EXAM PAPERS PRACTICE

IB Maths: AA HL

Differential Equations

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.10 Differential Equations
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Differential Equations

Question 1

Consider the first-order differential equation

$$\frac{dy}{dx} - 5x^4 = 3$$

Solve the equation given that $y = 40$ when $x = 2$, giving your answer in the form $y = f(x)$.

[5 marks]

Question 2

Use separation of variables to solve each of the following differential equations for y:

a)

$$\frac{dy}{dx} = \frac{4x^2}{y^4}$$

[4 marks]

b)

$$\frac{dy}{dx} = (x^2 + 1)e^{-y}$$

[1 mark]

Question 3

Use separation of variables to solve each of the following differential equations for which satisfies the given boundary condition:

a)

$$\frac{dy}{dx} = xy^2; \quad y(2) = 1$$

[1 mark]

b)

$$(x+3)\frac{dy}{dx} = \sec y; \quad y(-2) = \frac{3\pi}{2}$$

[5 marks]

Question 4

At any point in time, the rate of growth of a colony of bacteria is proportional to the current population size. At time $t = 0$ hours, the population size is 5000.

a)
Write a differential equation to model the size of the population of bacteria.

[1 mark]

After 1 hour, the population has grown to 7000.

b)
By first solving the differential equation from part (a), determine the constant of proportionality.

[6 marks]

c)
(i)
Show that, according to the model, it will take exactly $\frac{\ln 20}{\ln 7 - \ln 5}$ hours (from $t = 0$) for the population of bacteria to grow to $100\,000$.

(ii)
Confirm your answer to part (c)(i) graphically.

[5 marks]

Question 5

After clearing a large forest of malign influences, a wizard introduces a population of 100 unicorns to the forest. According to the wizard's mathemagicians, the population of unicorns in the forest may be modelled by the logistic equation

$$\frac{dP}{dt} = 0.0006\,P(250 - P)$$

where t is the time in years after the unicorns were introduced to the forest.

a)
Show that the population of unicorns at time t years is given by

$$P(t) = \frac{500e^{0.15t}}{3 + 2e^{0.15t}}$$

[8 marks]

b)
Find the length of time predicted by the model for the population of unicorns to double in size.

[3 marks]

c)
Determine the maximum size that the model predicts the population of unicorns can grow to.

[2 marks]

Question 6

a)
Show that

$$x^2 \frac{dy}{dx} = xy + 2x^2$$

is a homogeneous differential equation.

[2 marks]

b)

Using the substitution $v = \frac{y}{x}$, show that the solution to the differential equation in part (a) is

$$y = 2x\ln|x| + cx$$

where c is a constant of integration.

[4 marks]

Question 7

a)

Use the substitution $v = \frac{y}{x}$ to show that the differential equation

$$y' = \frac{y^2}{x^2} - \frac{y}{x} + 1$$

may be rewritten in the form

$$v' = \frac{(v-1)^2}{x}$$

[3 marks]

b)

Hence use separation of variables to solve the differential equation in part (a) for which satisfies the boundary condition $y(1) = \frac{2}{3}$. Give your answer in the form $y = f(x)$.

[5 marks]

Question 8

Consider the differential equation

$$y' + 2xy = (4x + 2)e^x$$

a)

Explain why it would be appropriate to use an integrating factor in attempting to solve the differential equation.

[2 marks]

b)
Show that the integrating factor for this differential equation is e^{x^2}.

[2 marks]

c)
Hence solve the differential equation.

[5 marks]

Question 9

Use an integrating factor to solve the differential equation

$$(x+3)\frac{\mathrm{d}y}{\mathrm{d}x} - 4y = (x+3)^6$$

for y which satisfies the boundary condition $y(-2)=0$.

[7 marks]

Question 10

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x} + 1$$

with the boundary condition $y(1)=0$.

a)
Apply Euler's method with a step size of $h=0.2$ to approximate the solution to the differential equation at $x=2$.

[3 marks]

b)

(i)

Explain what method you could use to solve the above differential equation analytically (i.e., exactly).

(ii)

The exact solution to the differential equation with the given boundary condition is $y = x \ln x$. Compare your approximation from part (a) to the exact value of the solution at $x = 2$.

[4 marks]

c)

Explain how the accuracy of the approximation in part (a) could be improved.

[1 mark]

Question 11

A particle moves in a straight line, such that its displacement x at time t is described by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{te^{3t^2} + 1}{4x^2}, \quad t \geq 0$$

At time $t = 0$, $x = \frac{1}{2}$.

(a) By using Euler's method with a step length of 0.1, find an approximate value for x at time $t = 0.3$.

[3 marks]

(b)

(i)

Solve the differential equation with the given boundary condition to show that

$$x = \frac{1}{2}\sqrt[s]{e^{3t^2} + 6t}$$

(ii)

Hence find the percentage error in your approximation for x at time $t = 0.3$.

[5 marks]