A Level Physics Edexcel

11. Nuclear Radiation

CONTENTS

Nuclear Fusion & Fission

11.1 Nuclear Binding Energy & Mass Deficit

Nuclear Binding Energy

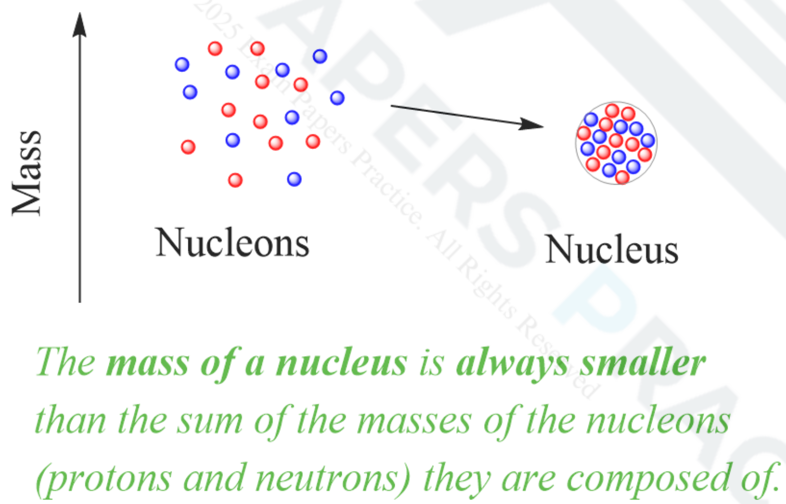
- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
  - This difference in mass is known as the **mass defect** or **mass deficit**
  - Mass defect is defined as:

The difference between the measured mass of a nucleus and the sum total of the masses of its constituents

- The mass defect Δm of a nucleus can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

- Where:
  - Z = proton number
  - A = nucleon number
  - m_p = mass of a proton (kg)
  - m_n = mass of a neutron (kg)
  - m_{total} = measured mass of the nucleus (kg)



A system of separated nucleons has a greater mass than a system of bound nucleons

- Due to mass-energy equivalence, this decrease in mass implies that energy is released
- Energy and mass are proportional, so, the total energy of a nucleus is **less than the sum of the energies** of its constituent nucleons

- Binding energy is defined as:

The energy required to break a nucleus into its constituent protons and neutrons

- The formation of a nucleus from a system of isolated protons and neutrons therefore **releases energy**, making it an **exothermic** reaction
  - This can be calculated using the equation:

$$\Delta E = \Delta mc^2$$

Mass-Energy Equivalence

- Einstein showed in his Theory of Relativity that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$\Delta E = \Delta mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})
- Some examples of mass-energy equivalence are:
  - The **fusion** of hydrogen into helium in the centre of the sun
  - The **fission** of uranium in nuclear power plants
  - Nuclear **weapons**
  - High-energy **particle collisions** in particle accelerators

Worked Example

The binding energy per nucleon is 7.98 MeV for an atom of Oxygen-16 (^{16}O).

Determine an approximate value for the energy required, in MeV, to completely separate the nucleons of this atom.

Step 1: List the known quantities

- Binding energy per nucleon, E = 7.98 MeV

Step 2: State the number of nucleons

- The number of nucleons is 8 protons and 8 neutrons, therefore 16 nucleons in total

Step 3: Find the total binding energy

- The binding energy for oxygen-16 is:

$$7.98 \times 16 = 127.7 \text{ MeV}$$

Step 4: State the final answer

- The approximate total energy needed to completely separate this nucleus is **127.7 MeV**

Exam Tip

Binding energy is named in a confusing way, so be careful!

Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put **into** the nucleus to **pull it apart.**

Atomic Mass Unit

- The unified atomic mass unit **(u or sometimes a.m.u)** is roughly equal to the mass of one proton or neutron:
  - **1 u = 1.66 × 10^{-27} kg**
  - This value is provided on the exam data sheet

- The a.m.u is commonly used in nuclear physics to express the mass of subatomic particles. It is defined as

The mass of exactly one-twelfth of an atom of carbon-12

- Therefore, one atom of carbon-12 has a mass of exactly 12 u
- Since mass and energy are interchangeable, the a.m.u can also be expressed in MeV
  - **1 u is equivalent to 931.5 MeV**

Table of common particles with mass in a.m.u

Particle	Mass / u
Proton	1
Neutron	1
Electron	0.0005
Alpha (α)	4

- The mass of an atom in a.m.u is roughly equal to the sum of its protons and neutrons (nucleon number)
  - For example, the mass of Uranium-235 is roughly equal to 235u
  - However, note that the actual mass is slightly lower than the expected mass, due to mass-energy equivalence

- a.m.u might be quoted in kg or MeV since mass and energy are equivalent via $\Delta E = \Delta mc^2$
  - MeV is a unit of energy whilst kg is a unit of mass

Worked Example

Estimate the mass of the nucleus of the element copernicium-285 in kg. Give your answer to 2 decimal places.

1. An atom's mass in atomic mass units (a.m.u) is approximately equal to its nucleon number.
2. Copernicium-285 has a mass of about 285 a.m.u.
3. To convert this to kilograms:
   - 1 a.m.u = 1.66×10^{-27} kg
   - So, $285 \times 1.66 \times 10^{-27} = 4.73 \times 10^{-25}$ kg (rounded to 2 decimal places).

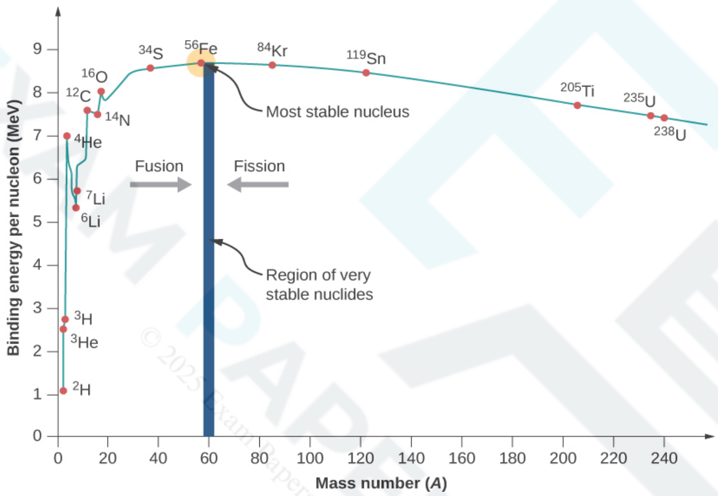
11.3 Binding Energy per Nucleon Graph

Binding Energy per Nucleon Graph

- When comparing the stability of different nuclei, it is useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability since it requires more energy to pull the nucleus apart
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the most stable of all the elements



By plotting a graph of binding energy per nucleon against nucleon number, the stability of elements can be inferred

Key Features of the Graph

- At low values of A:
  - Nuclei tend to have a lower binding energy per nucleon, hence, they are generally less stable
  - This means the lightest elements have weaker electrostatic forces and are the most likely to undergo **fusion**
- Helium (^{4}He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend
  - Helium-4 is a particularly stable nucleus hence it has a high binding energy per nucleon
  - Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together
- At high values of A:
  - The general binding energy per nucleon is high and gradually decreases with A
  - This means the heaviest elements are the most unstable and likely to undergo **fission**

Worked Example

Determine the binding energy per nucleon of Iron-56 ($^{56}_{26}Fe$) in MeV

Mass of a neutron = 1.675×10^{-27} kg

Mass of a proton = 1.673×10^{-27} kg

Mass of a $^{56}_{26}Fe$ nucleus = 9.228×10^{-26} kg

Step 1: Calculate the mass defect

Number of protons, Z = 26

Number of neutrons, A – Z = 56 – 26 = 30

Mass defect, $\Delta m = Zm_p + (A - Z)m_n - m_{total}$

$$\Delta m = (26 \times 1.673 \times 10^{-27}) + (30 \times 1.675 \times 10^{-27}) - (9.288 \times 10^{-26})$$

$$\Delta m = 8.680 \times 10^{-28} \text{ kg}$$

Step 2: Calculate the binding energy of the nucleus

Binding energy, $\Delta E = \Delta mc^2$

$$E = (8.680 \times 10^{-28}) \times (3.00 \times 10^8)^2 = 7.812 \times 10^{-11} \text{ J}$$

Step 3: Calculate the binding energy per nucleon

$$\text{Binding energy per nucleon} = \frac{E}{A}$$

$$\frac{E}{H} = \frac{7.812 \times 10^{-11}}{56} = 1.395 \times 10^{-12}\ J$$

Step 4: Convert to MeV

J → eV: divide by 1.6×10^{-19}

eV → MeV: divide by 10^6

$$\text{binding energy per nucleon} = \frac{1.395 \times 10^{-12}}{1.6 \times 10^{-19}} = 8\,718\,750\ eV = 8.7\ MeV\,(2\ s.f)$$

Exam Tip

Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

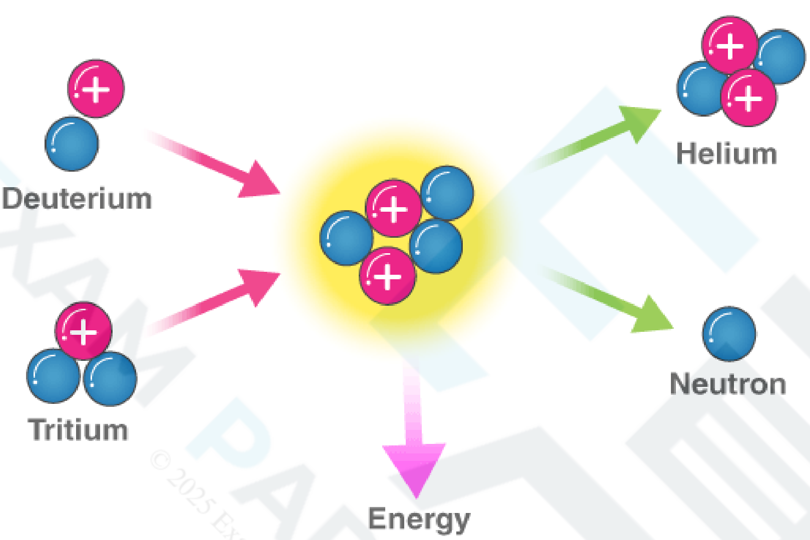
- You will be expected to draw the best fit curve AND a cross to show the **anomaly that is Helium**
- Do not begin your curve at A = 0, this is not a nucleus!
- Make sure to correctly label both axes AND units for binding energy **per nucleon**
- You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

Nuclear Fusion

- Fusion is defined as:

  Small nuclides that combine together to make larger nuclei, releasing energy

- Low mass nuclei (such as hydrogen and helium) can undergo fusion and release energy



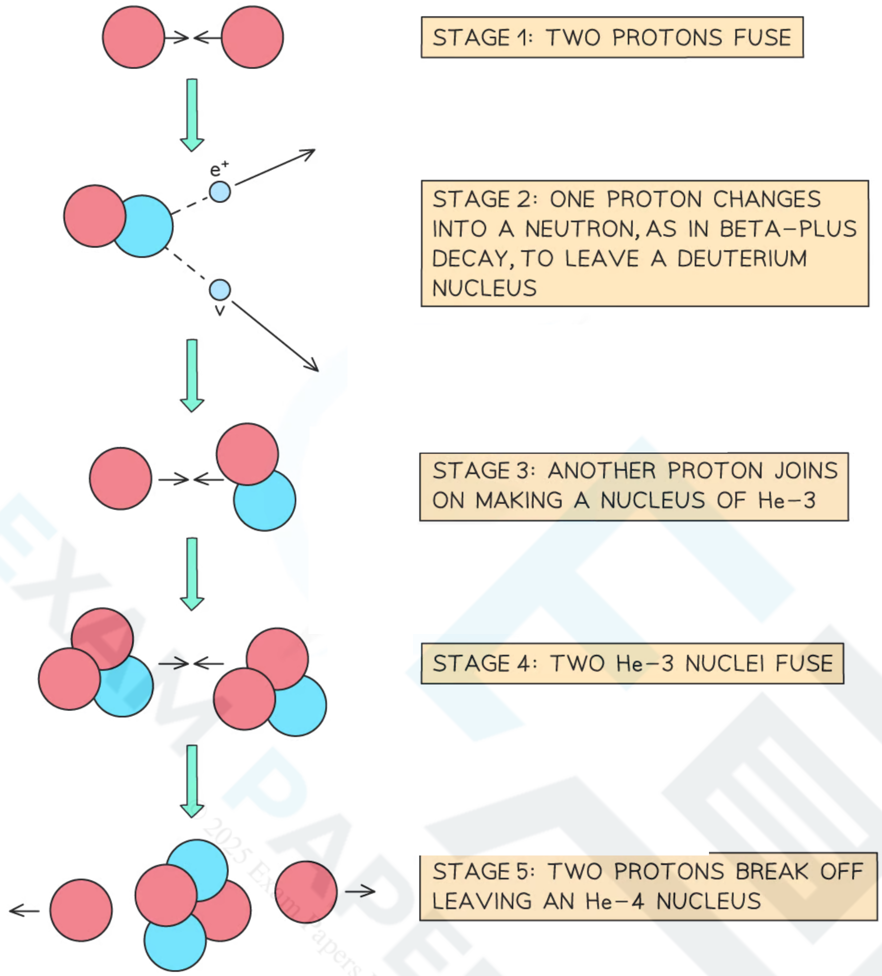
The fusion of deuterium and tritium to form helium with the release of energy

Conditions for Nuclear Fusion

- For two nuclei to fuse;
- Both must have high kinetic energy, to overcome the electrostatic repulsion between protons
  - This can only be achieved in an extremely high-energy environment, such as star's core
  - To achieve high enough kinetic energy the temperature must be on the scale of 100×10^6 Kelvin
- Very high density is also required

Fusion Products

- When two protons fuse, the element deuterium is produced
- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning



Exam Tip

In the fusion process, the mass of the new heavier nucleus is less than the mass of the constituent parts of the nuclei fused together, as some mass is converted into energy.

Not all of this energy is used as binding energy for the new larger nucleus, so energy will be released from this reaction. The binding energy per nucleon afterwards is higher than at the start.

Radioactivity Decay

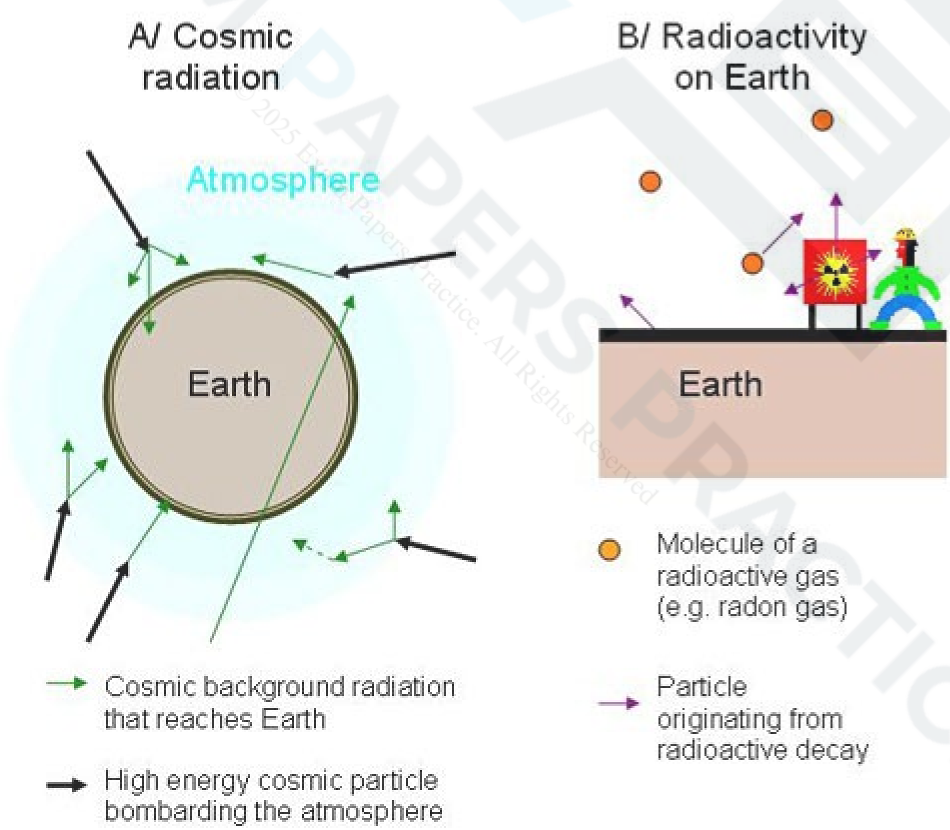
11.5 Background Radiation

Background Radiation

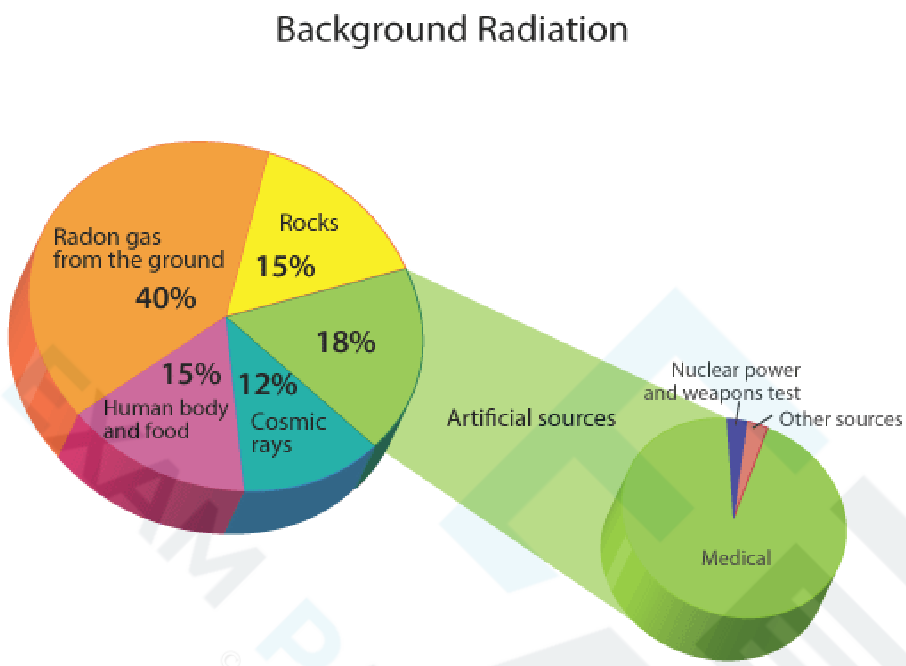
- Radiation is a natural phenomenon, with radioactive elements having always existed on Earth and in outer space
  - However, human activity has added to the amount of radiation that humans are exposed to in various ways
- Background radiation is defined as:

Low levels of radiation from environmental sources, which are always present around us

- Radiation is measured in **counts per second** in a unit called **Becquerel (Bq)**
- Different amounts of radiation are present in different places around the world, including in the UK.



* There are two types of background radiation:
  * Natural sources
  * Man-made sources



Background radiation is the radiation that is present all around the environment. Radon gas is given off from some types of rock

* Every second of the day there is some radiation emanating from **natural sources** such as:
  * Rocks
  * Cosmic rays from space
  * Foods

Natural Sources

* **Radon gas from rocks and soil**
  * Heavy radioactive elements, such as uranium and thorium, occur naturally in rocks in the ground
  * Uranium decays into radon gas, which is an alpha emitter
  * This is particularly dangerous if inhaled into the lungs in large quantities
* **Cosmic rays from space**
  * The sun emits an enormous number of protons every second
  * Some of these enter the Earth's atmosphere at high speeds
  * When they collide with molecules in the air, this leads to the production of gamma radiation
  * Other sources of cosmic rays are supernovae and other high energy cosmic events

- **Carbon-14 in biological material**
  - All organic matter contains a tiny amount of carbon-14
  - Living plants and animals constantly replace the supply of carbon in their systems hence the amount of carbon-14 in the system stays almost constant
- **Radioactive material in food and drink**
  - Naturally occurring radioactive elements can get into food and water since they are in contact with rocks and soil containing these elements
  - Some foods contain higher amounts such as potassium-40 in bananas
  - However, the amount of radioactive material is minuscule and is not a cause for concern

Man-Made Sources

- **Medical sources**
  - In medicine, radiation is commonly used in X-rays, CT scans, radioactive tracers, and radiation therapy
- **Nuclear waste**
  - While nuclear waste itself does not contribute much to background radiation, it can be dangerous for the people handling it
- **Nuclear fallout from nuclear weapons**
  - Fallout is the residue radioactive material that is thrown into the air after a nuclear explosion, such as the bomb that exploded at Hiroshima
  - While the amount of fallout in the environment is presently very low, it increases significantly in areas where nuclear weapons are tested
- **Nuclear accidents**
  - Accidents such as that in Chernobyl contributed a large dose of radiation into the environment
  - While these accidents are now extremely rare, they can be catastrophic

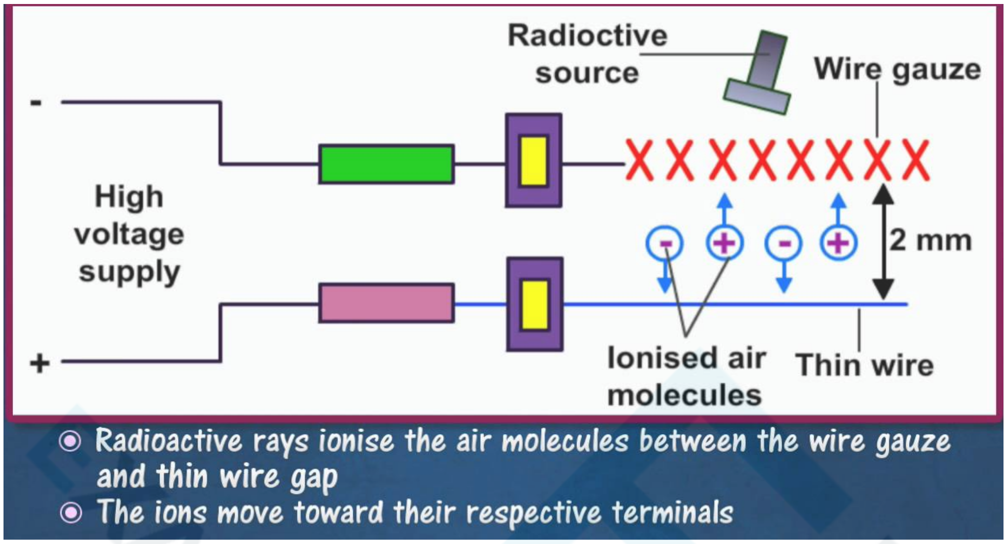
Corrected Count Rate

- Background radiation must be accounted for when taking readings in a laboratory
- This can be done by taking readings with no radioactive source present and then subtracting this from readings with the source present
  - This is known as the **corrected count rate**

Detecting Radiation

- When alpha or beta radiation pass close to an atom, they can deliver enough energy to remove electrons, ionising the atom
- Radiation detectors work by detecting the presence of either these ions, or the chemical changes that they produce
- Examples of radiation detectors include:
  - **Photographic film** (often used in badges)
  - **Geiger-Muller** (GM) tubes
  - **Ionisation chambers**
  - **Scintillation counters**

- Spark counters



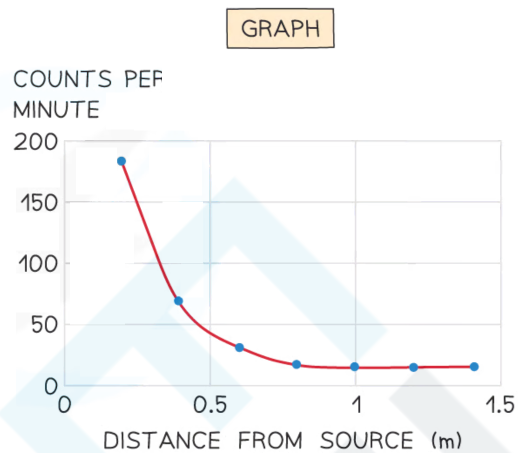
A Geiger-Muller tube (or Geiger counter) is a common type of radiation detector

Worked Example

A student is using a Geiger-counter to measure the counts per minute at different distances from a source of radiation. Their results and a graph of the results are shown here.

RESULTS TABLE

Distance from source (m)	Counts per minute
0.2	180
0.4	67
0.6	29
0.8	17
1.0	15
1.2	15
1.4	15



Determine the background radiation count.

Step 1: Determine the point at which the source radiation stops being detected

- The background radiation is the amount of radiation received all the time
- When the source and detector are far enough apart, the radiation is absorbed by the air before reaching the Geiger-counter
- Results after 1 metre should not change
- Therefore, the amount after 1 metre is only due to background radiation

Step 2: State the background radiation count

- The background radiation count is **15 counts per minute**

Exam Tip

Exam questions may expect you to remember about the existence of background radiation without mentioning it. Look out for count rates that do not drop to zero, or half-life graphs with a line that tends towards a value higher than zero.

When memorising lists of the causes of background radiation, make sure to choose at least one natural and one man-made cause as these are thought of quite separately.

11.6 Alpha, Beta & Gamma Radiation

Alpha, Beta & Gamma Particles

- Some elements have nuclei that are unstable
  - This tends to be when the number of nucleons does not balance
- In order to become more stable, they emit particles and/or electromagnetic radiation
  - These nuclei are said to be **radioactive**
- There are three different types of radioactive emission: Alpha, Beta and Gamma

Alpha Particles

- **Alpha (α) particles** are high energy particles made up of **2 protons and 2 neutrons** (the same as a helium nucleus)
- They are usually emitted from nuclei that are too large

$$^{4}_{2}\alpha \quad \text{OR} \quad ^{4}_{2}\text{He}$$

Beta Particles

- **Beta (β⁻) particles are high energy electrons** emitted from the nucleus
  - β⁻ particles are emitted by nuclei that have too many **neutrons**

$$^{0}_{-1}\beta$$

BETA MINUS

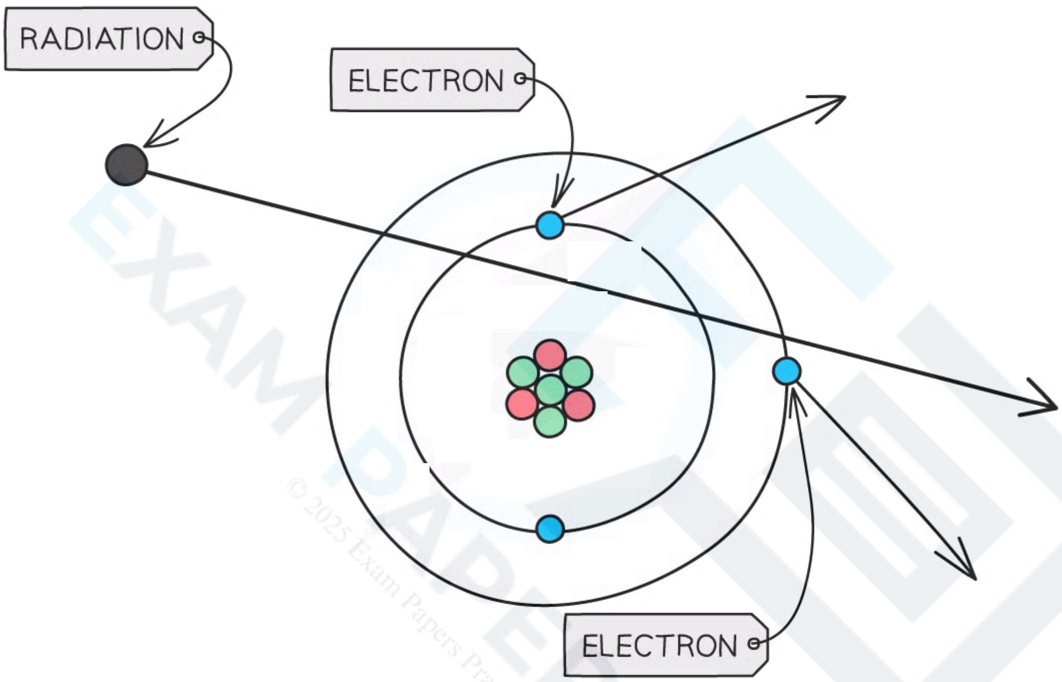
- Beta is a **moderately** ionising type of radiation
  - This is due to it having a charge of +1e
  - This means it is able to do some slight damage to cells (less than alpha but more than gamma)
- Beta is a **moderately** penetrating type of radiation
  - Beta particles have a range of around 20 cm - 3 m in air, depending on their energy
- Beta can be stopped by a few millimetres of **aluminium** foil

Gamma Rays

- **Gamma (γ) rays are high energy electromagnetic waves**
- They are emitted by nuclei that need to lose some energy

$$^{0}_{0}\gamma$$

- If these particles hit other atoms, they can knock out electrons, **ionising the atom**
- This can cause chemical changes in materials and can damage or kill living cells



When radiation passes close to atoms, it can knock out electrons, ionising the atom

- The properties of the different types of radiation are summarised in the table below

Name	Symbol	What is it?	Penetration depth in air	What blocks it?	Charge	Deflected by Magnetic Field?
Alpha	α or $^{4}{}_{2}He$	Helium nucleus: 2 protons and 2 neutrons	8cm	paper	Positive	Yes, less than beta particles because they have a higher mass
Beta	$^{0}_{-1}\beta$ or e^{-}	High energy electron	1m	3mm aluminium	Negative	Yes, more than alpha, and in the opposite direction because they have the opposite charge
Gamma	γ	Part of the Electromagnetic Spectrum	Forever	Several m of concrete or lead	None	No

- *u* is the atomic mass unit (see "Atomic Mass Unit (u)")
- *e* is the charge of the electron: 1.60×10^{-19} C
- c is the speed of light: 3×10^{8} m s^{-1}

Nuclear Decay Equations

Alpha Decay

- An alpha particle consists of **2 protons and 2 neutrons**

  (It is emitted from large unstable nuclei)
- When an alpha particle is emitted from a nucleus:
  - The nucleus loses 2 protons:

    The proton (atomic) number decreases by 2
  - The nucleus loses 4 particles (nucleons) in total:

    The nucleon (mass) number decreases by 4

- Equation for alpha emission:

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

$$\text{e.g.} \quad {}^{212}_{84}Po \longrightarrow {}^{208}_{82}Y + {}^{4}_{2}\alpha$$

- **Nuclear equations, just like chemical equations, balance**:
  - The sum of the upper (mass) numbers on the left of each equation should equal the sum on the right
  - The sum of the lower (atomic) numbers should also balance

Beta Emission

- Equation for beta emission:

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta$$

$$\text{e.g.} \quad {}^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta$$

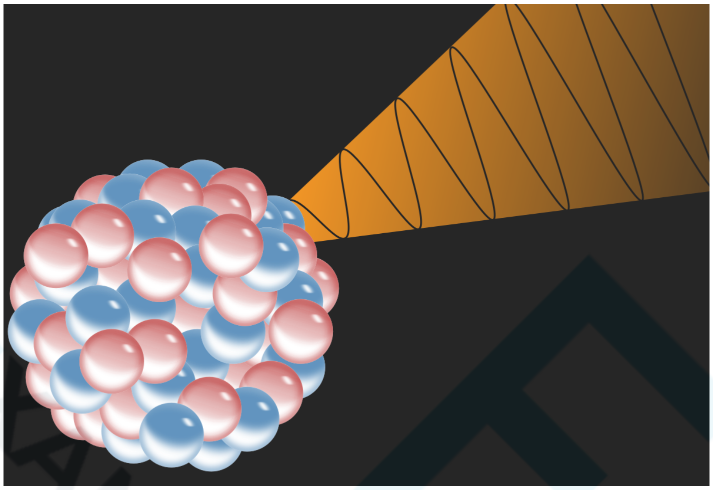
- Note that the beta particle is given an atomic number of **–1** in the above examples

  This is because the atomic number is being used to measure charge in this case: Protons, being positive particles, have positive atomic numbers

Electrons, being negative, have a negative number

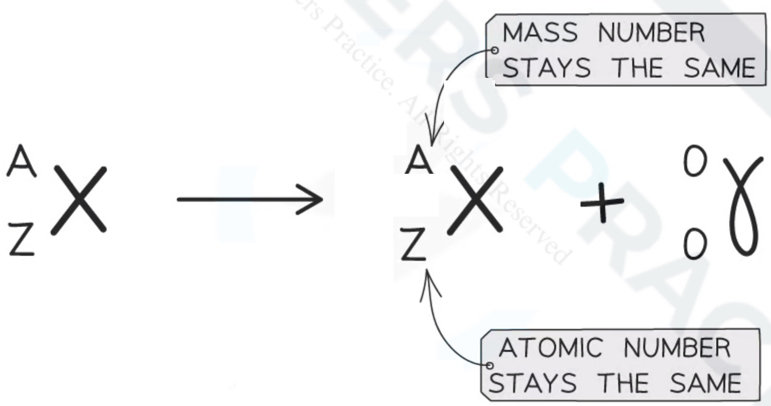
Gamma Decay

- During gamma decay, a gamma ray is emitted from an unstable nucleus
- The process that makes the nucleus less energetic but does not change its structure



Gamma decay does not affect the mass number or the atomic number of the radioactive nucleus, but it does reduce the energy of the nucleus

- The gamma ray that is emitted has a lot of energy, but no mass or charge
- Here is an example of Uranium-238 undergoing gamma decay
  - Notice that the mass number and atomic number of the unstable nuclei remains the same during the decay



Gamma decay equation

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z}X + \gamma$$

$$\text{e.g. } {}^{137}_{56}Ba \longrightarrow {}^{137}_{56}Ba + \gamma$$

An example of Barium decay through the release of a gamma ray

Step 1:

In alpha decay, the parent atom **loses 2 protons** and **2 neutrons**, meaning its **nucleon number decreases by 4** and **proton number by 2**.

Step 2:

To find the number of neutrons:

Neutrons = Nucleon number – Proton number

= 222 – 86 = 136

Step 3:

A helium nucleus (alpha particle) has **2 protons** and **2 neutrons**.

Step 4:

After alpha decay, the atom **moves 2 steps down in proton number** and **2 steps left in neutron number**, reaching **Point C** on the chart.

	Neutron Number →				
Proton ↓	**132**	**133**	**134**	**135**	**136**
87		**A**			**Rn (Radon)**
86					
85				**C ← (After decay)**	
84					
83			**B**		
82					

- The original nucleus is **Rn at (86, 136)**
- After alpha decay, it moves to **C at (84, 134)**

Core Practical 15: Investigating Gamma Radiation Absorption

Aim of the Experiment

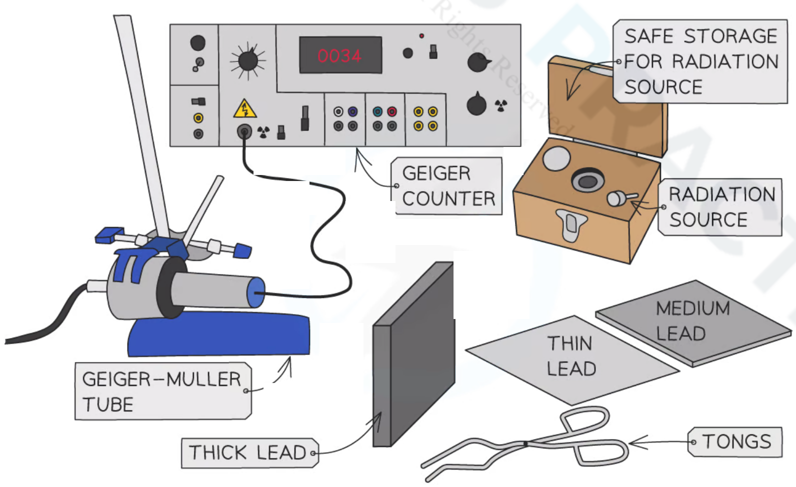
- To investigate the absorption of gamma rays by different thicknesses of lead

Variables:

- Independent variable = Thickness of lead
- Dependent variable = Count rate
- Control variables:
  - Radioactive source
  - Distance of GM tube to source
  - Location / background radiation

Equipment List

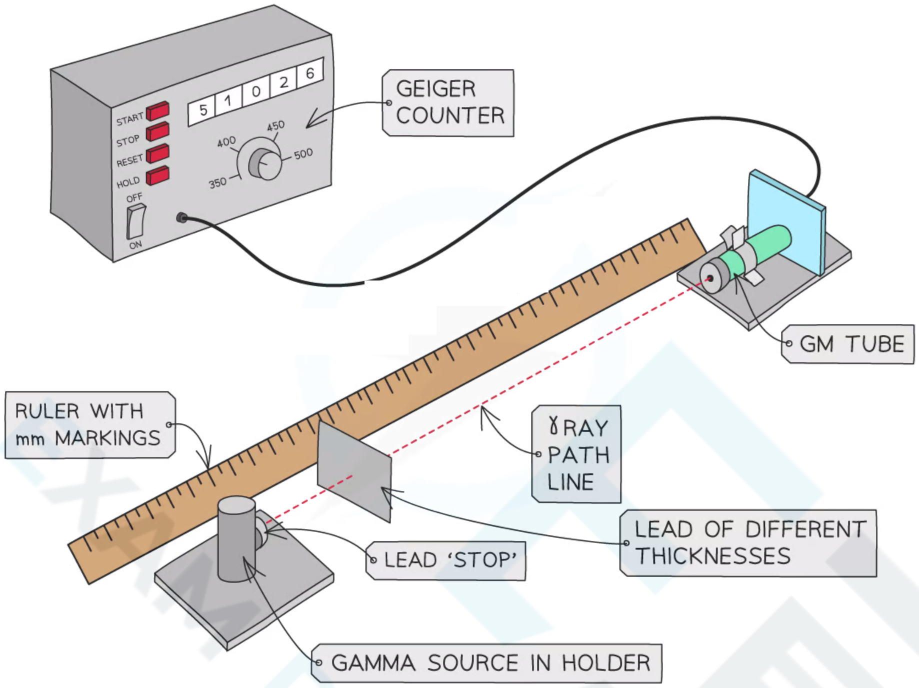
Equipment	Purpose
Radioactive gamma source	Used for testing
Ruler	To control the distance between the sources and the GM tube
Mount for radioactive sources	To hold the sources securely
Geiger–Muller tube and counter	For measuring count rate
Tongs	For safe handling of the sources
Different thicknesses of lead	To be changed as the independent variable. Increasing intervals of 0.5 mm
Lead lined container for storing sources when not in use	For safety and to ensure results are not affected by sources not being tested



- Resolution of measuring equipment:
  - Ruler = 1 mm

- Geiger-Müller tube = 0.01 µS/hr

Method



1. Connect the **Geiger-Müller tube** to the **counter** and, without any sources present, **measure background radiation** over a five-minute period
    - Record this value
    - Calculate the average background rate per minute
2. Measure the thickness of the lead absorbers using Vernier calipers at three points on each sheet.
    - For each sheet record the average thickness
3. Place the radioactive source **a fixed distance** of 10 cm away from the tube
4. Record the count rate over **one minute**
5. Repeat steps 3 and 4 a further two times, recording the count rate each time
6. Place the thinnest absorber directly in front of the gamma ray source
7. Repeat steps 3–5
8. Replace the sheet with another thickness and continue taking three readings per thickness

Analysis of Results

- **If the count over that interval falls to background levels** (allow for a little random variation), then the radiation has all been absorbed
- You will be able to determine the thickness of the lead required to absorb gamma radiation

Evaluating the Experiment

Systematic Errors:

- Make sure that the source is stored well away from the counter during the experiment

- Conduct all runs of the experiment in the same location to avoid changes in background radiation levels

Random Errors:

- The accuracy of such an experiment is improved with using a reliable source of radiation with a long half-life and an activity well above the natural background level

Safety Considerations

- When not using a source, keep it in a lead-lined container
- When in use, try and keep a good distance (a metre or so) between yourself and the source
- When handling the source, do so using tweezers (or tongs) and point the source away from you
- Wash your hands and remove your outer layer of clothing after handling a radioactive source

Exam Tip

When answering questions about the core practicals you could try to remember the acronym SCREAMS:

- S: Which variable will you keep the **same**
- C: which variable should you **change**
- R: what will you do to make your experiment **reliable**
- E: what special **equipment and equations** are required
- A: how will you **analyse** your results
- M: which variable will you **measure**
- S: what **safety** precautions will you take?

The Random Nature of Nuclear Decay

- Radioactive decay is defined as:

  The spontaneous disintegration of a nucleus to form a more stable nucleus, resulting in the emission of an alpha, beta or gamma particle

- The random nature of radioactive decay can be demonstrated by observing the count rate of a Geiger-Muller (GM) tube
  - When a GM tube is placed near a radioactive source, the counts are found to be irregular and cannot be predicted
  - Each count represents a decay of an unstable nucleus
  - These fluctuations in count rate on the GM tube **provide evidence for the randomness of radioactive decay**

The variation of count rate over time of a sample radioactive gas. The fluctuations show the randomness of radioactive decay

- Radioactive decay is both **spontaneous** and **random**
- A spontaneous process is defined as:

  A process which cannot be influenced by environmental factors

- This means radioactive decay is not affected by environmental factors such as:
  - Temperature
  - Pressure

- Chemical conditions

- A random process is defined as:

A process in which the exact time of decay of a nucleus cannot be predicted

- Instead, the nucleus has a **constant probability,** i.e.. the same chance, of decaying in a given time
  - Therefore, with large numbers of nuclei, it **is** possible to statistically predict the behaviour of the **entire group**

Exam Tip

Make sure you can define what constitutes a radioactive decay, a random process and a spontaneous decay - these are all very common exam questions!

Activity & The Decay Constant

- Since radioactive decay is spontaneous and random, it is useful to consider the average number of nuclei which are expected to decay per unit time
  - This is known as the **average decay rate**
- As a result, each radioactive element can be assigned a **decay constant**
- The decay constant λ is defined as:

The probability, per second, that a given nucleus will decay

- When a sample is highly radioactive, this means the number of decays per unit time is very high
  - This suggests it has a high level of **activity**
- Activity, or the number of decays per unit time can be calculated using:

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

- Where:
  - A = activity of the sample (Bq)
  - ΔN = number of decayed nuclei
  - Δt = time interval (s)
  - λ = decay constant (s^{-1})
  - N = number of nuclei remaining in a sample
- The activity of a sample is measured in **Becquerels** (Bq)
  - An activity of 1 Bq is equal to one decay per second, or 1 s^{-1}
- This equation shows:
  - The greater the decay constant, the greater the activity of the sample
  - The activity depends on the number of undecayed nuclei remaining in the sample
  - The minus sign indicates that the number of nuclei remaining decreases with time - however, for calculations it can be omitted

Worked Example

Americium-241 is an artificially produced radioactive element that emits α-particles. A sample of americium-241 of mass 5.1 μg is found to have an activity of 5.9×10^5 Bq.

(a)
Determine the number of nuclei in the sample of americium-241.
(b)
Determine the decay constant of americium-241.

Part (a)

Step 1: Write down the known quantities

- Mass = 5.1 μg = 5.1 × 10^{-6} g
- Molecular mass of americium = 241
- N_A = Avogadro constant

Step 2: Write down the equation relating number of nuclei, mass and molecular mass

$$\text{Number of nuclei} = \frac{\text{mass} \times N_A}{\text{molecular mass}}$$

Step 3: Calculate the number of nuclei

$$number\ of\ nuclei = \frac{(5.1 \times 10^{-6}) \times (6.02 \times 10^{23})}{241} = 1.27 \times 10^{16}$$

Part (b)

Step 1: Write the equation for activity

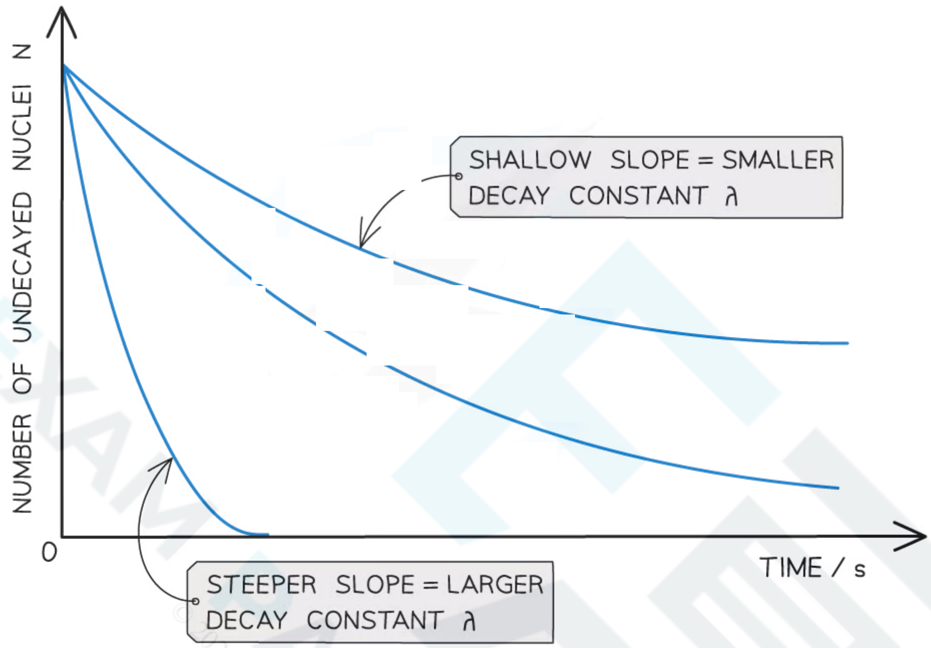
Activity, A = λN

Step 2: Rearrange for decay constant λ and calculate the answer

$$\lambda = \frac{A}{N} = \frac{5.9 \times 10^{5}}{1.27 \times 10^{16}} = 4.65 \times 10^{-11}\ s^{-1}$$

Exponential Decay

- In radioactive decay, the number of nuclei falls very rapidly, without ever reaching zero
  - Such a model is known as **exponential decay**
- The graph of number of undecayed nuclei and time has a very distinctive shape



Radioactive decay follows an exponential pattern. The graph shows three different isotopes each with a different rate of decay

Radioactive Decay Equation

- The number of undecayed nuclei N can be represented in exponential form by the equation:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

The exponential function e

- The symbol e represents the exponential constant
  - It is approximately equal to e = 2.718
- On a calculator it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if $e^x = y$, then $x = \ln(y)$

Worked Example

Strontium-90 decays with the emission of a β-particle to form Yttrium-90. The decay constant of Strontium-90 is 0.025 $year^{-1}$.

Determine the activity A of the sample after 5.0 years, expressing the answer as a fraction of the initial activity A_0

Step 1: Write out the known quantities

Decay constant, λ = 0.025 $year^{-1}$

Time interval, t = 5.0 years

Both quantities have the same unit, so there is no need for conversion

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange the equation for the ratio between A and A_0

$$\frac{A}{A_0} = e^{-\lambda t}$$

Step 4: Calculate the ratio A/A_0

$$\frac{A}{A_0} = e^{-(0.025 \times 5)} = 0.88$$

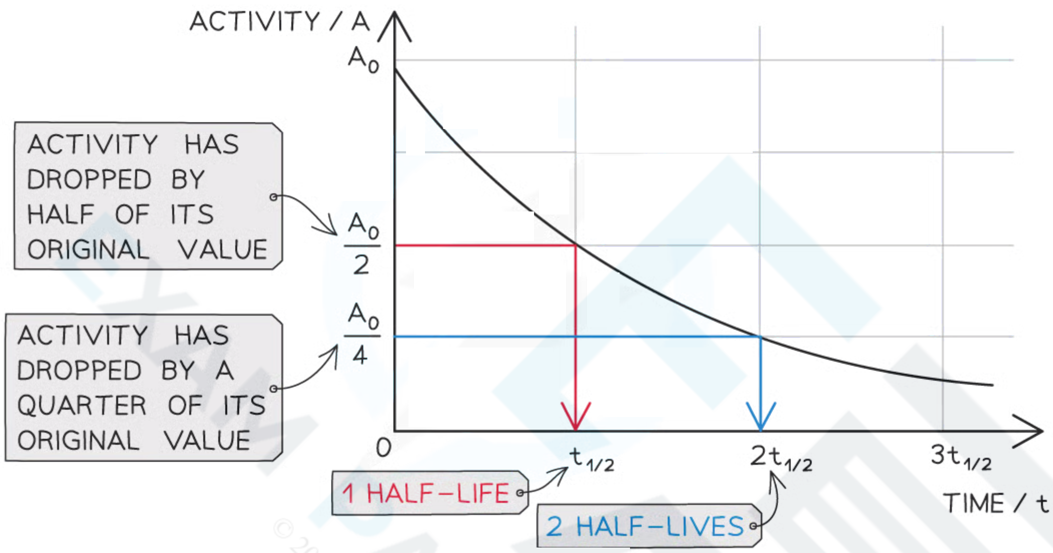
Therefore, the activity of Strontium-90 decreases by a factor of 0.88, or 12%, after 5 years

Half Life

- Half-life is defined as:

The time taken for half the number of nuclei in a sample to decay

- This means when a time equal to the half-life has passed, the **activity** of the sample will also half
- This is because the activity is proportional to the number of undecayed nuclei, $A \propto N$



When a time equal to the half-life passes, the activity falls by half, when two half-lives pass, the activity falls by another half (which is a quarter of the initial value)

- To find an expression for half-life, start with the equation for exponential decay:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N = number of nuclei remaining in a sample
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

- When time t is equal to the half-life $t_{½}$, the activity N of the sample will be half of its original value, so $N = ½ N_0$

$$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{½}}$$

- The formula can then be derived as follows:

Divide both sides by N_0: $\frac{1}{2} = e^{-\lambda t_{½}}$

Take the natural log of both sides: $\ln\left(\frac{1}{2}\right) = -\lambda t_{½}$

Apply properties of logarithms: $\lambda t_{½} = \ln(2)$

- Therefore, half-life $t_{½}$ can be calculated using the equation:

$$t_{½} = \frac{\ln 2}{\lambda} \simeq \frac{0.693}{\lambda}$$

- This equation shows that half-life $t_{½}$ and the radioactive decay rate constant λ are inversely proportional
  - Therefore, the **shorter** the half-life, the **larger** the decay constant and the **faster** the decay

Worked Example

Strontium-90 is a radioactive isotope with a half-life of 28.0 years.A sample of Strontium-90 has an activity of 6.4×10^9 Bq. Calculate the decay constant λ, in s^{-1}, of Strontium-90.

Step 1: Convert the half-life into seconds

- $t_{½}$ = 28 years = 28 × 365 × 24 × 60 × 60 = 8.83×10^8 s

Step 2: Write the equation for half-life

$$t_{½} = \frac{\ln 2}{\lambda}$$

Step 3: Rearrange for λ and calculate

$$\lambda = \frac{\ln 2}{t_{½}} = \frac{\ln 2}{8.83 \times 10^8} = \mathbf{7.85 \times 10^{-10}\ s^{-1}}$$

Exam Tip

Although you may not be expected to derive the half-life equation, make sure you're comfortable with how to use it in calculations such as that in the worked example.