EXAM PAPERS PRACTICE

IB Maths: AA HL

Differentiation

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Differentiation

Question 1

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$.

[4 marks]

Question 2

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$.

[1 mark]

(b) Find the gradient of the graph of f at $x = 0$.

[2 marks]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3 marks]

Question 3

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3 marks]

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3 marks]

Question 4

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5},\ x \neq 0$.

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3 marks]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

[3 marks]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2 marks]

Question 5

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

[1 mark]

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2 marks]

(c) Calculate the value of c and write down the function $f(x)$.

[3 marks]

Question 6

The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1, 13)$, and a gradient of -3 at the point $(1, 3)$.

(a) By considering $\frac{\mathrm{d}y}{\mathrm{d}x}$ show that $2a + b = -3$ and $-2a + b = -7$.

[2 marks]

(b) Hence find the values of a and b.

[1 mark]

(c) By considering a point that you know to be on the curve, find the value of c.

[2 marks]

Question 7

The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1, 1)$ lies on C.

(a) Find an expression for $\frac{dy}{dx}$.

[2 marks]

(b) Show that an equation of the normal to C at point P is $x + 2y = 3$.

[3 marks]

This normal cut the x-axis at the point Q.

(c) Find the length of PQ, giving your answer as an exact value.

[2 marks]

Question 8

Find the values of x for which $\mathrm{f}(x) = -9x^2 + 5x - 3$ is an increasing function.

[3 marks]

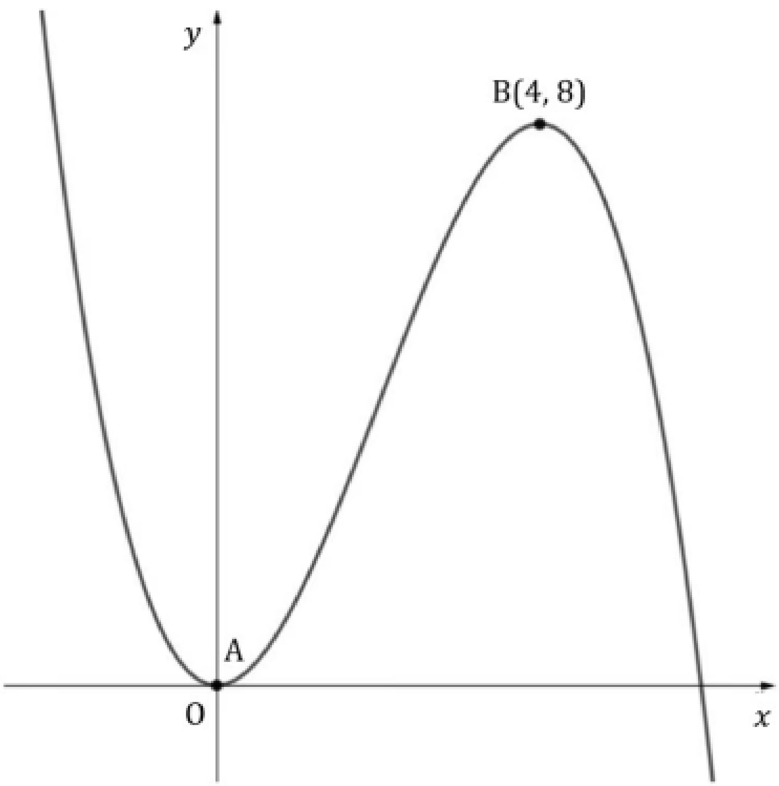
Question 9

Show that the function $\mathrm{f}(x) = x^3 - 3x^2 + 6x - 7$ is increasing for all $x \in \mathbb{R}$.

[3 marks]

Question 10

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point $(4,8)$.



(a) State the equations of the horizontal tangent to the curve.

[2 marks]

(b) Write down the value of x where the point of inflection is located.

[1 mark]

(c) Find the intervals where f is decreasing.

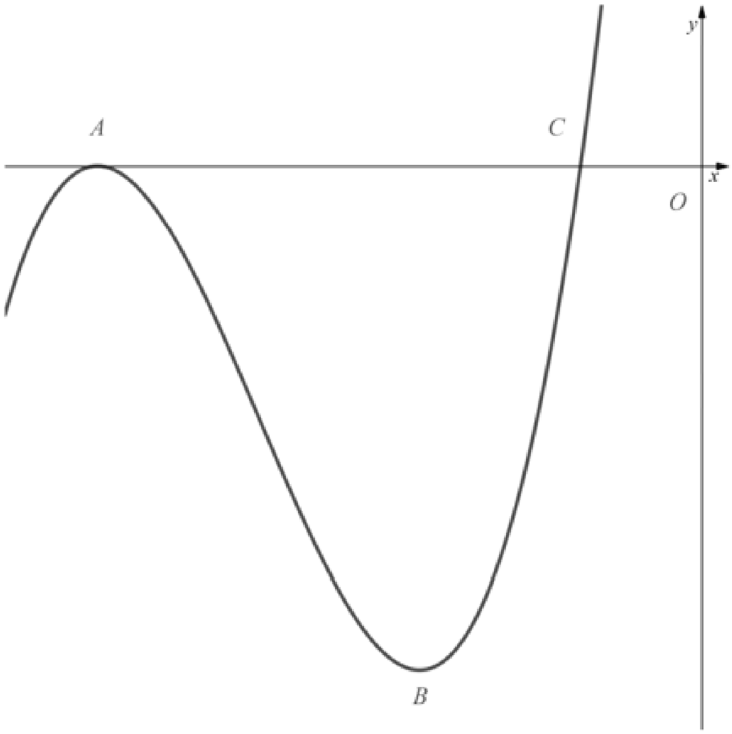
[2 marks]

(d) Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

[3 marks]

Question 11

The diagram below shows part of the curve with equation $y = x^3 + 11x^2 + 35x + 25$. The curve touches the x-axis at A and cuts the x-axis at C. The points A and B are stationary points on the curve.



(a) Using calculus, and showing all your working, find the coordinates of A and B.

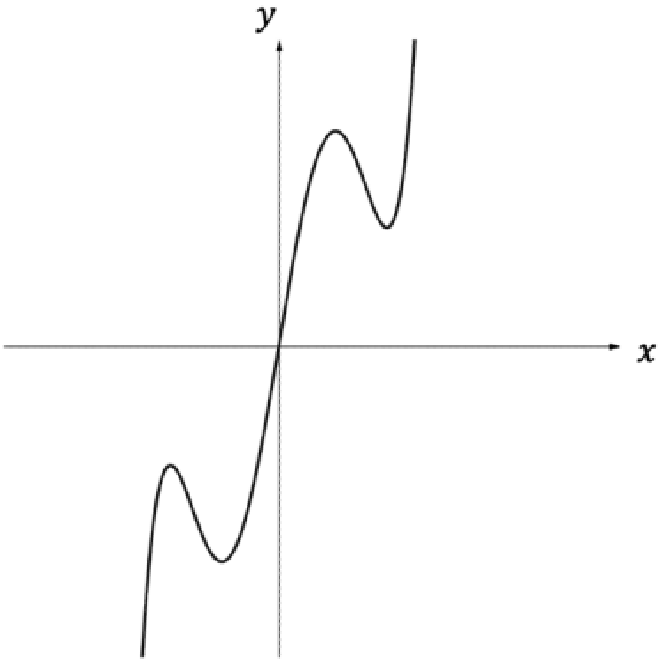
[5 marks]

(b) Show that $(-1, 0)$ is a point on the curve and explain why those must be the coordinates of point C.

[2 marks]

Question 12

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

There are two points, R and S, along the curve C at which the gradient of the tangent to the curve C is equal to 10.

(b) Calculate the x-coordinates of points R and S.

[4 marks]

Question 13

(a) Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

[4 marks]

(b) Find the nature of the stationary points found in part (a).

[3 marks]

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

[3 marks]

(d) Explain why, in this case, the point of inflection is not a stationary point.

[1 mark]

Question 14

The graph of a continuous function has the following properties:

The function is concave in the interval $(-\infty, a)$.

The function is convex in the interval (a, ∞).

The graph of the function intercepts the x-axis at the points $(b, 0)$, $(c, 0)$ and $(d, 0)$, where b, c and d are such that $d > c > b > 0$.

The x-coordinates of the turning points of the function are e and f, which are such that $f > e$.

The graph of the function intercepts the y-axis at $(0, g)$

Given that the value of the function is positive when $x = a$, sketch a graph of the function. Be sure to label the x-axis with the x-coordinates of the stationary points and the point of inflection, and also to label the points where the graph crosses the coordinate axes.

[4 marks]