

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel Level 3 GCE

Wednesday 3 June 2026

Paper reference
 Afternoon (Time: 2 hours) **9MA0/01**

Mathematics
Advanced
PAPER 1: Pure Mathematics 1

You must have:
 Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks



Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
 Answer the questions in the spaces provided
 – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
 Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
 There are 15 questions in this question paper. The total mark for this paper is 100.
 The marks for **each** question are shown in brackets
 – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
 Try to answer every question.
 Check your answers if you have time at the end.

Turn over ►

1. The point $P(-4, 5)$ lies on the continuous curve with equation $y = f(x)$. Find the coordinates of the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = 4f\left(\frac{x}{2}\right)$

(2)

(b) $y = f(x - 2)$

(2)

$$(a) \quad (-4, 5) \rightarrow \left(\frac{-4}{2}, 4 \times 5\right)$$

$$\therefore (-2, 20) //$$

$$(b) \quad (-7, 3) //$$

Question

1

continued

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(Total for Question 1 is 4 marks)

2.

$$f(x) = e^{2x-1} - 3x - 7$$

The equation $f(x) = 0$ has a single root α

(a) Show that α lies in the interval $[1.1, 1.2]$

(2)

The iterative formula

$$x_{n+1} = \frac{1 + \ln 7 - 3x_n}{2}$$

is used to find an approximate value for α

Starting with $x_1 = 1.1$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of α

(3)

$$(a) f(1.1) = e^{2(1.1)-1} + 3(1.1) - 7 = -0.38 < 0$$

$$f(1.2) = e^{2(1.2)-1} + 3(1.2) - 7 = 0.66 > 0$$

Since $f(1.1) < 0$ and $f(1.2) > 0$, and $f(x)$ is continuous

α lies in $[1.1, 1.2]$

$$(b)(i) x_2 = 1.1542$$

$$(ii) \alpha = 1.1384$$

Question

2

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(Total for Question 2 is 5 marks)

3. The curve C has equation

$$y = x^3 - 2x^2 + 5x - 7$$

(a) Find $\frac{dy}{dx}$ giving the answer in simplest form.

(2)

(b) Hence find the equation of the tangent to C at the point where $x=2$.
Give the answer in the form $y = mx + c$, where m and c are integers.

(Solutions relying on calculator technology are not acceptable.)

(3)

$$(a) \frac{dy}{dx} = 3x^2 - 4x + 5$$

$$(b) m = 3(2)^2 - 4(2) + 5$$

$$= 12 - 8 + 5$$

$$= 9$$

$$\text{when } x=2; y = 2^3 - 2(2^2) + 5(2) - 7$$

$$= 8 - 8 + 10 - 7 = 3$$

tangent passes through $(2, 3)$ with gradient 9

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 9(x - 2)$$

$$y - 3 = 9x - 18$$

$$y = 9x - 15$$

$$\underline{\underline{y = 9x - 15}}$$

Question

3

continued

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(Total for Question 3 is 5 marks)

4. The functions f and g are defined by

$$f(x) = 3x^2 - 2x - 4$$

$$g(x) = \frac{5x-1}{x-4}$$

- (a) State the range of f (1)
- (b) Find $gf^{-1}(2)$ (2)
- (c) Find $g^{-1}(x)$ (2)

$$(a) [-2, \infty)$$

$$(b) \text{ Since } f(2) = 10$$

$$g(f(2)) = g(10) = \frac{49}{6} //$$

$$(c) y = \frac{5x-1}{x-4}$$

$$y(x-4) = 5x-1$$

$$yx - 4y = 5x - 1$$

$$yx - 5x = 4y - 1$$

$$x(y-5) = 4y-1$$

$$g^{-1}(x) = \frac{4x-1}{x-5}$$

Question

4

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(Total for Question 4 is 5 marks)

5



Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The graph shown in Figure 1 shows the flight path of a model helicopter.

The helicopter travels in a vertical plane above horizontal ground.

The helicopter

- starts its flight from the point on the ground represented by O
- lands on the ground at the point represented by A

The height, y metres, of the helicopter above the ground has been plotted against the horizontal distance, x metres, measured from O .

The flight path of the helicopter is modelled by the curve with parametric equations

$$x = 100t \sqrt{y} = 10 \sin \frac{t^2}{4} \quad t \geq 0$$

where t seconds is the time after the helicopter starts its flight.

- (a) Write down, according to the model, the maximum height of the helicopter above the ground. (1)

- (b) Find, according to the model, the horizontal distance the helicopter has travelled when it is 5 m above the ground for the **second** time. (4)

$$(a) \text{ Max. height when } \sin\left(\frac{t^2}{4}\right) = 1$$

$$\text{Max height} = \underline{\underline{10\text{m}}}$$

$$(b) 10 \sin\left(\frac{t^2}{4}\right) = 5$$

$$\sin\left(\frac{t^2}{4}\right) = \frac{1}{2}$$

$$\text{Let } \theta = \frac{t^2}{4}$$

$$\theta = \frac{\pi}{6} \text{ or } \theta = \frac{5\pi}{6}$$

$$\text{Second time } (\theta = \frac{5\pi}{6})$$

$$\frac{t^2}{4} = \frac{5\pi}{6}$$

$$t^2 = \frac{10\pi}{3}$$

$$t = \sqrt{\frac{10\pi}{3}}$$

$$x = 100\sqrt{t} = 100\left(\frac{10\pi}{3}\right)^{1/4}$$

$$\text{Horizontal distance} = \underline{\underline{180\text{m}}}$$

(Total for Question 5 is 5 marks)

6. (a) Find

$$\frac{3}{x^2} dx \quad (2)$$

(b) Hence find, in simplest form, the exact value of the constant p for which

$$\int_2^p \frac{3}{7x+1} dx = \frac{6}{7} \quad p > 2 \quad (3)$$

$$(a) \int \frac{3}{7x+1} dx = \frac{3}{7} \ln |7x+1| + C$$

$$(b) \left[\frac{3}{7} \ln |7x+1| \right]_2^p = \frac{6}{7}$$

$$\frac{3}{7} (\ln |7p+1| - \ln 15) = \frac{6}{7}$$

$$\ln |7p+1| - \ln 15 = 2$$

$$\ln \left(\frac{7p+1}{15} \right) = 2$$

$$\frac{7p+1}{15} = e^2$$

$$7p+1 = 15e^2$$

$$p = \frac{15e^2 - 1}{7} //$$

Question

6

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(Total for Question 6 is 5 marks)

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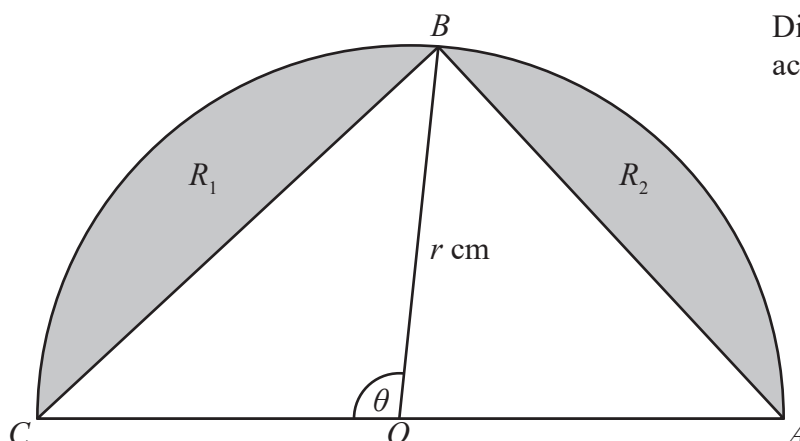


Figure 2

Figure 2 shows a semicircle $OABCO$ with centre O and radius r cm.

Given that

- angle BOC is θ radians
- the region R_1 , shown shaded in Figure 2, is bounded by the arc BC and the chord BC
- the region R_2 , also shown shaded in Figure 2, is bounded by the arc AB and the chord AB
- the area of R_1 is twice the area of R_2

(a) show that θ satisfies the equation

$$p \sin \theta \approx q\theta \approx 2\pi \approx 0$$

where p and q are integers to be found.

(4)

(b) Hence, taking 1.3 as a first approximation for θ , apply the Newton-Raphson method once to obtain a second approximation for θ . Give the answer to 3 decimal places.

(3)

$$(a) \text{ Area } R_1 = \frac{1}{2}r^2(\theta - \sin\theta)$$

$$\text{Area } R_2 = \frac{1}{2}r^2(\pi - \theta - \sin\theta)$$

$$\text{Given } R_1 = 2R_2;$$

$$\theta - \sin\theta = 2(\pi - \theta - \sin\theta)$$

$$\theta - \sin\theta = 2\pi - 2\theta - 2\sin\theta$$

$$3\theta + \sin\theta - 2\pi = 0$$

$$\text{So } p=1, q=3 //$$

$$(b) f(\theta) = \cos \theta + 3$$

$$\theta = 1.3$$

$$f_1(1.3) \approx -1.4196$$

$$f'(1.3) \approx 3.2675$$

$$\theta_2 = 1.3 - \frac{f(1.3)}{f'(1.3)} \approx \underline{\underline{1.735}}$$

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Question

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(Total for Question 7 is 7 marks)

8.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

(a) Express $\lim_{\delta x \rightarrow 0} \sum_{x=0}^3 x e^{2x} \delta x$ as an integral. (1)

(b) Hence show that

$$\lim_{\delta x \rightarrow 0} \sum_{x=0}^3 x e^{2x} \delta x = A e^6 + B$$

where A and B are constants to be found. (4)

$$(a) \lim_{\delta x \rightarrow 0} \sum_{x=0}^3 x e^{2x} \delta x = \int_0^3 x e^{2x} dx$$

$$(b) \int x e^{2x} dx = \left[\frac{1}{2} x e^{2x} \right] - \int \frac{1}{2} e^{2x} dx$$

$$\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$$

Evaluating from 0 to 3:

$$\left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_0^3$$

$$\left(\frac{1}{2} (3) e^{2(3)} - \frac{1}{4} e^{2(3)} \right) - \left(\frac{1}{2} (0) e^0 - \frac{1}{4} e^0 \right)$$

$$= \left(\frac{3}{2} e^6 - \frac{1}{4} e^6 \right) - \left(0 - \frac{1}{4} \right)$$

$$= \frac{5}{4}e^6 + \frac{1}{4}$$

$$\therefore A = \frac{5}{4} \text{ and } B = \frac{1}{4}$$

(Total for Question 8 is 5 marks)

9.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos \theta + \sin \theta \csc \theta - \sec \theta = k \cot 2\theta \quad \theta \neq 90^\circ + n^\circ$$

where k is an integer to be found.

(3)

(b) Hence solve, for $90^\circ < x < 90^\circ$, the equation

$$5 \cos x \sin x \csc x \sec x = 4 \csc 2x$$

Give each answer, in degrees, to one decimal place.

(5)

$$(a) \text{ Let } (\cos \theta + \sin \theta)(\csc \theta - \sec \theta)$$

$$= (\cos \theta + \sin \theta) \left(\frac{1}{\sin \theta} - \frac{1}{\cos \theta} \right)$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} = \frac{\cos(2\theta)}{\frac{1}{2} \sin(2\theta)} = 2 \cot(2\theta)$$

$$\underline{\underline{k=2}}$$

(b) Solutions:

$$2u = 53.13^\circ \text{ or } 126.87^\circ$$

$$u = 13.3^\circ \text{ or } 31.7^\circ$$

$$\therefore \underline{\underline{13.3^\circ, 31.7^\circ, -76.7^\circ, -58.3^\circ}}$$

Question

9

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(Total for Question 9 is 8 marks)

10.

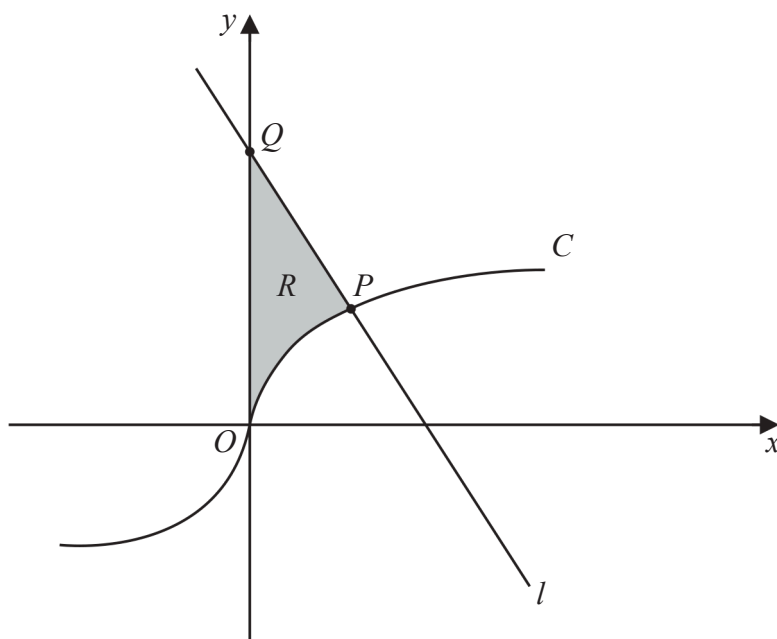


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{7x}{\sqrt{3x^2 + 4}}$$

The point P lies on C and has x coordinate 2

The line l , also shown in Figure 3, is the normal to C at P .

(a) Use calculus to find an equation for l .

Give the answer in the form $ax + by + c = 0$, where a , b and c are integers.

(5)

The line l intersects the y -axis at the point Q .

(b) Write down the exact y coordinate of Q .

(1)

The region R , shown shaded in Figure 3, is bounded by C , l and the y -axis.

(c) Use algebraic integration to find the exact area of R .

(5)

$$1a) \text{ At } x=2, y = \frac{7 \times 2}{\sqrt{3(2^2)+4}} = \frac{14}{\sqrt{16}} = \frac{14}{4} = 3.5$$

$$\frac{dy}{dx} = 7(3x^2+4)^{-\frac{1}{2}} + 7x(-\frac{1}{2})(3x^2+4)^{-\frac{3}{2}}(6x)$$

$$\frac{dy}{dx} = \frac{7}{16} \text{ at } x=2 \text{ so m' of normal} = -\frac{16}{7}$$

$$\text{Equation: } y - \frac{7}{2} = -\frac{16}{7}(x-2)$$

$$32x + 14y - 113 = \underline{\underline{0}}$$

$$(b) \text{ At } x=0: 14y = 113 \rightarrow y = \frac{113}{14}$$

$$(c) \quad \text{Area} = \int_0^2 \left(-\frac{16}{7}x + \frac{113}{14} - \frac{7x}{\sqrt{3x^2+4}} \right) dx$$

$$\int_4^{16} \frac{7}{6\sqrt{u}} du = \frac{7}{6} [2\sqrt{u}]_4^{16} = \frac{7}{3} (\sqrt{16} - \sqrt{4})$$

$$= \frac{7}{3} (4-2) = \frac{14}{3}$$

$$\text{Area } R = \frac{81}{7} - \frac{14}{3} = \frac{243-98}{21} = \underline{\underline{\frac{145}{21}}}$$

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Question

10

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(Total for Question 10 is 11 marks)

11.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

- (a) Express $4\cos x - 13\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

The demand for electricity, D gigawatts (GW), in the UK on a particular day, is modelled by the equation

$$D = 30.4\cos\left(\frac{t}{12}\right) - 0.213\sin\left(\frac{t}{12}\right) \quad 0 < t < 24$$

where t is the number of hours after midnight.

Use the equation of the model and the answer to part (a), to answer parts (b) and (c).

- (b) (i) Deduce the minimum demand for electricity during this day.

- (ii) Find the time of the day when this minimum demand occurs.

(4)

At 3 pm on this day, the demand for electricity is increasing at a rate of k GW per hour.

- (c) Find the value of the constant k , giving the answer to 3 significant figures.

(3)

- (d) Give a reason why it would not be sensible to use the same equation to model the demand for electricity in the UK on every day of the year.

(1)

$$(a) R = \sqrt{4^2 + (-13)^2} = \sqrt{185}$$

$$\cos \alpha = \frac{4}{\sqrt{185}}, \sin \alpha = \frac{13}{\sqrt{185}}$$

$$\alpha = 1.272 //$$

$$(b) (i) \text{ Minimum} = 30 - R = 30 - \sqrt{185} \approx 16.4 \text{ GW}$$

$$(ii) \text{ Min occurs when } \cos\left(\frac{\pi t}{12} + 0.2 + \alpha\right) = -1$$

$$\frac{\pi t}{12} + 0.2 + \alpha = \pi$$

$$\frac{\pi t}{12} = \pi - 0.2 - \alpha$$

$$t = \frac{12}{\pi} (\pi - 0.2 - \alpha)$$

$$\text{With } \alpha = 1.267$$

$$t = 6.38 //$$

$$(c) D = 30 + \sqrt{185} \cos\left(\frac{\pi t}{12} + 0.2 + 1.272\right) = 30 + \sqrt{185} \cos\left(\frac{\pi t}{12} + 1.472\right)$$

$$\frac{dD}{dt} = -\sqrt{185} \cdot \frac{\pi}{12} \sin\left(\frac{\pi t}{12} + 1.472\right)$$

$$\frac{dD}{dt} = -\frac{\sqrt{185}\pi}{12} \sin\left(\frac{15\pi}{12} + 1.472\right)$$

$$\frac{dD}{dt} \approx -3.561... \times (-0.771)$$

$$K \approx 2.747...$$

$$\therefore K = \underline{\underline{2.75}}$$

(d) Equation does not take into account seasonal variations

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Question

11

continued

Lined area for writing the answer to Question 11.

(TotalforQuestion11 is 11 marks)

12. (a) Sketch the graph with equation

$$y = |px - q|$$

where p and q are **positive** constants.

State, on your sketch, the coordinates of all points where the graph meets or cuts the coordinate axes.

(3)

The curve C has equation

$$y = \frac{k}{x}$$

where k is a **positive** constant.

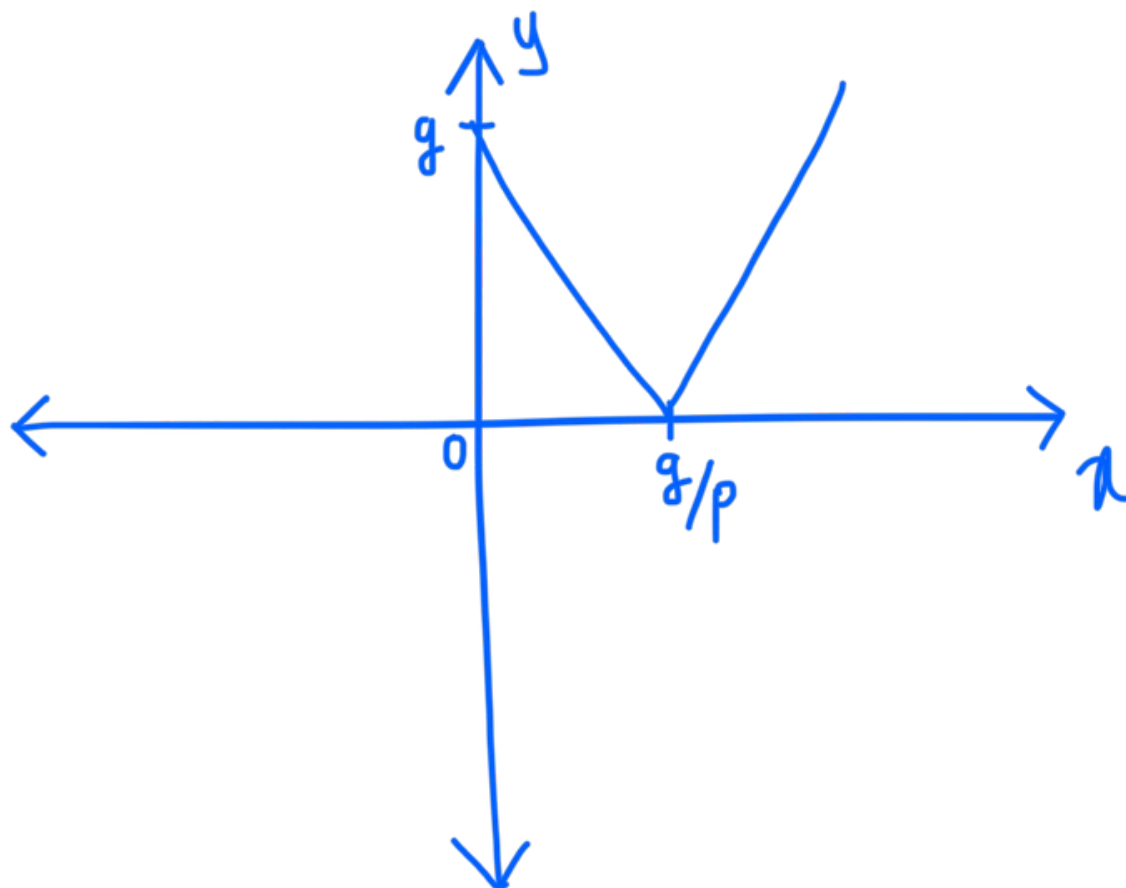
Given that C intersects the graph with equation $y = |px - q|$ at 3 distinct points,

(b) find, in terms of p and q , the range of values of k .

(4)

(a) V-Shaped graph with Vertex at $(\frac{q}{p}, 0)$

Intercepts axes at $(0, q)$ and $(\frac{q}{p}, 0)$



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(b) Range of k: $0 < k < \frac{g^2}{4p}$

(Total for Question 12 is 7 marks)

13.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

The 3rd, 4th and 5th terms of a geometric sequence are $\sin \theta$, $\sqrt{2}\cos \theta$ and $\sqrt{3}\cot \theta$ respectively, where θ is a constant, $0 < \theta < \frac{\pi}{2}$

(a) Find the exact value of the common ratio of this sequence.

(4)

(b) Hence find, in simplest form, the exact value of the sum of the first 3 terms of the sequence.

(3)

$$(a) \quad r = \frac{T_4}{T_3} = \frac{\sqrt{2} \cos \theta}{\sin \theta} = \frac{\sqrt{2} \left(\frac{\sqrt{3}}{2} \right)}{\frac{1}{2}} = \sqrt{6}$$

$$r = \sqrt{6} //$$

$$(b) \quad \text{Since } T_3 = \frac{1}{2}$$

$$T_2 = \frac{T_3}{r} = \frac{1/2}{\sqrt{6}} = \frac{1}{2\sqrt{6}}$$

and

$$T_1 = \frac{T_2}{r} = \frac{1}{2\sqrt{6}} \cdot \frac{1}{\sqrt{6}} = \frac{1}{12}$$

$$S_3 = T_1 + T_2 + T_3 = \frac{1}{12} + \frac{1}{2\sqrt{6}} + \frac{1}{2}$$

$$\frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$S_3 = \frac{1}{12} + \frac{\sqrt{6}}{12} + \frac{6}{12} = \frac{7 + \sqrt{6}}{12}$$

$$S_3 = \frac{7 + \sqrt{6}}{12} //$$

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Question

13

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Lined area for writing the answer to Question 13.

Question

13

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Question

13

continued

Handwriting practice area with horizontal lines.

(TotalforQuestion 13 is 7 marks)

14

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The concentration of a chemical in a lake is being monitored.

The concentration, x mg L⁻¹, of the chemical in the lake, t hours after monitoring began, is modelled by the differential equation

$$\frac{dx}{dt} = x(A - t), \quad t \geq 0$$

where A is a positive constant.

Given that

- initially $x=0.3$
- $x=21$ when $t=4$

- (a) find the particular solution of the differential equation, giving the answer in the form $x = f(t)$.

(5)

Use the model to answer parts (b) and (c).

- (b) Find, to 3 significant figures, the maximum concentration of the chemical in the lake.

(3)

When $t=T$, the concentration of the chemical in the lake is 0.1 mg L^{-1} .

- (c) Find the value of T , giving the answer to 3 significant figures.

(3)

$$(a) \quad \frac{1}{x} \frac{dx}{dt} = A - t$$

$$\int \frac{1}{x} dx = \int (A - t) dt$$

$$\ln x = At - \frac{1}{2}t^2 + C$$

$$x = C e^{At - \frac{1}{2}t^2}$$

$$0.3 = C$$

$$x = 0.3 e^{At - \frac{1}{2}t^2}$$

Using $x=21$ when $t=4$

$$21 = 0.3e^{4A-8}$$

$$70 = e^{4A-8}$$

$$\ln 70 = 4A - 8$$

$$A = \frac{\ln 70 + 8}{4} \quad \therefore A \approx 3.0621$$

$$\therefore x = 0.3e^{3.0621t - \frac{1}{2}t^2}$$

(b) $t = A \approx 3.0621$

$$x_{\max} = 0.3e^{A^2 - \frac{1}{2}A^2} = 0.3e^{A^2/2}$$

(Then Substitute $A = 3.0621$)

$$\therefore x = 32.6 \text{ mg L}^{-1}$$

(c) $0.1 = 0.3e^{At - \frac{1}{2}t^2}$

$$-\ln 3 = At - \frac{1}{2}t^2$$

Substitute $A = 3.0621$

$$\frac{1}{2}t^2 - 3.0621t - \ln 3 = 0$$

$$t^2 - 6.1242t - 2.1972 = 0$$

Use quadratic formula

$$t = \frac{6.1242 \pm \sqrt{6.1242^2 + 4(2.1972)}}{2}$$

$$t = 6.4676 \text{ or } -0.343$$

$$\therefore T = \underline{6.47} \text{ hours}$$

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Question

14

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(Total for Question 14 is 11 marks)

15. A student attempts to answer the following question:

Given that

$$a^2 + b^2 = c^2 \quad a, b, c \text{ are odd}$$

use algebra to prove by contradiction that a and b cannot **both** be odd.

The student starts the proof with:

Assume that a and b are both odd so that

$$a = 2m + 1 \text{ and } b = 2n + 1 \text{ where } m, n \in \mathbb{N}$$

The start of the student's proof is reprinted below.

Use detailed **algebraic** reasoning to complete the proof.

(You may assume without proof that if k^2 is even then k is even, where $k \in \mathbb{N}$)

(4)

Assume that a and b are both odd so that

$$a = 2m + 1 \text{ and } b = 2n + 1 \text{ where } m, n \in \mathbb{N}$$

$$\text{Assume } a \text{ and } b \text{ are odd;} \\ a = 2m + 1 \quad b = 2n + 1$$

$$\text{where } m, n \in \mathbb{N}$$

$$a^2 + b^2 = (2m + 1)^2 + (2n + 1)^2$$

$$\begin{aligned} c^2 &= 4m^2 + 4m + 1 + 4n^2 + 4n + 1 \\ &= 4(m^2 + m + n^2 + n) + 2 \end{aligned}$$

$$c^2 = 2(2m^2 + 2m + 2n^2 + 2n + 1)$$

$$\therefore c^2 \text{ is even}$$

$$c = 2r$$

$$a^2 + b^2 = 4r^2$$

$$a^2 + b^2 = 4(m^2 + m + n^2 + n) + 2$$

$$4(m^2 + m + n^2 + n) + 2 = 4r^2$$

$$2 = 4(r^2 - m^2 - m - n^2 - n)$$

RHS is $\times 4$ but LHS is $\times 2$

Assumption is false

Hence a and b cannot be both odd

Question

15

continued

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(Total for Question 15 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS