A Level Physics Edexcel

12. Gravitational Fields

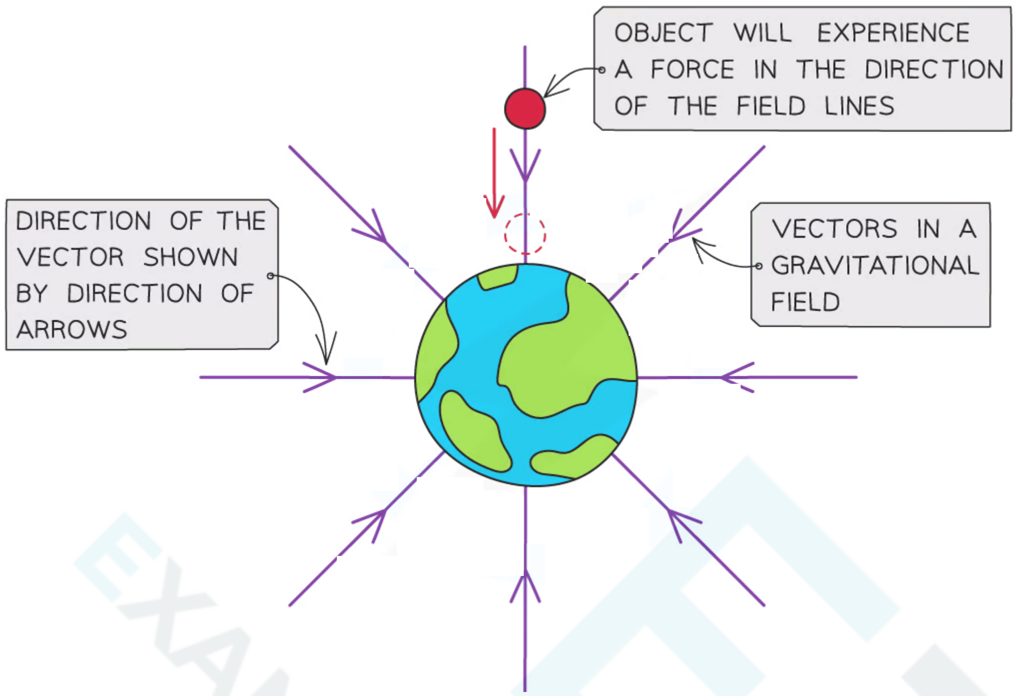
CONTENTS

Gravitational Fields

- Generally, a **force field** is a region of space in which an object will experience a **force**
- A **gravitational field,** therefore, is a region of space in which any object with **mass** experiences a **gravitational force**
- The Sun, for example, creates a gravitational field around it
  - The Earth, which has mass, experiences the gravitational force due to the Sun
  - This gravitational force keeps the Earth in orbit around the Sun
- Additional effects of the Moon and Sun's gravitational fields can be seen on Earth, such as the cause of tides

Direction of a Gravitational Field

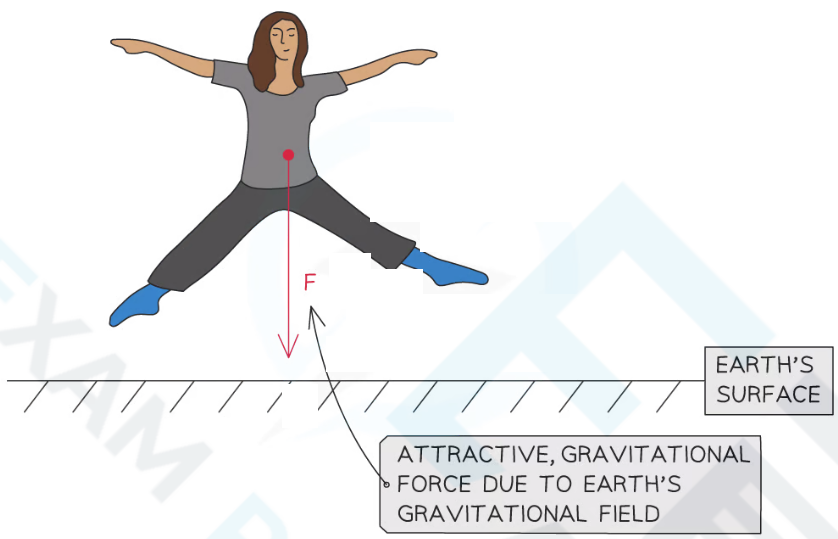
- The direction of a gravitational field can be represented as a **vector**, the direction of which must be determined by inspection
- The direction of the vector shows the direction of the gravitational force that would be exerted on a mass if it was placed at that position in the field
- These vectors are known as **field lines** (or 'lines of force'), which are represented by arrows
  - Therefore, gravitational field lines also show the direction of **acceleration** of a mass placed in the field
- Gravitational field lines are therefore directed toward the centre of mass of a body
  - This is because the gravitational force is **attractive**
  - Therefore, masses always attract each other via the gravitational force
- The gravitational field around a point mass will be **radial** in shape and the field lines will always point towards the centre of mass



The direction of the gravitational field is shown by the vector field lines

Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
  - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth



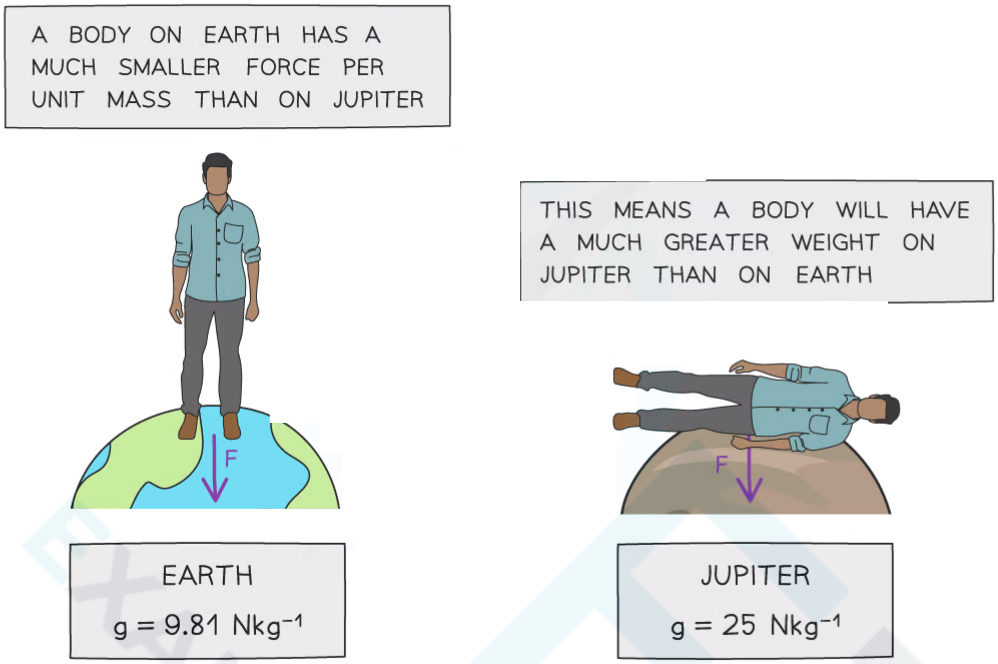
The Earth's gravitational field produces an attractive force F, which is equal and opposite to the attractive force produced by the person's gravitational field

- The **gravitational field strength** *g* at a point is defined as force *F* per unit mass *m* of an object at that point:

$$g = \frac{F}{m}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - F = force due to gravity, or weight (N)
  - m = mass (kg)

- This equation shows that:
  - The larger the mass of an object, the greater its pull on another mass
  - On planets with a large value of *g*, the gravitational force per unit mass is **greater** than on planets with a smaller value of *g*
- An object's mass remains the same at all points in space
  - However, on planets such as Jupiter, the weight of an object will be a lot greater than on a less massive planet, such as Earth
  - This means the gravitational force would be so high that humans, for example, would not be able to fully stand up (or, even worse...)



The weight force on Jupiter would be so large that even standing upright would be difficult

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** (or diameter) of the planet
  - The **mass** (or density) of the planet

Worked Example

Calculate the mass of an object with weight 10 N on Earth.

STEP 1 GRAVITATIONAL FIELD STRENGTH EQUATION

$$g = \frac{F_g}{m}$$

STEP 2 REARRANGE FOR MASS m

$$m = \frac{F_g}{g}$$

STEP 3 SUBSTITUTE IN VALUES

$$m = \frac{10}{9.81} = 1.0 \text{ kg}$$

g ON EARTH

Exam Tip

There is a big difference between *g* and *G* (sometimes referred to as 'little g' and 'big G' respectively).

- *g* is the gravitational field strength, which varies depending on the size of the mass *M* producing the gravitational field and the distance *r* from its centre of mass
- *G* is Newton's Universal Gravitational Constant, which always has a value of 6.67×10^{-11} N m^2 kg^{-2}

In addition, remember the equation density ρ = mass *m* ÷ volume *V*, which may come in handy with calculations where the density of some object is given, rather than its mass!

Newton's Law of Universal Gravitation

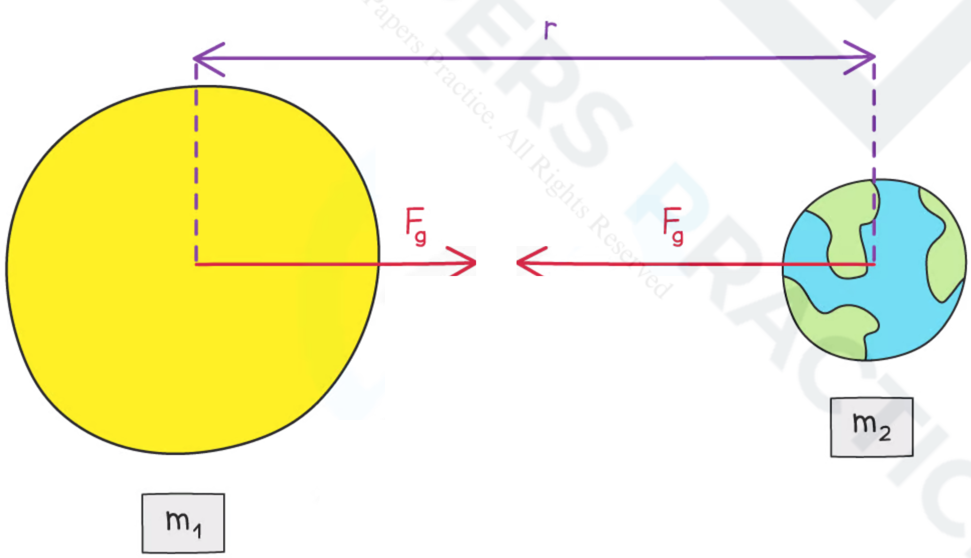
- The gravitational force between two masses, e.g., between the Earth and the Sun, is defined by Newton's Law of Universal Gravitation
- Newton's Law of Universal Gravitation states:

  The gravitational force F_G between two masses m_1 and m_2 is proportional to the product of their masses and inversely proportional to the square of their separation, r

- In equation form, this is written as:

$$F_G = \frac{Gm_1m_2}{r^2}$$

- Where:
  - F_G = gravitational force between two masses (N)
  - G = Newton's gravitational constant
  - m_1 and m_2 = mass of body 1 and mass of body 2 (kg)
  - r = distance between the centre of the two masses (m)
- The $1/r^2$ relation is called the 'inverse square law'
  - This means that if the distance between two masses **doubles**, r becomes $2r$
  - Therefore, $1/r^2$ becomes $1/(2r)^2$, which is equal to $1/4r^2$
  - Hence, the gravitational force between the two masses **reduces** by a factor of **four**



The gravitational force between two masses is defined by Newton's Law of Universal Gravitation

Worked Example

A satellite with mass 6500 kg is orbiting the Earth at 2000 km above the Earth's surface. The gravitational force between them is 37 kN.

Calculate the mass of the Earth.

(Radius of the Earth = 6400 km)

STEP 1

NEWTON'S LAW OF GRAVITATION

$$F_G = \frac{Gm_1m_2}{r^2}$$

m_1 IS THE MASS OF THE SATELLITE

m_2 IS THE MASS OF THE EARTH

THESE CAN BE ANY WAY AROUND

STEP 2

REARRANGE FOR m_2 (MASS OF EARTH)

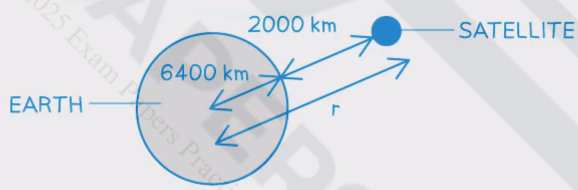
$$\frac{r^2F_G}{Gm_1} = m_2$$

STEP 3

CALCULATE THE DISTANCE r

r IS THE DISTANCE BETWEEN THE CENTRE OF THE EARTH AND SATELLITE

r = DISTANCE OF SATELLITE ABOVE THE SURFACE + RADIUS OF THE EARTH



$r = 2000 + 6400 = 8400 \text{ km} = 8400 \times 10^3 \text{ m}$

STEP 4

SUBSTITUTE IN VALUES

$$\frac{(8400 \times 10^3)^2 \times 37 \times 10^3}{6.67 \times 10^{-11} \times 6500} = 6.0 \times 10^{24} \text{ kg} \quad (2 \text{ s.f.})$$

37 kN

NEWTON'S GRAVITATIONAL CONSTANT

Exam Tip

A common mistake in exams is to forget to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured between the **centre** of each mass, which is from the **centre** of the planet to the centre of the satellite!

Another common mistake is forgetting that the distance between masses m_1 and m_2 is **squared**. Remember this whenever you use Newton's Law of Universal Gravitation!

Gravitational Field due to a Point Mass

- The gravitational field strength at a point describes how much gravitational force is experienced by a test mass at that point
- The strength of a gravitational field caused by a point mass can be derived using Newton's Law of Universal Gravitation
  - For calculations involving gravitational forces, a spherical mass can be treated as a point mass at the centre of the sphere

- Newton's Law of Universal Gravitation states that the magnitude of the attractive gravitational force *F* between two masses *M* and *m* with separation *r* is equal to:

$$F_G = \frac{GMm}{r^2}$$

- The gravitational field strength at a point is defined as the force *F* per unit mass *m*

$$g = \frac{F}{m}$$

- Substituting the force *F* with the gravitational force F_G leads to:

$$g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m}$$

- Cancelling mass *m*, the equation becomes:

$$g = \frac{GM}{r^2}$$

- Where:
  - *g* = gravitational field strength due to a point mass *M* ($N\ kg^{-1}$)
  - *G* = Newton's Gravitational Constant ($N\ m^2\ kg^{-2}$)
  - *M* = mass of the body producing the gravitational field (kg)
  - *r* = distance from the centre of mass *M* to a chosen point in the field (m)

Worked Example

The Earth's gravitational field strength at its surface is 9.81 $N\ kg^{-1}$.

Calculate the Earth's gravitational field strength 1 million km away from its surface.

Use the following data:

- Mass of Earth = 6.0×10^{24} kg
- Radius of Earth = 6400 km

Step 1: Write the known quantities

- Mass of Earth = 6.0×10^{24} kg

- Radius of Earth = 6400 km = 6400×10^3 m
- Newton's Universal Gravitation Constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
- Distance from the Earth's surface = 1 million km = $(1 \times 10^6) \times 10^3$ m = 1×10^9 m

Step 2: Write the equation for gravitational field strength

- The gravitational field strength *g* at some distance *r* due to a mass *M* is given by the equation:

$$g = \frac{GM}{r^2}$$

Step 3: Determine the distance *r* from the centre of mass *M*

- The Earth is causing the gravitational field in the question
- 1 million km from the Earth's surface means the distance from Earth's centre of mass $r = (6400 \times 10^3) + (1 \times 10^9) = 1.0064 \times 10^9$ m

Step 4: Substitute values and calculate the gravitational field strength

- Therefore, the gravitational field strength is given by:

$$g = \frac{(6.67 \times 10^{-11}) \times (6.0 \times 10^{24})}{(1.0064 \times 10^9)^2} = 4.0 \times 10^{-4} \text{ N kg}^{-1}$$

Exam Tip

When using the equation for gravitational field strength, remember that the mass *M* is the mass **causing** the gravitational field. The mass *m* is the object that **experiences** the gravitational field of *M* : hence, you may see *m* referred to as a 'test mass'.

It should make sense, therefore, that *g* is defined as the 'force experienced per unit mass' in a gravitational field. The force experienced is by the 'test mass', in the field, *m*.

Gravitational Potential for a Radial Field

Near the Earth's Surface

- The gravitational potential energy (G.P.E) is the energy an object has when lifted off the ground given by the familiar equation:

$$G.P.E = mg\Delta h$$

- When using this equation, the G.P.E on the surface of the Earth is taken to be zero
  - This means **work is done** to lift the object
- This equation is **only** used for objects that are near the Earth's surface
  - This is because, near Earth's surface, the gravitational field is approximated to be **uniform**
  - Far away from the Earth's surface, the gravitational field is **radial** because the Earth is a **sphere**

In a Radial Field

- In a radial field, G.P.E is defined as:

The energy an object possesses due to its position in a gravitational field

- The gravitational potential at a point is the **gravitational potential energy per unit mass** at that point
- Gravity is always attractive, so work must be done on a mass to move it away to a point infinitely far away from every other mass
  - 'Infinity' is the point at which the gravitational potential is **zero**
  - Therefore, since the potential energy of all masses **increases** as work is done on them to move them infinitely far away, the value of the potential is always **negative**
- Gravitational potential energy is defined as:

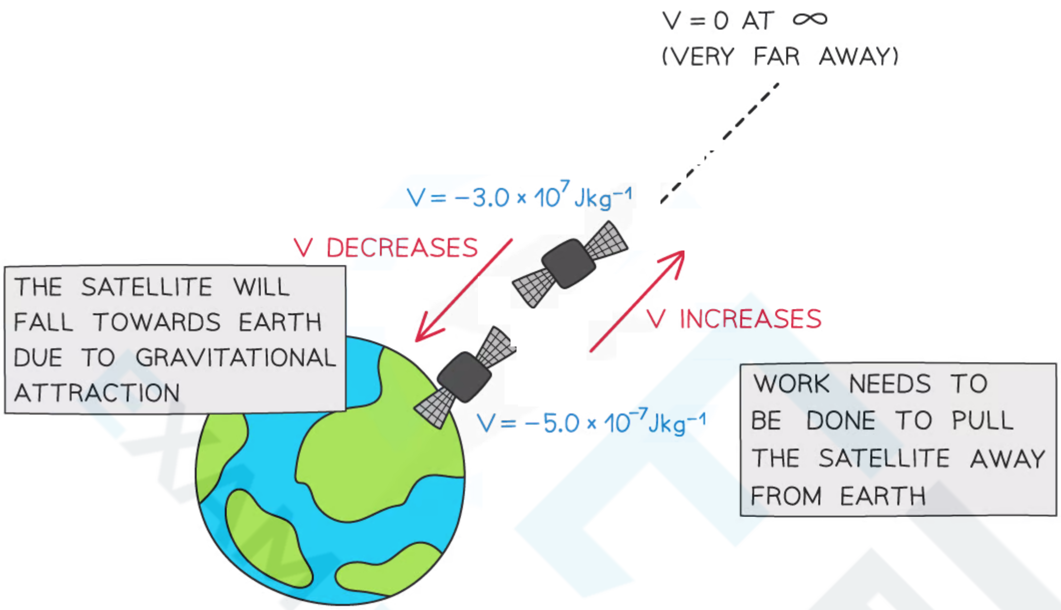
The work done per unit mass in bringing a test mass from infinity to a defined point

- It is represented by the symbol, V and is measured in **J kg^{-1}**
- Gravitational potential V_{grav} can be calculated at a distance r from a mass M using the equation:

$$V_{grav} = -\frac{GM}{r}$$

- Where:
  - V_{grav} = gravitational potential (J kg^{-1})
  - G = Newton's gravitational constant
  - M = mass of the body producing the gravitational field (kg)
  - r = distance from the centre of the mass to the point mass (m)
- This means that the gravitational potential is negative on the surface of a mass (such as a planet), and **increases** with distance from that mass (becomes less negative)

- Work has to be done **against** the gravitational pull of the planet to take a unit mass **away** from the planet
- The gravitational potential at a point depends on the mass of the object producing the gravitational field and the distance the point is from that mass



Gravitational potential decreases as the satellite moves closer to the Earth

Worked Example

Calculate gravitational potential at the surface of Mars.

Radius of Mars = 3400 km

Mass of Mars = 6.4×10^{23} kg

Step 1: Write the gravitational potential equation

$$V_{grav} = -\frac{GM}{r}$$

Step 2: Substitute known quantities

$$V_{grav} = -\frac{(6.67 \times 10^{-11}) \times (6.4 \times 10^{23})}{3400 \times 10^{3}} = -1.3 \times 10^{7} \text{ J kg}^{-1}$$

Exam Tip

The equation for gravitational potential in a radial field looks very similar to the equation for gravitational field strength in a radial field, but there is a **very important difference**! Remember, for gravitational potential:

$$V_{grav} = -\frac{GM}{r} \text{ so } V_{grav} \propto \frac{1}{r}$$

However, for gravitational field strength:

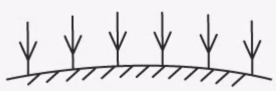
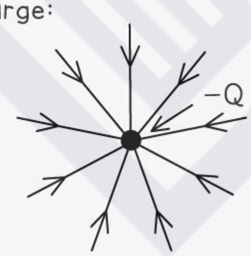
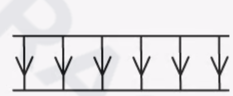
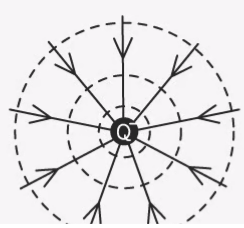
$$g = \frac{GM}{r^2} \text{ so } g \propto \frac{1}{r^2}$$

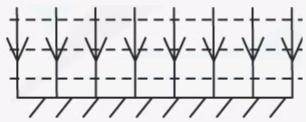
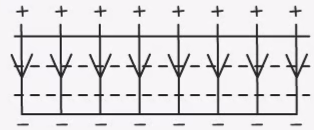
Additionally, remember that both V_{grav} and g are measured from the **centre of the mass *M*** causing the field!

Comparing Electric & Gravitational Fields

- The similarities and differences between gravitational and electrostatic forces are listed in the table below:

Table Comparing G and E Fields

	Gravitational Fields	Electric Fields
Origin of the force	Mass	Charge
Force between two point masses/ charges	$F_G = \frac{GM_1GM_2}{r^2}$	$F_E = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
Type of Force	Attractive force	Attractive force (opposite charges) Repulsive force (like charges)
Field Strength	$g = \frac{F}{M}$	$E = \frac{F}{Q}$
Field strength due to a point mass/ charge	$g = \frac{GM}{r^2}$	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$
Field Lines	Around a point mass:  In a uniform field (surface of a planet): 	Around a (negative) point charge:  In a uniform field (between charged) parallel plates: 
Potential	$V = -\frac{GM}{r}$	$V = \frac{Q}{4\pi\varepsilon_0 r}$
Equipotential Surfaces	Around a point mass: 	Around a point charge: 

	In a uniform field (surface of a planet): 	In a uniform field (between charged) parallel plates 
Work Done on a Mass or Charge	$\Delta W = M\Delta V$	$\Delta W = Q\Delta V$

Similarities between Gravitational & Electric Fields

- The key **similarities** are:
  - The magnitude of the gravitational and electrostatic force between two point masses or charges follow **inverse square law** relationships
  - The field lines around a **spherical masses** and **spherical charges** are **radial**
  - The field lines in a **uniform** gravitational and electric field are identical (they are **parallel** and **equally spaced**)
  - The **gravitational field strength** and **electric field strength** both have a $\frac{1}{r^2}$ relationship in a **radial field**
  - The **gravitational potential** and **electric potential** both have a $\frac{1}{r}$ relationship

Differences between Gravitational & Electric Fields

- The key **differences** are:
  - The gravitational force acts on particles with **mass** whilst the electrostatic force acts on particles with **charge**
  - The gravitational force is **always** attractive whilst the electrostatic force can be attractive **or** repulsive
  - The gravitational potential is **always** negative whilst the electric potential can be either negative **or** positive

Exam Tip

This topic could require a long structured-answer, so practice summing up this information as you would for a 6 mark question.

Orbital Motion

Newton's Law of Gravitation & Orbits

- Since most planets and satellites have a near circular orbit, the gravitational force F_G between the sun or another planet provides the centripetal force needed to stay in an orbit
  - This centripetal force is **perpendicular** to the velocity of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*
  - Equating the gravitational force from Newton's Law of Gravitation to the centripetal force for a planet or satellite in orbit gives:

$$F_G = F_{centripetal}$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

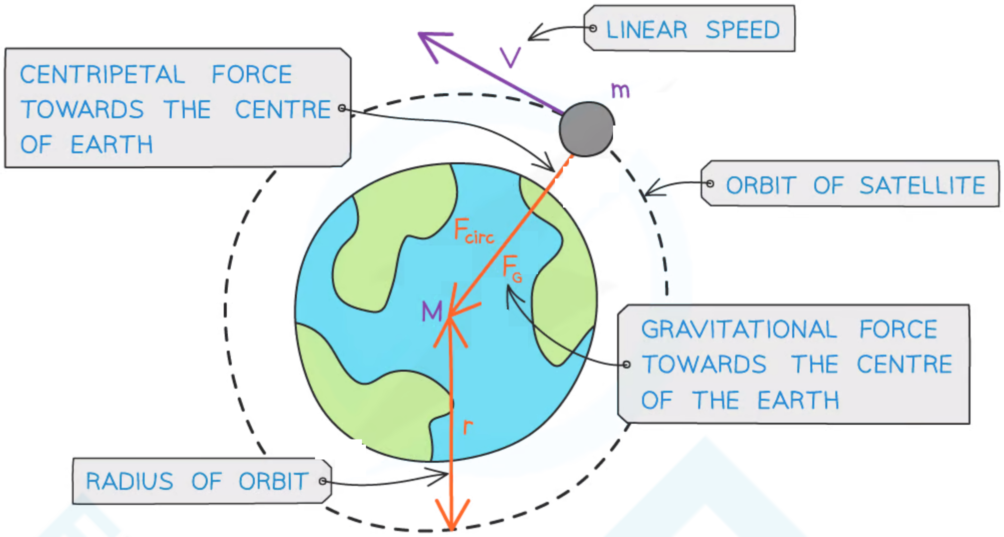
- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{GM}{r}$$

- Where:
  - v = linear speed of the mass in orbit (m s^{-1})
  - G = Newton's Gravitational Constant (N m^2 kg^{-2})
  - M = mass of the object being orbited (kg)
  - r = orbital radius (m)
- This means that all satellites, whatever their mass, will travel at the same speed *v* in a particular orbital radius *r*

Newton's Laws of Motion & Orbits

- Newton's first law of motion states that a body remain at rest or at constant velocity unless a **resultant force** acts on it
- Bodies in orbit do have a resultant force acting on them
  - This is the gravitational force due to the mass *M* being orbited
  - This force is a **centripetal force** because it acts towards the centre of *M*, perpendicular to the velocity of the planet or satellite
- Therefore, since the direction of a planet or satellite orbiting in circular motion is constantly changing, it must be **accelerating**
  - This is called **centripetal acceleration**



A satellite in orbit around the Earth travels in circular motion

Time Period & Orbital Radius Relation

- A planet or a satellite orbits in circular motion
    - Therefore, its orbital time period *T* is the time taken to travel the circumference of the orbit 2π*r*
    - This means the linear speed, or orbital speed *v* is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation speed = distance / time
- Equating the two equations for orbital speed gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period *T* and orbital radius *r*:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

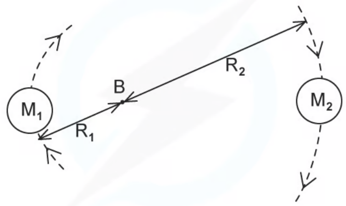
- Where:
    - *T* = time period of the orbit (s)
    - *r* = orbital radius (m)
    - *G* = Newton's Gravitational Constant (N m^2 kg^{-2})
    - *M* = mass of the object being orbited (kg)
- The equation shows the relationship between the **orbital period** and the **orbital radius** for any planet or satellite in orbit

- It is summarised mathematically as:

$$T^2 \propto r^3$$

Worked Example

A binary star system constant of two stars orbiting about a fixed point **B**.The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed v and an angular speed ω about **B**.



State the following formula, in terms of G, M_2, R_1 and R_2

(i) The angular speed ω of M_1

(ii) The time period T for each star in terms of angular speed ω

(i) The angular speed ω of M_1

Step 1: Equate the centripetal force to the gravitational force

$$M_1 R_1 \omega^2 = \frac{GM_1M_2}{(R_1+R_2)^2}$$

Step 2: M_1 cancels on both sides

$$R_1 \omega^2 = \frac{GM_2}{(R_1+R_2)^2}$$

Step 3: Rearrange for angular velocity ω

$$\omega^2 = \frac{GM_2}{R_1(R_1+R_2)^2}$$

Step 4: Square root both sides

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}}$$

(ii) The time period T for each star in terms of angular speed ω

Step 1: Write down the angular speed ω equation with time period T

$$\omega = \frac{2\pi}{T}$$

Step 2: Rearrange for T

$$T = \frac{2\pi}{\omega}$$

Step 3: Substitute in ω from part (i)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{GM_2}}$$

Exam Tip

This worked example helps you practise two crucial techniques that are often examined:

1. The centripetal force is expressed as $\frac{mv^2}{r}$ or equivalently as $m\omega^2 r$. In our case the angular speed is given in the question, so it is best to use the latter expression $m\omega^2 r$ when equating the centripetal force to the gravitational force.
2. The gravitational force is given as $\frac{GMm}{r^2}$ but note that the distance r in this question is given as a sum, $R_1 + R_2$. You should remember that r is defined as the distance between the **centre of masses** of M and m, therefore, $r = R_1 + R_2$ and so $r^2 = (R_1 + R_2)^2$.