Question 1

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{dy}{dx}$. [2]

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$. [4]

a) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$y = \frac{3}{2}x^2 - 15x + 2$

Apply formula

$\boxed{\frac{dy}{dx} = 3x - 15}$

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{dy}{dx}$. [2]

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$. [4]

b) i) Set $\frac{dy}{dx} = -3$ and solve for x.

$3x - 15 = -3$

add 15 then divide by 3

$x = 4$

Sub $x = 4$ into y.

$y = \frac{3}{2}(4)^2 - 15(4) + 2$

$y = -34$

$\boxed{\therefore A(4, -34)}$

ii) Sub A and $m = -3$ into $y - y_1 = m(x - x_1)$.

$y - (-34) = -3(x - 4)$

expand and rearrange

$\boxed{y = -3x - 22}$

Question 2

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$.

[1]

(b) Find the gradient of the graph of f at $x = 0$.

[2]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3]

a) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$f(x) = 3x^7 - 12x$

Apply formula

$\boxed{f'(x) = 21x^6 - 12}$

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$. $\boxed{f'(x) = 21x^6 - 12}$

[1]

(b) Find the gradient of the graph of f at $x = 0$.

[2]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3]

b) Sub $x = 0$ into $f'(x)$.

$f'(0) = 21(0)^6 - 12.$

$\boxed{f'(0) = -12}$

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$. $\boxed{f'(x) = 21x^6 - 12}$

[1]

(b) Find the gradient of the graph of $f(x)$ at $x = 0$.

[2]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3]

c) The tangent and normal are perpendicular.

If $m_{normal} = 4$ then $m_{tangent} = -\frac{1}{4}$

Set $f'(x) = -\frac{1}{4}$ and solve for x.

$21x^6 - 12 = -\frac{1}{4}$

add 12, divide by 21 then $\sqrt[6]{\ }$.

$x = \pm\sqrt[6]{\frac{11.75}{21}} = \pm 0.9072...$

$x = \pm 0.908$ (3sf)

$\therefore y = \pm 9.3694... = \pm 9.37$ (3sf)

$\boxed{(0.908, -9.37) \text{ and } (-0.908, 9.37)}$

Question 3

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3]

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3]

a) Find $\frac{dy}{dx}$

$y = 4 - \frac{4}{x} = 4 - 4x^{-1}$

$\frac{dy}{dx} = \frac{4}{x^2} = 4x^{-2}$

Sub $x = 2$ into $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{4}{(2)^2} = \frac{4}{4} = 1$ $\therefore m = 1$

Sub $x = 2$ into y.

$y = 4 - \frac{4}{(2)} = 2$ $\therefore$ point $(2, 2)$

Sub m and the point into $y - y_1 = m(x - x_1)$.

$y - 2 = 1(x - 2)$

$\boxed{y = x}$

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3]

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3]

b) Set $\frac{dy}{dx} = 16$ and solve for x.

$\frac{4}{x^2} = 16$ reciprocate and multiply by 4

$x^2 = \frac{1}{4}$ $\sqrt{\ }$

$x = \pm\frac{1}{2}$

$\therefore y = -4$ and 12.

$\boxed{\left(\frac{1}{2}, -4\right) \text{ and } \left(-\frac{1}{2}, 12\right)}$

Question 4

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5},\ x \neq 0.$

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

[3]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2]

a) i) Sub $x = 2$ into $f(x)$.

$f(2) = \frac{4}{(2)} + \frac{2(2)^4}{5} - \frac{2}{5}$

$\boxed{f(2) = 8}$

ii) Find $f'(x)$

$f(x) = 4x^{-1} + \frac{2}{5}x^4 - \frac{2}{5}$

$f'(x) = -4x^{-2} + \frac{8}{5}x^3$

Sub $x = 2$ into $f'(x)$.

$f'(2) = -4(2)^{-2} + \frac{8}{5}(2)^3$

$\boxed{f'(2) = 11.8}$

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5},\ x \neq 0.$

(a) Calculate

(i) $f(2)$ $\boxed{f(2) = 8}$

(ii) $f'(2)$. $\boxed{f'(2) = 11.8}$

[3]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

[3]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2]

b) point $(2, 8)$ $m = 11.8$

Sub m and the point into $y - y_1 = m(x - x_1)$.

$y - 8 = 11.8(x - 2)$ expand and rearrange

$y = 11.8x - 15.6$

$\boxed{\text{The equation of } l \text{ is } y = 11.8x - 15.6.}$

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5}$, $x \neq 0$.

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

The equation of l is $y = 11.8x - 15.6$

[3]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2]

c) Graph $f(x)$ and l on your GDC and find their intersection.

$\boxed{A(-0.222, -18.2)}$

Question 5

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

[1]

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2]

(c) Calculate the value of c and write down the function $f(x)$.

[3]

a) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$f(x) = x^2 - bx + c$

Apply formula

$\boxed{f'(x) = 2x - b}$

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

$\boxed{f'(x) = 2x - b}$

[1]

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2]

(c) Calculate the value of c and write down the function $f(x)$.

[3]

b) Tangent equation at $x = 2$ is $y = x - 1$.

$\therefore f'(2) = 1$

$2(2) - b = 1$

$\boxed{b = 3}$

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

[1]

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2]

(c) Calculate the value of c and write down the function $f(x)$.

[3]

c) Sub $x = 2$ into $y = x - 1$.

$y = 2 - 1$

$y = 1$

$\therefore f(x)$ passes through $(2, 1)$.

$f(2) = 1$

$(2)^2 - 3(2) + c = 1$

$\boxed{c = 3}$

$\boxed{f(x) = x^2 - 3x + 3}$

Question 6

The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1, 13)$, and a gradient of -3 at the point $(1, 3)$.

(a) By considering $\frac{dy}{dx}$ show that $2a + b = -3$ and $-2a + b = -7$.

[2]

(b) Hence find the values of a and b.

[1]

(c) By considering a point that you know to be on the curve, find the value of c.

[2]

a) $\frac{dy}{dx} = 2ax^{2-1} + bx^{1-1} + 0$

$= 2ax + b$

Sub in each gradient and its corresponding x value

$-7 = 2a(-1) + b$

$\boxed{-7 = -2a + b}$

$-3 = 2a(1) + b$

$\boxed{-3 = 2a + b}$

The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1, 13)$, and a gradient of -3 at the point $(1, 3)$.

(a) By considering $\frac{dy}{dx}$ show that $2a + b = -3$ and $-2a + b = -7$.

1 2

[2]

(b) Hence find the values of a and b.

[1]

(c) By considering a point that you know to be on the curve, find the value of c.

[2]

b) Find 1 + 2 to eliminate a

$2a + b - 2a + b = -3 - 7$

$2b = -10$

$\boxed{b = -5}$

Sub $b = -5$ into 1

$2a - 5 = -3$

$2a = 2$

$\boxed{a = 1}$

The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1, 13)$, and a gradient of -3 at the point $(1, 3)$.

(a) By considering $\frac{dy}{dx}$ show that $2a + b = -3$ and $-2a + b = -7$.

[2]

(b) Hence find the values of a and b.

$a = 1$ $b = -5$

[1]

(c) By considering a point that you know to be on the curve, find the value of c.

[2]

c) Sub in values of x, y, a and b into eqn to find c

$(13) = (1)(-1)^2 + (-5)(-1) + c$

$13 = 1 + 5 + c$

$\boxed{c = 7}$

Question 7

The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1, 1)$ lies on C.

(a) Find an expression for $\frac{dy}{dx}$.

[2]

(b) Show that an equation of the normal to C at point P is $x + 2y = 3$.

[3]

This normal cut the x-axis at the point Q.

(c) Find the length of PQ, giving your answer as an exact value.

[2]

a) $y = 3x^2 - 6 + 4x^{-1}$

$\boxed{\frac{dy}{dx} = 6x - 4x^{-2}}$

EXAM PAPERS PRACTICE

The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1, 1)$ lies on C.

x_1 y_1

(a) Find an expression for $\frac{dy}{dx}$. [2]

$\frac{dy}{dx} = 6x - 4x^{-2}$

(b) Show that an equation of the normal to C at point P is $x + 2y = 3$. [3]

This normal cut the x-axis at the point Q.

(c) Find the length of PQ, giving your answer as an exact value. [2]

b) Find gradient at P by subbing $x = 2$ into $\frac{dy}{dx}$ eqn.

$\boxed{y - y_1 = \frac{-1}{m}(x - x_1)}$

$m = \frac{dy}{dx} = 6(1) - 4(1)^{-2} = 2$

$y - 1 = \frac{-1}{2}(x - 1)$

$2y - 2 = -x + 1$

$\boxed{x + 2y = 3}$

The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1, 1)$ lies on C.

(a) Find an expression for $\frac{dy}{dx}$. [2]

(b) Show that an equation of the normal to C at point P is $x + 2y = 3$. [3]

$y = -\frac{1}{2}x + \frac{3}{2}$

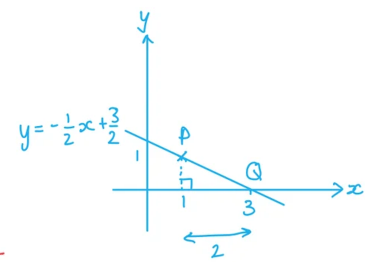
This normal cut the x-axis at the point Q.

(c) Find the length of PQ, giving your answer as an exact value. [2]

c) At Q, $y = 0$

$x + 2(0) = 3$

$x = 3$



Length $PQ = \sqrt{1^2 + 2^2}$

$= \boxed{\sqrt{5}}$

Question 8

Find the values of x for which $\mathrm{f}(x) = -9x^2 + 5x - 3$ is an increasing function.

[3]

$f'(x) \geqslant 0$ for the function to be increasing

$f'(x) = -9(2)x^1 + 5 \geqslant 0$

$-18x + 5 \geqslant 0$

$\boxed{x \leqslant \frac{5}{18}}$

Question 9

Show that the function $\mathrm{f}(x) = x^3 - 3x^2 + 6x - 7$ is increasing for all $x \in \mathbb{R}$.

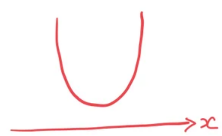
[3]

$f'(x) \geq 0$

$f'(x) = 3x^2 - 6x + 6 \geqslant 0$

positive quadratic with no real roots

$\boxed{\therefore f'(x) \text{ is always } > 0}$



This curve never crosses the x axis, so $f'(x) > 0$

OR

...An alternative way to deal with the quadratic:

1) Add 3 to both sides of inequality

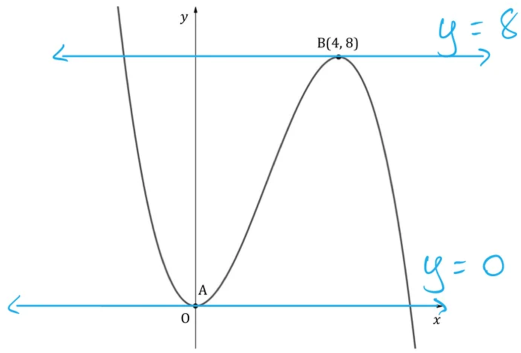
$3x^2 - 6x + 9 \geqslant 3$

2) Complete the square

$3\left(\underbrace{(x-1)^2}_{\geqslant 0 \text{ for all } x \in \mathbb{R}} + 1\right) \geqslant 3$

Question 10

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point (4,8).



(a) State the equations of the horizontal tangent to the curve.

[2]

(b) Write down the value of x where the point of inflection is located.

[1]

(c) Find the intervals where f is decreasing.

[2]

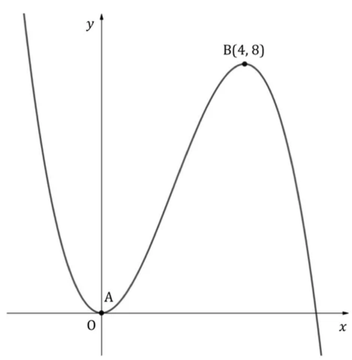
(d) Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

[3]

a) Horizontal tangents are at the local minimum and maximum (points A and B), where the gradient is 0.

$\boxed{y = 0 \quad \text{and} \quad y = 8}$

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point (4,8).



(a) State the equations of the horizontal tangent to the curve.

[2]

(b) Write down the value of x where the point of inflection is located.

[1]

(c) Find the intervals where f is decreasing.

[2]

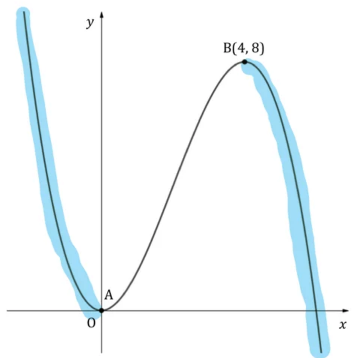
(d) Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

[3]

b) Point of inflection is half way between the local min and max.

$\boxed{\therefore x = 2}$

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point (4,8).



(a) State the equations of the horizontal tangent to the curve.

[2]

(b) Write down the value of x where the point of inflection is located.

[1]

(c) Find the intervals where f is decreasing.

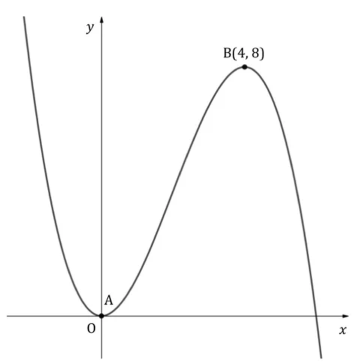
[2]

(d) Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

[3]

c) f is decreasing in the intervals $x < 0$ and $x > 4$.

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point (4,8).



(a) State the equations of the horizontal tangent to the curve.

[2]

(b) Write down the value of x where the point of inflection is located.

[1]

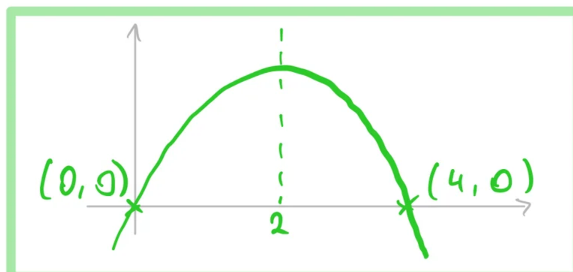
(c) Find the intervals where f is decreasing.

[2]

(d) Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

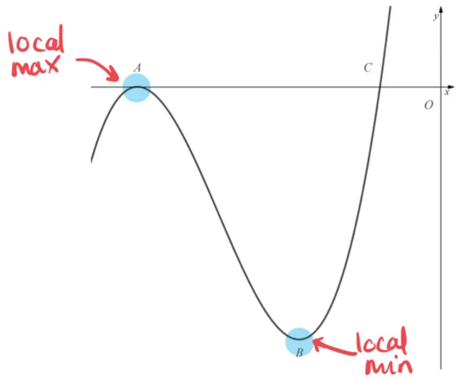
[3]

d) The graph of $f'(x)$ has zeros at $(0, 0)$ and $(4, 0)$ and a max when $x = 2$.



Question 11

The diagram below shows part of the curve with equation $y = x^3 + 11x^2 + 35x + 25$. The curve touches the x-axis at A and cuts the x-axis at C. The points A and B are stationary points on the curve.



(a) Using calculus, and showing all your working, find the coordinates of A and B.

[5]

(b) Show that $(-1, 0)$ is a point on the curve and explain why those must be the coordinates of point C.

[2]

a) 1. Find x coordinates of stationary points.

$$\frac{dy}{dx} = 0 = 3x^2 + 22x + 35$$
$$(3x+7)(x+5)$$
$$x = -\frac{7}{3}, -5$$

2. Classify the stationary points.

$$\frac{d^2y}{dx^2} = 6x + 22$$

At $x = -\frac{7}{3}$ $\quad \frac{d^2y}{dx^2} = 6\left(-\frac{7}{3}\right) + 22 = 8 > 0 \therefore$ local min $\rightarrow B$

At $x = -5$ $\quad \frac{d^2y}{dx^2} = 6(-5) + 22 = -8 < 0 \therefore$ local max $\rightarrow A$

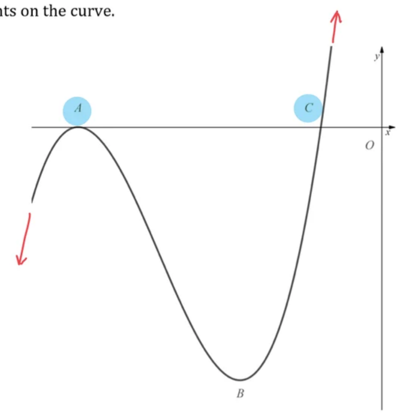
3. Find corresponding y values.

At $x = -\frac{7}{3}$ $\quad y = \left(-\frac{7}{3}\right)^3 + 11\left(-\frac{7}{3}\right)^2 + 35\left(-\frac{7}{3}\right) + 25 = -\frac{256}{27}$

At $x = -5$ $\quad y = (-5)^3 + 11(-5)^2 + 35(-5) + 25 = 0$

$$\boxed{A(-5, 0) \quad B\left(-\frac{7}{3}, -\frac{256}{27}\right)}$$

The diagram below shows part of the curve with equation $y = x^3 + 11x^2 + 35x + 25$. The curve touches the x-axis at A and cuts the x-axis at C. The points A and B are stationary points on the curve.



(a) Using calculus, and showing all your working, find the coordinates of A and B.

[5]

(b) Show that $(-1, 0)$ is a point on the curve and explain why those must be the coordinates of point C.

[2]

b) When $x = -1$,

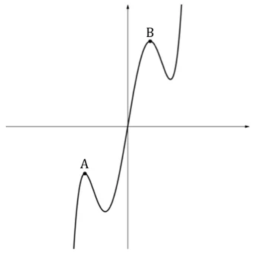
$$y = (-1)^3 + 11(-1)^2 + 35(-1) + 25$$
$$= -1 + 11 - 35 + 25$$
$$= 0$$

$\therefore$ $(-1, 0)$ Lies on the curve

This has to be C, because the only other point where $y = 0$ is $A(-5, 0)$, and the curve is a cubic $\therefore$ it's not going to double back and cross the x axis again!

Question 12

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

[2]

Points A and B represent the local maximums on the diagram above.

(b) Write down the coordinates of

(i) A

(ii) B.

[4]

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

(c) Calculate the x-coordinates of points R and S.

[2]

a) Derivative of x^n formula (in formula booklet)

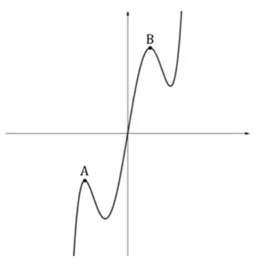
$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$

Apply formula

$\boxed{\frac{dy}{dx} = \frac{1}{7}x^4 - \frac{9}{4}x^2 + 6}$

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

[2]

Points A and B represent the local maximums on the diagram above.

(b) Write down the coordinates of

(i) A

(ii) B.

[4]

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

(c) Calculate the x-coordinates of points R and S.

[2]

b) Graph y on your GDC and find its maximums.

Note that A has a negative x and y coordinate and B has a positive x and y coordinate.

i) $\boxed{A(-3.51, -3.85)}$

ii) $\boxed{B(1.84, 6.97)}$

Question 13

(a) Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$. [4]

(b) Find the nature of the stationary points found in part (a). [3]

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$. [3]

(d) Explain why, in this case, the point of inflection is not a stationary point. [1]

a) Stationary points occur where the gradient, $\frac{dy}{dx} = 0$.

$\frac{dy}{dx} = 3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\boxed{x = 3, 1}$

(a) Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

$x = 3, 1$ $\frac{dy}{dx} = 3x^2 - 12x + 9$

[4]

(b) Find the nature of the stationary points found in part (a). [3]

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$. [3]

(d) Explain why, in this case, the point of inflection is not a stationary point. [1]

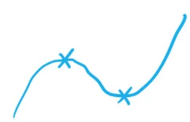
b) $\frac{d^2y}{dx^2} = 6x - 12$

When $x = 1$

$\frac{d^2y}{dx^2} = 6(1) - 12 = -6 < 0$ $\therefore$ maximum point

When $x = 3$

$\frac{d^2y}{dx^2} = 6(3) - 12 = 6 > 0$ $\therefore$ minimum point



This can even be predicted by considering where the stationary points are on a positive cubic curve!

$\boxed{\text{There is a maximum point at } x = 1 \text{ and a minimum point at } x = 3.}$

(a) Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$. [4]

(b) Find the nature of the stationary points found in part (a).

Maximum point at $x=1$, minimum point at $x=3$. [3]

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$. [3]

(d) Explain why, in this case, the point of inflection is not a stationary point. [1]

c) For a point of inflection, $\frac{d^2y}{dx^2} = 0$, and $\frac{d^2y}{dx^2}$ changes sign either side.

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$\frac{d^2y}{dx^2} = 6x - 12 = 0$

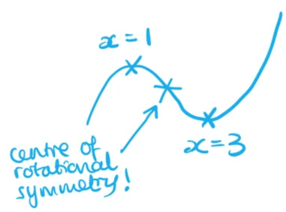
$x = 2$

When $x=0$ $\quad \frac{d^2y}{dx^2} = 6(0) - 12 = -12 < 0$

When $x = 3$ $\quad \frac{d^2y}{dx^2} = 6(3) - 12 = 6 > 0$

Change of sign indicates $x=2$ is a POI.

OR



Since this is a +ve cubic, the POI lies midway between the stationary points $\therefore x = 2$.

(a) Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

$x = 1, 3$ [4]

(b) Find the nature of the stationary points found in part (a).

Maximum point at $x=1$, minimum point at $x=3$. [3]

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

$x=2$ [3]

(d) Explain why, in this case, the point of inflection is not a stationary point. [1]

d) Several possible answers...

- It was not a solution in part (a)
- When $x = 2$, $\frac{dy}{dx} \neq 0$

Question 14

The graph of a continuous function has the following properties:

The function is concave in the interval $(-\infty, a)$.

The function is convex in the interval (a, ∞).

The graph of the function intercepts the x-axis at the points $(b, 0)$, $(c, 0)$ and $(d, 0)$, where b, c and d are such that $d > c > b > 0$.

The x-coordinates of the turning points of the function are e and f, which are such that $f > e$.

The graph of the function intercepts the y-axis at $(0, g)$

Given that the value of the function is positive when $x = a$, sketch a graph of the function. Be sure to label the x-axis with the x-coordinates of the stationary points and the point of inflection, and also to label the points where the graph crosses the coordinate axes.

[4]

A stationary point is a point where a function's gradient is zero. This includes (but is not limited to) turning points, i.e. local maximums and minimums.

A point of inflection is where a function changes from concave to convex or vice versa.

concave (sometimes called 'concave down')

convex (sometimes called 'concave up')