IB Maths: AI HL

Complex Numbers

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions

Course	IB Maths
Section	1. Number & Algebra
Topic	1.5 Complex Numbers
Difficulty	Medium

Level: IB Maths

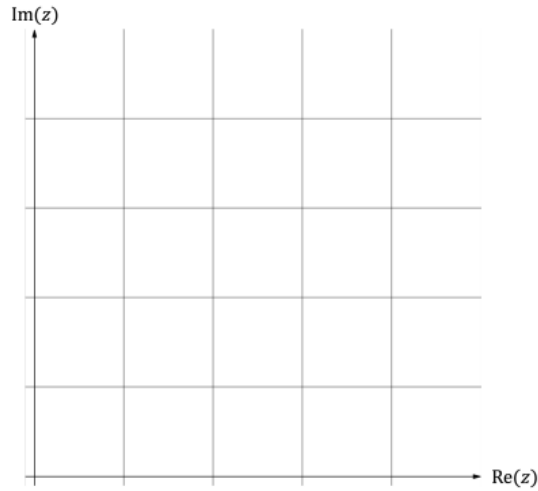**Subject: IB Maths AI HL**

Board: IB Maths

Topic: Complex Numbers

Question 1

Consider the complex numbers $z_1 = 2 + 2i$ and $z_2 = 2 + 2\sqrt{3}i$.

a)
Sketch z_1 and z_2 on the Argand diagram below, be sure to include an appropriate scale.

Im(z)

Re(z)

[2 marks]

b)
Find the modulus of z_1 and z_2.

[3 marks]

c)
Find the argument of z_1 and z_2.

[3 marks]

Question 2

Solve the following equations for x

(i)
$x^2 + 4x + 5 = 0$

(ii)
$x^2 = -625$

(iii)
$x^4 = 24 - 2x^2.$

[7 marks]

Question 3

Let $w_1 = z_1 z_2$, where $z_1 = 5 + i$ and $z_2 = 1 + 2i$.

a)
Express w in the form $w = a + bi$.

[2 marks]

b)
Find the modulus and argument for w

[4 marks]

Question 4

Let $z = \dfrac{w_1}{w_2}$, where $w_1 = 4 - i$ and $w_2 = 1 - 2i$.

a)
Express z in the form $z = a + bi$.

[3 marks]

b)
Find the modulus and argument for z.

[4 marks]

Question 5

Consider the complex numbers $z = 3 - 4i$ and $w = 7 - 2i$.

a)
Find

(i)
$z + w$

(ii)
$w - z$.

[2 marks]

Let z^* and w^* represent the complex conjugates of z and w, respectively.

b)
Write down z^* and w^*, giving your answers in the form $a + bi$.

[2 marks]

c)
Find

(i)
$z^* w$

(ii)
$\dfrac{w^*}{z}$.

[4 marks]

Question 6

Find all possible real values for a and b such that

(i)
$a + b\text{i} = 8\text{i}$

(ii)
$(2 + 3\text{i})(a + b\text{i}) = 13$

(iii)
$(a + \text{i})(2 + b\text{i}) = -6 + 22\text{i}.$

[7 marks]

Question 7

Consider the complex numbers $w = \text{i}z$ and $w + 2z = 7 + 6\text{i}$.

Find

(i)
$\text{Re}(w)$

(ii)
$\text{Im}(w)$

(iii)
$\text{Re}(z)$

(iv)
$\text{Im}(z).$

[7 marks]

Question 8

It is given that $z_1 = 3 + 4i$ and $z_2 = -2 + 2i$.

Find

(i)
$iz_1 + z_2$

(ii)
$\dfrac{z_1}{iz_2}$

(iii)
$i(z_1 z_2)$.

[7 marks]

Question 9

Find the complex numbers z and w such that
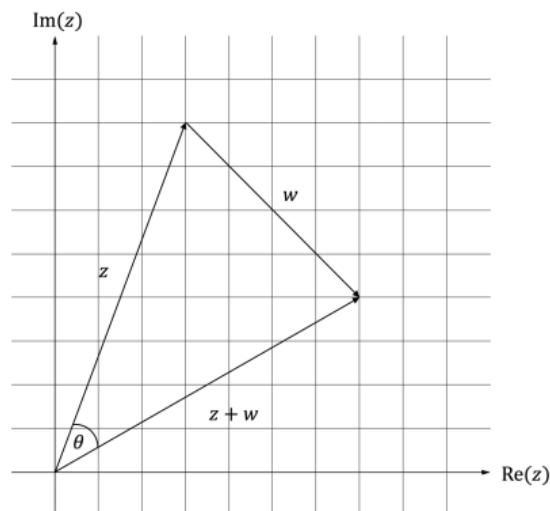
$$2z - iw^* = 5 + 7i$$
$$w + iz^* = 5 + 16i$$

[8 marks]

Question 10

Let $z = 3 + 8i$ and $w = 4 - 4i$.

a)
Find θ, the angle shown on the diagram below.

[5 marks]

b)
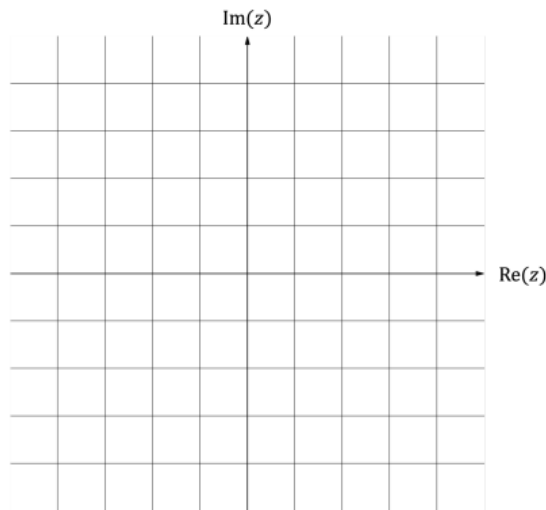Find the area of the triangle formed in the diagram above.

[3 marks]

Question 11

Let $z = -1 - 3i$ and $w = 1 + i$.

a)
Find zw.

[2 marks]

b)
Sketch z, w and zw on the Argand diagram below.

[3 marks]

Let θ be the angle between z and zw and ϕ be the angle between w and zw.

c)
Find the angles θ and ϕ, giving your answers in degrees.

[4 marks]

Question 12

Let $w = \dfrac{z+1}{z^* +1}$, where $z = a + bi$, $a, b \in \mathbb{R}$.

a)
Write w in the form $x + yi$, $x, y \in \mathbb{R}$.

[4 marks]

b)
Determine the conditions under which w is purely imaginary.

[3 marks]

Question 13

Consider the equation $x^2 + bx + c = 0$.

(a) Write down an inequality, in terms of b and c, that shows the equation has no real solutions.

[1 mark]

$5 - 3i$ is one solution to the equation $x^2 + bx + c = 0$.

(b) Find the values of b and c.

[4 marks]

Let $z = c + bi$.

(c) Find z^5 using technology.

[1 mark]