Question 1

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{dy}{dx}$. [2]

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A. Give your answer in the form $y = mx + c$. [4]

a) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$y = \frac{3}{2}x^2 - 15x + 2$

Apply formula

$\boxed{\frac{dy}{dx} = 3x - 15}$

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{dy}{dx}$. [2]

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A. Give your answer in the form $y = mx + c$. [4]

b)i) Set $\frac{dy}{dx} = -3$ and solve for x.

$3x - 15 = -3$

$x = 4$

add 15 then divide by 3

Sub $x = 4$ into y.

$y = \frac{3}{2}(4)^2 - 15(4) + 2$

$y = -34$

$\boxed{\therefore A(4, -34)}$

ii) Sub A and $m = -3$ into $y - y_1 = m(x - x_1)$.

$y - (-34) = -3(x - 4)$

expand and rearrange

$\boxed{y = -3x - 22}$

Question 2

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$.

[1]

(b) Find the gradient of the graph of f at $x = 0$.

[2]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3]

a) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$f(x) = 3x^7 - 12x$

Apply formula

$\boxed{f'(x) = 21x^6 - 12}$

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$. $\boxed{f'(x) = 21x^6 - 12}$

[1]

(b) Find the gradient of the graph of f at $x = 0$.

[2]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3]

b) Sub $x = 0$ into $f'(x)$.

$f'(0) = 21(0)^6 - 12.$

$\boxed{f'(0) = -12}$

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$. $\boxed{f'(x) = 21x^6 - 12}$

[1]

(b) Find the gradient of the graph of $f(x)$ at $x = 0$.

[2]

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3]

c) The tangent and normal are perpendicular.

If $m_{normal} = 4$ then $m_{tangent} = -\frac{1}{4}$

Set $f'(x) = -\frac{1}{4}$ and solve for x.

$21x^6 - 12 = -\frac{1}{4}$

add 12, divide by 21 then $\sqrt[6]{}$.

$x = \pm \sqrt[6]{\frac{11.75}{21}} = \pm 0.9077...$

$x = \pm 0.908$ (3sf)

$\therefore y = \pm 9.3694... = \pm 9.37$ (3sf)

$\boxed{(0.908, -9.37) \text{ and } (-0.908, 9.37)}$

Question 3

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3]

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3]

a) Find $\frac{dy}{dx}$

$y = 4 - \frac{4}{x} = 4 - 4x^{-1}$

$\frac{dy}{dx} = \frac{4}{x^2} = 4x^{-2}$

Sub $x = 2$ into $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{4}{(2)^2} = \frac{4}{4} = 1 \qquad \therefore m = 1$

Sub $x = 2$ into y.

$y = 4 - \frac{4}{(2)} = 2 \qquad \therefore \text{point } (2, 2)$

Sub m and the point into $y - y_1 = m(x - x_1)$.

$y - 2 = 1(x - 2)$

$\boxed{y = x}$

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3]

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3]

b) Set $\frac{dy}{dx} = 16$ and solve for x.

$\frac{4}{x^2} = 16$

reciprocate and multiply by 4

$x^2 = \frac{1}{4}$

$\sqrt{\ }$

$x = \pm\frac{1}{2}$

$\therefore y = -4$ and 12.

$\boxed{\left(\frac{1}{2}, -4\right) \text{ and } \left(-\frac{1}{2}, 12\right)}$

Question 4

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5},\ x \neq 0.$

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

[3]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2]

a) i) Sub $x = 2$ into $f(x)$.

$$f(2) = \frac{4}{(2)} + \frac{2(2)^4}{5} - \frac{2}{5}$$

$$\boxed{f(2) = 8}$$

ii) Find $f'(x)$

$$f(x) = 4x^{-1} + \frac{2}{5}x^4 - \frac{2}{5}$$

$$f'(x) = -4x^{-2} + \frac{8}{5}x^3$$

Sub $x = 2$ into $f'(x)$.

$$f'(2) = -4(2)^{-2} + \frac{8}{5}(2)^3$$

$$\boxed{f'(2) = 11.8}$$

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5},\ x \neq 0.$

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

$\boxed{f(2) = 8}$

$\boxed{f'(2) = 11.8}$

[3]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

b) point $(2, 8)$ $m = 11.8$

Sub m and the point into $y - y_1 = m(x - x_1)$.

$y - 8 = 11.8(x - 2)$ expand and rearrange

$y = 11.8x - 15.6$

$$\boxed{\text{The equation of } l \text{ is } y = 11.8x - 15.6}$$

[3]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2]

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5}$, $x \neq 0$.

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3]

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

The equation of l is $y = 11.8x - 15.6$

[3]

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2]

c) Graph $f(x)$ and l on your GDC and find their intersection.

$\boxed{A(-0.222, -18.2)}$

Question 5

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

[1]

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2]

(c) Calculate the value of c and write down the function $f(x)$.

[3]

a) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$f(x) = x^2 - bx + c$

Apply formula

$\boxed{f'(x) = 2x - b}$

EXAM PAPERS PRACTICE

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$. [1]

$\boxed{f'(x) = 2x - b}$

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b. [2]

(c) Calculate the value of c and write down the function $f(x)$. [3]

b) Tangent equation at $x = 2$ is $y = x - 1$.

$\therefore f'(2) = 1$

$2(2) - b = 1$

$\boxed{b = 3}$

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$. [1]

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b. [2]

(c) Calculate the value of c and write down the function $f(x)$. [3]

c) Sub $x = 2$ into $y = x - 1$.

$y = 2 - 1$

$y = 1$

$\therefore f(x)$ passes through $(2, 1)$.

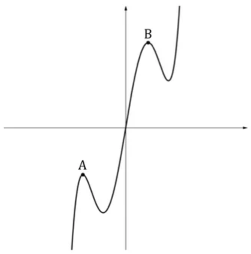
$f(2) = 1$

$(2)^2 - 3(2) + c = 1$

$\boxed{\begin{array}{l} c = 3 \\ f(x) = x^2 - 3x + 3 \end{array}}$

Question 6

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

[2]

Points A and B represent the local maximums on the diagram above.

(b) Write down the coordinates of

(i) A

(ii) B.

[4]

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

(c) Calculate the x-coordinates of points R and S.

[2]

a) Derivative of x^n formula (in formula booklet)

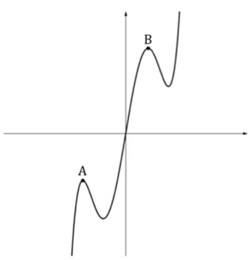
$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$

Apply formula

$$\boxed{\frac{dy}{dx} = \frac{1}{7}x^4 - \frac{9}{4}x^2 + 6}$$

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

[2]

Points A and B represent the local maximums on the diagram above.

(b) Write down the coordinates of

(i) A

(ii) B.

[4]

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

(c) Calculate the x-coordinates of points R and S.

[2]

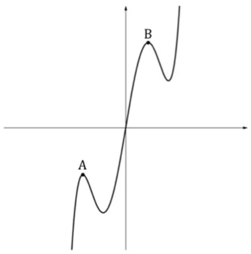
b) Graph y on your GDC and find its maximums.

Note that A has a negative x and y coordinate and B has a positive x and y coordinate.

i) $\boxed{A(-3.51, -3.85)}$

ii) $\boxed{B(1.84, 6.97)}$

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

$\boxed{\frac{dy}{dx} = \frac{1}{7}x^4 - \frac{9}{4}x^2 + 6}$

[2]

Points A and B represent the local maximums on the diagram above.

(b) Write down the coordinates of

(i) A

(ii) B.

[4]

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

(c) Calculate the x-coordinates of points R and S.

[2]

c) The tangent and normal line are perpendicular.

$m_{normal} = -\frac{1}{10}$ $\quad \therefore m_{tangent} = 10$

Set $\frac{dy}{dx} = 10$ and solve for x.

$\frac{1}{7}x^4 - \frac{9}{4}x^2 + 6 = 10$

$x = \pm 4.1668... = \pm 4.17$ (3sf)

x-coordinate for R is 4.17 and the x-coordinate for S is -4.17.

Question 7

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \le x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

a) $x = 0$ when no shoes are produced.

$C(0) = 1225 + 11(0) - 0.009(0)^2 - 0.0001(0)^3$

$\boxed{C(0) = 1225 \text{ USD}}$

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \leq x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

b) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3$

Apply formula

$\boxed{C'(x) = 11 - 0.018x - 0.0003x^2}$

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \leq x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

$\boxed{C'(x) = 11 - 0.018x - 0.0003x^2}$

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

(3)

c) i) Sub $x = 40$ into $C'(x)$

$C'(40) = 11 - 0.018(40) - 0.0003(40)^2$

$\boxed{C'(40) = 9.80 \text{ USD}}$

ii) Sub $x = 90$ into $C'(x)$

$C'(90) = 11 - 0.018(90) - 0.0003(90)^2$

$\boxed{C'(90) = 6.95 \text{ USD}}$

The daily cost function of a company producing pairs of running shoes is modelled by the cubic function

$$C(x) = 1225 + 11x - 0.009x^2 - 0.0001x^3, \qquad 0 \le x < 160$$

where x is the number of pairs of running shoes produced and C the cost in USD.

(a) Write down the daily cost to the company if no pairs of running shoes are produced.

[1]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find an expression for the marginal cost, $C'(x)$, of producing pairs of running shoes.

$\boxed{C'(x) = 11 - 0.018x - 0.0003x^2}$

[2]

(c) Find the marginal cost of producing

(i) 40 pairs of running shoes

(ii) 90 pairs of running shoes.

[2]

The optimum level of production is when marginal revenue equals marginal cost. The marginal revenue, $R'(x)$, is equal to 4.5.

(d) Find the optimum level of production.

[3]

d) Optimum level of production is when

$R'(x) = C'(x)$

$4.5 = 11 - 0.018x - 0.0003x^2$

Solve for x on your GDC

$x = 120.222...$

$\boxed{\text{120 pairs of running shoes.}}$

Question 8

A cyclist riding over a hill can be modelled by the function

$$h(t) = -\frac{1}{24}t^2 + 3t + 12, \qquad 0 \le t \le 70$$

where h is the cyclist's altitude above mean sea level, in metres, and t is the elapsed time, in seconds.

(a) Calculate the cyclist's altitude after a minute.

[2]

(b) Find $h'(t)$.

[2]

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude.

[3]

a) Sub $t = 60$ into $h(t)$.

$h(60) = -\frac{1}{24}(60)^2 + 3(60) + 12$

$\boxed{h(60) = 42 \text{ m}}$

EXAM PAPERS PRACTICE

A cyclist riding over a hill can be modelled by the function

$$h(t) = -\frac{1}{24}t^2 + 3t + 12, \qquad 0 \le t \le 70$$

where h is the cyclist's altitude above mean sea level, in metres, and t is the elapsed time, in seconds.

(a) Calculate the cyclist's altitude after a minute. [2]

(b) Find $h'(t)$. [2]

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude. [3]

b) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$h(t) = -\frac{1}{24}t^2 + 3t + 12$

$\boxed{h'(t) = -\frac{1}{12}t + 3}$

A cyclist riding over a hill can be modelled by the function

$$h(t) = -\frac{1}{24}t^2 + 3t + 12, \qquad 0 \le t \le 70$$

where h is the cyclist's altitude above mean sea level, in metres, and t is the elapsed time, in seconds.

(a) Calculate the cyclist's altitude after a minute. [2]

(b) Find $h'(t)$. $\boxed{h'(t) = -\frac{1}{12}t + 3}$ [2]

(c) Calculate the cyclist's maximum altitude and the time it takes to reach this altitude. [3]

c) Set $h'(t) = 0$ and solve for t.

$-\frac{1}{12}t + 3 = 0$

$\boxed{t = 36\text{s}}$

Sub $t = 36$ into $h(t)$.

$h(36) = -\frac{1}{24}(36)^2 + 3(36) + 12$

$\boxed{h(36) = 66\text{m}}$

Question 9

A company produces and sells cricket bats. The company's daily cost, C, in hundreds of Australian dollars (AUD), changes based on the number of cricket bats they produce per day. The daily cost function of the company can be modelled by

$$C(x) = 6x^3 - 10x^2 + 10x + 4$$

where x hundred cricket bats is the number of cricket bats produced on a particular day.

(a) Find the cost to the company for any day zero cricket bats are produced.

[1]

The company's daily revenue, in hundreds of AUD, from selling x hundred cricket bats is given by the function $R(x) = 42x$.

(b) Given that profit = revenue – cost, determine a function for the profit, $P(x)$, in hundreds of AUD from selling x hundred cricket bats.

[2]

(c) Find $P'(x)$.

[2]

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3]

a) Sub $x = 0$ into $C(x)$.

$C(0) = 6(0)^3 - 10(0)^2 + 10(0) + 4$

$C(0) = 4$

$\boxed{400 \text{ AUD}}$

A company produces and sells cricket bats. The company's daily cost, C, in hundreds of Australian dollars (AUD), changes based on the number of cricket bats they produce per day. The daily cost function of the company can be modelled by

$$C(x) = 6x^3 - 10x^2 + 10x + 4$$

where x hundred cricket bats is the number of cricket bats produced on a particular day.

(a) Find the cost to the company for any day zero cricket bats are produced.

[1]

The company's daily revenue, in hundreds of AUD, from selling x hundred cricket bats is given by the function $R(x) = 42x$.

(b) Given that profit = revenue – cost, determine a function for the profit, $P(x)$, in hundreds of AUD from selling x hundred cricket bats.

[2]

(c) Find $P'(x)$.

[2]

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3]

b) $P(x) = R(x) - C(x)$

$P(x) = 42x - (6x^3 - 10x^2 + 10x + 4)$

$\boxed{P(x) = -6x^3 + 10x^2 + 32x - 4}$

A company produces and sells cricket bats. The company's daily cost, C, in hundreds of Australian dollars (AUD), changes based on the number of cricket bats they produce per day. The daily cost function of the company can be modelled by

$$C(x) = 6x^3 - 10x^2 + 10x + 4$$

where x hundred cricket bats is the number of cricket bats produced on a particular day.

(a) Find the cost to the company for any day zero cricket bats are produced.

[1]

The company's daily revenue, in hundreds of AUD, from selling x hundred cricket bats is given by the function $R(x) = 42x$.

(b) Given that profit = revenue – cost, determine a function for the profit, $P(x)$, in hundreds of AUD from selling x hundred cricket bats.

$\boxed{P(x) = -6x^3 + 10x^2 + 32x - 4}$

[2]

(c) Find $P'(x)$.

[2]

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3]

c) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$P(x) = -6x^3 + 10x^2 + 32x - 4$

$\boxed{P'(x) = -18x^2 + 20x + 32}$

A company produces and sells cricket bats. The company's daily cost, C, in hundreds of Australian dollars (AUD), changes based on the number of cricket bats they produce per day. The daily cost function of the company can be modelled by

$$C(x) = 6x^3 - 10x^2 + 10x + 4$$

where x hundred cricket bats is the number of cricket bats produced on a particular day.

(a) Find the cost to the company for any day zero cricket bats are produced.

[1]

The company's daily revenue, in hundreds of AUD, from selling x hundred cricket bats is given by the function $R(x) = 42x$.

(b) Given that profit = revenue – cost, determine a function for the profit, $P(x)$, in hundreds of AUD from selling x hundred cricket bats.

$\boxed{P(x) = -6x^3 + 10x^2 + 32x - 4}$

[2]

(c) Find $P'(x)$.

$\boxed{P'(x) = -18x^2 + 20x + 32}$

[2]

The derivative of $P(x)$ gives the marginal profit. The production of bats will reach its profit maximising level when the marginal profit equals zero and $P(x)$ is positive.

(d) Find the profit maximising production level and the expected profit.

[3]

d) Set $P'(x) = 0$ and solve for x.

$-18x^2 + 20x + 32 = 0$

$x = 2$ $\quad$ $\cancel{x = -0.889}$

Reject as $P(x) > 0$ and $x \geq 0$

Sub $x = 2$ into $P(x)$.

$P(2) = -6(2)^3 + 10(2)^2 + 32(2) - 4$

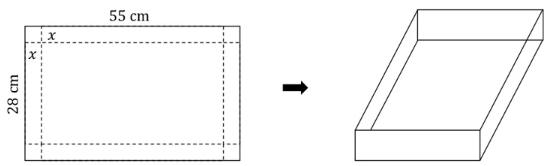
$P(2) = 52$

$\boxed{\text{∴ The profit maximising production level is 200 bats and the expected profit is 5200 AUD.}}$

EXAM PAPERS PRACTICE

Question 10

Dora decides to build a cardboard container for when she goes strawberry picking from a rectangular piece of cardboard, 55 cm $\times$ 28 cm. She cuts squares with side length x cm from each corner as shown in the diagram below.



(a) Show that the volume, V cm^3, of the container is given by

$$V = 4x^3 - 166x^2 + 1540x$$

[2]

(b) Find $\frac{dV}{dx}$.

[2]

(c) Find

(i) the value of x that maximises the volume of the container

(ii) the maximum volume of the container. Give your answer in the form $a \times 10^k$, where $1 \le a \le 10$ and $k \in \mathbb{Z}$.

[4]

a) Volume of a cuboid formula.

$V = lwh$ (in formula booklet)

where l is the length, w is the width and h is the height.

$l = 55 - 2x$ $\quad w = 28 - 2x$ $\quad h = x$

Sub l, w and h into formula.

$V = (55-2x)(28-2x)x$

expand and rearrange

$$\boxed{V = 4x^3 - 166x^2 + 1540x}$$

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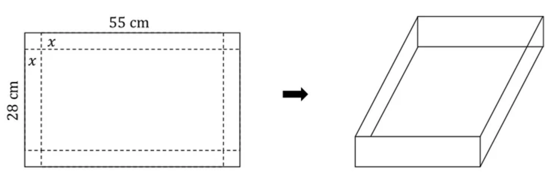

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b) Derivative of x^n formula (in formula booklet)

$f(x) = y = x^n \longrightarrow f'(x) = \frac{dy}{dx} = nx^{n-1}$

$V = 4x^3 - 166x^2 + 1540x$

$$\boxed{\frac{dV}{dx} = 12x^2 - 332x + 1540}$$

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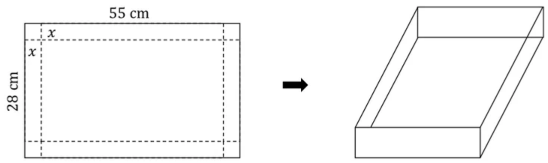

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[4]

c) i) Set $\frac{dV}{dx} = 0$ and solve for x.

$12x^2 - 332x + 1540 = 0$

$x = 5.8943...$ $\qquad$ ~~$x = 21.772...$~~

Reject as $l > 0$

$$\boxed{x = 5.89 \text{ cm} \quad (3sf)}$$

ii) Sub $x = 5.8943...$ into V.

$V = 4(5.8943)^3 - 166(5.8943)^2 + 1540(5.8943)$

$V = 4129.059...$

$= 4130 \quad (3sf)$

$$\boxed{V = 4.13 \times 10^3 \text{ cm}^3}$$