EXAM PAPERS PRACTICE

Co-ordinate Geometry

Model Answer

A line has gradient 5.
M and N are two points on this line.
M is the point $(x, 8)$ and N is the point $(k, 23)$.

Find an expression for x in terms of k. [3]

The gradient of a line is given by the formula:

Gradient $= \frac{\text{Change in } y}{\text{Change in } x}$

For the given line with a gradient of 5 and points $M(x, 8)$ and $N(k, 23)$, we can write:

$5 = \frac{23-8}{k-x}$

Now, solve for x :

$5 = \frac{15}{k-x}$

Cross-multiply:

$5(k-x) = 15$

Distribute 5:

$5k - 5x = 15$

Add $5x$ to both sides:

$5k = 5x + 15$

Subtract 15 from both sides:

$5k - 15 = 5x$

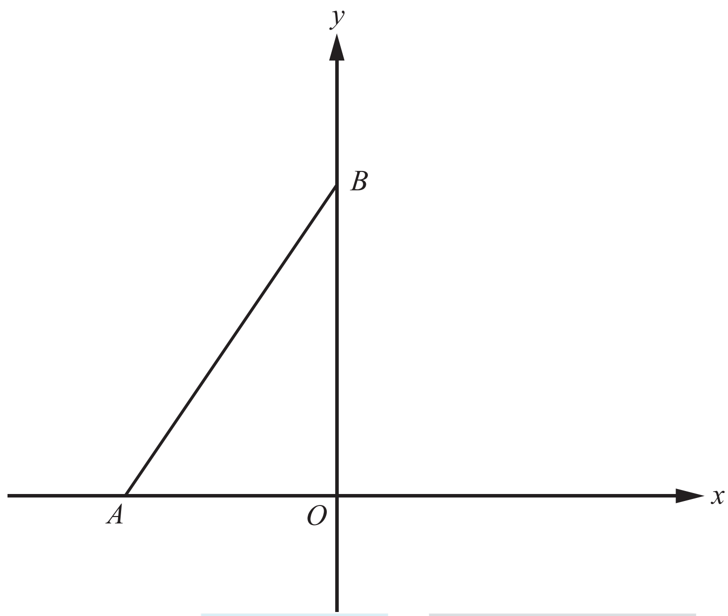
Divide by 5 :

$x = \frac{5k-15}{5}$

Simplify:

$x = k - 3$

So, the expression for x in terms of k is $x = k - 3$.



NOT TO SCALE

A is the point $(-2, 0)$ and B is the point $(0, 4)$.

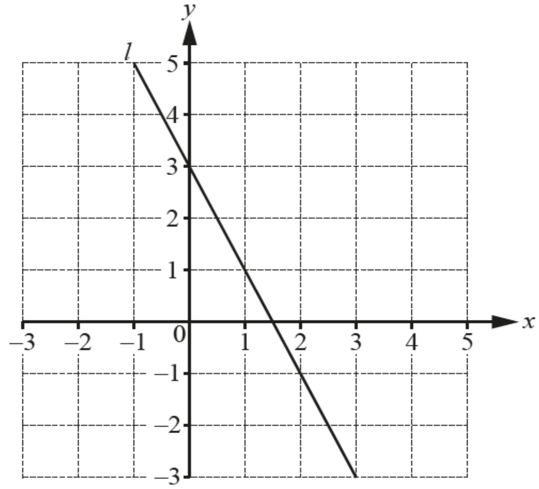
(a) Find the equation of the straight line joining A and B. [3]

The equation of the straight line joining A and B is:
$y = 2x$

(b) Find the equation of the perpendicular bisector of AB. [4]

The equation of the perpendicular bisector of AB is:
$y = (-5/2)x + 5/4$
The perpendicular bisector of AB has a slope of $-5/2$ and a y-intercept of $5/4$.
This can be found by using the following steps:
1. Find the midpoint of AB. This is the point where the perpendicular bisector intersects AB.
2. Find the slope of AB.
3. The perpendicular bisector will have a slope that is the negative reciprocal of the slope of AB.
4. Use the midpoint of AB and the slope of the perpendicular bisector to find the equation of the perpendicular bisector in point-slope form.

Question 3



(a) Find the equation of the line l.
Give your answer in the form $y = mx + c$.

the equation of the line l is $y = \frac{3}{2}x - 1$.

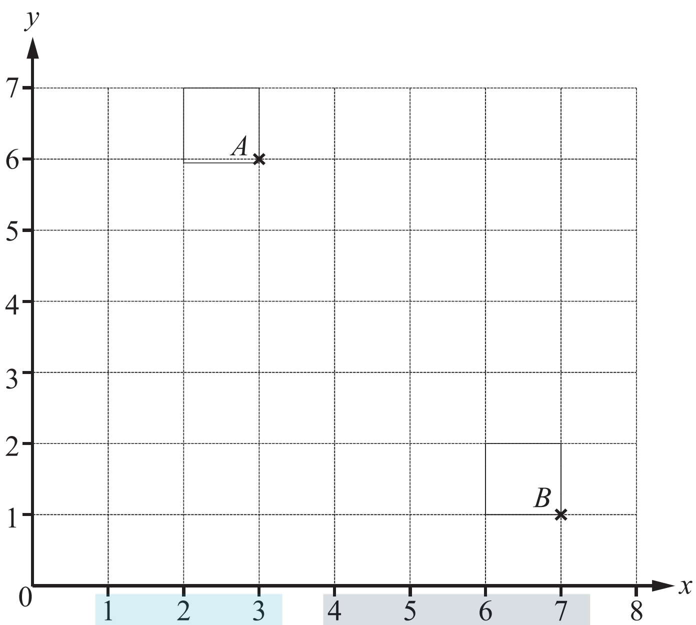
[3]

(b) A line perpendicular to the line l passes through the point $(3, -1)$.

Find the equation of this line.

The slope of line l is -1 , so the slope of the perpendicular line is 1 . The equation of the perpendicular line passing through $(3, -1)$ is:
$y + 1 = 1(x - 3)$
$y = x - 4$
This can be found by using the following steps:
1. Find the slope of line l.
2. Find the negative reciprocal of the slope of line l to find the slope of the perpendicular line.
3. Use the point-slope form of linear equations to find the equation of the perpendicular line passing through$(3, -1)$.

[3]



Point A has co-ordinates (3, 6).

(a) Write down the co-ordinates of point B. [1]

The coordinates of point B are $(-2, 3)$.

(b) Find the gradient of the line AB. [2]

The gradient of the line AB is -2.

(c) Find the equation of the line that

- is perpendicular to the line AB

and [3]

- passes through the point $(0, 2)$.

The equation of the line that is perpendicular to AB and passes through the point $(0, 2)$ is$y = -\frac{2}{5}x + 2$.

A is the point (8, 3) and B is the point (12, 1).

Find the equation of the line, perpendicular to the line AB, which passes through the point (0, 0).

[3]

The equation of the line that is perpendicular to AB and passes through the point $(0, 0)$ is$y = -\frac{3}{8}x$.

A is the point (4, 1) and B is the point (10, 15).

Find the equation of the perpendicular bisector of the line AB. [6]

The equation of the perpendicular bisector of line AB is $y = (-3/7)x + 11$.
Explanation:
The midpoint of AB is $(7, 8)$, and the slope of AB is $7/3$. The slope of the perpendicular bisector is $-3/7$, so its equation is $y = (-3/7)x + 11$.

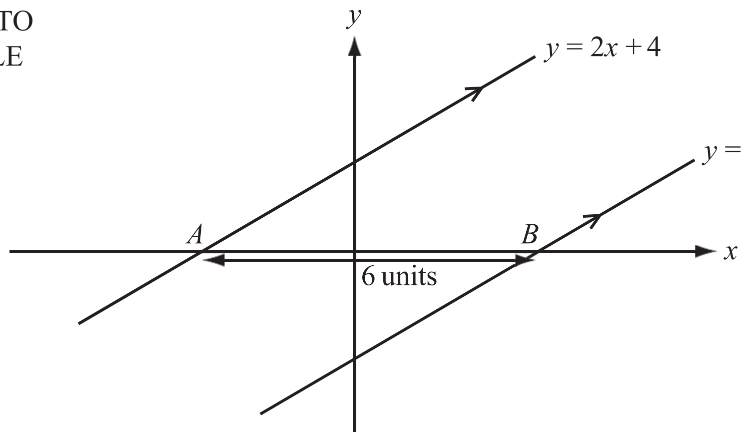
Find the equation of the line that

- is perpendicular to the line $y = 3x - 1$

and

- passes through the point (7, 4). [3]

The equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y -intercept.
The given line is $y = 3x - 1$, and it has a slope m of 3 .
The slope of a line perpendicular to this line is the negative reciprocal of the original slope. So, the slope m_{perp} of the perpendicular line is $-1/3$.
Now, using the point-slope form of the equation of a line $(y - y_1) = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope:
$(y - 4) = -\frac{1}{3}(x - 7)$
Multiply through by -3 to get rid of the fraction:
$-3(y - 4) = (x - 7)$
Distribute:
$-3y + 12 = x - 7$
Bring like terms to one side:
$x + 3y = 19$
So, the equation of the line perpendicular to $y = 3x - 1$ and passing through the point $(7, 4)$ is $x + 3y = 19$.



The line $y = mx + c$ is parallel to the line $y = 2x + 4$.
The distance AB is 6 units.

Find the value of m and the value of c. [4]

The value of m is 2 and the value of c is 4 .
Explanation:
The equation of the line is $y = mx + c$, where m is the slope of the line and c is the y-intercept. In this case, the slope of the line is 2 and the y-intercept is 4 . Therefore, the value of m is 2 and the value of c is 4 .

Find the co-ordinates of the mid-point of the line joining the points $A(2, –5)$ and $B(6, 9)$. [2]

The coordinates of the midpoint (M_x, M_y) of a line segment with endpoints (x_1, y_1) and (x_2, y_2) can be found using the midpoint formula:
$M_x = \frac{x_1+x_2}{2}$
$M_y = \frac{y_1+y_2}{2}$
For the given points $A(2, -5)$ and $B(6, 9)$, the midpoint coordinates (M_x, M_y) are:
$M_x = \frac{2+6}{2} = 4$
$M_y = \frac{(-5)+9}{2} = 2$
So, the coordinates of the midpoint of the line joining points A and B are $(4, 2)$.

A straight line passes through two points with co-ordinates (6, 8) and (0, 5).
Work out the equation of the line. [3]

The equation of a straight line in slope-intercept form $y = mx + b$ can be found using the following steps:

1. Find the slope (m) :

The slope (m) is given by the formula:

$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

Using the points $(6, 8)$ and $(0, 5)$:

$m = \frac{8-5}{6-0} = \frac{3}{6} = \frac{1}{2}$

2. Use the slope-intercept form $(y = mx + b)$:

Now that we have the slope (m), we can use one of the points to solve for the y-intercept (b).

Using the point $(6, 8)$:

$8 = \frac{1}{2}(6) + b$

Solve for b :

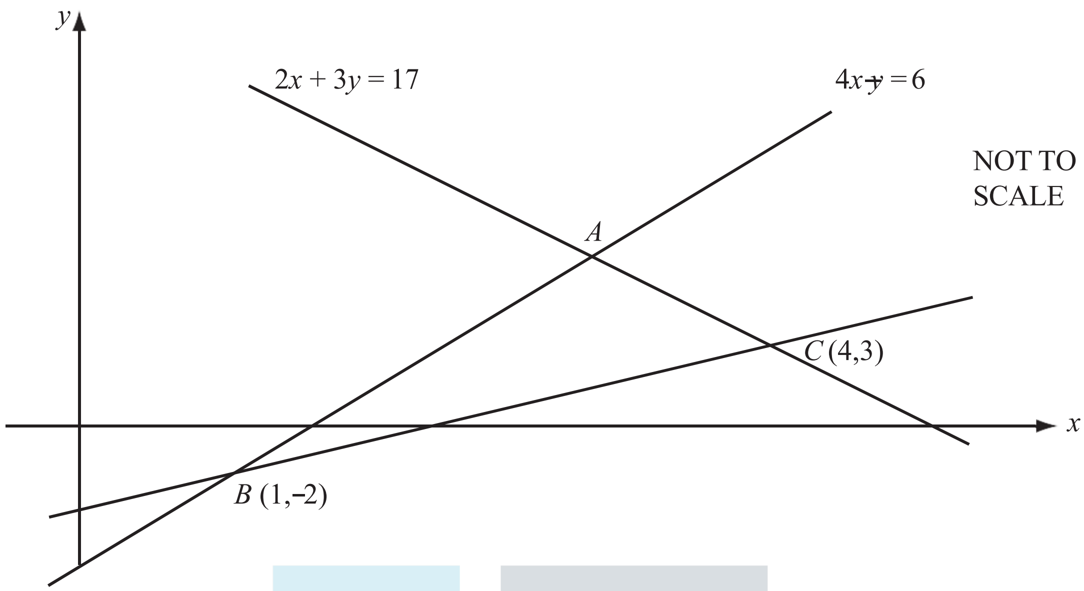
$8 = 3 + b \Longrightarrow b = 5$

3. Write the equation:

Substitute the values of m and b into the slope-intercept form:

$y = \frac{1}{2}x + 5$

So, the equation of the line passing through the points $(6, 8)$ and $(0, 5)$ is $y = \frac{1}{2}x + 5$.



In the diagram, the line AC has equation $2x + 3y = 17$ and the line AB has equation $4x - y = 6$.
The lines BC and AB intersect at $B(1, -2)$.
The lines AC and BC intersect at $C(4, 3)$.

(a) Use algebra to find the coordinates of the point A. [3]

To find the coordinates of the point A using algebra, we substitute x = 1.5 into either equation and find that the y-coordinate of A is 4.67. Therefore, the coordinates of the point A are (1.5, 4.67).

(b) Find the equation of the line BC. [3]

Answer

The coordinates of the point A are (1.5, 4.67).

Explanation:

The equations of lines BC and AB are 2x + 3y = 17 and 4x - y = 6, respectively.

Solving the system of equations, we get x = 1.5 and y = 4.67. Therefore, the coordinates of A are (1.5, 4.67).

The points $A(6,2)$ and $B(8,5)$ lie on a straight line.

(a) Work out the gradient of this line. [1]

The gradient (m) of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by the formula:
$m = \frac{y_2 - y_1}{x_2 - x_1}$
For the given points $A(6,2)$ and $B(8,5)$, the gradient m is calculated as follows:
$m = \frac{5-2}{8-6} = \frac{3}{2}$
So, the gradient of the line passing through points $A(6,2)$ and $B(8,5)$ is $\frac{3}{2}$.

(b) Work out the equation of the line, giving your answer in the form $y = mx + c$. [2]

1. Find the slope (m) :
$m = \frac{y_2 - y_1}{x_2 - x_1}$
Using points $A(6,2)$ and $B(8,5)$:
$m = \frac{5-2}{8-6} = \frac{3}{2}$
2. Use the slope-intercept form $(y = mx + c)$:
Now that we have the slope (m), we can use one of the points to solve for the y-intercept (c). Using point $A(6,2)$:
$2 = \frac{3}{2}(6) + c$
Solve for c :
$2 = 9 + c \Longrightarrow c = -7$
3. Write the equation:
Substitute the values of m and c into the slope-intercept form:
$y = \frac{3}{2}x - 7$
So, the equation of the line passing through points $A(6,2)$ and $B(8,5)$ is $y = \frac{3}{2}x - 7$.

Question 13

The points (2, 5), (3, 3) and (k, 1) all lie in a straight line.

(a) Find the value of k. [1]

If three points $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) lie on a straight line, the slopes of the line segments formed by consecutive points are equal. Let's use the points $(2,5), (3,3)$, and $(k,1)$. The slope between the first two points is:

$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{3-5}{3-2} = -2$

Now, find the slope between the second and third points:

$m_2 = \frac{y_3-y_2}{x_3-x_2} = \frac{1-3}{k-3}$

Since the points lie on the same line, $m_1 = m_2$:

$-2 = \frac{1-3}{k-3}$

Now, solve for k :

$-2(k-3) = 1-3$

$-2k+6 = -2$

$-2k = -8$

$k = 4$

So, the value of k is 4 .

(b) Find the equation of the line. [3]

To find the equation of the line passing through the points $(2,5), (3,3)$, and $(k,1)$, follow these steps:

1. Find the slope (m) :

$m = \frac{w_2-y_1}{x_2-x_1}$

Using the points $(2,5)$ and $(3,3)$:

$m = \frac{3-5}{3-2} = -2$

2. Use the point-slope form $(y - y_1 = m(x - x_1))$:

Using one of the points, let's use $(2,5)$:

$y-5 = -2(x-2)$

3. Simplify the equation:

$y-5 = -2x+4$

$y = -2x+9$

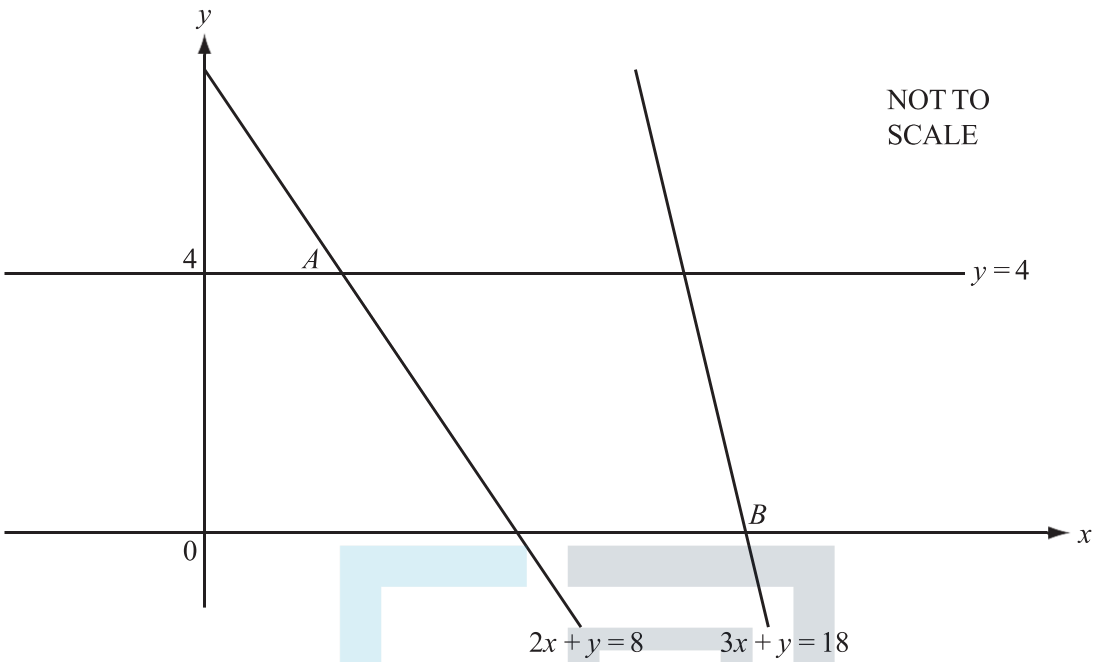
Now, check if the third point $(k,1)$ lies on the line. Substitute $x = k$ and $y = 1$ into the equation:

$1 = -2k+9$

Solve for k :

$k = 4$

So, the equation of the line is $y = -2x+9$.



(a) The line $y = 4$ meets the line $2x + y = 8$ at the point A.
Find the co-ordinates of A.

The coordinates of point A are $(2, 4)$.

[1]

(b) The line $3x + y = 18$ meets the x axis at the point B.
Find the co-ordinates of B.

The coordinates of point B are $(6, 0)$.

[1]

(c) (i) Find the co-ordinates of the mid-point M of the line joining A to B.

The coordinates of the midpoint, M, are $(-1, 2)$.

[1]

(ii) Find the equation of the line through M parallel to $3x + y = 18$.

the equation of the line through M parallel to $3x + y = 18$ is $y = -3x + 7$.

[2]

Find the length of the line joining the points $A(-4, 8)$ and $B(-1, 4)$. [2]

We use the distance formula:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

where $(x_1, y_1) = (-4, 8)$ and $(x_2, y_2) = (-1, 4)$. Plugging in, we get:

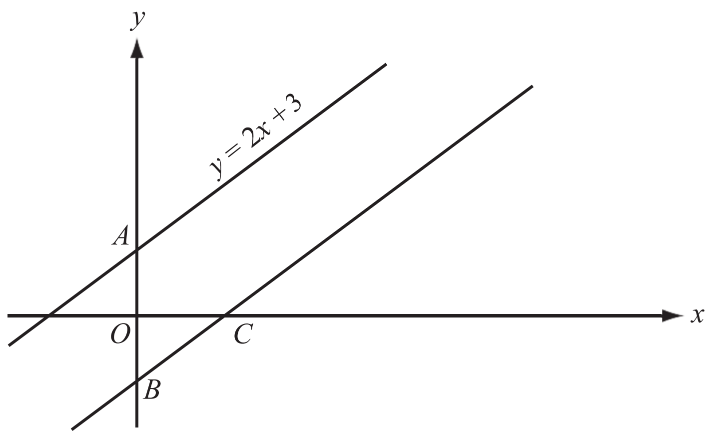
$d = \sqrt{(-4 - (-1))^2 + (8 - 4)^2}$

$d = \sqrt{(-3)^2 + (4)^2}$

$d = \sqrt{25}$

$d = 5$

Therefore, the length of the line joining the points A and B is 5 .



NOT TO SCALE

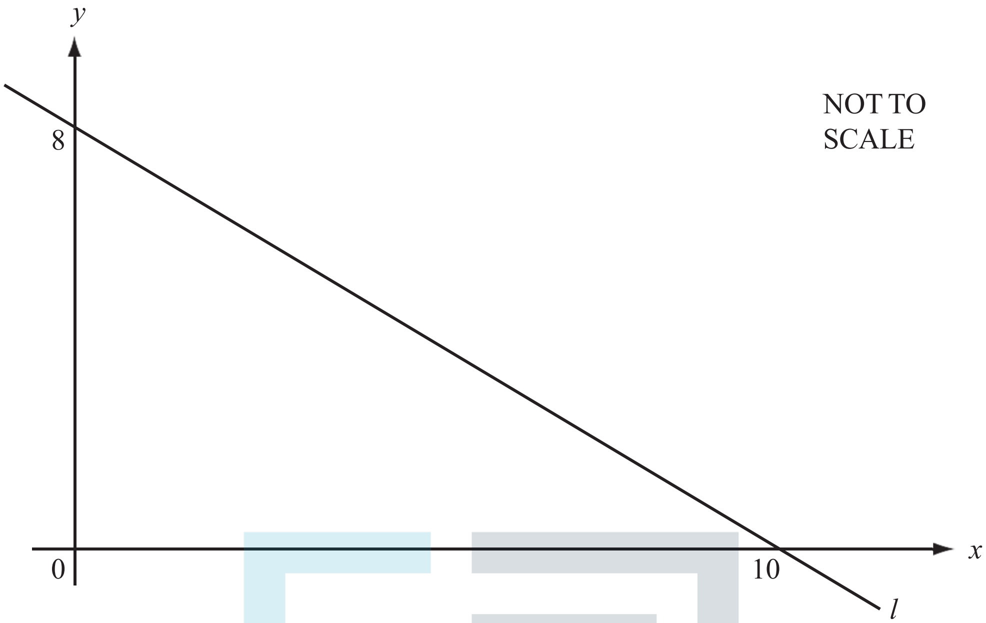
The distance *AB* is 7 units.

(a) Write down the equation of the line through *B* which is parallel to $y = 2x + 3$. [2]

The equation of the line through B which is parallel to $y = 2x + 3$ is $y = 2x - 5$.

(b) Find the co-ordinates of the point *C* where this line crosses the *x* axis. [1]

The line crosses the x-axis at the point $(-3/2, 0)$.



The line l passes through the points (10, 0) and (0, 8) as shown in the diagram.

(a) Find the gradient of the line as a fraction in its simplest form. [1]

Since the change in the y-coordinate is zero, the gradient of the line is 0 as a fraction in its simplest form.

(b) **Write down** the equation of the line parallel to l which passes through the origin. [1]

The slope of line l is equal to $\frac{0-8}{10-0} = -\frac{4}{5}$. Since parallel lines have equal slopes, the equation of the line parallel to l passing through the origin is:
$y = -4/5{}^*x$
or, equivalently, $4x + 5y = 0$.

(c) Find the equation of the line parallel to l which passes through the point (3, 1). [2]

Line l is vertical, since it passes through two points with the same x-coordinate.
Therefore, the equation of any parallel line is of the form x = C for some constant C.

Since we want the parallel line to pass through the point (3,1), we must have 3 = C.
Therefore, the equation of the parallel line is x = 3.

The equation of a straight line can be written in the form $3x + 2y - 8 = 0$.

(a) Rearrange this equation to make y the subject. [2]

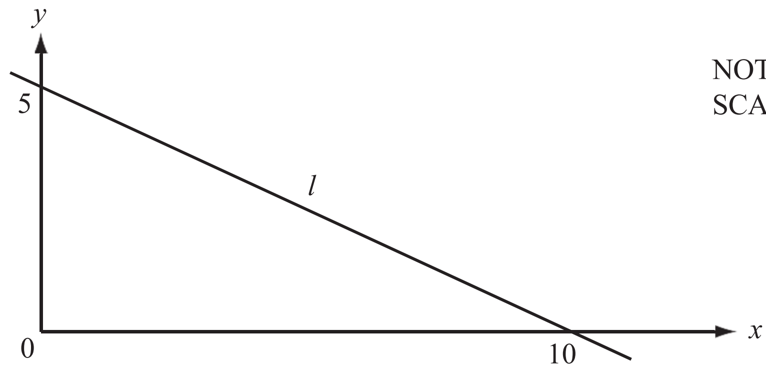
Step 1: Isolate the y term
$3x + 2y - 8 = 0$
$2y = 8 - 3x$
Step 2: Divide both sides by 2
$2y = 8 - 3x$
$y = 4 - 1.5x$
Step 3: Write the equation in slope-intercept form
$y = -1.5x + 4$
Therefore, the equation of the line with y as the subject is $y = -1.5x + 4$.

(b) Write down the gradient of the line. [1]

To find the gradient of the line, we need to rewrite the equation in slope-intercept form, which is $y = \mathrm{mx} + \mathrm{b}$, where m is the gradient and b is the y-intercept.
To do this, we need to isolate y on one side of the equation:
$3x + 2y - 8 = 0$
$2y = -3x + 8$
$y = (-3/2)x + 4$
Therefore, the gradient of the line is $-3/2$.

(c) Write down the co-ordinates of the point where the line crosses the y axis. [1]

To find the y-intercept, let's substitute $x = 0$ into the equation.
$3(0) + 2y - 8 = 0$
$2y - 8 = 0$
$2y = 8$
$y = 4$
Therefore the line intercepts the y axis at the point $(0, 4)$.



NOT TO SCALE

(a) Calculate the gradient of the line l.

To calculate the gradient of the line l, we can use the following formula:

gradient $= (y2 - y1)/(x2 - x1)$

where $(x1, y1)$ and $(x2, y2)$ are any two points on the line.

Since the line l intersects the y-axis at the point $(0, 4)$ and passes through the point $(3, 1)$, we can use these two points to calculate the gradient:

gradient $= (1 - 4)/(3 - \varnothing)$

gradient $= -3/3$

gradient $= -1$

Therefore, the gradient of the line l is -1 .

Answer: The gradient of the line l is -1 .

[2]

(b) Write down the equation of the line l. [2]

The equation of the line l is y = x.

This is because the line passes through the origin (0, 0) and has a slope of 1. The slope of a line is calculated by taking the difference in the y-coordinates of two points on the line and dividing it by the difference in the x-coordinates of those same two points. In this case, the two points are the origin and (1, 1), which both lie on the line l.

The equation of a line in slope-intercept form is y = mx + b, where m is the slope and b is the y-intercept. In this case, the slope is 1 and the y-intercept is 0, so the equation of the line is y = x.

Another way to think about this is that the line l is simply a diagonal line that passes through the first and third quadrants of the graph. The equation of a diagonal line is always y = x, regardless of where it is located on the graph.

16 The straight line graph of $y = 3x - 6$ cuts the x-axis at A and the y-axis at B.

(a) Find the coordinates of A and the coordinates of B.

1. For point A (x-axis):

When a point lies on the x-axis, the y-coordinate is 0 . Set $y = 0$ and solve for x :

$0 = 3x - 6$ [2]

Solve for x :

$x = 2$

So, point A is $(2, 0)$.

2. For point **B** (y-axis):

When a point lies on the y-axis, the x-coordinate is 0 . Set $x = 0$ and solve for y :

$y = 3(0) - 6$

Solve for y : [2]

$y = -6$

So, point B is $(0, -6)$.

Therefore, the coordinates of point A are $(2, 0)$ and the coordinates of point B are $(0, -6)$.

(b) Calculate the length of AB.

The x-axis intercept (A) occurs when $y = 0$, and the y-axis intercept (B) occurs when $x = 0$.

1. Finding A : [1]

Set y to 0 in the equation $y = 3x - 6$:

$0 = 3x - 6$

Solve for x :

$x = 2$

So, the point A is $(2, 0)$.

2. Finding B :

Set x to 0 in the equation $y = 3x - 6$:

$y = 3(0) - 6$

$y = -6$

So, the point B is $(0, -6)$.

3. Calculating the length of AB :

Use the distance formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(2-0)^2 + (0-(-6))^2}$

$AB = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$

Therefore, the length of AB is $2\sqrt{10}$.

(c) M is the mid-point of AB.

Find the coordinates of M.

The x-axis intercept (A) occurs when $y = 0$ in the equation $y = 3x - 6$:

$0 = 3x - 6$

Solving for x :

$x = 2$

So, the coordinates of A are $(2, 0)$.

The y-axis intercept (B) occurs when $x = 0$:

$y = 3 \cdot 0 - 6$

Solving for y :

$y = -6$

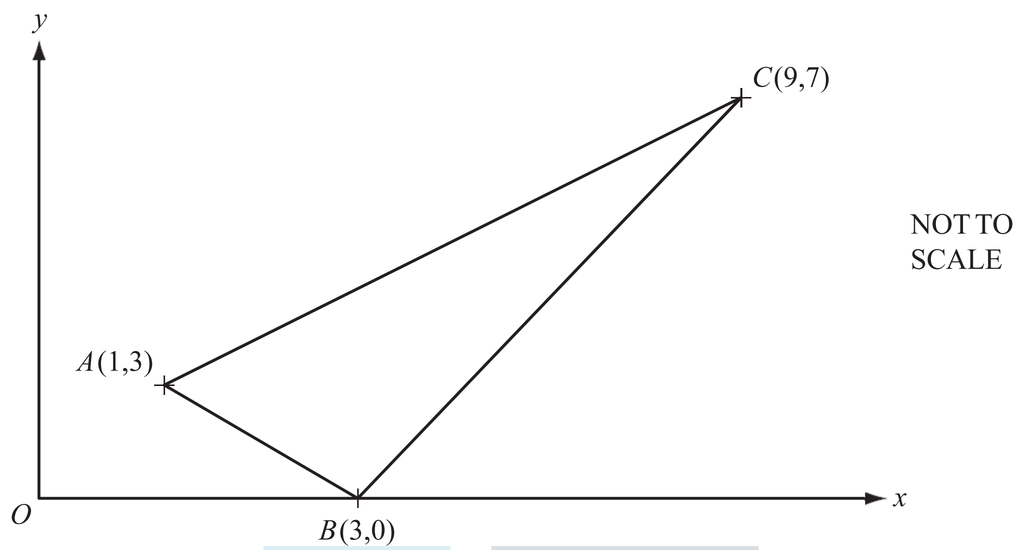
So, the coordinates of B are $(0, -6)$.

The midpoint (M) of AB is the average of the x-coordinates and the y-coordinates:

$M_x = \frac{2+0}{2} = 1$

$M_y = \frac{0+(-6)}{2} = -3$

So, the coordinates of M are $(1, -3)$.



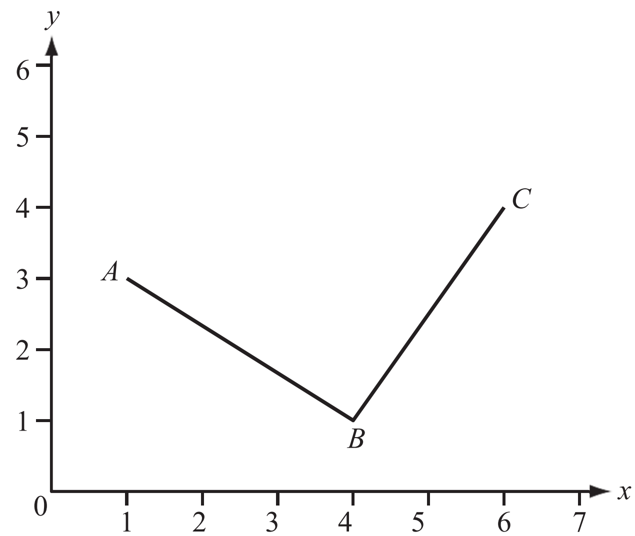
The co-ordinates of A, B and C are shown on the diagram, which is not to scale.

(a) Find the length of the line AB. [3]

The length of line AB is 3

(b) Find the equation of the line AC. [3]

$y = x + 2$



$A(1, 3)$, $B(4, 1)$ and $C(6, 4)$ are shown on the diagram.

(b) Work out the equation of the line BC. [3]

The equation of the line BC is $y = -\frac{1}{2}x + 3$.

(c) ABC forms a right-angled isosceles triangle of area 6.5 cm^2.

Calculate the length of AB. [2]

The length of AB is 5 units.

Find the length of the straight line from Q (–8, 1) to R (4 , 6). [3]

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For $Q(-8, 1)$ and $R(4, 6)$:

Length of QR $= \sqrt{(4 - (-8))^2 + (6 - 1)^2}$

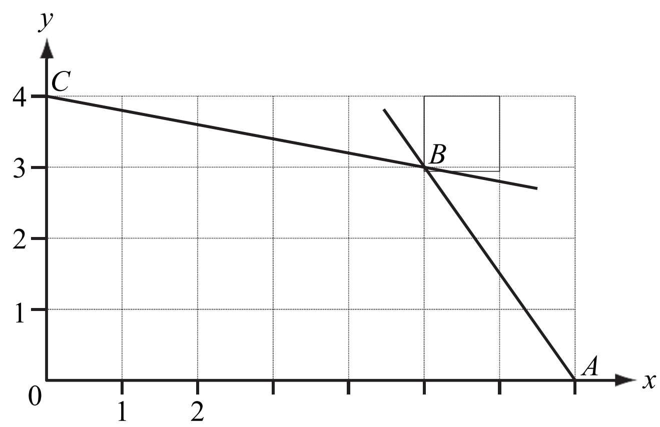
Length of QR $= \sqrt{12^2 + 5^2}$

Length of QR $= \sqrt{144 + 25}$

Length of QR $= \sqrt{169}$

Length of QR $= 13$

So, the length of the straight line from Q to R is 13 units.



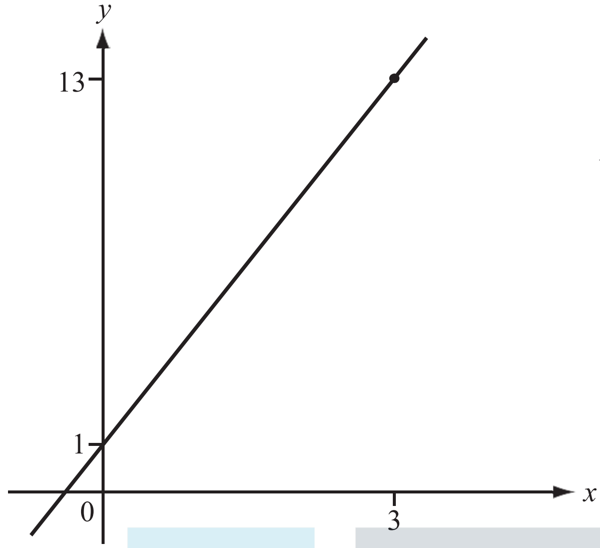
The lines *AB* and *CB* intersect at *B*.

(a) Find the co-ordinates of the midpoint of *AB*. [1]

The coordinates of the midpoint of AB are $\left(\frac{5}{2}, 2\right)$.

(b) Find the equation of the line *CB*. [3]

To find the equation of the line CB, you need the slope and the y-intercept.
Given points C(6, 4) and B, you can find the slope (m) using the formula:
$m = \frac{y_2 - y_1}{x_2 - x_1}$
Let's take B(x, y). Using B(4, 1) and C(6, 4) :
$m = \frac{1-4}{4-6}$
$m = \frac{-3}{-2}$
$m = \frac{3}{2}$
Now that you have the slope (m), you can use the point-slope form of the equation of a line:
$y - y_1 = m(x - x_1)$
Using point C(6, 4):
$y - 4 = \frac{3}{2}(x - 6)$
Now, simplify this equation to put it in the slope-intercept form $(y = mx + b)$:
$y - 4 = \frac{3}{2}x - 9$
$y = \frac{3}{2}x - 5$
So, the equation of the line CB is $y = \frac{3}{2}x - 5$.

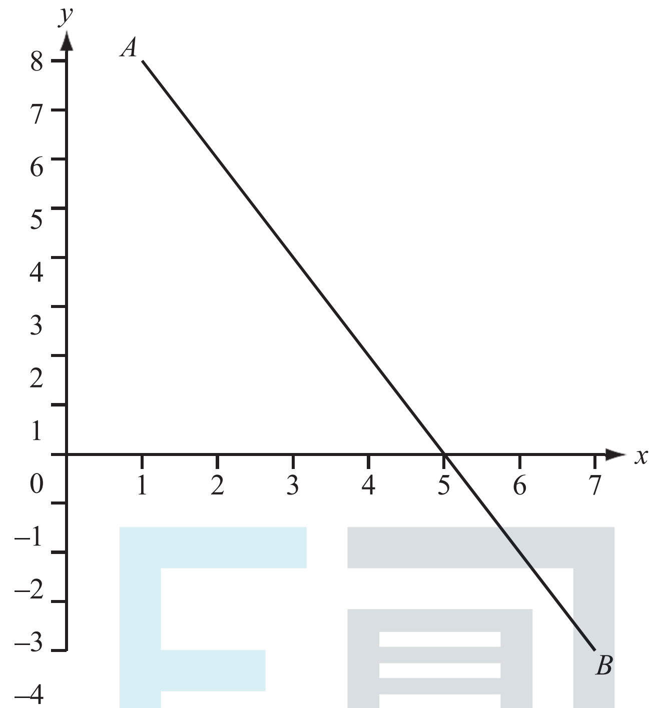


NOT TO SCALE

The diagram shows the straight line which passes through the points (0, 1) and (3, 13).

Find the equation of the straight line. [3]

$y - y_1 = m(x - x_1)$
where (x_1, y_1) is a point on the line, and m is the slope.
First, find the slope (m) using the given points $(0, 1)$ and $(3, 13)$:
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{13 - 1}{3 - 0}$
$m = \frac{12}{3}$
$m = 4$
Now that you have the slope, choose one of the points, let's say $(0, 1)$, and substitute the values into the point-slope form:
$y - 1 = 4(x - 0)$
Simplify:
$y - 1 = 4x$
Now, bring y to one side:
$y = 4x + 1$
So, the equation of the straight line is $y = 4x + 1$.



(a) **Using a straight edge and compasses only**, construct the perpendicular bisector of *AB* on the diagram above. [2]

To construct the perpendicular bisector of AB using only a straight edge and compasses, draw arcs above and below AB from points A and B with a radius greater than half the length of AB. Then, draw a straight line through the points where the arcs intersect.

[1]

(b) Write down the co-ordinates of the midpoint of the line segment joining $A(1, 8)$ to $B(7, –4)$.

The coordinates of the midpoint are $M\left(\frac{1+7}{2}, \frac{8-4}{2}\right) = (4, 2)$.

(c) Find the equation of the line *AB*.

The equation of the line AB is $y = 2x + 3$.

[3]

(a) The line $y = 2x + 7$ meets the y-axis at A.

Write down the co-ordinates of A.

To find the coordinates where the line $y = 2x + 7$ meets the y-axis, you set $x = 0$. The y-axis has the equation $x = 0$.
Substitute $x = 0$ into $y = 2x + 7$:
$y = 2(0) + 7$
$y = 7$
So, the coordinates where the line meets the y-axis (point A) are $(0, 7)$.

[1]

(b) A line parallel to $y = 2x + 7$ passes through $B(0, 3)$.

(i) Find the equation of this line.

The equation of the line parallel to $y = 2x + 7$ passing through $B(0, 3)$ is $y = 2x + 3$.

[2]

(ii) C is the point on the line $y = 2x + 1$ where $x = 2$.

Find the co-ordinates of the midpoint of BC.

The line parallel to $y = 2x + 7$ passing through $B(0, 3)$ has the equation $y = 2x + 3$.
For $y = 2x + 1$, when $x = 2$, $y = 2(2) + 1 = 5$.
The midpoint of BC is $(1, 4)$.

[3]

Find the equation of the straight line which passes through the points (0, 8) and (3, 2).

To find the equation of the straight line passing through the points $(0,8)$ and $(3,2)$, you can use the point-slope form of the equation:
$y-y_1=m\left(x-x_1\right)$
where $\left(x_1,y_1\right)$ is a point on the line, and m is the slope.
First, find the slope (m) using the given points $(0,8)$ and $(3,2)$:
$m=\frac{y_2-y_1}{x_2-x_1}$
$m=\frac{2-8}{3-0}$
$m=\frac{-6}{3}$
$m=-2$
Now that you have the slope, choose one of the points, let's say $(0,8)$, and substitute the values into the point-slope form:
$y-8=-2(x-0)$
Simplify:
$y-8=-2x$
Now, bring y to one side:
$y=-2x+8$
So, the equation of the straight line is $y=-2x+8$. [3]