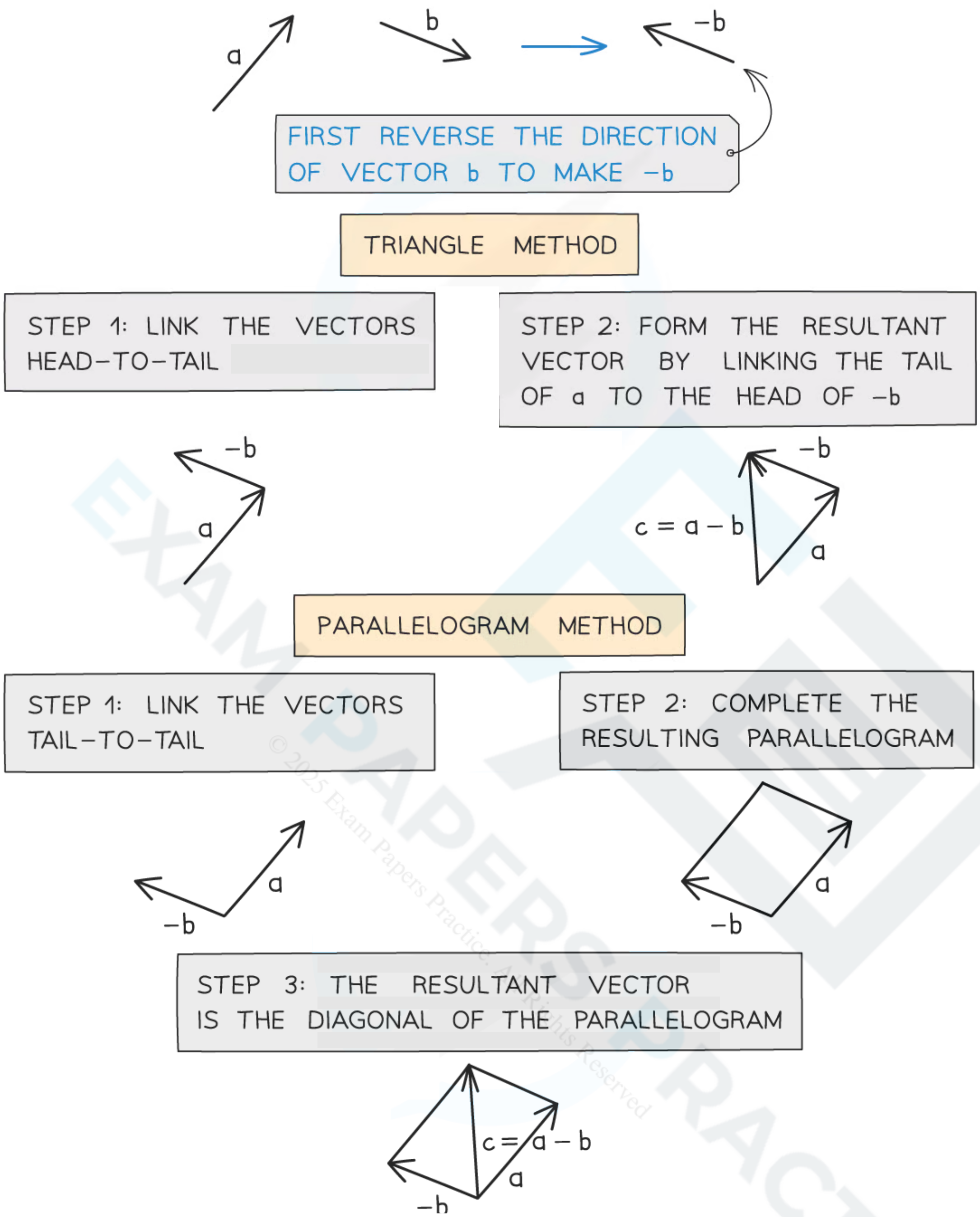
Worked Example

Draw the vector c = a + b



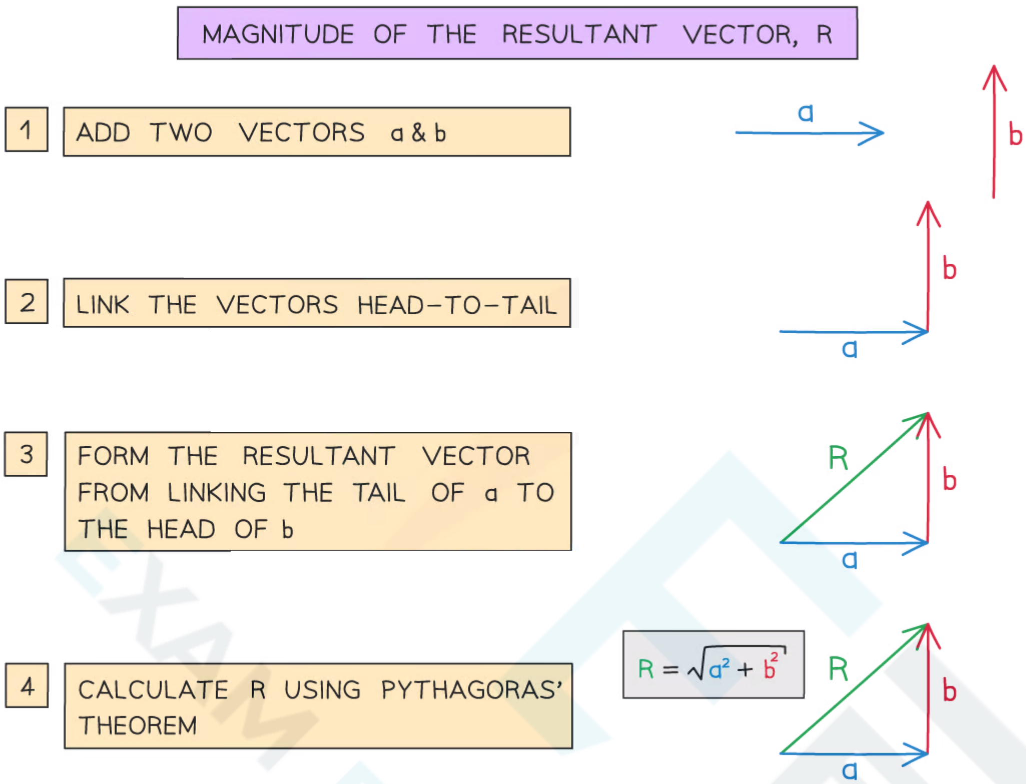
Worked Example

Draw the vector c = a – b



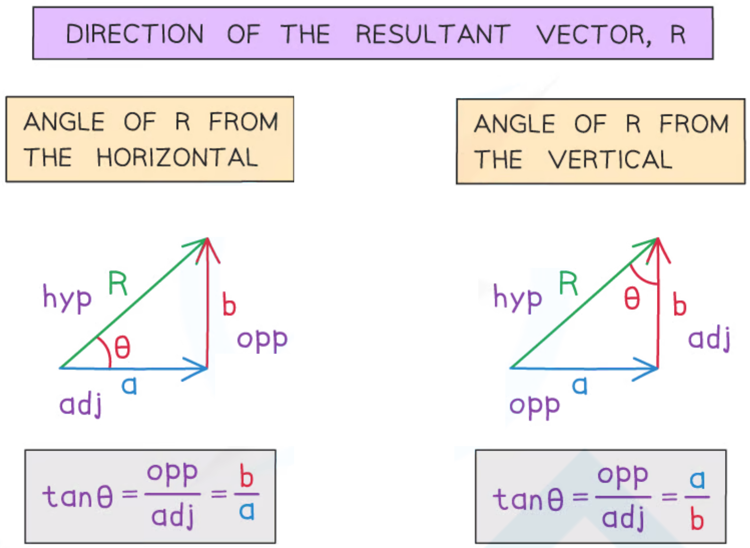
Combining Vectors by Calculating

- Combining vectors by calculation is a two-step process
  - Finding the direction of the resultant using trigonometry
  - Finding the magnitude of the resultant using Pythagoras



The magnitude of the resultant vector is found by using Pythagoras' Theorem

- The direction of the resultant vector is found from the angle it makes with the horizontal or vertical
  - The question should imply which angle it is referring to (ie. Calculate the angle from the x-axis)
- Calculating the angle of this resultant vector from the horizontal or vertical can be done using **trigonometry**
  - Either the sine, cosine or tangent formula can be used depending on which vector magnitudes are calculated



The direction of vectors is found by using trigonometry

Worked Example

A swimmer is crossing a river by swimming due north at 2 m s^{-1}. The current flows east at 5 m s^{-1}.

Determine the resultant velocity of the swimmer's motion.

Step 1: Sketch a diagram, including all known values

Step 2: Calculate the magnitude of the resultant using Pythagoras

$$R = \sqrt{V_s^2 + V_c^2} = \sqrt{4 + 25} = 5.4$$

Step 3: Calculate the angle using trigonometry

$$tan\theta = \frac{2}{5} = 0.4$$

$$\theta = \tan^{-1}(0.4)$$

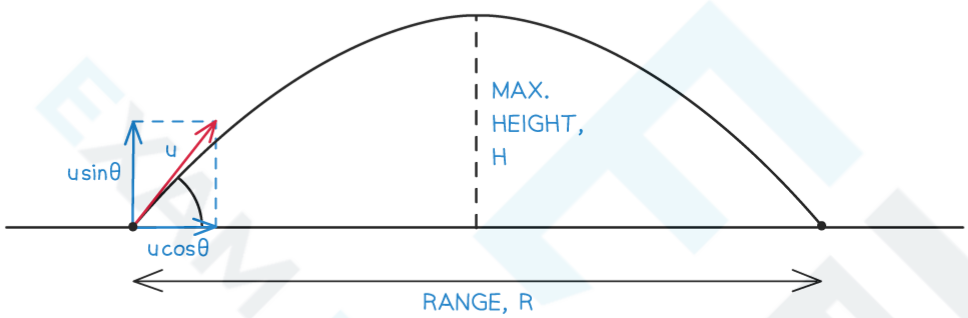
$$\theta = 21.8$$

Step 4: Write the answer in full giving both magnitude and direction of the velocity and all units

The swimmer's velocity is 5.4 ms^{-1} at 22° to the horizontal direction

Components of Velocity

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance traveled by the projectile



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ \text{ms}^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \dfrac{u\sin\theta}{g}$

$2t = \dfrac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

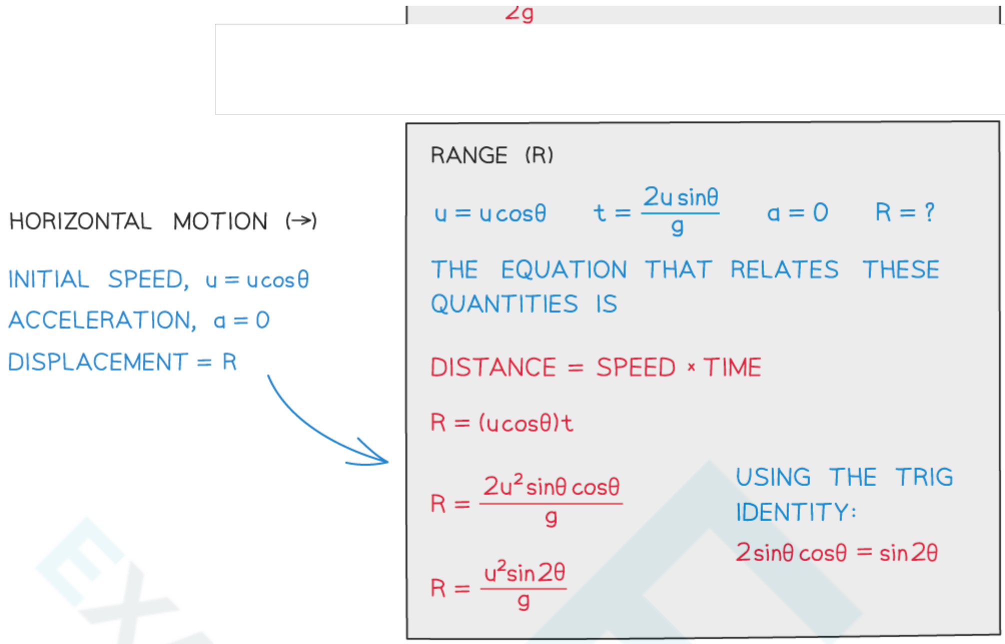
MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$



How to find the time of flight, maximum height and range

Problems Involving Projectile Motion

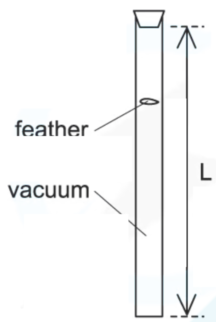
- There are two main considerations for solving problems involving two-dimensional motion of a projectile
  - Constant velocity in the horizontal direction
  - Constant acceleration in a perpendicular direction
- The only force acting on the projectile, after it has been released, is **gravity**
- There are three possible scenarios for projectile motion:
  - **Vertical** projection
  - **Horizontal** projection
  - **Projection** at an **angle**

Worked Example

To Calculate Vertical Projection (Free Fall)

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.



What is the length of the tube, L?

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81\ ms^{-2}$ $\quad u = 0 \quad t = 0.5\ s \quad L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

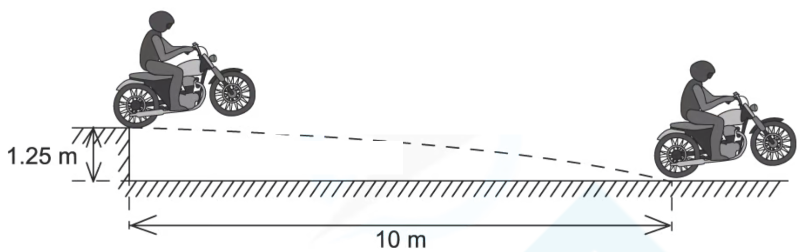
$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2\ m$$

Worked Example

To Calculate Horizontal Projection

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off? (ignoring air resistance)



IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $a = 9.81$ ms^{-2} $u = 0$ $t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $a = 0$ $t = 0.5$ s $u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

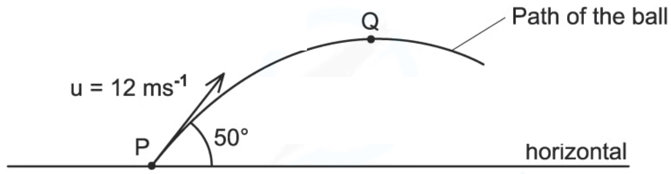
$$v = \frac{10}{0.5} = 20\ \text{ms}^{-1}$$

Worked Example

To calculate projection at an angle

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q? (ignoring air resistance)



IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $\quad a = -9.81\ \text{ms}^{-2}$ $\quad v = 0$ $\quad H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ \text{m}$$

Exam Tip

In almost every question using the SUVAT equations one component of velocity will have constant acceleration (for example, the vertical component of projectile motion) and the other will have no acceleration (for example the horizontal component of projectile motion when we ignore air resistance).

The trick which you nearly always have to remember is find the common value - time - first, then substitute it into the second part of your calculation.

As long as your working is clear, even if you forget, you are likely to still achieve half marks. But writing good clear Maths and remembering this tip should get you to the end of your calculation - and full marks!

And finally.... don't forget that deceleration is **negative** as the object rises.

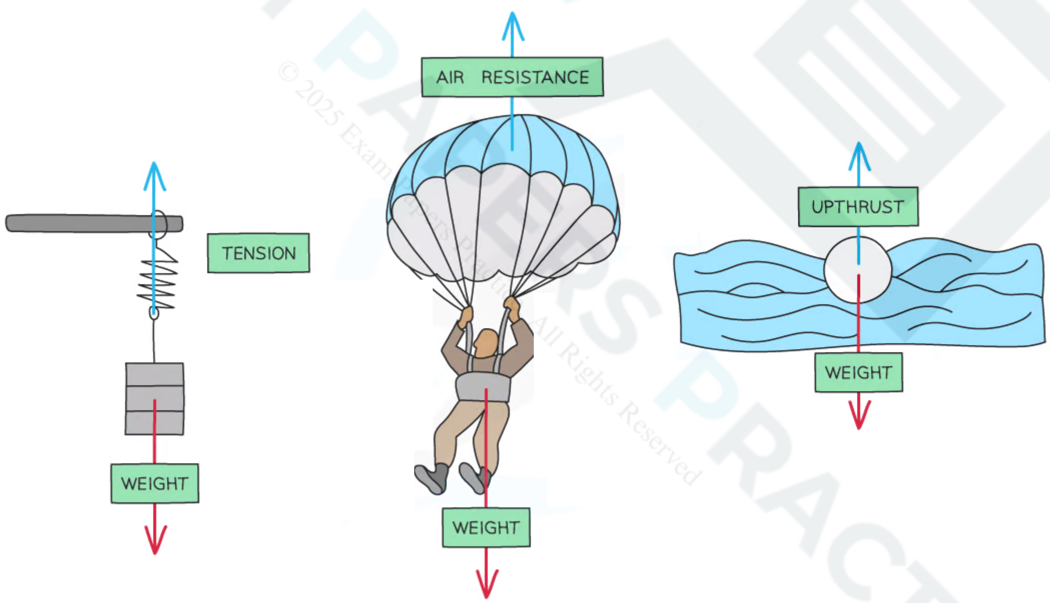
Free-body Force Diagrams

- Free body diagrams are useful for modeling the forces that are acting on an object
- Each force is represented as a **vector** arrow, where each arrow:
  - Is scaled to the **magnitude** of the force it represents
  - Points in the **direction** that the force acts
  - Is **labelled** with the name of the force it represents or an appropriate symbol

- Free body diagrams can be used:
  - To identify which forces act in which plane
  - To resolve the net force in a particular direction

Simple Free Body Diagrams

- The rules for drawing a free-body diagram are the following:
  - **Rule 1:** Draw a point in the centre of mass of the body
  - **Rule 2:** Draw the body free from contact with any other object
  - **Rule 3:** Draw the forces acting on that body using vectors with correct direction and proportional length

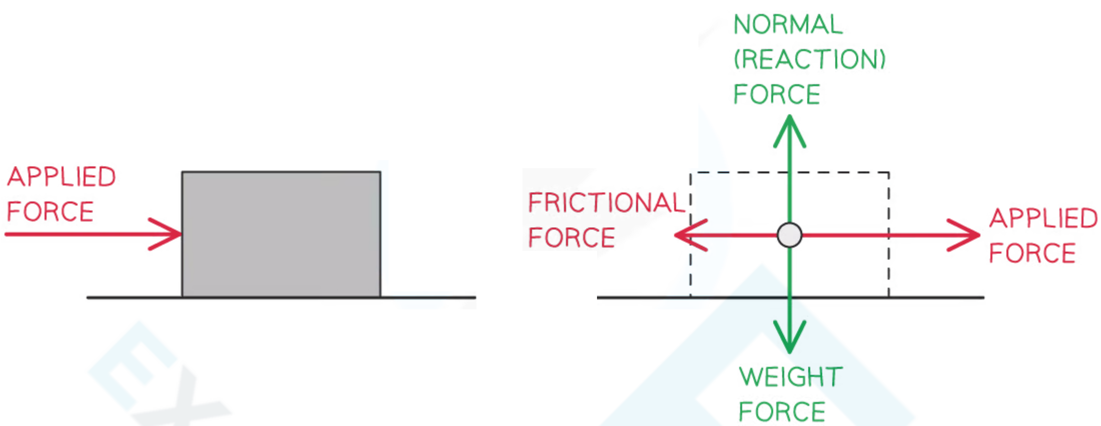


Free body diagrams can be used to show the various forces acting on objects

Forces on a Particle or an Extended but Rigid Body

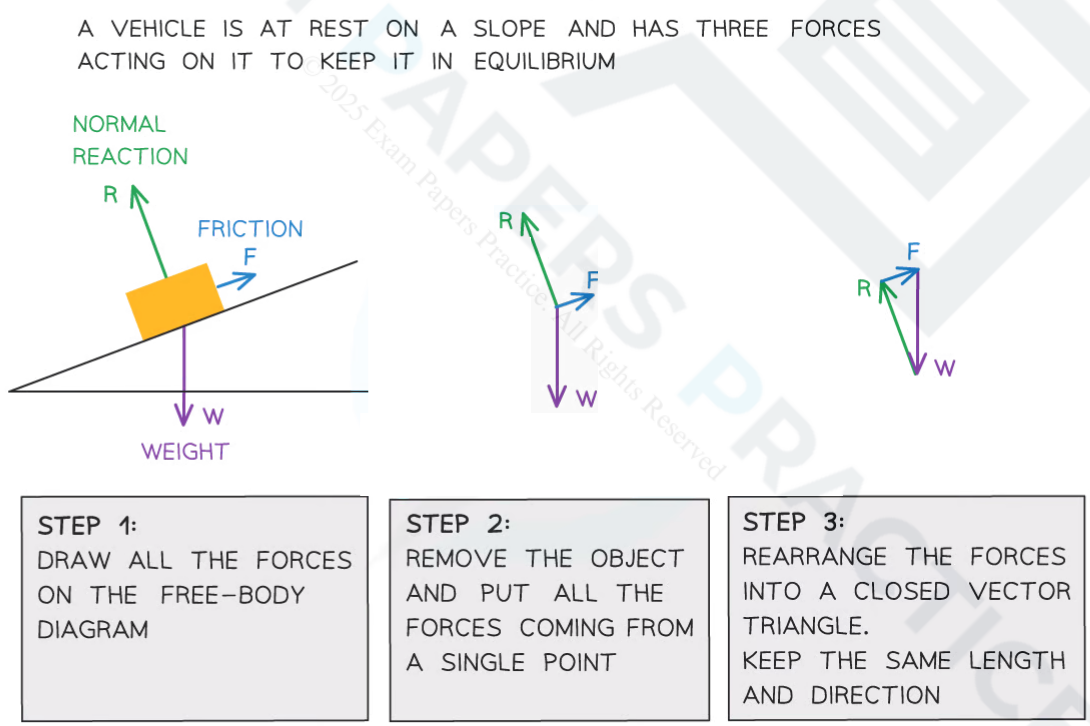
- Free body diagrams simplify problems by reducing complex shapes into simple one
- Any object can be thought of as a particle

- Even something as large as the Earth can be modelled as a point mass for most calculations
- Objects can also be thought of as being **extended but rigid bodies**
  - This simply means that all parts stay in the same position relative to each other when the object moves



Point particle representation of the forces acting on a moving object

- The below example shows an object sitting on a slope in equilibrium



Three forces on an object in equilibrium form a closed vector triangle

Worked Example

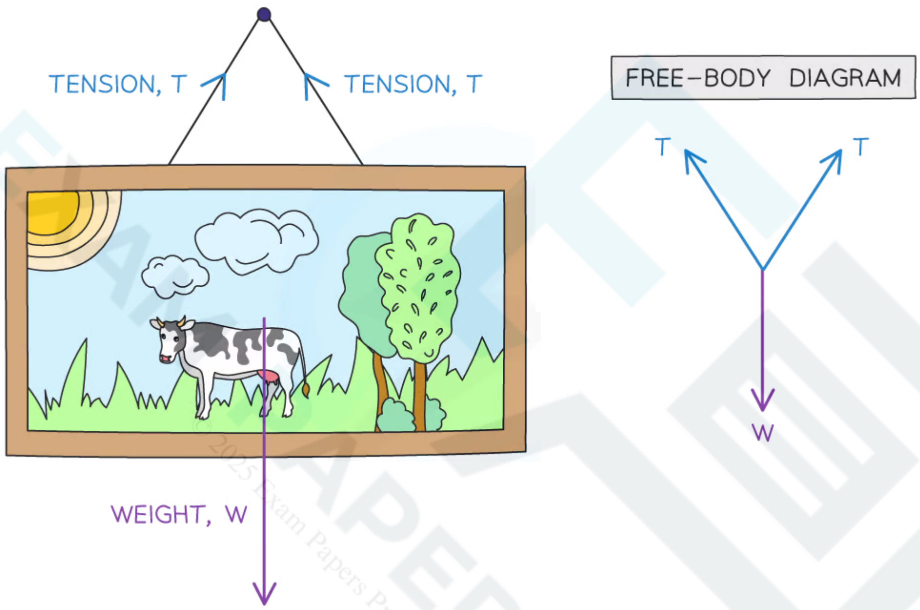
Draw free-body diagrams for the following scenarios:

a) A picture frame hanging from a nail

b) A box sliding down a slope

Part (a)

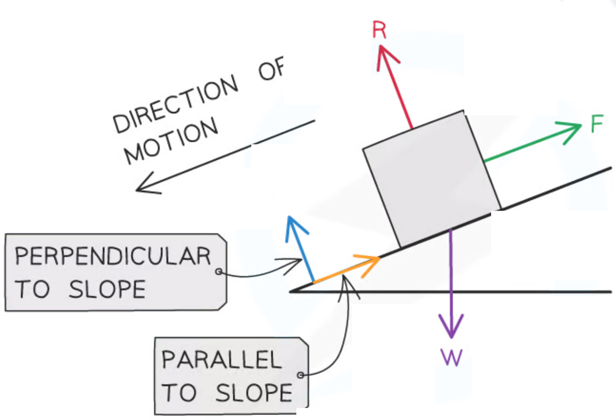
A picture frame hanging from a nail:



- The size of the arrows should be such that the 3 forces would make a closed triangle as they are **balanced**

Part (b)

A box sliding down a slope:



- There are three forces acting on the box:
  - The **normal contact force**, *R*, acts perpendicular to the slope
  - **Friction**, *F*, acts parallel to the slope and in the opposite direction to the direction of motion
  - **Weight**, *W*, acts down towards the Earth

Exam Tip

When labeling force vectors, it is important to use conventional and appropriate naming or symbols such as:

- ***w*** or Weight force or ***mg***
- ***N*** *or* ***R*** for normal reaction force (depending on your local context either of these could be acceptable)

Getting used to this notation will make labelling free-body diagrams automatic - leaving you time to focus on the Physics rather than the process. This is crucial as a good diagram will show you exactly what to include in your calculations.

Newton's First Law of Motion

- Newton's First Law states:

A body will remain at rest or move with constant velocity unless acted on by a resultant force

- The law is used to explain why things move with a **constant** (or **uniform**) **velocity**
- If the **forces** acting on an object are **balanced**, then the **resultant force** is **zero**
  - The forces to the left = the forces to the right
  - The forces up = the forces down
- The velocity (i.e. speed and direction) **can only change** if a **resultant force** acts on the object

Worked Example

If there are no external forces acting on the car, other than friction, and it is moving at a constant velocity, what is the value of the frictional force F?

Step 1: Identify the prompt (or 'clue') word in the question to choose which rules to apply

- The clue in this question is 'constant velocity'
- This means that forces are perfectly balanced in every direction

Step 2: Calculate the missing force

- For constant velocity sum of forces,

$$\Sigma F = 0$$

$$F = D$$

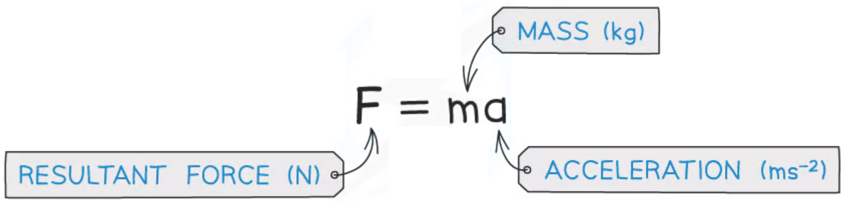
Step 3: Write the answer including units

$$F = 6\ kN$$

Newton's Second Law of Motion

- Newton's Second Law states that:

    The acceleration of an object with constant mass is directly proportional to the resultant force on it



- An unbalanced force on a body means it experiences a resultant force
    - If the resultant force is **along the direction of motion**, the body will speed up (**accelerate**) or slow down (**decelerate**)
    - If the resultant force is at an **angle**, the body will change **direction**
- Since resultant force is being considered, the equation is often written

$$\Sigma F = ma$$

- The use of Σ (Sigma) signifies that F is the **sum** of the forces rather than just one force acting alone

Worked Example

An object with a mass of 750 g accelerates in a straight line at 11 m s^{-2}.

Determine the resultant force acting on the object.

Step 1: Write down the known quantities

- Mass, m = 750 g = 0.750 kg
- Acceleration, a = 11 m s^{-2}.

Step 2: Identify the equation needed and substitute in the values

$$\Sigma F = ma = 0.75 \times 11 = 8.25$$

Step 3: Write the final answer using the correct significant figures and units

$$\Sigma F = 8.3 \text{ N}$$

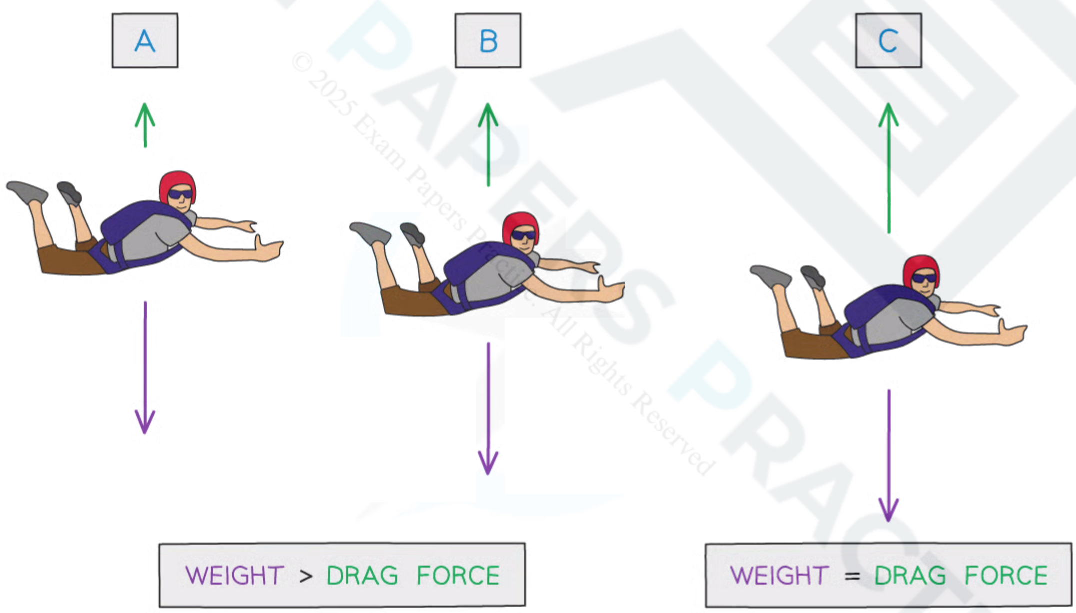
Terminal Velocity

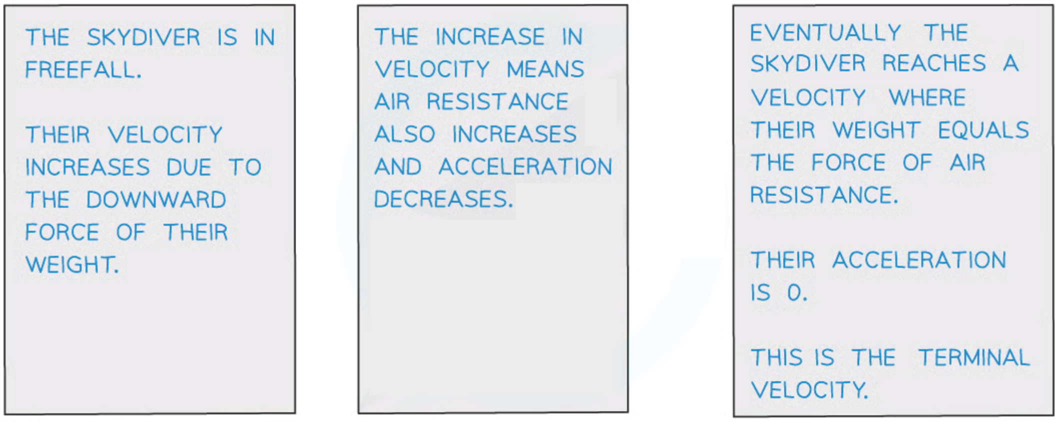
- In reality moving bodies always do have **some resultant force** acting on them, for example, due to friction caused by;
  - Movement through air or water (drag forces)
  - Movement across a surface (such as tyres on the road)
  - Moving parts within the object itself (such as the engine of a car, or the bearings in a wheel)

- When the drag forces become equal to the driving force the velocity reaches its **maximum** value and the object is said to be moving at **terminal velocity**

  - 'Terminal' means final, meaning there can be no more increases in velocity after it has been reached

- Terminal velocity is reached when the forces in the direction of motion are balanced by the forces opposing motion
  - This is often used in relation to the case of a skydiver falling in freefall, although it can apply to **any** situation where drag forces apply
- In the case of the skydiver the force in the direction of motion is their weight, which remains constant
  - As the skydiver accelerates air resistance increases, until the opposing forces balance





Worked Example

Suggest two ways in which a designer could increase the maximum velocity of a car.

Step 1: Identify exactly what the question is asking for

- The question asks about 'maximum' (terminal) velocity, this occurs when all forces are balanced
- To increase the terminal velocity the car either needs to
  - Increase the forces in the forwards direction, or
  - Decrease the forces in the backwards direction (the ones which are opposing motion)

Step 2: Write down the forces affecting the motion

- In the forwards direction the only force is the thrust from the engine
- The forces opposing motion include:
  - Friction between the tyres and the road
  - Air resistance
  - Friction in the moving parts of the engine
  - Friction in the moving parts of the wheel system (bearings)

Step 3: Suggest changes the designer could make

- Increase the forwards thrust by increasing power from the engine
- Reduce opposing forces by
  - Using tyres with less grip
  - Making the car body more streamlined
  - Reducing friction in the engine (using better oil for example)
  - Reduce friction in the moving parts with smoother surfaces or better lubricants

Exam Tip

The direction you consider positive is your choice, as long as the signs of the numbers (positive or negative) are consistent throughout your answer. A clear, labelled diagram will tell both you and the Examiner which direction is positive, so nothing gets mixed up.

Having said that, there is a general rule to consider the direction the object is initially travelling in as positive. Therefore all vectors in the direction of motion will be positive and opposing vectors, such as drag forces, will be negative.

If you do get an acceleration in your answer with a negative sign, you have found that the object is slowing down. This is the Super Power of Physics equations!

Mass, Weight & Gravitational Field Strength

Mass

- **Mass** is the measure of the amount of matter in an object
    - Consequently, this is the property of an object that resists change in motion
- The greater the mass of a body, the smaller the change produced by an applied force
    - The SI unit for mass is the **kilogram** (kg)

Weight

- **Weight** is the force a body experiences due to being in a gravitational field, it is given by the equation

$$W = mg$$

- Where
    - m = mass (kg)
    - g = gravitational field strength (N kg^{-1})

Gravitational Field Strength

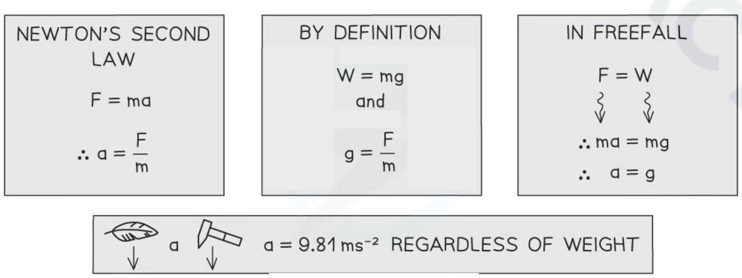
- Gravitational field strength is the force per kilogram that acts on an object, it is found using the equation

$$g = \frac{F}{m}$$

- Where
    - F = weight (N)
    - m = mass (kg)
- On the Earth's surface the average gravitational field strength = **9.81 N kg^{-1}**

Constant Acceleration in Freefall

- **Freefall** is used to describe falling objects where the only force is weight
    - Drag forces are ignored
    - By considering freefall we can see that all objects must fall with exactly the same value for acceleration, regardless of their mass or weight



Acceleration of Freefall Using Electromagnets & Light Gates

Aim of the Investigations

- The overall aim of these investigations is to calculate the value of the acceleration due to gravity, *g*
- The first two experiments both use the method of dropping an object and either timing its fall, or finding the final velocity
  - Both use the SUVAT equations to produce a straight line graph
- The third experiment, using the **ramp and trolley**, is based on the inclined ramp experiment done by Galileo when he proved that all objects fall at the same rate, regardless of weight

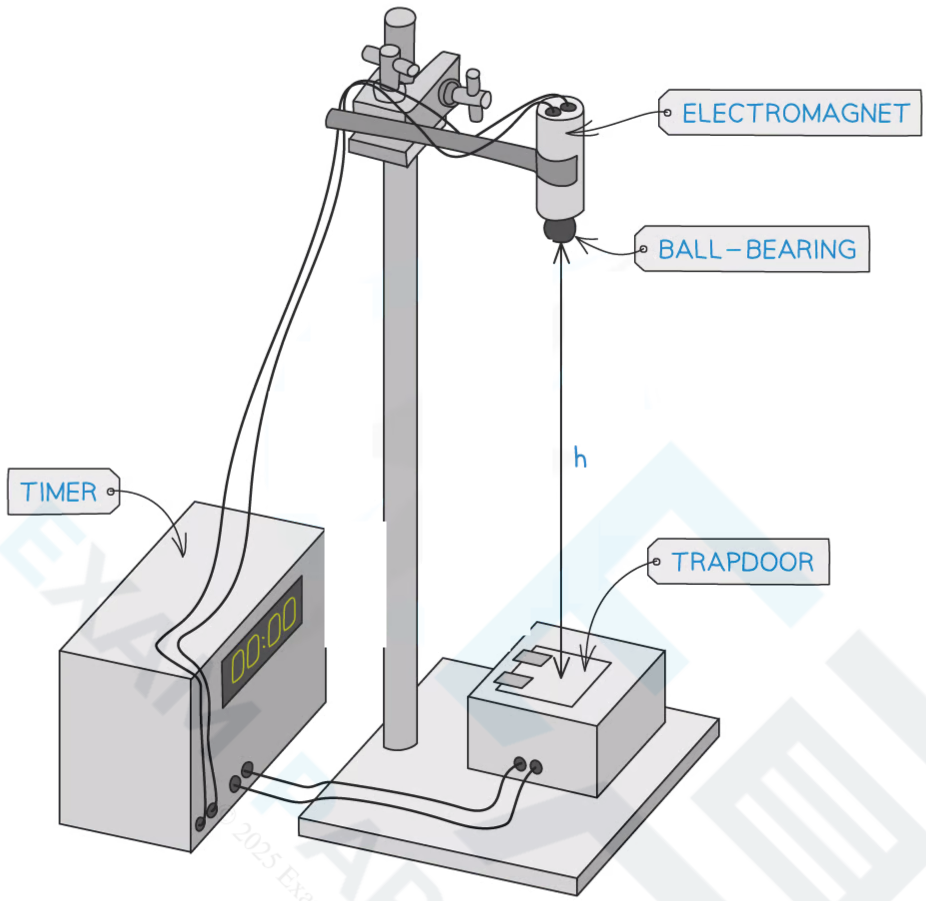
Electromagnet Method

Variables

- Independent variable = height, *h*
- Dependent variable = time, *t*
- Control variables:
  - Same object being dropped
  - Same electromagnet and trap door switching system

Apparatus

- Metre rule, ball bearing, electromagnet, electronic timer, trapdoor, plumb line



Apparatus used to measure g using the electromagnet method

- Resolution of measuring equipment:
  - Metre ruler = 1 mm
  - Timer = 0.01 s

Method

- By using the plumb line to find the vertical drop, position the trap door switch directly underneath the electromagnet.
- Check that the ball bearing triggers both the trap door switch and the timer when it is released.
- When the equipment is set up correctly;
  - As the current to the magnet switches off, the ball drops and the timer starts
  - When the ball hits the trapdoor, the timer stops
- The reading on the timer indicates the time it takes for the ball to fall a distance, *h*
- Measure the distance from the **bottom** of the ball bearing to the trap door switch with a metre ruler and record this distance as height, *h*
- Increase *h* (eg. by 5 cm) and repeat the experiment. At least 5 – 10 values for *h* should be used
- Repeat this method at least 3 times for each value of *h* and calculate an average *t* for each

Table of Results

HEIGHT h/m	TIME t_1/s	TIME t_2/s	TIME t_3/s	AVERAGE TIME t/s
0.10				
0.15				
0.20				
0.25				
0.30				
0.35				

Analysis of Results

- The acceleration is found by using one of the SUVAT equations and rearranging it to create a straight line graph (y = mx + c)
- The known quantities are
  - Displacement $s = h$
  - Time taken = t
  - Initial velocity = u
  - Acceleration $a = g$
- The missing SUVAT value is final velocity, v
- Therefore use

$$s = ut + \frac{1}{2}at^2$$

- Replace a with g and s with h and then rearrange to fit the equation of a straight line

$$h = \frac{1}{2}gt^2 \text{ (since initial velocity, u = 0)}$$

- The above equation shows that if h is plotted on the y-axis and t^2 on the x-axis the graph will produce a straight line with gradient = $\frac{1}{2}g$

Evaluating the experiment

Systematic Errors:

- Residue magnetism after the electromagnet is switched off may cause t to be recorded as longer than it should be

Random Errors:

- Large uncertainty in *h* from using a metre rule with a precision of 1 mm
- Parallax error from reading *h*
- The ball may not fall accurately down the centre of the trap door
- Random errors are reduced through repeating the experiment for each value of *h* at least 3–5 times and finding an average time, *t*

Safety Considerations

- The electromagnetic requires current
  - Care must be taken to not have any water near it
  - To reduce the risk of electrocution, only switch on the current to the electromagnet once everything is set up
- A cushion or a soft surface must be used to catch the ball-bearing so it doesn't roll off / damage the surface
- The tall clamp stand needs to be attached to a surface with a G clamp to keep it stable

Card and Light Gates Method

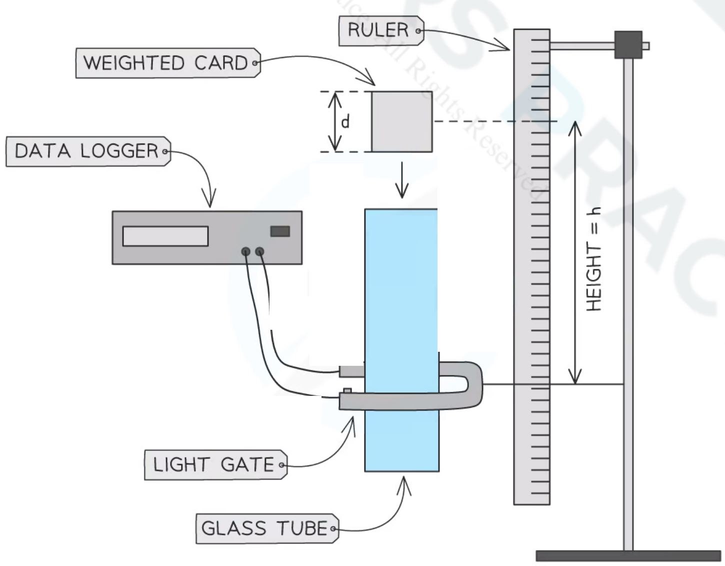
Variables

- Independent variable = height, *h*
- Dependent variable = final velocity, *v*
- Control variables:
  - Same card being dropped
  - All other equipment is the same

Apparatus

- Metre rule, clear tube with large enough diameter for card to fall cleanly through it, card, blu-tack, light gate, data logger, plumb line

Method



- Clamp the clear tube vertically using the plumb line as a guide

- Attach the light gate about 20 cm above the bench
- Clamp the metre ruler vertically next to the tube so that the vertical distance from the top of the tube to the light gate can be accurately measured
- Record the distance between the light gate and the top of the tube as height, *h*
- Cut a piece of card to approximately 10 cm, measure this length precisely and enter it into the data logger as the distance
  - Weight the card slightly at one end (a large paperclip or small pieces of blu-tack can be used)
- Hold the card at the top of the tube and release it so that it falls inside the tube
  - The data logger will record velocity
- Repeat this measurement from the same height two more times
- Move the light gate up by 5 cm, record the new height, h, and drop the card three more times, recording the velocity each time
- Repeat for five more values of height

h / m	V_1 / m s^{-1}	V_2 / m s^{-1}	V_3 / m s^{-1}	V_{av} / m s^{-1}

Analysis of Results

- The acceleration is found by using one of the SUVAT equations and rearranging it to create a straight line graph (y = mx + c)
- The known quantities are
  - Displacement s = *h*
  - Initial velocity = *u*
  - Final velocity = *v*
  - Acceleration *a* = *g*
- The missing SUVAT value is time, *t*
- Therefore use

$$v^2 = u^2 + 2as$$

- Replace *a* with *g* and *s* with *h* and then rearrange to fit the equation of a straight line

$$v^2 = 2gh \text{ (since initial velocity, u = 0)}$$

- The above equation shows that if v^2 is plotted on the y-axis and 2*h* on the x-axis the graph will produce a straight line with gradient = *g*

Evaluating the experiment

Systematic Errors:

- The metre ruler needs to be fixed vertically and close to the tube
- All height measurements are taken at eye level to avoid parallax errors

Random Errors:

- Large uncertainty in h from using a metre rule with a precision of 1 mm
- Parallax error from reading h
- The card may fall against the sides of the tube, slowing it down
- Dropping the card from the top of the tube can introduce parallax errors
- Random errors are reduced through repeating the experiment for each value of h at least 3–5 times and finding an average time, t

Safety Considerations

- The tall clamp stand needs to be attached to a surface with a G clamp to keep it stable

Acceleration of Freefall Using a Ramp & Trolley

- This method of finding acceleration due to freefall uses the SUVAT equations, but applies them to a trolley rolling down an inclined ramp.

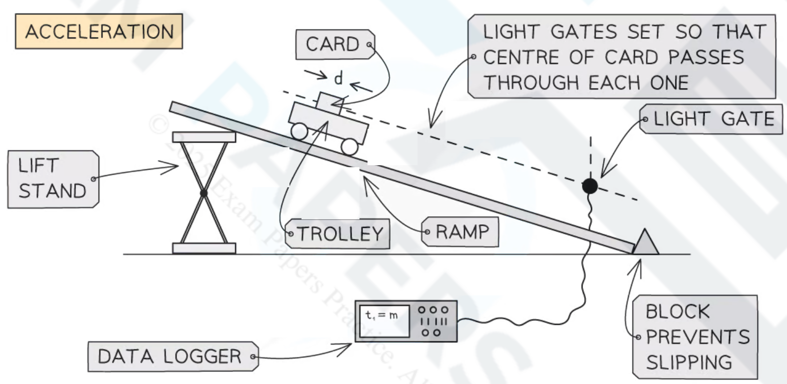
Variables

- Independent variable = velocity of the trolley, *v*
- Dependent variable = time, *t*
- Control variables:
  - Height of ramp must be constant
  - Same trolley being used

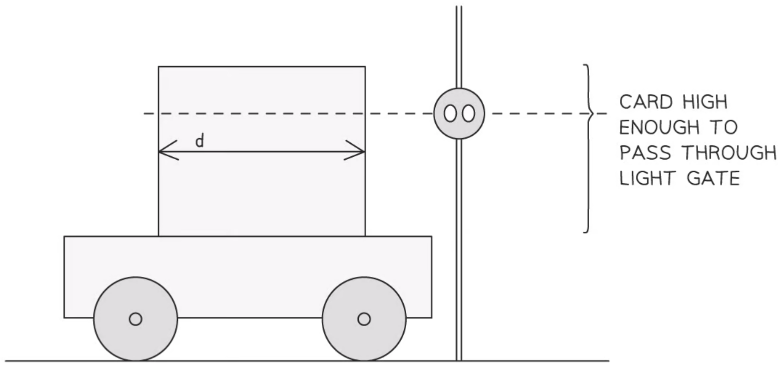
Apparatus

- Inclined ramp
- Trolley with ≅10 cm card attached
- Light gate and computer or datalogger
- Stopwatch
- Block to prevent slipping

Method



- Carefully cut a piece of card so that it is between 5 – 10 cm in length, and has a height which can break the beam of a light gate as the trolley passes through.
- Measure and record the length, d, of the card
  - Record this in the datalogging software
- Attach the card to the trolley and roll the trolley past the light gate checking the beam is broken by the card
  - Adjust the height of the light gate as needed.



- Start the timing on the software, making sure it is set to record instantaneous velocity
- Release the trolley and simultaneously start the stopwatch.
- As the card passes the light gate stops the stopwatch
  - record the time, *t*
- Repeat procedure 3 times, discard anomalies and calculate mean *t*
  - This reduces errors
- Repeat the procedure at least 5 times, varying the height the trolley is dropped from for each reading
  - This causes a variation in v which is recorded by the light gate, and t which is recorded using the stopwatch.

AS RECORDED BY THE LIGHT GATE AND DATA LOGGING SOFTWARE

VELOCITY / m s^{-1}	TIME t_1/s	TIME t_2/s	TIME t_3/s	AVERAGE TIME t/s

Analysis of Results

- The acceleration is found by using one of the SUVAT equations and rearranging it to create a straight line graph (y = mx + c)
- The known quantities are
  - Time taken, *t* = average *t*
  - Initial velocity, *u* = 0 (the trolley starts from rest)
  - Final velocity *v* = *v* (recorded by the light-gate)
  - Acceleration *a* = *g*
- The missing SUVAT value is displacement, *s*
  - Therefore use

$$v = u + at$$

- This matches the equation of a straight line
  - y = velocity, *v*
  - x = average time, *t*
  - gradient = acceleration, *a*
  - y-intercept = initial velocity, *u*
- Plot a graph of v against average t
  - The gradient will be the acceleration
  - This acceleration is provided by gravity, and so will give a value for *g*

Evaluating the experiment

Systematic Errors:

- Make sure for each repeat reading the trolley is released from the same point
- The card should be measured carefully so value *d* is accurate

Random Errors:

- Large uncertainty in *d* from using a ruler with a precision of 1 mm
- Reaction time when starting and stopping the stopwatch
  - Random errors are reduced through repeating the experiment for each value of *v* at least 3 times and finding an average time, *t*
- The card may hit the light gate
  - Discard a result where this occurs
- They trolley may not travel straight down the ramp
  - Discard a result where this occurs

Safety Considerations

- The trolley may fly off the end of the ramp
  - Use a block or tray at the bottom of the ramp to prevent this

Exam Tip

This experiment can be modified by using the light gate to record time through the gate.

You can then use the time from the light gate to calculate the velocity, *v* of the trolley by calculating with *v* = *d*/*t* where d is the length of the card and the time is the time on the light gate.

However, most light-gate software should allow you to eliminate this step.

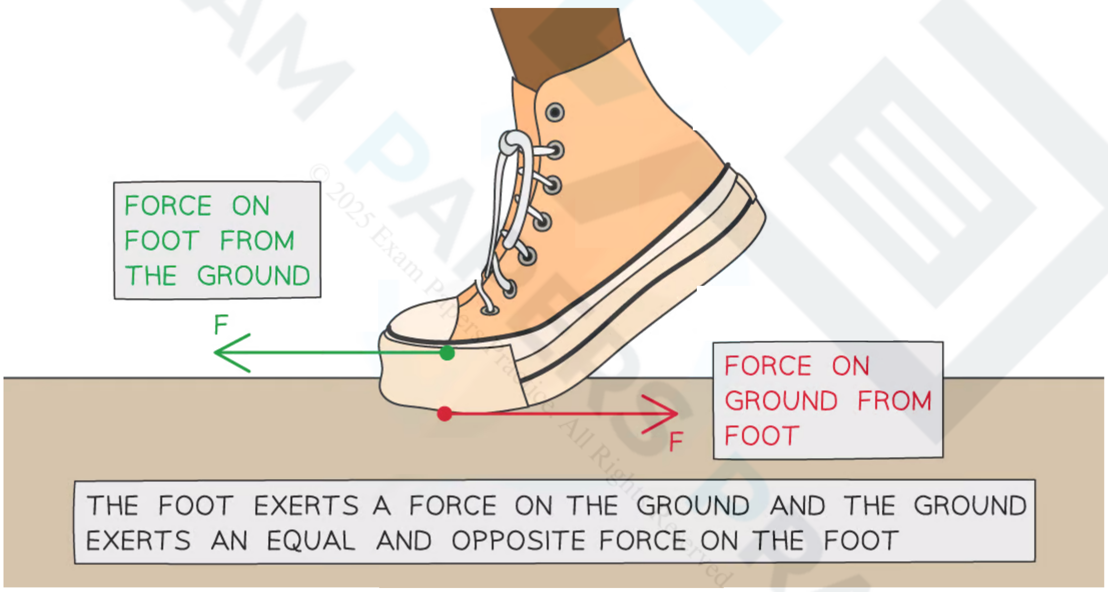
Newton's Third Law of Motion

- Newton's **third law of motion** states:

  Whenever two bodies interact, the forces they exert on each other are equal in size, act in opposite directions, and are of the same type

- Newton's third law explains the following important principles about forces:
  - All forces arise in **pairs** – if object A exerts a force on object B, then object B exerts an **equal** and **opposite** force on object A
  - Force pairs are of the **same type** – for example, if object A exerts a **gravitational force** on object B, then object B exerts an equal and opposite **gravitational force** on object A

- Newton's third law explains the forces that enable someone to walk
- The image below shows an example of a pair of equal and opposite forces acting on two objects (the ground and a foot):



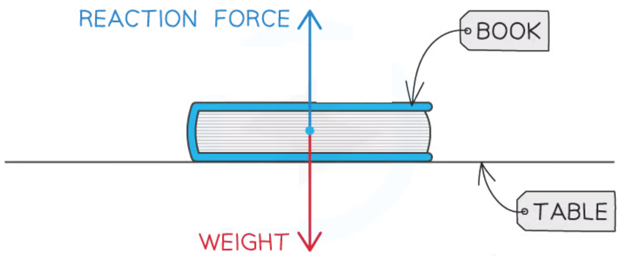
Newton's Third Law: The foot pushes the ground backwards, and the ground pushes the foot forwards

- One force is from the foot that pushes the ground backwards
- The other is an **equal and opposite** force from the ground that pushes the foot forwards

Worked Example

A physics textbook is at rest on a dining room table.

Eugene draws a free body force diagram for the book and labels the forces acting on it.



Eugene says the diagram is an example of Newton's third law of motion. William disagrees with Eugene and says the diagram is an example of Newton's first law of motion.By referring to the free-body force diagram, state and explain who is correct.

Step 1: State Newton's first law of motion

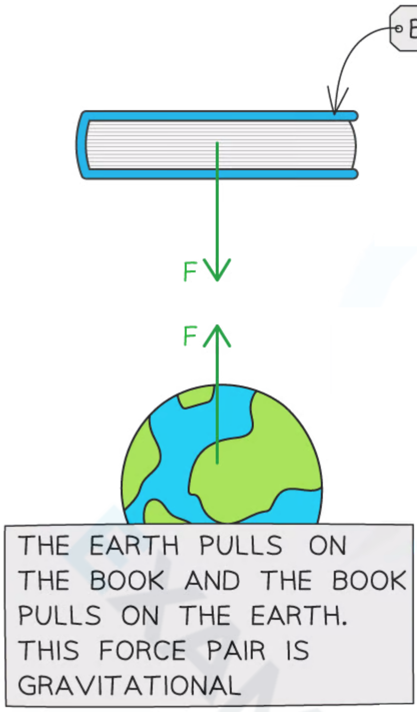
- Objects will remain at rest, or move with a constant velocity unless acted on by a resultant force

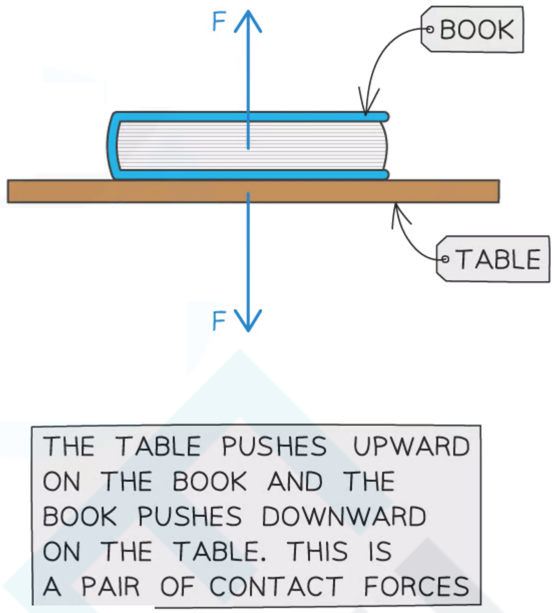
Step 2: State Newton's third law of motion

- Whenever two bodies interact, the forces they exert on each other are equal and opposite

Step 3: Check if the diagram satisfies the two conditions for identifying Newton's third law

- In each case, Newton's third law identifies pairs of equal and opposite forces, of the same type, acting on two different objects
- The diagram only involves **one object**
- Furthermore, the forces acting on the object are different types of force - one is a **contact force** (from the table) and the other is a **gravitational force** on the book (from the Earth) - its **weight**
- The image below shows how to apply Newton's third law correctly in this case, considering the pairs of forces acting:





Step 4: Conclude which person is correct

- In this case, **William is correct**
- The free-body force diagram in the question is an example of **Newton's first law**
- The book is **at rest** because the two forces acting on it are **balanced** - i.e. there is no **resultant force**

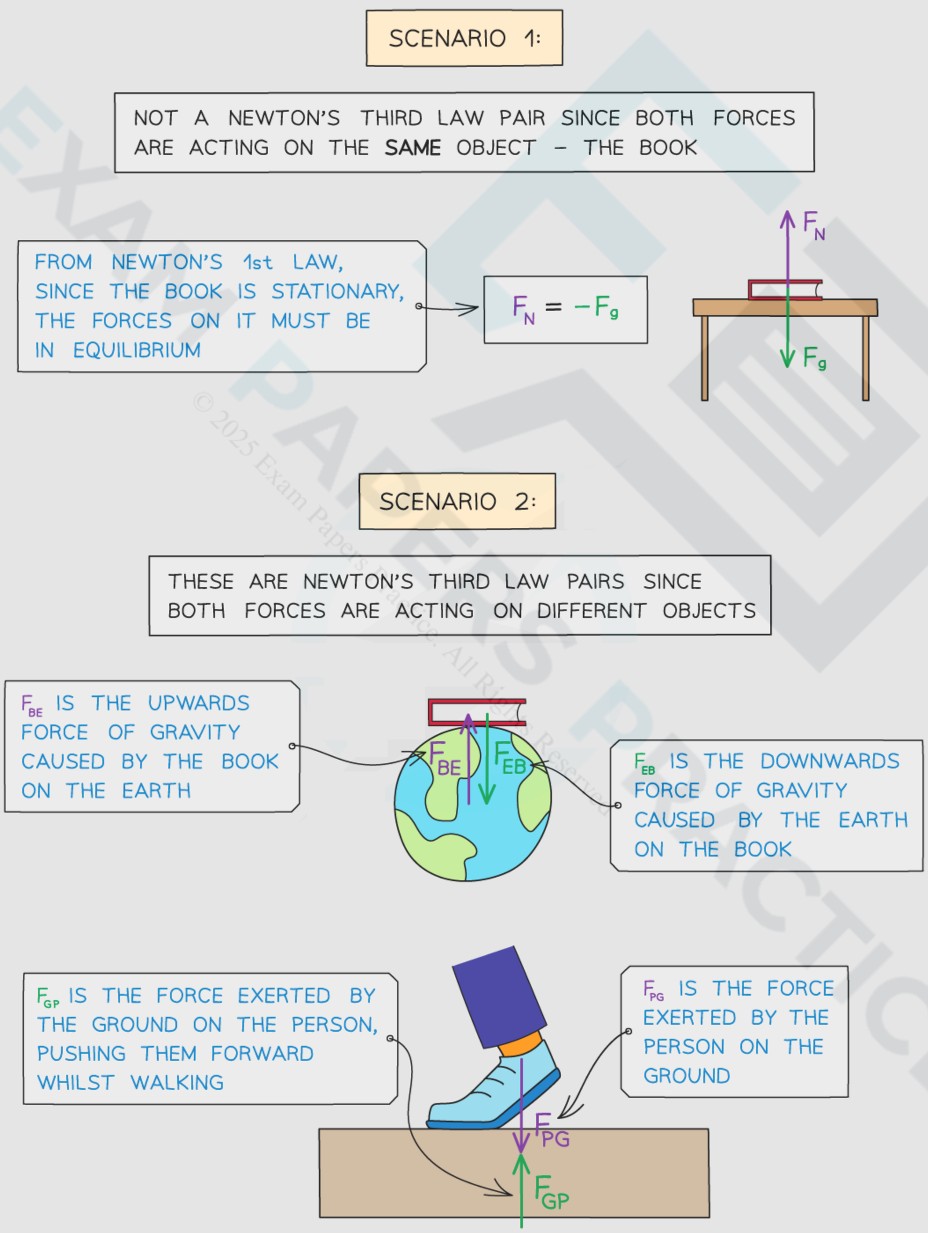
Exam Tip

Remember that pairs of equal and opposite forces in Newton's third law act on **two different objects.** It's a really common mistake to confuse Newton's third law with Newton's first law. That also means it's a really common exam question, which often uses a version of thee example below.

Applying this check will help you distinguish between them.

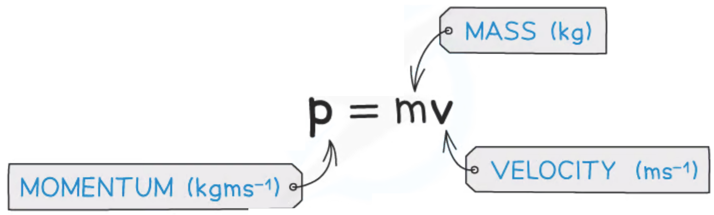
Newton's first law involves forces acting on a **single** object.

These differences are shown in Scenario 1 (Newton's first law) vs. Scenario 2 (Newton's third law)



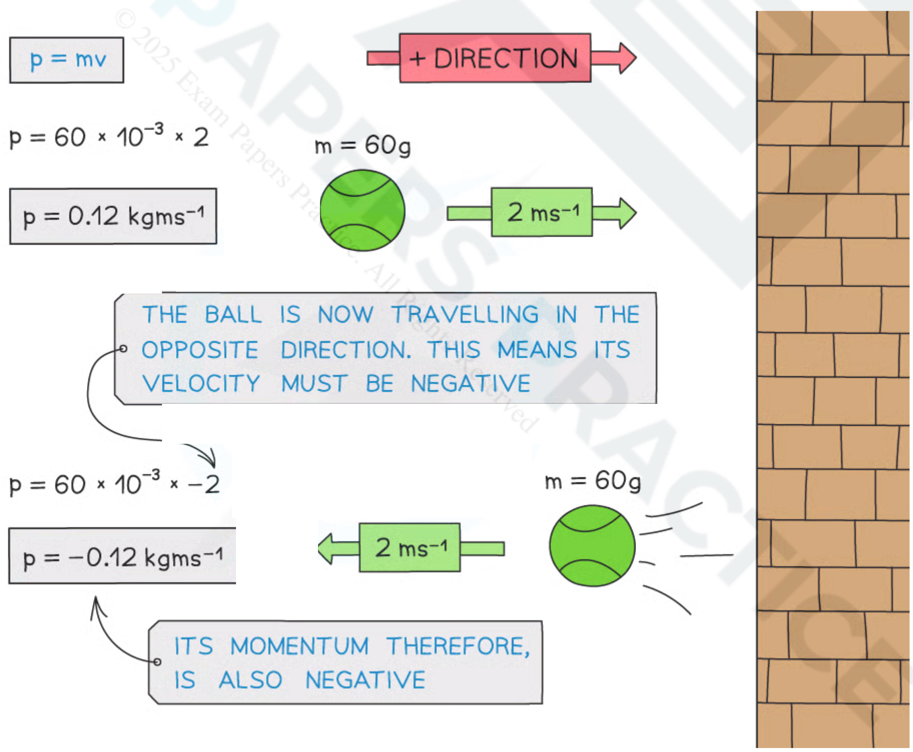
Defining Momentum

- Linear momentum (*p*) is defined as the product of mass and velocity



Momentum is the product of mass and velocity

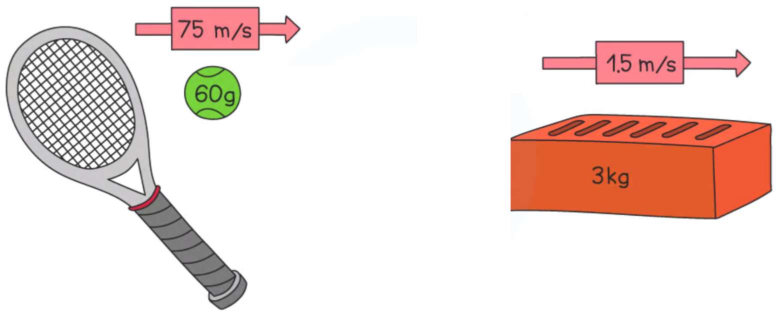
- Momentum is a vector quantity - it has both a magnitude and a direction
- This means it can have a negative or positive value
  - If an object travelling to the right has positive momentum, an object travelling to the left (in the opposite direction) has a negative momentum
- The SI unit for momentum is **kg m s**$^{-1}$



When the ball is travelling in the opposite direction, its velocity is negative. Since momentum = mass × velocity, its momentum is also negative

Worked Example

Which object has the most momentum?



MOMENTUM = MASS × VELOCITY

MOMENTUM = 0.06 kg × 75 m/s
= 4.5 kg m/s

MOMENTUM = MASS × VELOCITY

MOMENTUM = 3 kg × 1.5 m/s
= 4.5 kg m/s

- Both the tennis ball and the brick have the same momentum
- Even though the brick is much heavier than the ball, the ball is travelling much faster than the brick
- This means that on impact, they would both exert a similar force (depending on the time it takes for each to come to rest)

Exam Tip

Since momentum is in **kg m s^{-1}**:

- If the mass is given in grams, make sure to convert to kg by dividing the value by 1000.
- If the velocity is given in km s^{-1}, make sure to convert to m s^{-1} by multiplying the value by 1000

The direction you consider positive is your choice, as long the signs of the numbers (positive or negative) are consistent throughout the question.

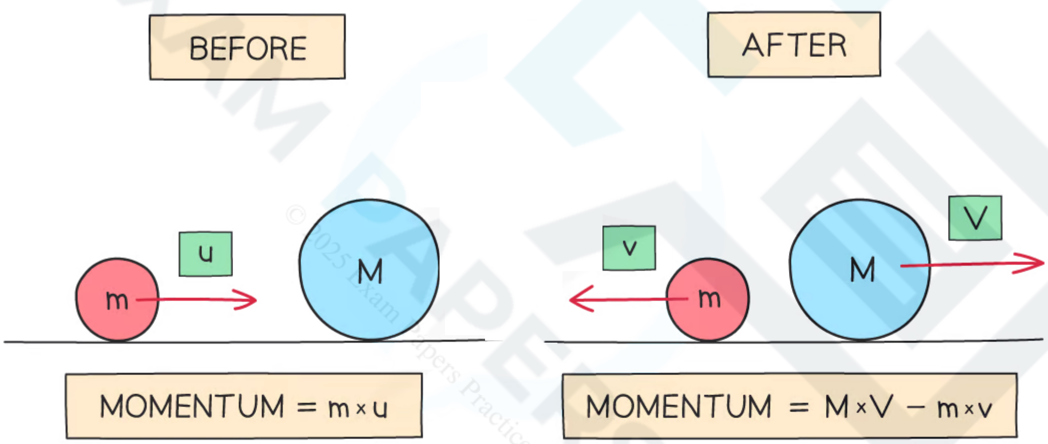
Sketching a diagram **which includes the signs on positive and negative values** will help you avoid mistakes when calculating

The Principle of Conservation of Linear Momentum

- The principle of conservation of momentum states that in a closed system, the total momentum before an event is equal to the total momentum after the event
- Momentum is **always** conserved in collisions where no external forces act
- This is usually written as:

Total momentum before a collision = Total momentum after a collision

- Since momentum is a **vector** quantity, a system of objects moving in opposite directions can have an overall momentum of 0
    - This applies to objects moving towards each other or away from each other
- The diagram below shows two masses *m* with velocity *u* and *M* at rest (M has zero velocity)



The momentum of a system before and after a collision

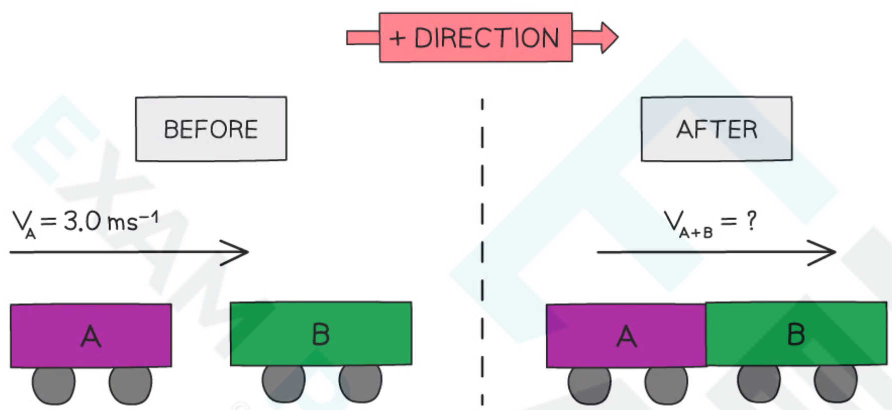
- Before the collision:
    - The momentum is only of mass *m* which is moving
    - If the right is taken as the positive direction, the total momentum of the system is $\boldsymbol{m \times u}$
- After the collision:
    - Mass *M* also now has momentum
    - The velocity of *m* is now $-v$ (since it is now travelling to the left) and the velocity of *M* is *V*
    - The total momentum is now the momentum of *M* + momentum of *m*
    - This is $(M \times V) + (m \times -v)$ or $\boldsymbol{(M \times V) - (m \times v)}$

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** whilst travelling at

3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. On impact, the trolleys stick together.

Using the conversation of momentum, calculate the common velocity of both trolleys after the collision.



MOMENTUM BEFORE $= (M_A \times V_A) + (M_B \times V_B)$

$= (0.8 \text{ kg} \times 3.0 \text{ ms}^{-1}) + 0$

$= 2.4 \text{ kgms}^{-1}$

SINCE TROLLEY B IS STATIONARY, V = 0 THEREFORE ITS MOMENTUM IS 0

MOMENTUM AFTER $= (M_A + M_B) \times V_{A+B}$

$= (0.8 \text{ kg} + 1.60 \text{ kg}) \times V_{A+B}$

$= 2.4 \text{ kg} \times V_{A+B}$

TROLLEY B HAS TWICE THE MASS OF TROLLEY A

THE PRINCIPLE OF CONVERSATION OF MOMENTUM STATES THAT THE TOTAL MOMENTUM OF A SYSTEM REMAINS CONSTANT PROVIDED NO EXTERNAL FORCE ACTS ON IT

MOMENTUM BEFORE = MOMENTUM AFTER

$2.4 \text{ kgms}^{-1} = 2.4 \text{ kg} \times V_{A+B}$

$$V_{A+B} = \frac{2.4 \text{ kgms}^{-1}}{2.4 \text{ kg}}$$

REARRANGING FOR V_{A+B}

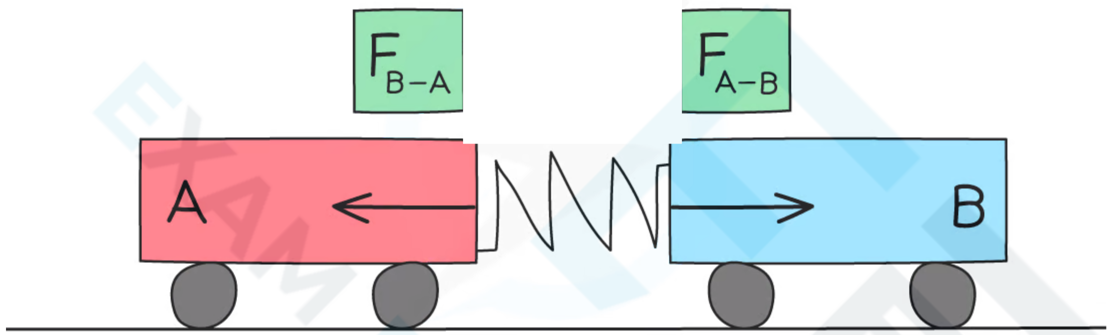
$V_{A+B} = 1.0 \text{ ms}^{-1}$

Conservation of Linear Momentum & Newton's Third Law

- Newton's third law of motion states:

  Whenever two bodies interact, the forces they exert on each other are equal and opposite

- This means:
  - When one object exerts a force on another object, the second object will exert an equal force on the first object in the opposite direction
  - When two objects collide, both objects will react, generally causing one object to speed up (gain momentum) and the other object to slow down (lose momentum)



Newton's third law can be applied to collisions

- Consider the collision between two trolleys, **A** and **B**:
  - When trolley **A** exerts a force on trolley **B**, trolley **B** will exert an equal force on trolley **A** in the opposite direction

- In this case:

$$F_{B-A} = -F_{A-B}$$

- While the forces are equal in magnitude and opposite in direction, the accelerations of the objects are not necessarily equal in magnitude
- From Newton's second law, acceleration depends upon both force and mass, this means:
  - For objects of equal mass, they will have equal accelerations
  - For objects of unequal mass, they will have unequal accelerations

Exam Tip

Momentum questions are often very long and wordy. Even if you are given a diagram, make a quick sketch representing all the bodies as point masses. Mark the velocities with arrows and include positive and negative signs.

The Maths of momentum is straightforward, so as long as you have your vector directions clear in your mind, nothing can go wrong!

Moments

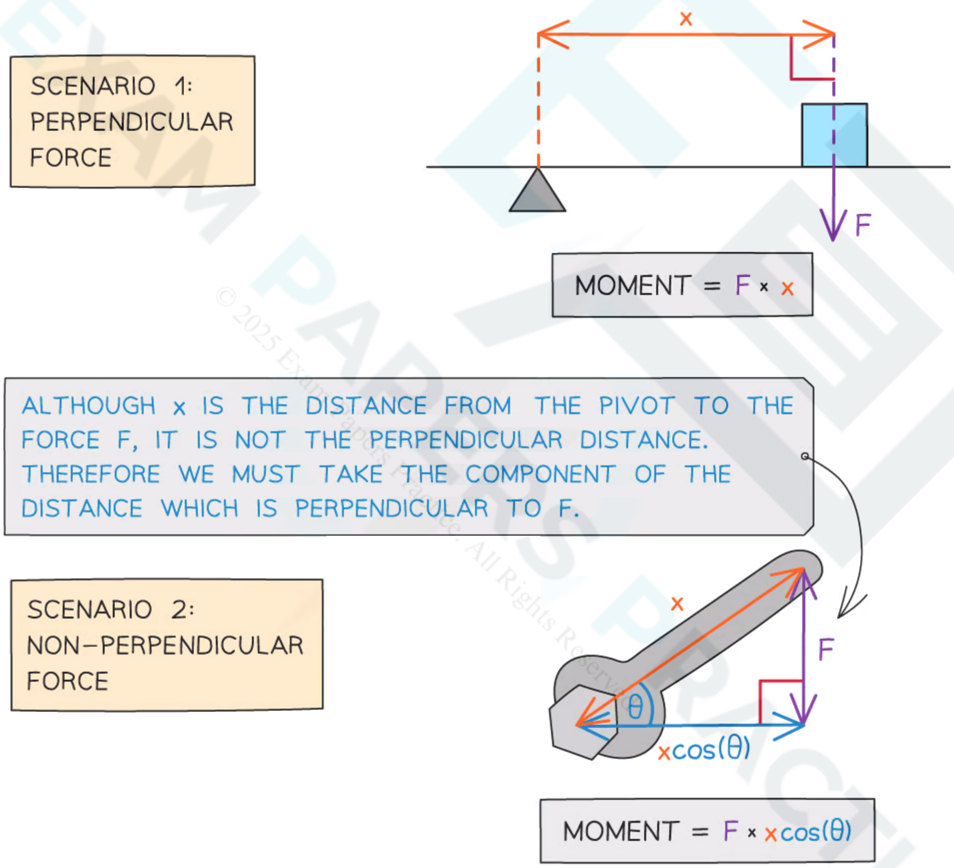
2.15 Moments

Calculating the Moment of a Force

- A moment is the **turning effect of a force**
- Moments occur when forces cause objects to **rotate** about some pivot
- The moment of a force is given by

Moment (**N m**) = Force (**N**) × perpendicular distance from the pivot (**m**)

- The SI unit for the moment is Newton metres (**N m**). This may also be Newton centimetres (**N cm**) depending on the units given for the distance



The force might not always be perpendicular to the distance

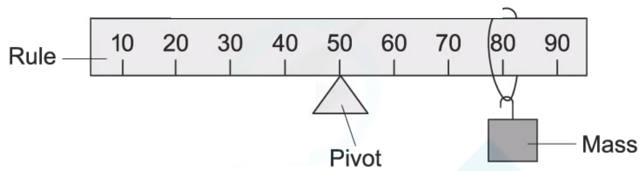
- An example of moments in everyday life is opening a door
- The door handle is placed on the other side of the door to the hinge (the pivot) to **maximise** the distance for a given force and therefore provides a greater moment (turning force)
    - This makes it easier to push or pull

Worked Example

A uniform metre rule is pivoted at the 50 cm mark.

A 0.5 kg weight is suspended at the 80 cm mark, causing the rule to rotate about the pivot.

Assuming the weight of the rule is negligible, what is the turning moment about the pivot?



STEP 1

MOMENT = FORCE × PERPENDICULAR DISTANCE FROM THE PIVOT

STEP 2

IDENTIFY THE FORCE REQUIRED

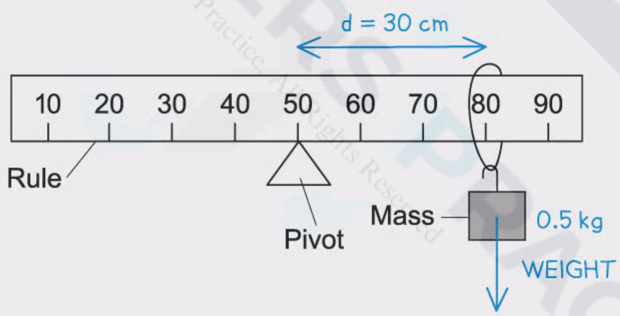
THE ONLY FORCE IS THE WEIGHT OF THE MASS ACTING DOWNWARDS

WEIGHT = mg = 0.5 × 9.81 = 4.905 N = 5 N

STEP 3

IDENTIFY THE PERPENDICULAR DISTANCE

FROM THE RULE: 80 cm − 50 cm = 30 cm



STEP 4

SUBSTITUTE VALUES INTO MOMENT EQUATION

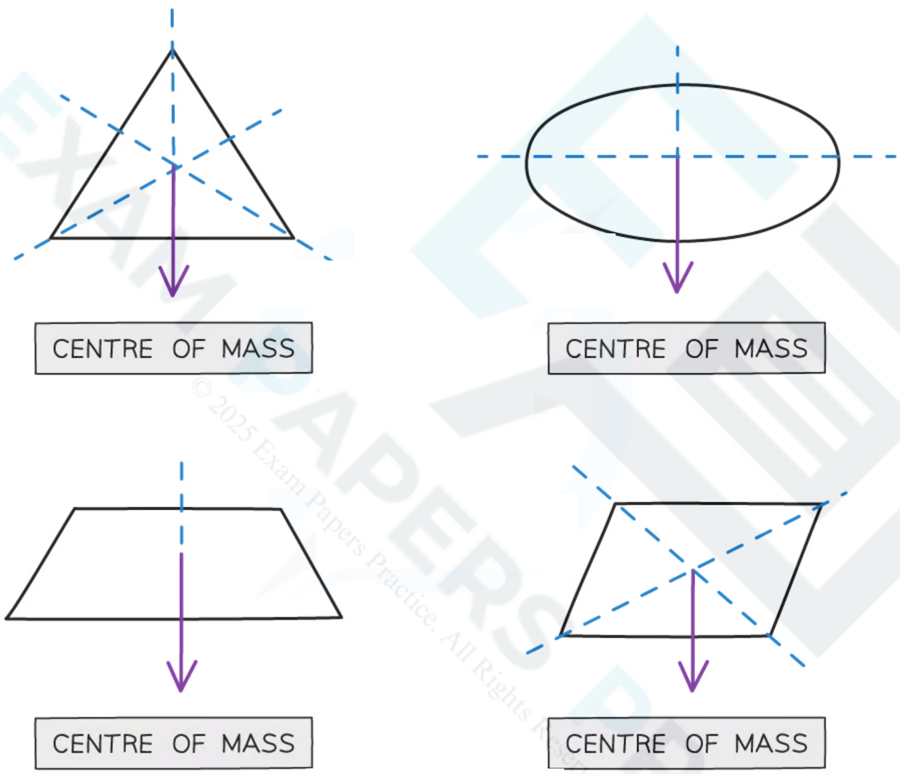
MOMENT = 5 N × 30 cm = 150 Ncm

Exam Tip

If not already given, drawing all the forces on an object in the diagram will help you see which ones are perpendicular to the distance from the pivot. Not all the forces will provide a turning effect and it is not unusual for a question to provide more forces than required to throw you off!

Centre of Gravity

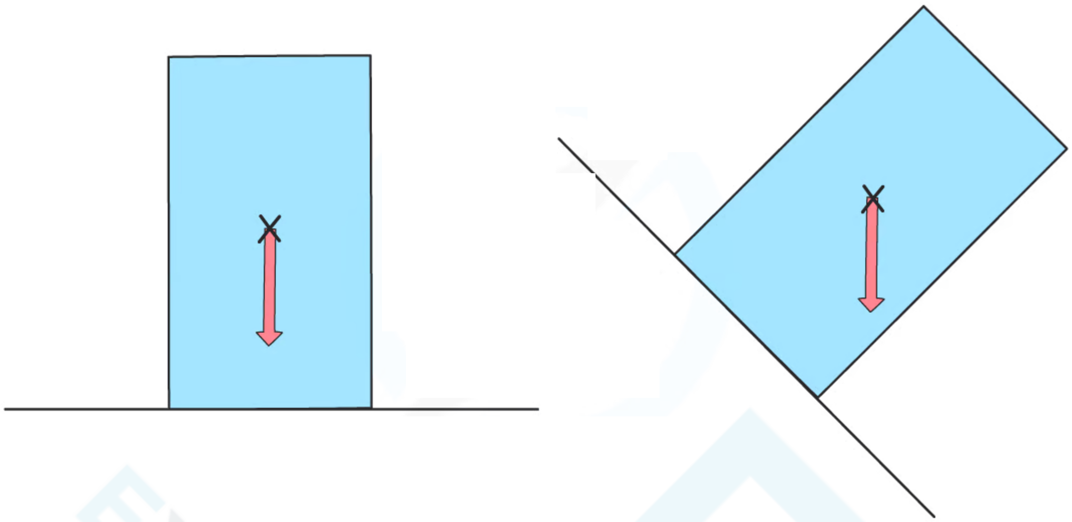
- The centre of gravity (sometimes called the centre of mass) of an object is **the point through which all the weight can be considered to act**
- The position of the centre of gravity of uniform regular solid is at its centre
  - For example, for a person standing upright, their centre of gravity is roughly in the middle of the body behind the navel, and for a sphere, it is at the centre
- For symmetrical objects with uniform density, the centre of gravity is located at the **point of symmetry**



The centre of mass of a shape can be found by symmetry

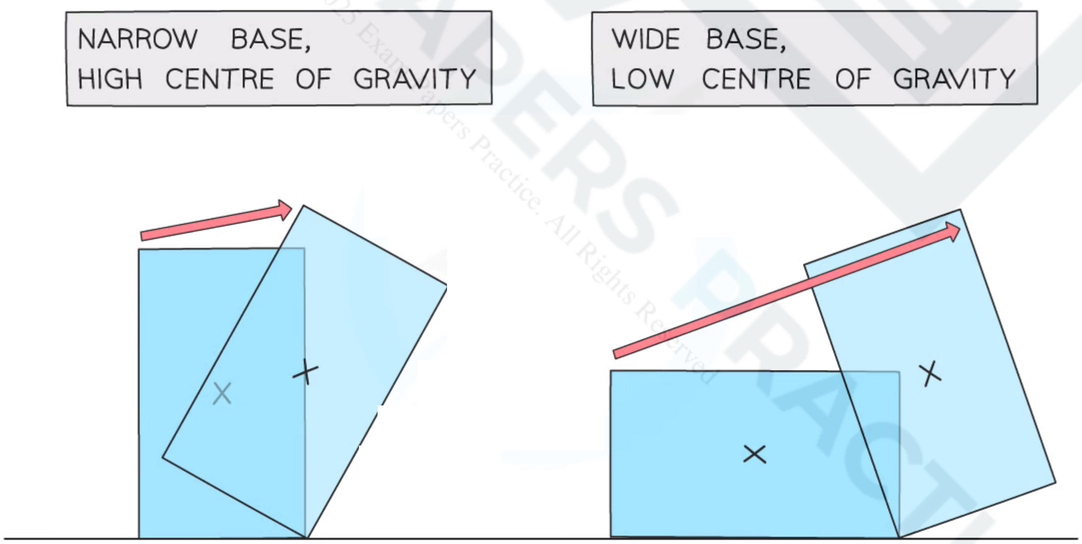
Stability

- The position of the centre of gravity of an object affects its stability
- An object is stable when its centre of gravity lies above its base



The object on the right will topple, as its centre of gravity is no longer over its base

- The **wider** base an object has, the **lower** its centre of gravity and it is more **stable**
- The **narrower** base an object has, the **higher** its centre of gravity and the object is more likely to topple over if pushed



The most stable objects have wide bases and low centres of mass

The Principle of Moments

- The principle of moments states:

For a system to be in equilibrium, the sum of clockwise moments about a point must be equal to the sum of the anticlockwise moments (about the same point)

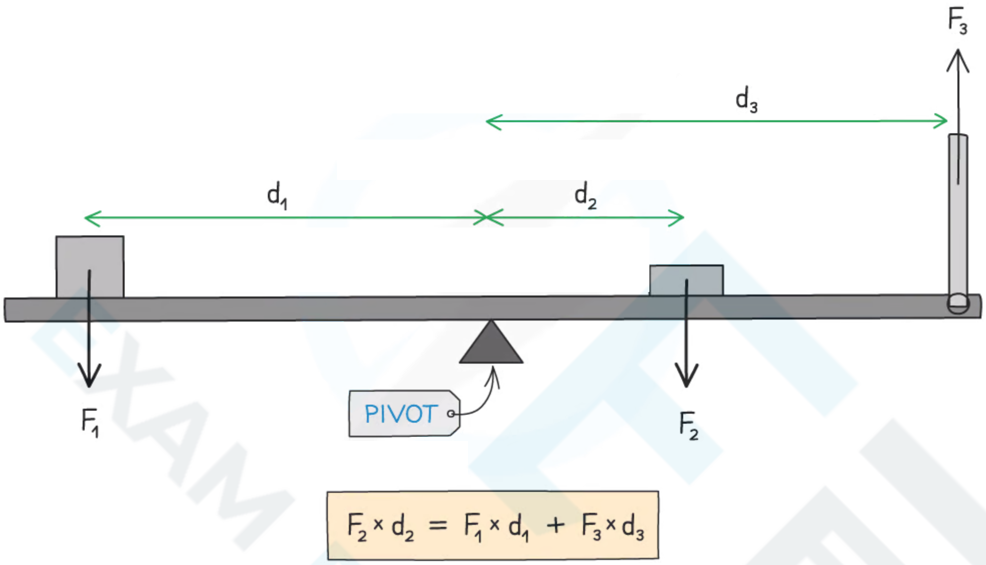


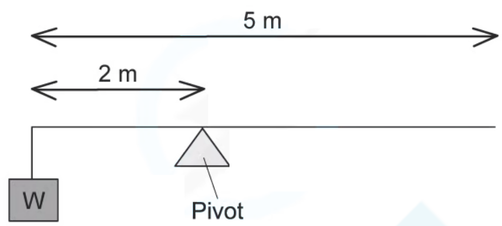
Diagram showing the moments acting on a balanced beam

- In the above diagram:
  - Force F_2 is supplying a clockwise moment;
  - Forces F_1 and F_3 are supplying anticlockwise moments
- Hence: $F_2 \times d_2 = (F_1 \times d_1) + (F_3 \times d_3)$

Worked Example

A uniform beam of weight 40 N is 5 m long and is supported by a pivot situated 2 m from one end.

When a load of weight W is hung from that end, the beam is in equilibrium, as shown in the diagram.



What is the value of W?

A 10 N **B** 50 N **C** 25 N **D** 30 N

EXAM PAPERS PRACTICE

ANSWER: A

STEP 1

PRINCIPLE OF MOMENTS STATES THAT

CLOCKWISE MOMENTS = ANTICLOCKWISE MOMENTS

STEP 2

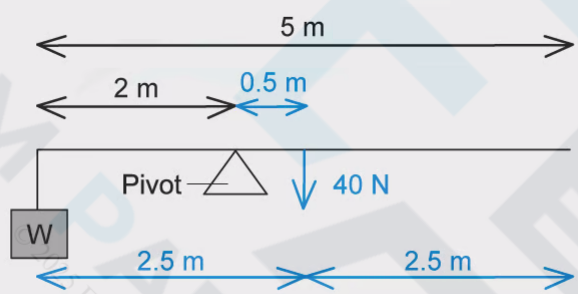
CALCULATE THE CLOCKWISE MOMENT

SINCE THE BEAM IS UNIFORM, ITS WEIGHT WILL ACT AT ITS CENTRE OF GRAVITY (THE MIDDLE)

THIS IS $5 \div 2 = 2.5$ m FROM THE END

SINCE THE PIVOT IS 2m FROM THE END, THIS FORCE IS 0.5 m FROM THE PIVOT

CLOCKWISE MOMENT = 40 N × 0.5 m = 20 Nm



STEP 3

CALCULATE THE ANTICLOCKWISE MOMENT

ANTICLOCKWISE MOMENT = W × 2m

STEP 4

EQUATE BOTH THESE MOMENTS

20 Nm = W × 2m

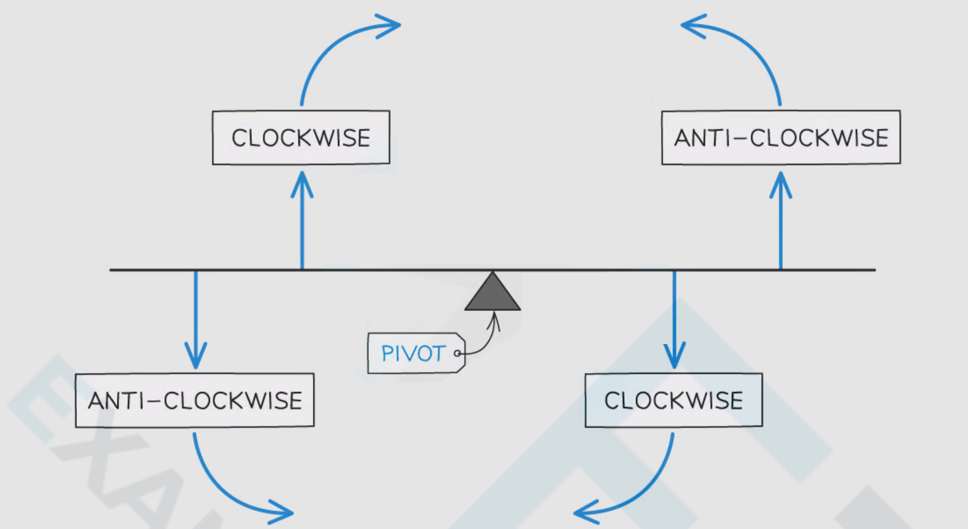
STEP 5

REARRANGE FOR W

$$W = \frac{20\text{ Nm}}{2\text{m}} = 10\text{ N}$$

Exam Tip

Make sure that all the distances are in the same units and you're considering the correct forces as clockwise or anticlockwise, as seen in the diagram below



Work, Energy & Power

2.17 Work

Work

- Work is defined as

  The amount of energy transferred when an external force causes an object to move over a certain distance

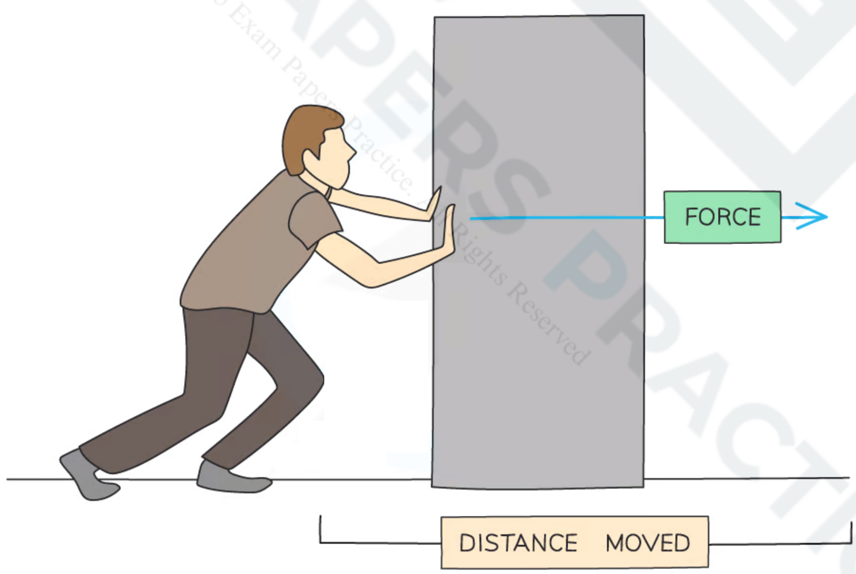
- If the force is **parallel** to the direction of the object's displacement, the work done can be calculated using the equation:

$$\Delta W = F\Delta s$$

- Where:
  - ΔW = change in work done (J)
  - F = average force applied **in the direction of the motion** (N)
  - s = displacement (m)

- Force **in the direction of the motion** means that when a force is applied at an angle the **component** in the direction of motion is used to calculate, rather than the whole force

- In the diagram below, the man's pushing force on the block is doing work as it is transferring energy to the block



Work is done when a force is used to move an object over a distance

- When pushing a block, **work is done against friction** to give the box kinetic energy to move
  - The kinetic energy is transferred to other forms of energy such as heat and sound

- Usually, if a force acts **in** the direction that an object is moving then the object will **gain energy**
- If the force acts in the **opposite** direction to the movement then the object will **lose energy**

Calculating with Force at an Angle

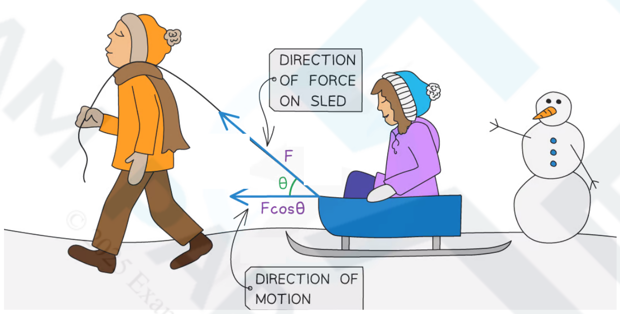
- Sometimes the direction of motion of an object is not parallel to the direction of the force
- If the force is at an angle θ to the object's displacement, the work done is calculated by:

$$W = Fs \cos \theta$$

or

$$W = Fs \sin \theta$$

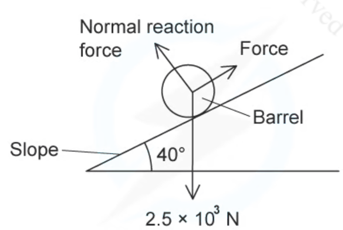
- Where θ is the angle, in degrees, between the direction of the force and the motion
- When the angle is between the force and the horizontal use cosine
- When the angle is between the force and the vertical, use sine
    - The component needed is the one that is **parallel to the displacement**



When the force is at an angle, only the component of the force in the direction of motion is considered for the work done

Worked Example

The diagram shows a barrel of weight 2.5 × 10^3 N on a frictionless slope inclined at 40° to the horizontal.



A force is applied to the barrel to move it up the slope at a constant speed. The force is parallel to the slope.

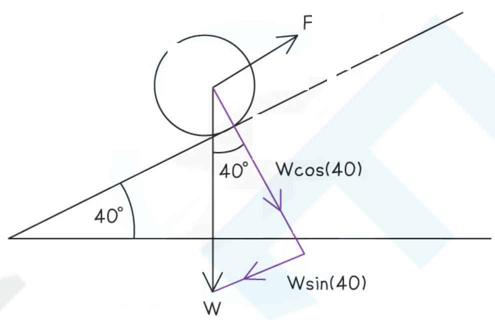
What is the work done in moving the barrel a distance of 6.0 m up the slope?

Step 1: Write the known values from the question

- Force acting downwards, $F = 2.5 \times 10^3$ N
- Angle of the slope, $\theta = 40°$
- Displacement, $s = 6.0$ m

Step 2: Find the force in the direction of motion by resolving the forces

- Draw a diagram, showing the weight acting downwards
- Resolve the weight into two components
- The first component is parallel to the slope - the same as the direction of the motion; this is **W sin θ**
- The second is at right angles to the slope, showing the normal reaction force (this component, **W cos θ** is not used in this answer)



Force acting along the slope, $F = W \sin(40) = 2.5 \times 10^3 \times \sin(40) = 1607$ N

Step 3: Write the equation for work done and substitute in the values

$$\Delta W = F\Delta s = 1607 \times 6.0 = 9642$$

Step 4: Give the answer to the correct number of significant figures and with units

$$\Delta W = F\Delta s = 1607 \times 6.0 = 9.6 \times 10^3 \text{ J}$$

Exam Tip

A common exam mistake is choosing the incorrect force which is not parallel to the direction of movement of an object.

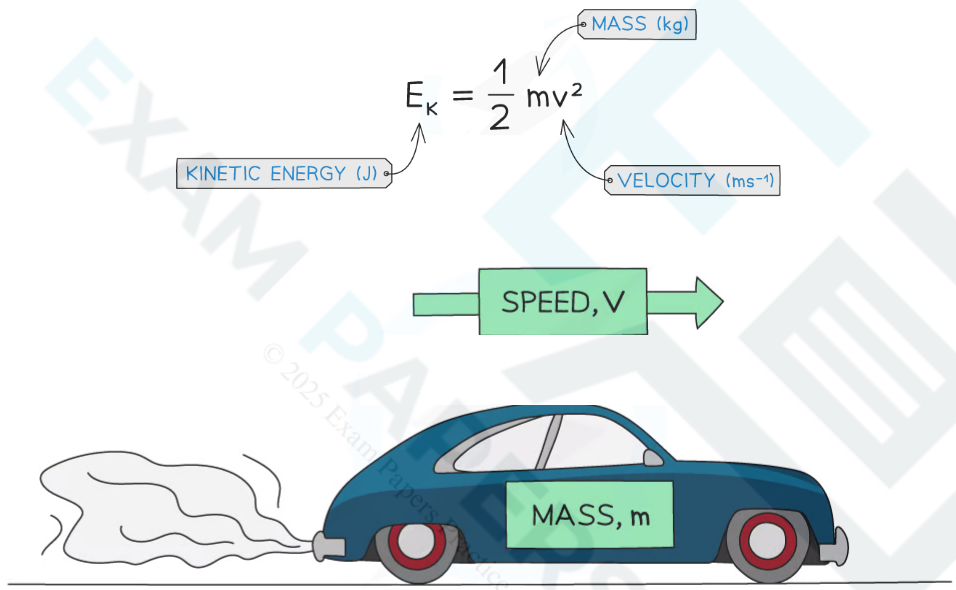
You may have to resolve the force vector first in order to find the correct, parallel component.

The applied force does **not** have to be in the same direction as the movement, as shown in the worked example. In fact, in most cases we apply a force at an angle to move something, because it saves us having to lean down to the height of whatever we are pulling or pushing!

2.18 Kinetic Energy

Kinetic Energy

- Kinetic energy (usually written E_k and sometimes KE) is the energy an object has due to its **motion** (or velocity)
    - The faster an object moves, the greater its kinetic energy
- When an object is falling, it is **gaining** kinetic energy since it is gaining speed
    - This energy transferred from the gravitational potential energy it is losing
    - An object will maintain this kinetic energy unless its speed changes
- Kinetic energy can be calculated using the following equation:



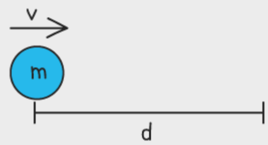
Kinetic energy (KE): The energy an object has when it is moving

Derivation of Kinetic Energy Equation

- A force can make an object accelerate; work is done by the force and energy is transferred to the object
- Using this concept of work done and an equation of motion, the extra work done due to an object's speed can be derived
- The derivation for this equation is shown below:

Derivation of KE = $\frac{1}{2}$mv^2

CONSIDER A MASS m AT REST WHICH ACCELERATES TO A SPEED v OVER A DISTANCE d



WORK DONE IN ACCELERATING THE MASS

$$W = F \times d$$

AND $F = ma$ FROM NEWTON'S SECOND LAW

RECALL THE SUVAT EQUATION

$$v^2 = u^2 + 2as$$

IF $u = 0$ AND $s = d$

$$v^2 = 2ad$$

REARRANGING FOR a

$$a = \frac{v^2}{2d}$$

SUBSTITUTE BACK INTO $F = ma$

$$F = ma = \frac{mv^2}{2d}$$

SUBSTITUTE THIS FORCE F INTO THE WORK DONE EQUATION

$$W = \frac{mv^2}{2d} \times d = \frac{1}{2}mv^2$$

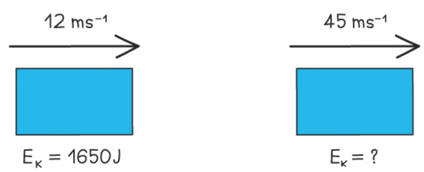
THE MASS IS NOW ABLE TO DO EXTRA WORK $= \frac{1}{2}mv^2$ DUE TO ITS SPEED

IT HAS KINETIC ENERGY $= \frac{1}{2}mv^2$

Worked Example

A body travelling with a speed of 12 m s^{-1} has kinetic energy 1650 J.

If the speed of the body is increased to 45 m s^{-1}, estimate its new kinetic energy.



STEP 1 EQUATION FOR KINETIC ENERGY

$$E_K = \frac{1}{2} mv^2$$

STEP 2 MASS WILL NOT CHANGE, SO CAN BE CALCULATED FROM ITS INITIAL KINETIC ENERGY

REARRANGE FOR MASS m

$$m = \frac{2 \times KE}{v^2} = \frac{2 \times 1650}{12^2} = 23 \text{ kg}$$

STEP 3 SUBSTITUTE INTO KINETIC ENERGY EQUATION

USING VALUE OF MASS AND NEW VALUE OF VELOCITY

$$E_K = \frac{1}{2} \times 23 \times 45^2 = 23000 \text{ J} \quad (2 \text{ s.f})$$

Exam Tip

When using the kinetic energy equation, note that only the speed is squared, not the mass or the ½.

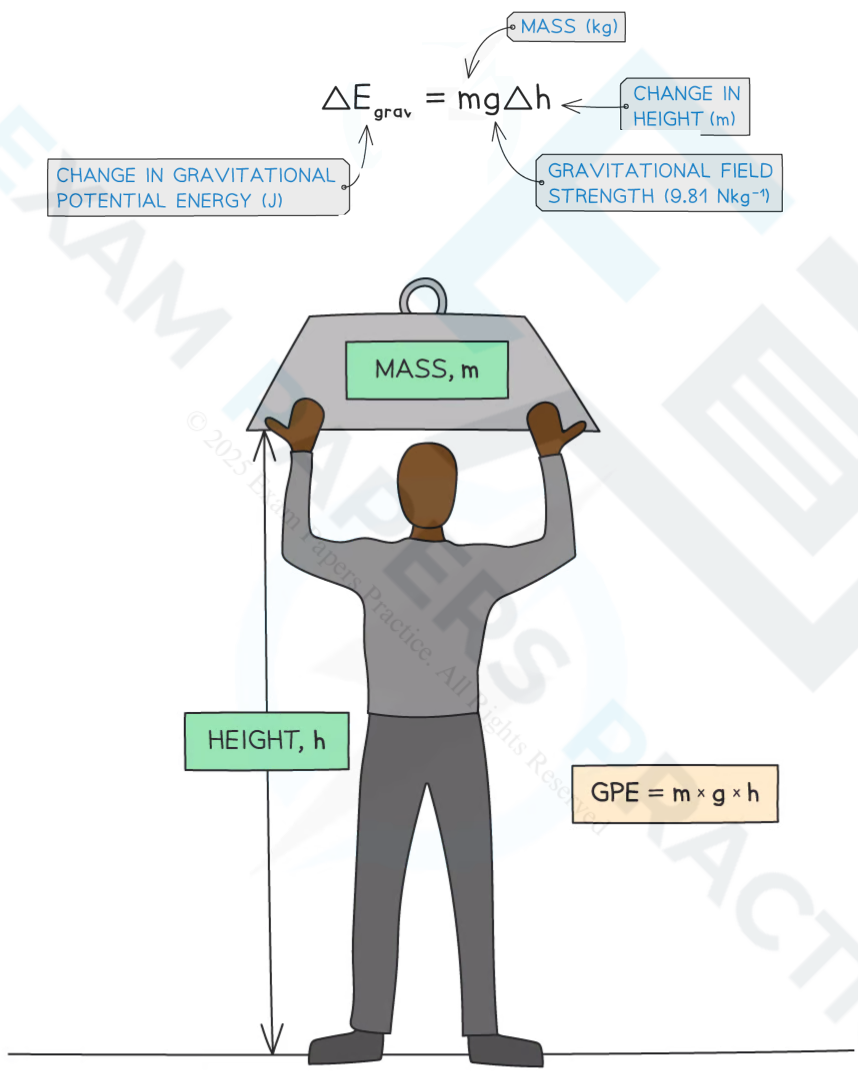
If a question asks about the 'loss of kinetic energy', remember **not** to include a negative sign since energy is a **scalar** quantity.

Gravitational Potential Energy

- Gravitational potential energy (usually written E_p, but sometimes GPE) is energy stored in a mass due to its position in a gravitational field
  - If a mass is **lifted** up, it will **gain** E_p (converted **from** other forms of energy)
  - If a mass **falls**, it will **lose** E_p (and be converted **to** other forms of energy)

- The equation for gravitational potential energy for energy changes in a **uniform gravitational field** is:

$$\Delta E_{grav} = mg\Delta h$$

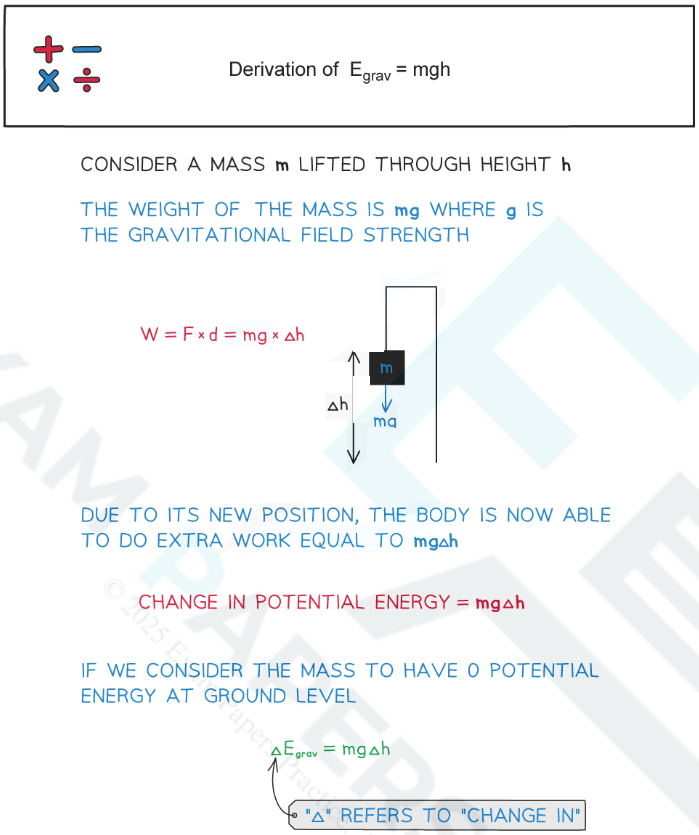


Gravitational potential energy (GPE): The energy an object has when lifted up

- The potential energy on the Earth's surface at ground level is taken to be equal to 0
- This equation is only relevant for energy changes in a **uniform gravitational field** (such as near the Earth's surface)

Derivation of GPE Equation

- When a heavy object is lifted, work is done since the object is provided with an upward force against the downward force of gravity
  - Therefore, **energy is transferred to the object**
- This equation can therefore be derived from the work done



Worked Example

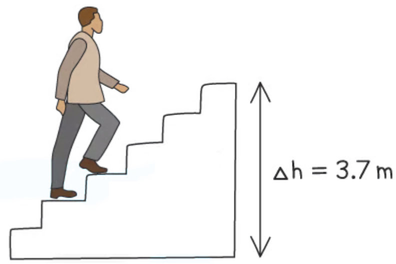
To get to his apartment a man has to climb five flights of stairs.

The height of each flight is 3.7 m and the man has a mass of 74 kg.

What is the approximate gain in the man's gravitational potential energy during the climb?

A. 13 000 J **B.** 2700 J **C.** 1500 J **D.** 12 500 J

ANSWER: A



STEP 1

GPE EQUATION

$\Delta GPE = mg\Delta h$

STEP 2

FIND h

$\Delta h = 5 \times 3.7\text{ m} = 18.5\text{ m}$

5 FLIGHTS OF STAIRS

STEP 3

SUBSTITUTE VALUES INTO GPE EQUATION

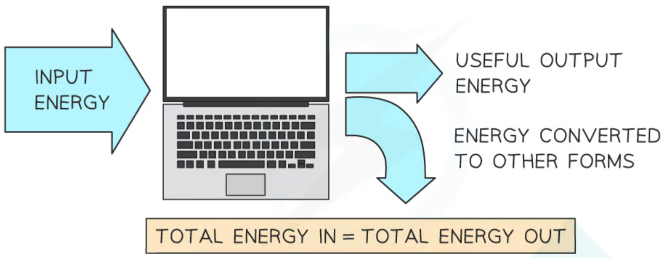
$\Delta GPE = 74 \times 9.81 \times 18.5 = 13000\text{ J}$ (2 s.f.)

Exam Tip

Gravitational potential energy questions often use falling objects, where you are expected to realise that, since energy is conserved, the gravitational potential energy **at the start** is equal to the kinetic energy **at the end**.

The Principle of Conservation of Energy

- The principle of conservation of energy is a law of Physics which always applies to a closed system



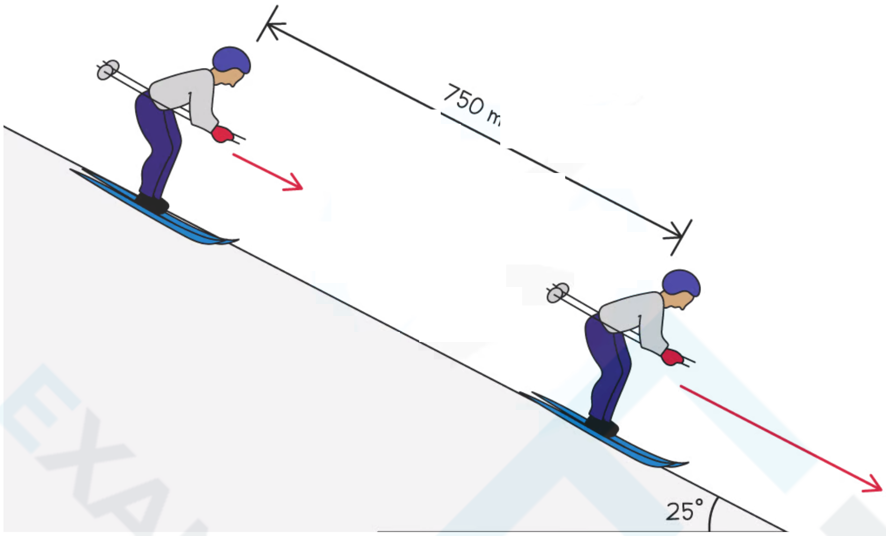
- To apply conservation of energy, heat losses are usually ignored during the calculation stage
- In reality there are always some energy losses from the system
  - These should be mentioned when comparing calculated, ideal values to real-life situations

- Conservation of energy is often applied in questions about exchanges between **kinetic energy** and **gravitational energy**
- Common examples include:
  - A swinging pendulum
  - Objects in free fall
  - Sports such as skiing or skydiving where gravity is causing motion and few drag forces apply

- The gravitational potential energy stored initially is transferred to kinetic energy, or vice versa
- This allows either;
  - Final velocity to be found from the distance the object moved, or
  - Height of a drop from the final velocity

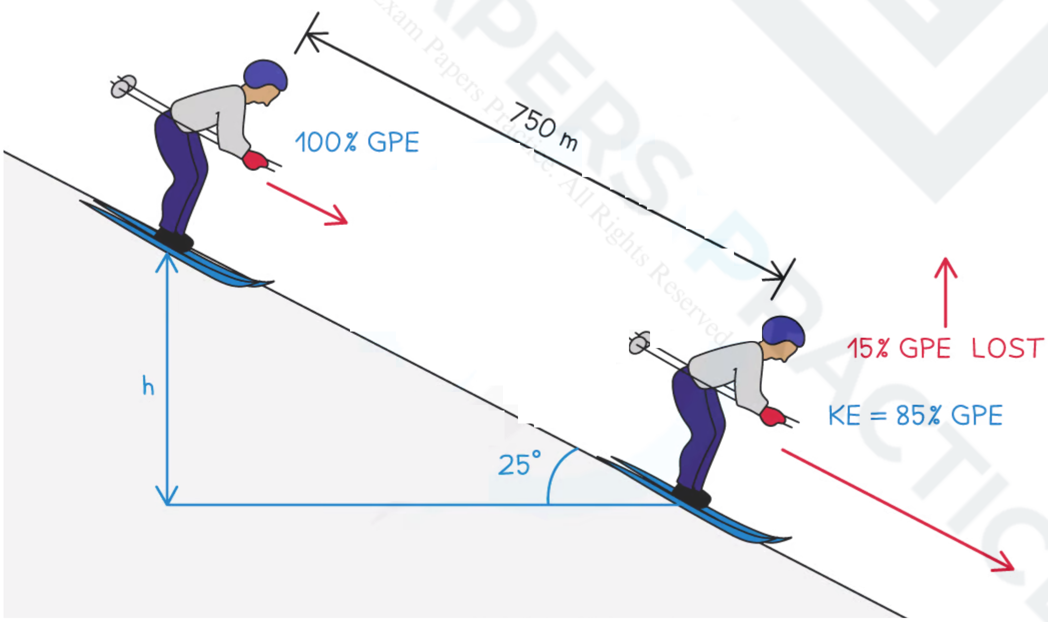
Worked Example

The diagram below shows a skier on a slope descending 750 m at an angle of 25° to the horizontal.



Calculate the final speed of the skier, assuming that he starts from rest and 15% of his initial gravitational potential energy is **not** transferred to kinetic energy.

Step 1: Write down the known quantities



- Vertical height, $h = 750 \sin 25°$
- $E_k = 0.85\ E_p$

Step 2: Equate the equations for E_k and E_{grav}

$$E_k = 0.85\ E_{grav}$$

$$½\ mv^2 = 0.85 \times mgh$$

Step 3: Rearrange for final speed, *v*

$$\frac{1}{2}\cancel{m}v^2 = 0.85 \times \cancel{m}gh$$

$$v^2 = 0.85 \times 2gh$$

$$v = \sqrt{0.85 \times 2gh}$$

Step 4: Calculate the final speed, *v*

$$v = \sqrt{0.85 \times 2 \times 9.81 \times 750 \sin 25°} = 72.7$$

$$\text{Final speed, } v = 73\ \text{m s}^{-1}$$

Exam Tip

Gravitational energy:

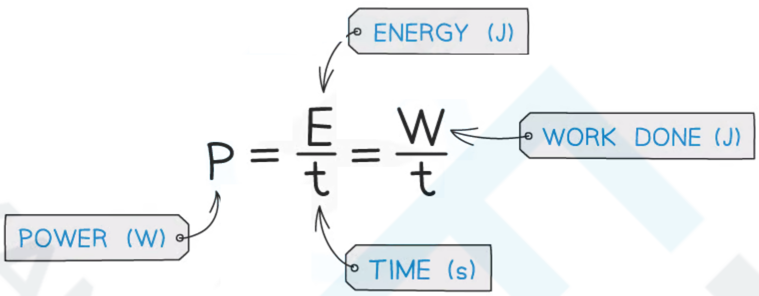
- This equation only works for objects close to the Earth's surface where we can consider the gravitational field to be uniform.

Kinetic energy:

- When using the kinetic energy equation, note that only the speed is squared, not the mass or the ½.
- If a question asks about the 'loss of kinetic energy', remember **not** to include a negative sign since energy is a scalar quantity.

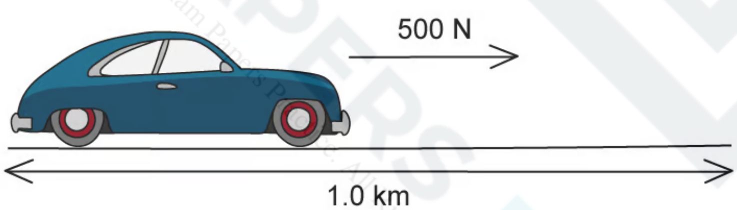
Power

- The power of a mechanical process is the **rate at which energy is transferred**
    - Since work done is equal to the energy transferred, power can also be defined as the rate of doing work or **the work done per unit time**
- Power is measured in **Watts (W)**
    - Since power is energy used per unit time, **1 W = 1 J s^{-1}**
- Power can be calculated using the equation:



Worked Example

A car engine exerts the following force for 1.0 km in 200 s.



Determine what is the average power developed by the engine.

STEP 1

EQUATION FOR POWER

$$\text{POWER} = \frac{\text{WORK DONE}}{\text{TIME}}$$

STEP 2

CALCULATE WORK DONE

$$W = F \times d$$

$$= 500\ \text{N} \times 1.0 \times 10^3\ \text{m}$$

$$= 5 \times 10^5\ \text{J}$$

STEP 3

SUBSTITUTE VALUES INTO POWER EQUATION

$$\text{POWER} = \frac{5 \times 10^5\ \text{J}}{200\ \text{s}} = 2500\ \text{W} = 2.5\ \text{kW}$$

Exam Tip

Power is also used in electricity, with labels on lightbulbs which indicate their power, such as 60 W or 100 W, which indicate the amount of energy transferred by an electrical current rather than by a force doing work. Just remembering 'energy per unit time' will help, as it doesn't matter what kind of energy is being transferred.

When working with power equations the numbers are often very large, so expect to see kW, MW and even GW in questions.

(In case you forgot... ***kW*** means $\times 10^3$. ***MW*** means $\times 10^6$, and ***GW*** means $\times 10^9$. If you weren't sure, go and revise S.I. Units!)

Efficiency

- The efficiency of a system is a measure of how well energy is transferred in a system
- Efficiency is defined as:

The ratio of the useful power or energy transfer output from a system to its total power or energy transfer input

- If a system has **high** efficiency, this means most of the energy transferred is **useful**
- If a system has **low** efficiency, this means most of the energy transferred is **wasted**
- Determining which type of energy is useful or wasted depends on the system
    - When electrical energy is converted to light in a lightbulb, the light energy is **useful** and the heat energy produced is **wasted**
    - When electrical energy is converted to heat for a heater, the heat energy is **useful** and the sound energy produced is **wasted**

- Efficiency can be given as a ratio (between 0 and 1) or a percentage (between 0 and 100%)
- Since efficiency is a ratio, it has **no units**

- Calculate energy efficiency and power efficiency in the same way, using one of the following equations;

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$

- The energy can be of any form e.g. gravitational potential energy, kinetic energy

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

- Where power is defined as the energy transferred per unit of time

$$\text{Power} = \frac{\text{Energy transferred}}{\text{Time}} = \frac{E}{t}$$

Worked Example

An electric motor has an efficiency of 35 %. It lifts a 7.2 kg load through a height of 5 m in 3 s.

Calculate the power of the motor.

Step 1: Write down the efficiency equation

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} \times 100$$

Step 2: Rearrange for the power input

$$\text{Power input} = \frac{\text{Power output} \times 100}{\text{Efficiency}}$$

Step 3: Calculate the power output

- The power output is equal to energy ÷ time
- The electric motor transferred electric energy into gravitational potential energy to lift the load

$$\text{Gravitational potential energy} = mgh = 7.2 \times 9.81 \times 5 = 353.16 \text{ J}$$

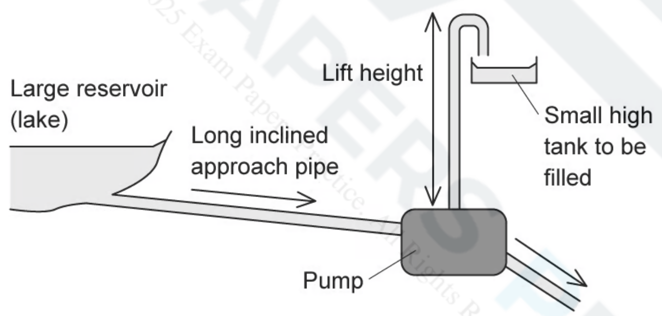
$$\text{Power} = 353.16 \div 3 = 117.72 \text{ W}$$

Step 4: Substitute values into power input equation

$$\text{Power input} = \frac{117.72 \times 100}{35} = 336 \text{ W}$$

Worked Example

The diagram shows a pump called a hydraulic ram.



In one such pump, the long approach pipe holds 700 kg of water. A valve shuts when the speed of this water reaches 3.5 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by height of 12m.

The efficiency of the pump is 20%.

Determine the mass of water which could be lifted 12 m

Step 1: Identify the energy conversions and write them in an equation

- The kinetic energy of the moving water is converted to gravitational potential energy as it is lifted
- The equation should be that;

$$initial\ E_k = final\ E_{grav} \rightarrow \frac{1}{2}mv^2 = mg\Delta h$$

Step 2: Include the efficiency in the equation

- Efficiency = 20% meaning 20% of the kinetic energy is converted;

$$0.2 \times \frac{1}{2}mv^2 = mg\Delta h$$

Step 3: Substitute in the values and calculate

$$0.2 \times \frac{1}{2} \times 700 \times (3.5^2) = m \times 9.81 \times 12$$

$$857.5 = 117.72\ m$$

$$m = 7.284$$

Step 4: Write the answer with the correct significant figures and units

- Mass of water lifted, m = 7.3 kg (2 s.f.)

Exam Tip

In efficiency calculations decide **before starting** where the energy is lost from the system.

In the example above, the pump is what converts the water's **kinetic energy** into **gravitational potential energy**, and it is the pump whose efficiency we are given. That means the losses are from the kinetic energy.

Don't just calculate then deduct the efficiency at the end - this can lead to lots of work for no marks. Which isn't very efficient!