Current & Circuits

Mark Schemes

Course	HL IB Physics
Section	The Particulate Nature of Matter
Topic	Current & Circuits
Difficulty	Medium

To be used by all students preparing for HL IB Physics
Students of other boards may also find this useful

1

The correct answer is **B** because:

- Current $I = \frac{\Delta q}{\Delta t}$
  - This is read as the rate of flow of electric charge

A is incorrect as while current $I = \frac{V}{R}$, this does not **define** the electric current (it **quantifies** it, since resistance R is defined as the ratio $\frac{V}{I}$)

C is incorrect as this is the definition of the potential difference across a component, $V = \frac{W}{q}$

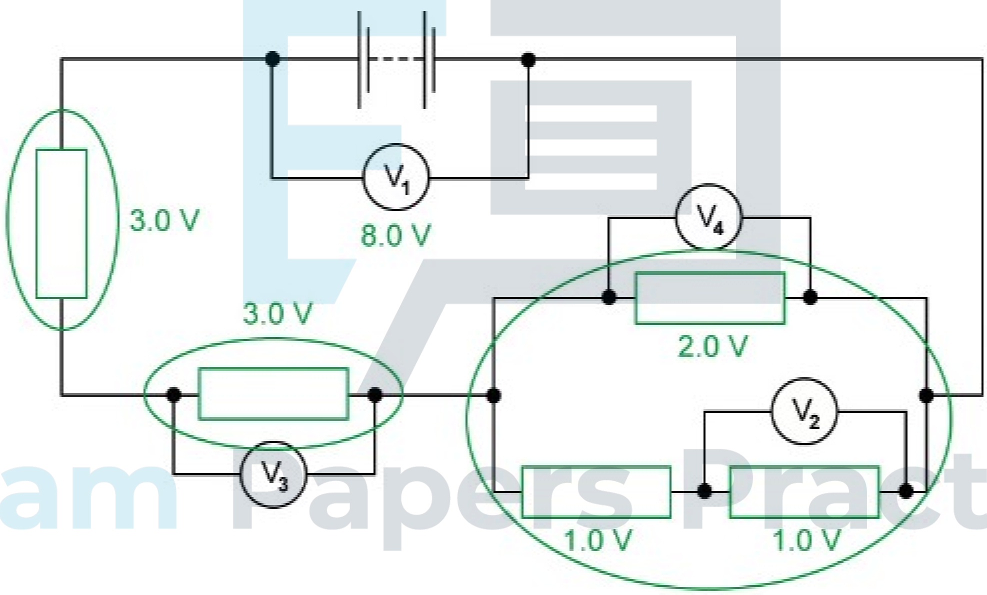
D is incorrect as this is the definition of the electromotive force ϵ of a cell

2

The correct answer is **B** because:

- Let the resistance of each resistor be R
- V_2 = 1.0 V
  - Therefore, the p.d. across the resistor just left of V_2 is also 1.0 V since the resistance is the same and they are in the same branch, so, the same current flows through them both
- So, the total p.d. in the lower branch of the parallel combination is equal to:
  - 0 + 1.0 = 2.0 V
  - The total p.d. across any branch of a parallel combination is the same according to Kirchhoff's Second Law
  - Therefore, V_4 = 2.0 V
- The complete circuit may be described as a series combination of the two single resistors and the parallel combination

- The p.d. across the vertical resistor is the same as V_3 since they have the same resistance and the same current through them, thefore:
  - V_1 = total e.m.f in circuit = 8.0 V
  - The sum of p.d's in any loop is equal to the e.m.f in the circuit
  - $V_3 + V_3 + 2.0 = 8.0$
  - $2V_3 = 6.0$ V
- So, $V_3 = 3.0$ V



The underlying principle that will help you understand this question is the conservation of charge. Charge is always carried in units of the elementary charge ($e = \pm 1.6 \times 10^{-19}$ C) so since the conducting spheres in this question are identical, when they are brought into contact a **finite** amount of charge will spread evenly across them. The total charge is therefore distributed symmetrically.

3

The correct answer is **B** because:

- From the data booklet:
  - $P = (IR)I = I^2R$
- The power dissipated in **X** is:
  - $P_x = I^2R$
- The power dissipated in **Y** is:
  - $P_y = \left(\frac{1}{2}\right)^2 (2R) = \frac{2I^2R}{4} = \frac{I^2R}{2}$
- Therefore, the ratio:
  - $\frac{power\ dissipated\ in\ Y}{power\ dissipated\ in\ X} = \frac{I^2R}{2} \times \frac{1}{I^2R} = \frac{1}{2}$

A is incorrect as the resistance in **Y** is twice the resistance in **X** so $P_y = \left(\frac{I}{2}\right)^2 (2R) = \frac{2I^2R}{4} = \frac{I^2R}{2}$ and not $(R) = \frac{I^2R}{4}$

C is incorrect as the equation is: $\frac{power\ dissipated\ in\ Y}{power\ dissipated\ in\ X}$ and not $\frac{power\ dissipated\ in\ X}{power\ dissipated\ in\ Y}$

D is incorrect as the equation is: $\frac{power\ dissipated\ in\ Y}{power\ dissipated\ in\ X}$ and not $\frac{power\ dissipated\ in\ X}{power\ dissipated\ in\ Y}$ and the resistance in **Y** is twice the resistance in **X**

4

The correct answer is **D** because:

- The cell has an e.m.f., *E*
- The negative terminal of the cell is conventionally taken to be at a potential of 0 V while the positive terminal is taken to be at a potential of *E*
- So, the positive terminal is at a higher potential than the negative terminal
- No component is connected between the positive terminal and point X, so they are at the same potential

- Therefore, point **X** is at potential *E*, while point **Y** is at the same potential as the negative terminal of the cell, so, the potential at **Y** is zero
    - This means we can eliminate graphs **A** and **B**
- Ohm's Law states that:
    - $V = IR$
    - Therefore, the p.d. across the 3 Ω resistor must be less than the p.d. across the 6 Ω resistor
- We can write $\frac{\Delta V}{\Delta x} = I \times \frac{\Delta R}{\Delta x}$ where $\frac{\Delta V}{\Delta x}$ represents the gradient of the graph, which is the change in potential difference with distance along XY
    - Since we can assume the length of wire in both resistors is the same, Δ is the same across both resistors
    - However, by Ohm's law, Δ*V* is less across the 3 Ω resistor than it is for the 6 Ω resistor
    - This means the gradient of the line is smaller across the 3 Ω resistor than it is across the 6 Ω resistor
    - This is illustrated in graph **D**

A & B are incorrect as the potential at point **X** is *E* and point **Y** is zero so the graph should start at a maximum and decrease to zero

C is incorrect as Ohm's Law tells us that, so the gradient should be steeper across the 6 Ω resistor and shallower across the 3 Ω resistor

5

The correct answer is **A** because:

- The 8 Ω resistors are connected in parallel
    - Therefore, the effective resistance can be calculated as $\frac{1}{R_T} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$
    - Hence, the effective resistance at this point in the circuit $R_T = \frac{8}{2} = 4\ \Omega$

- Therefore, the 2 Ω resistor is in series with an effective resistor of 4 Ω
    - Since they are in series, they can be combined into another effective resistance $R_T = 2 + 4 = 6$ Ω
- This 6 Ω effective resistance is in parallel with the 6 Ω resistor, so the total resistance in the circuit is calculated by:
    - $\frac{1}{R_T} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$
    - So, $R_T = \frac{6}{2} = 3$ Ω
- This is the total resistance in a circuit with e.m.f., $\epsilon = 6$ V
    - Therefore, the current in the battery $I = \frac{\varepsilon}{R} = \frac{6}{3} = 2$ A

Practice identifying where resistors can be combined into effective resistances, e.g., by identifying parallel and series networks. This circuit can be simplified in the following steps, sketched out below:

6

The incorrect answer is **B** because:

- The equation for resistivity is:
    - $\rho = \frac{RA}{L}$
- Using dimensional analysis (i.e., analysing the units):
    - $[\rho] = \frac{[R][A]}{[L]} = \frac{\Omega\, m^2}{m} = \Omega$ m
    - Therefore, option **D** is a possible unit (the standard unit!) of resistivity
- We can use the other electricity equations to provide substitutions for these units:
    - $[V] = [I][R] \rightarrow$ V = A Ω
    - $[Q] = [I][t] \rightarrow$ C = A s
    - $[V] = \frac{[W]}{[Q]} \rightarrow$ V = J C^{-1}

- From the above equations, we get:
  - $\Omega = \frac{V}{A}$
  - Hence, substituting this into [ρ] = $\frac{\Omega\, m^2}{m} = \frac{V\, m^2}{A\, m} = \frac{V\, m}{A} = \frac{J\, m}{C\, A} = \frac{J\, m}{A^2 s}$ = J m s^{-1} A^{-2}
  - Therefore, option **B** must be incorrect

A is incorrect as [ρ] = $\frac{V\, m}{A}$ = V m A^{-1}

C is incorrect as [ρ] = J m s^{-1} A^{-2}

D is incorrect as [ρ] = $\frac{[R][A]}{[L]} = \frac{\Omega\, m^2}{m}$ = Ω m

You must read this question carefully as it asks which statement is incorrect.

7

The correct answer is **D** because:

Method 1: Power Equation

- Use the equation for power $P = I^2R$
- First, consider the whole circuit:
  - V = e.m.f. = 10 V
  - $R_T = (\frac{1}{2} + \frac{1}{2}) + 4 = 5\ \Omega$
- Use Ohm's Law to calculate the current in the whole circuit:
  - $I = \frac{V}{R} = \frac{\text{e.m.f.}}{R_T} = \frac{10}{5}$ = 2 A

- So, $P = I^2R = 2^2 \times 5 = 4 \times 5 = 20$ W
- Now, consider the power across a 2 Ω resistor:
  - The 2 A are split evenly between the paths because the resistors are the same size.
  - So, 1 A of current passes through a 2 Ω resistor
  - Use the equation for power, $P = I^2R = 1^2 \times 2 = 1 \times 2 = 2$ W
- So, the correct answer is option **D**

A is incorrect as the resistance across $R_1 = 1$ Ω. This is not the same as the power across the resistor which is 2 W there are two 2 Ω resistors, each dissipating a power of 2 W and not just one, so total power dissipated in the circuit $P = (2 \times 2) + 16 = 20$ W and not $P = (1 \times 2) + 16 = 18$ W

B is incorrect as the resistance across $R_1 = 1$ Ω. This is not the same as the power across the resistor which is 2 W

C is incorrect as there are two 2 Ω resistors, each dissipating a power of 2 W and not just one, so total power dissipated in the circuit $P = (2 \times 2) + 16 = 20$ W and not $P = (1 \times 2) + 16 = 18$ W

8

The correct answer is **C** because:

- When the switch is opened, the loop containing lamp W and lamp X is not a complete circuit
- However, the loop containing lamp W and lamp Y is a complete circuit
  - Therefore, we can eliminate option **A** and option **D** since lamp X must be off when the switch is open
- When the switch is closed, lamp X and Y are in parallel with each other
  - Therefore, removing lamp X from the circuit by opening the switch increases the overall resistance of the circuit
  - Hence, the overall current in the circuit decreases
- We know each of the lamps are identical, hence, they each have the same resistance R

- Therefore, the power dissipated in lamp W, $P = I^2R$ must decrease since the resistance stays constant but the current through it decreases
- Hence, the brightness across lamp W must decrease

- Opening the switch will also increase the potential difference across lamp Y
  - Hence, the power dissipated in lamp Y, $P = \frac{V^2}{R}$ will increase, since the resistance stays constant
  - Hence, the brightness across lamp Y will increase
- Therefore, the correct answer is **C**

A is incorrect as lamp **X** is off when the switch is open, as the circuit for this lamp is no longer complete

B is incorrect as the brightness of lamp **Y** will increase when the switch is opened because the current is no longer split between the two parallel branches

D is incorrect as lamp **X** is off when the switch is open, as the circuit for this lamp is no longer complete

This question requires you to slowly eliminate the different options until you are left with the correct one.

9

The correct answer is **C** because:

- The power rating on devices is correct for the country / mains voltage where they are sold
  - Therefore the power dissipated in the UK is the same as the value written on the bulb
  - Only **B** or **C** can be correct
- To find the power dissipated when using a different voltage, use the equation $P = \frac{V^2}{R}$

- Since the same bulb is being used, R is constant, therefore set up a ratio to compare the UK and Canada
  - $\frac{V_{uk}^2}{P_{uk}} = \frac{V_c^2}{P_c}$ therefore $\frac{240^2}{60} = \frac{100^2}{P_c}$
- Power dissipated in Canada, $P_c = 10.42$ W

10

The correct answer is **C** because:

- A charge of 240 C flows through the resistor in a time of 2 minutes = 120 s
- Calculating current:
  - $I = \frac{\Delta Q}{\Delta t} = \frac{240}{120} = 2$ A
- Power dissipated $P = I^2R$ and power is defined as the rate of transfer of energy
  - Therefore, energy $E = P \times t = I^2R \times t$
  - So, $1440 = (2)^2 R \times 120$
  - Hence, $R = \frac{1440}{(2)^2 \times 120} = \frac{12}{480} = 3\ \Omega$
- The emf of a circuit $\epsilon = I(R + r) = 9.0$ V
  - Therefore, the total resistance in the circuit $(R + r) = \frac{9.0}{I} = \frac{9.0}{2} = 4.5\ \Omega$
- Therefore, since $R = 3$
  - $3 + r = 4.5$
  - Therefore $r = 4.5 - 3 = 1.5\ \Omega$

A is incorrect as this is the resistance of the resistor and not the internal resistance of the e.m.f

B is incorrect as this is the current in the circuit and not the internal resistance of the e.m.f.

D is incorrect as this is the total resistance in the circuit and not the internal resistance of the e.m.f.

The power dissipated is the energy per time from Power = $\frac{Energy}{Time}$. Look carefully that the units 1440 is measured in J, so to calculate the energy use Energy = Power × Time.

11

The correct answer is **D** because:

- The electromotive force ϵ is defined as the work done per unit charge in moving charge from one terminal of the battery to the other
  - The work, or energy transferred, between terminals is chemical
- The electromotive force can be written as:
  - $\epsilon = \frac{W}{Q}$ such that the energy transferred (or work done) W = ϵQ
- However, this chemical energy is deliverable as electrical energy to both the load resistance and the internal resistance
- Therefore, ϵQ is equal to the total energy dissipated in the battery due to the internal resistance as well is externally (the load resistance)

A is incorrect as the work done around the circuit is not chemical, it is electrical, and is equal to VQ, where V is the terminal potential difference

B is incorrect as the work done between the terminals is not electric, it is chemical

C is incorrect as the load resistance is usually given the symbol R. ϵQ is a quantity of energy, not resistance

Read each of the answer options carefully and don't just jump to a conclusion without considering them all!

12

The correct answer is **A** because:

- There are two unknowns, so we must set up two unique simultaneous equations

- The emf $\epsilon = I(R + r) = V + Ir$ where V is the terminal potential difference
  - Therefore, when $V = 5.0$ V and $I = 1.0$ A, $\epsilon = 5 + r$ (equation 1)
  - When $V = 4.0$ V and $I = 1.5$ A, $\epsilon = 4 + 1.5r$ (equation 2)
- Equating the two equations to eliminate ϵ gives:
  - $5 + r = 4 + 1.5r$
  - $1 = 0.5r$
  - Therefore, $r = \frac{1}{0.5} = 2.0\ \Omega$
- Using this value for r in equation 1 gives:
  - $\epsilon = 5 + 2 = 7.0$ V

B is incorrect as the internal resistance $r = 2\ \Omega$ because $1 = 0.5r$ so, $\frac{1}{0.5} = 2.0\ \Omega$ and not $0.5\ \Omega$

C is incorrect as $\epsilon = 5 + 2 = 7.0$ V and not $\epsilon = 5 - 2 = 3.0$ V

D is incorrect as the internal resistance $r = 2\ \Omega$ because $1 = 0.5r$ so, $\frac{1}{0.5} = 2.0\ \Omega$ and not $0.5\ \Omega$

$\epsilon = 5 + 2 = 7.0$ V and not $\epsilon = 5 - 2 = 3.0$ V

If there are **two** unknowns in each question, don't panic! This is a huge flag that you are required to set up **two** simultaneous equations and solve them. In more extended questions, you may even sometimes have **three** unknowns – but as you might expect, this just means you then need to set up **three** unique simultaneous equations and perform a bit more algebra to manipulate and solve them. Anytime you have the same number of unknowns as you have equations – think simultaneous equations!

13

The correct answer is **A** because:

- The cell emf $\epsilon = I(R + r)$
  - Therefore, this can be rearranged into the form of a straight line
  - $I = \frac{\varepsilon}{R + r}$ hence $\frac{1}{I} = \frac{R + r}{\varepsilon} = \frac{R}{\varepsilon} + \frac{r}{\varepsilon}$

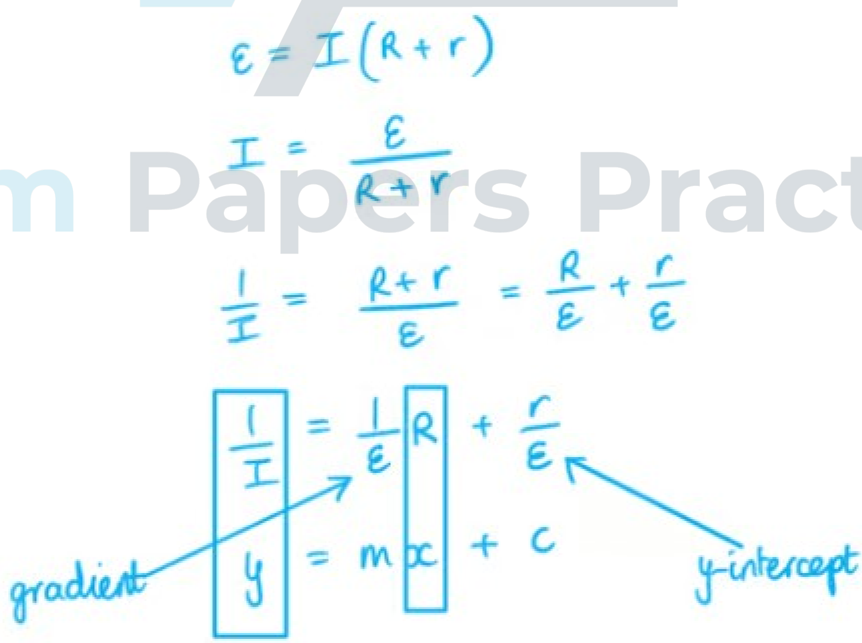
- If $y = mx + c$, then $\frac{1}{I}$ on the y–axis and R on the x–axis is a straight line with a gradient equal to $\frac{1}{\varepsilon}$

B is incorrect as $I = \frac{\varepsilon}{R + r}$ so $\frac{1}{I} = \frac{R + r}{\varepsilon} = \frac{R}{\varepsilon} + \frac{r}{\varepsilon}$ and not $\frac{1}{R}$. R is a quantity on the x-axis, so it is not possible to have a straight-line graph with a gradient of $\frac{1}{quantity}$

C is incorrect as $I = \frac{\varepsilon}{R + r}$ so $\frac{1}{I} = \frac{R + r}{\varepsilon} = \frac{R}{\varepsilon} + \frac{r}{\varepsilon}$ and not $\frac{1}{r}$

D is incorrect as $I = \frac{\varepsilon}{R + r}$ so $\frac{1}{I} = \frac{R + r}{\varepsilon} = \frac{R}{\varepsilon} + \frac{r}{\varepsilon}$ and not r

You should be comfortable, for the Standard Level Diploma Programme, with recognising linear equations and rearranging them so that they match a given graph. The method for this question is sketched out in more detail below:



14

The incorrect statement is **C** because:

- An ideal battery is one for which the internal resistance is zero
- Hence, since $\epsilon = I(R + r) = V + Ir$, where V is the terminal potential difference, I is the current and r is the internal resistance,
  - If the internal resistance is zero, then the terminal potential difference (i.e., that which is measurable) would be exactly equal to the emf ϵ

A is correct as non-ideal batteries have internal resistance. Hence, the terminal pd $V = \epsilon - Ir$, and so is always less than the emf ϵ

B is correct as $V = \epsilon - Ir$. Hence, if the current $I = 0$, then $V = \epsilon$

D is correct as the terminal potential difference for non-ideal batteries decreases with time as current is drawn from it, due to the battery's internal resistance

15

The incorrect answer is **C** because:

- The internal resistance of a cell must have units equivalent to the ohm, Ω
  - $[R] = \Omega = \frac{[V]}{[I]} = \text{V A}^{-1}$
  - Therefore, option A is correct
- The Volt, V is defined by the equation $V = \frac{W}{q}$:
  - $[V] = \text{V} = \frac{[W]}{[q]} = \text{J C}^{-1}$
  - Since $[Q] = [I][t]$, then C = A s
  - Substituting this into $\Omega = \text{V A}^{-1} = (\text{J C}^{-1})\,\text{A}^{-1} = \text{J}\,(\text{A}^{-1}\,\text{s}^{-1})\,\text{A}^{-1}$
  - Therefore, $\Omega = \text{J s}^{-1}\,\text{A}^{-2}$ so option B is correct
- The Joule, J is defined by the equation $W = Fd$:
  - $[W] = [F][d] = \text{N m}$
  - Since $[F] = [m][a]$ then $[\text{F}] = \text{N} = \text{kg m s}^{-2}$
  - Substituting this into $[W] = \text{J} = \text{N m} = (\text{kg m s}^{-2})\,\text{m} = \text{kg m}^2\,\text{s}^{-2}$
  - Therefore, $\Omega = (\text{kg m}^2\,\text{s}^{-2})\,\text{s}^{-1}\,\text{A}^{-2} = \text{kg m}^2\,\text{A}^{-2}\,\text{s}^{-3}$
- Therefore, option D is correct
  - So, option C cannot be units for internal resistance

Make sure you read this question carefully. It says incorrect and not correct. Do not just start and then assume A is correct.

16

The correct answer is **B** because:

- The terminal potential difference V_X and V_Y can be written in terms of the emf ϵ:
  - Treating the circuit as a potential divider, with the 3 Ω resistor in series with the internal resistance r = 0.5 Ω, then $V_X = \epsilon \times \frac{R}{R + r} = \frac{1.5 \times 3}{3 + 0.5} = \frac{9}{7}$
  - Similarly, $V_Y = \epsilon \times \frac{R}{R + r} = \frac{3 \times 1.5}{3 + 2} = \frac{9}{10}$
  - Therefore, $V_X > V_Y$ since the denominator is smaller for V_X
- The power dissipated across the resistor, P_X and P_Y, can be written in terms of the terminal potential differences V_X and V_Y:
  - $P_X = \frac{V_X^2}{R} = \frac{V_X^2}{3}$
  - $P_Y = \frac{V_Y^2}{R} = \frac{V_Y^2}{3}$
  - Since $V_X > V_Y$, $P_X > P_Y$
  - Therefore, row B is correct

A is incorrect as the power is greater in X than in Y. Consider the equation for power, $P = IV$ then if V is greater then P will also be greater.

C is incorrect as both the power and potential difference are greater in X than in Y, as $V_X > V_Y$ as $\frac{9}{7} > \frac{9}{10}$

and consider the equation for power, $P = IV$ then if V is greater then P will also be greater.

D is correct as the potential difference is greater in X than in Y, as $V_X > V_Y$ as $\frac{9}{7} > \frac{9}{10}$

17

The correct answer is **D** because:

- The terminal potential difference $V = \epsilon - Ir$
  - If the resistance R decreases, then current in the circuit increases
  - Therefore, the terminal potential difference V decreases as well
  - The power dissipated in the cell (that is, across the internal resistance)
    $P = I^2r$, which therefore becomes large
- Hence, statement (1) is correct
- If the resistance R is made very large, this causes a very small current in the circuit
  - The power supplied by the cell $P = \epsilon I = I^2(R + r) = I^2R + I^2r$
  - The power dissipated by the resistor R is given by the term I^2R, which dominates since R is now very large
  - Therefore, most of the power supplied by the cell is dissipated across the resistor R
- Hence, statement (2) is correct
- If the resistance of R is made very small, then the current in the circuit will increase
  - Hence, the charge delivered by the cell (its capacity) will rapidly deplete, resulting in a shorter lifetime

Walking through each step in this question is useful practise for you to understand the difference components of a circuit, in terms of the external (or load) resistance R and the internal resistance r. Imagining these as two distinct components, over which power supplied by the emf of the cell is dissipated, is very good exam technique.

18

The correct answer is **B** because:

- Kirchhoff's Circuit Laws tells us $\Sigma V = 0$ in a loop:
  - In each branch total potential difference = 4 × 12 = 48 V.
  - All potential differences in parallel are the same (option **A** or **B** must be correct)
- Kirchhoff's Circuit Laws tell us that $\Sigma I = 0$ at a junction
  - Current = 2.5 A in each branch
  - There are five branches, therefore the total current = 5 × 2.5 = 12.5 A
  - **B** is correct