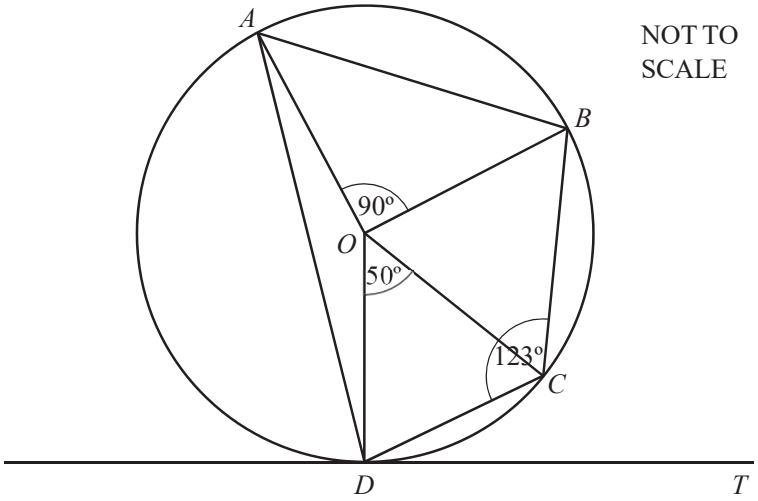
EXAM PAPERS PRACTICE

Circle Theorems

Model Answer



NOT TO SCALE

The points A, B, C and D lie on a circle centre O.
Angle $AOB = 90°$, angle $COD = 50°$ and angle $BCD = 123°$.
The line DT is a tangent to the circle at D.

Find

(a) angle OCD, [1]

$$\angle OCD = 65°$$

(b) angle TDC, [1]

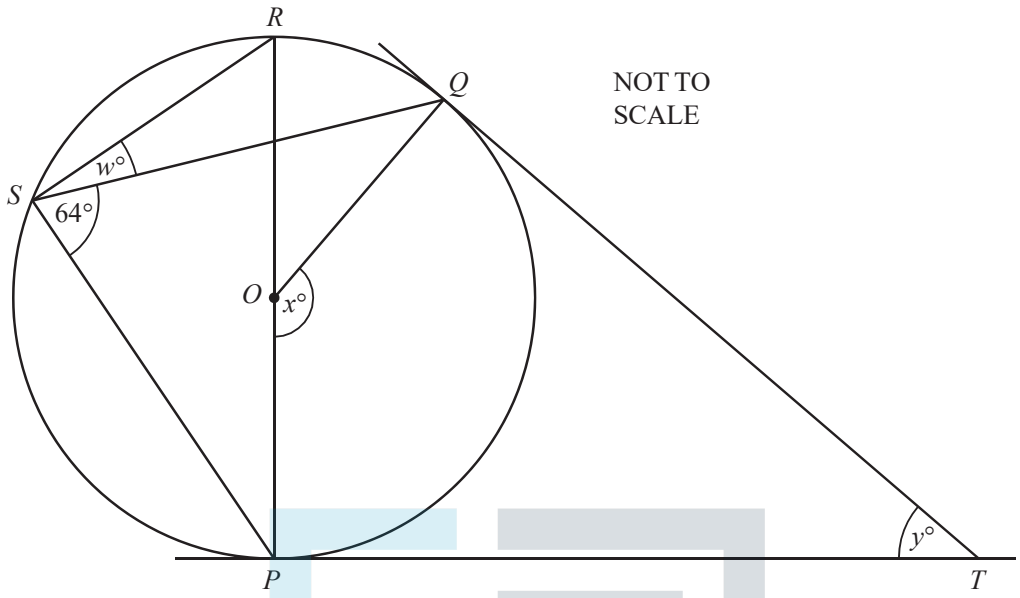
$$\angle TDC = 25°$$

(c) angle ABC, [1]

$$\angle ABC = 103°$$

(d) reflex angle AOC. [1]

$$\angle AOC = 154°$$



P, Q, R and S lie on a circle, centre O.
TP and TQ are tangents to the circle.
PR is a diameter and angle $PSQ = 64°$.

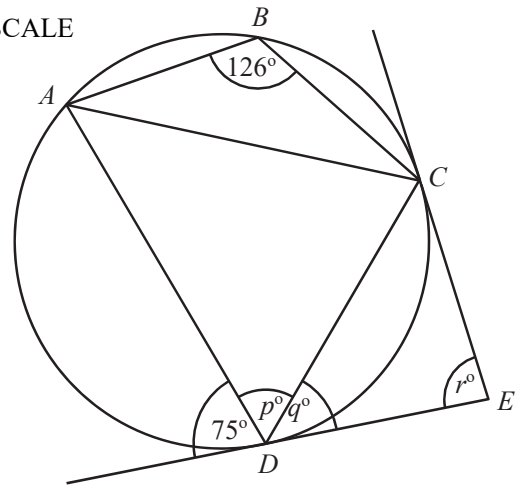
(a) Work out the values of w and x. [2]

$\because$ PR is diameter
$\therefore$ the angle $PSR = 90°$
$\because$ angle $PSQ = 64°$
$\therefore$ angle $RSQ =$ angle PSR-angle BSQ
$\therefore w = 26$
angle QSP and angle Qop corresponds to the same arc PQ
angle POQ is a central angle
ungle PSQ is a inscribed angle
angle $POQ =$ double PSQ
$x = 128$

(b) Showing all your working, find the value of y. [2]

Now, we can use the fact that angles QSP and QPO add up to 180 degrees to solve for y. We have:
QSP + QPO $= 180$ degrees
128 degrees $+ y = 180$ degrees
$y = 180$ degrees -128 degrees
$y = 52$ degrees
Therefore, the value of y is 52 .

NOT TO SCALE



ABCD is a cyclic quadrilateral.
The tangents at *C* and *D* meet at *E*.
Calculate the values of *p*, *q* and *r*.

[4]

Soluoton: From the frgure, we know $\angle ABC$ and $\angle ADC$ are opposite angles

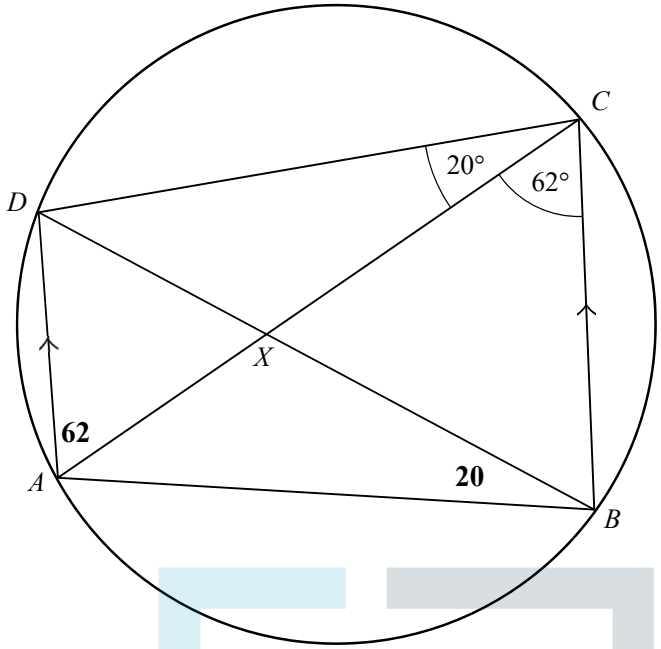
So $126^\circ + p^\circ = 180^\circ \Rightarrow p^\circ = 54^\circ \Rightarrow p = 54$

And $75^\circ + p^\circ + q^\circ = 180^\circ \Rightarrow q^\circ = 180^\circ - 75^\circ - p^\circ = 51^\circ \Rightarrow q = 51.$

And CE and DE are tangent. So $CE = DE$. Tangents length theorem.

So $\angle CDE = \angle DCE = q^\circ - 51^\circ$.

So $r^\circ = 180^\circ - 2q^\circ = 180^\circ - 2 \times 51^\circ = 78^\circ \Rightarrow r = 78.$



NOT TO SCALE

$ABCD$ is a cyclic quadrilateral.
AD is parallel to BC. The diagonals DB and AC meet at X.
Angle ACB = 62° and angle ACD = 20°.
Calculate

(a) angle DBA, [1]

angle DBA = angle DCA = 20°

(b) angle DAB, [1]

$DAB = 180° -$ angle $ADB -$ angle DBA
$= 180° - 62° - 20°$
$= 98°$

(c) angle DAC, [1]

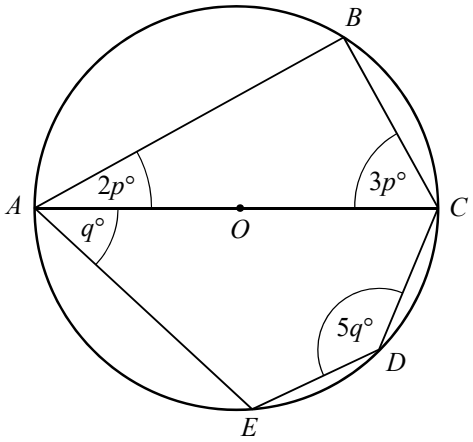
AD is parallel to BC
So angle DAC = angle ACB = 62°

(d) angle AXB, [1]

AXB is 62 degrees.

(e) angle CDB. [1]

CDB is 62 degrees.



NOT TO SCALE

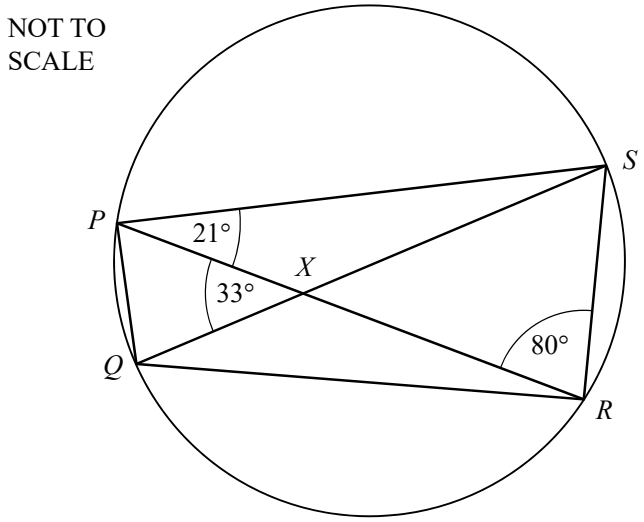
A, B, C, D and E lie on a circle, centre O. AOC is a diameter.
Find the value of

(a) p, [2]

$$2p^\circ + 3p^\circ = 90^\circ$$
$$p = 18$$

(b) q. [2]

$$q^\circ + 5q^\circ = 180^\circ$$
$$q = 30$$



PQRS is a cyclic quadrilateral. The diagonals *PR* and *QS* intersect at *X*.
Angle *SPR* = 21°, angle *PRS* = 80° and angle *PXQ* = 33°.
Calculate

(a) angle *PQS*,

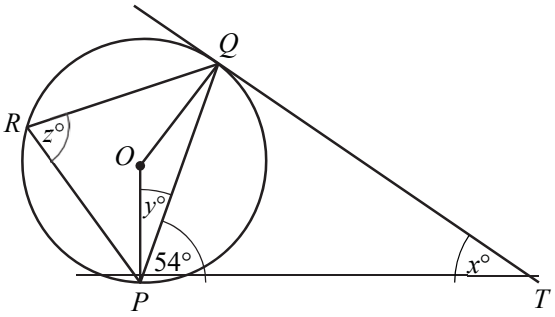
$$\angle PQS = \angle PRS = 80^\circ$$ [1]

(b) angle *QPR*,

$$\angle PQX + \angle PXQ + \angle QPX = 180^\circ$$
$$\angle QPR = 180^\circ - \angle PQX - \angle PXQ$$
$$= 180^\circ - 80^\circ - 33^\circ$$
$$= 67^\circ$$ [1]

(c) angle *PSQ*.

$$\angle PXQ = \angle PRQ + \angle RQS$$
$$\angle PRQ = \angle PXQ - \angle RQS$$
$$= \angle PXQ - \angle RPS$$
$$= 33^\circ - 21^\circ$$
$$= 12^\circ$$
$$\angle PSQ = \angle PRQ = 12^\circ$$ [1]



NOT TO SCALE

The points P, Q and R lie on a circle, centre O.
TP and TQ are tangents to the circle.
Angle $TPQ = 54°$.

Calculate the value of

(a) x, [1]

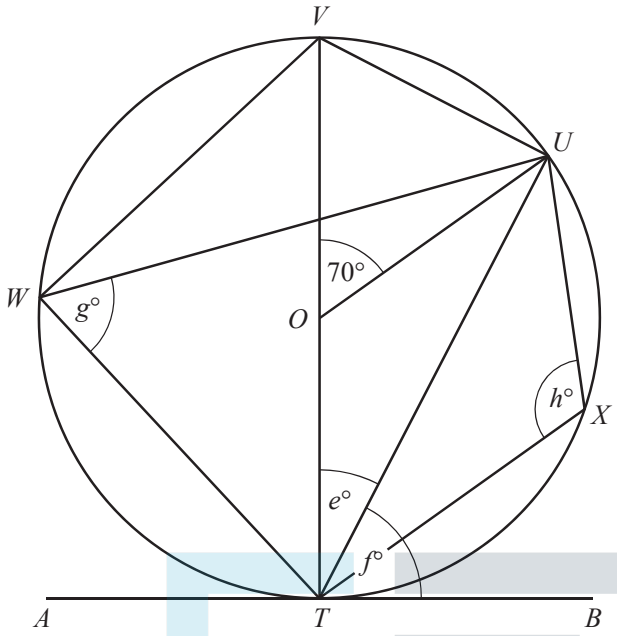
$x = 72°$

(b) y, [1]

$y = 36°$

(c) z. [2]

$z = 54°$



NOT TO SCALE

The diagram shows a circle, centre O.
VT is a diameter and ATB is a tangent to the circle at T.
U, V, W and X lie on the circle and angle $VOU = 70°$.

Calculate the value of

(a) e, [1]

$$e = \frac{70^\circ}{2} = 35^\circ$$

(b) f, [1]

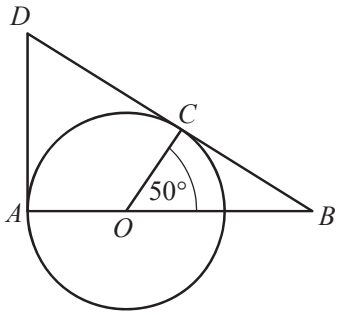
$$f = 90^\circ - 35^\circ = 55^\circ$$

(c) g, [1]

$$g = \frac{180^\circ - 70^\circ}{2} = 55^\circ.$$

(d) h. [1]

$$h = 125^\circ$$



NOT TO SCALE

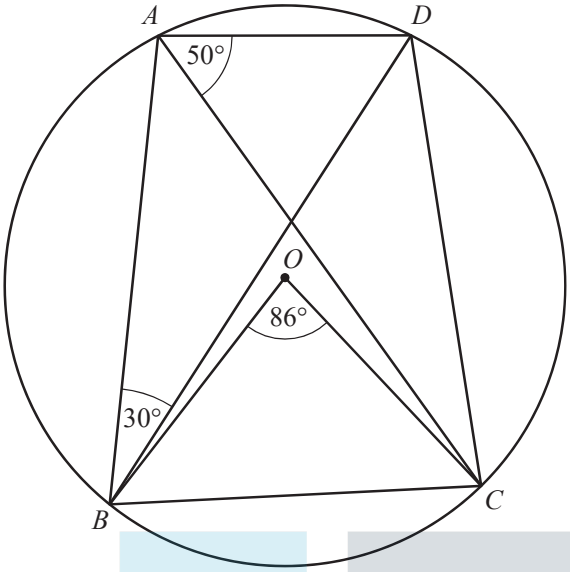
O is the centre of the circle.
DA is the tangent to the circle at A and DB is the tangent to the circle at C.
AOB is a straight line. Angle $COB = 50°$.
Calculate

(a) angle CBO,

$40°$ [1]

(b) angle DOC.

$65°$ [1]



NOT TO SCALE

The points A, B, C and D lie on the circumference of the circle, centre O.

Angle $ABD = 30°$, angle $CAD = 50°$ and angle $BOC = 86°$.

(a) Give the reason why angle $DBC = 50°$.

The reason why angle $DBC = 50°$ is because of the Angle Chaser Theorem [1]

(b) Find

(i) angle ADC,

Angle ADC = 94°

[1]

(ii) angle BDC,

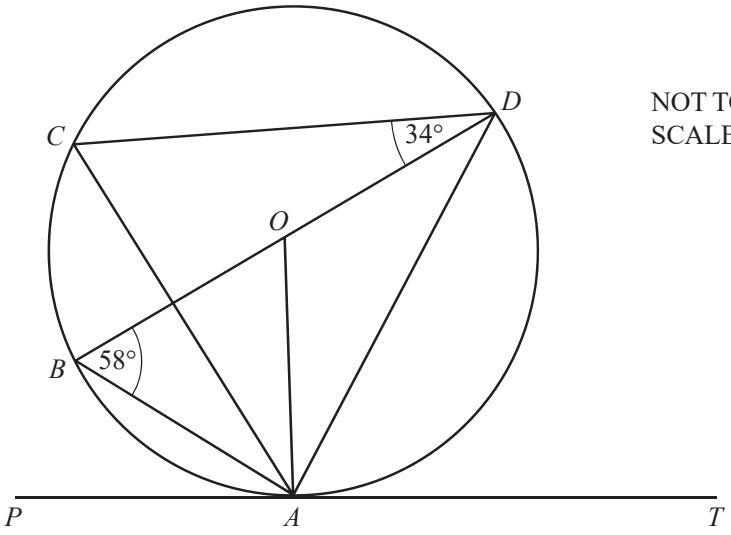
Angle BDC = 6°

[1]

(iii) angle OBD.

Angle OBD = 168°

[2]



NOT TO SCALE

A, B, C and *D* lie on the circle, centre *O*.
BD is a diameter and *PAT* is the tangent at *A*.
Angle *ABD* = 58° and angle *CDB* = 34°.

Find

(a) angle *ACD*,

$\angle \mathbf{ACD} = \mathbf{58^\circ}$

[1]

(b) angle *ADB*,

$\angle \mathbf{ADB} = \mathbf{32^\circ}$

[1]

(c) angle *DAT*,

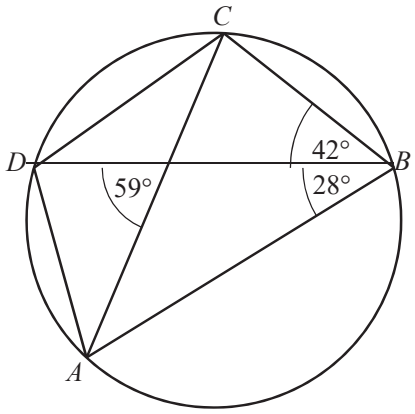
$\angle \mathbf{DAT} = \mathbf{58^\circ}$

[1]

(d) angle *CAO*.

angle CAO must also be 58°.

[2]



NOT TO SCALE

A, B, C and D lie on the circle.

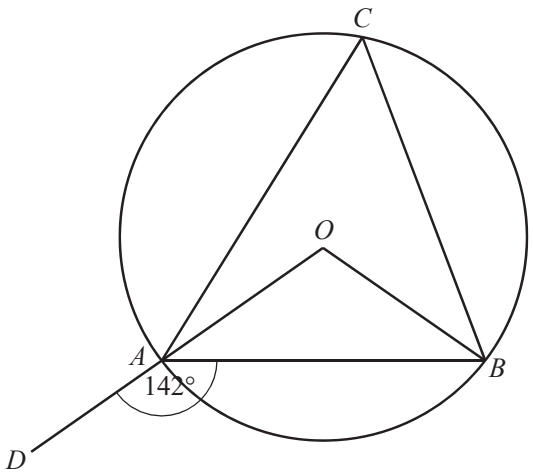
Find

(a) angle ADC, [1]

$\angle ADC = 110°$

(b) angle ADB. [2]

$\angle ADB = 180° - \angle ADC - \angle ACD = 180° - 45° - 59° = 76°.$



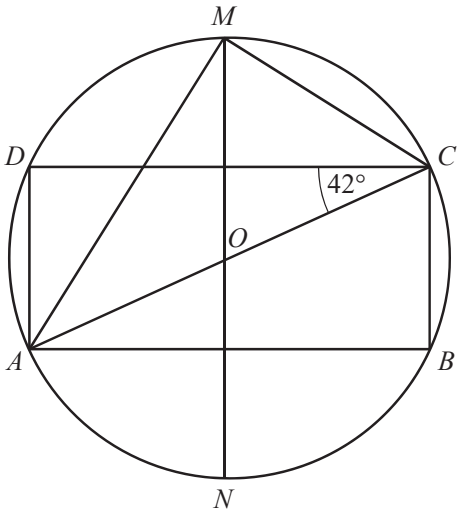
NOT TO SCALE

A, B and C are points on the circumference of a circle centre O.
OAD is a straight line and angle $DAB = 142°$.

Calculate the size of angle ACB. [3]

The size of angle ACB is 42 degrees.

Question 14



NOT TO SCALE

The vertices of the rectangle $ABCD$ lie on a circle centre O.
MN is a line of symmetry of the rectangle.
AC is a diameter of the circle and angle $ACD = 42°$.

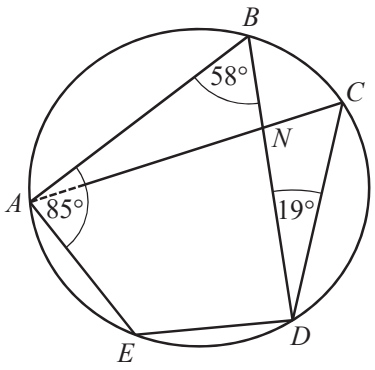
Calculate

(a) angle CAM, [2]

The angle of CAM is 42 degrees.

(b) angle DCM. [2]

The angle of DCM is 42 degrees.



NOT TO SCALE

A, B, C, D and E are points on a circle.
Angle $ABD = 58°$, angle $BAE = 85°$ and angle $BDC = 19°$.
BD and CA intersect at N.

Calculate

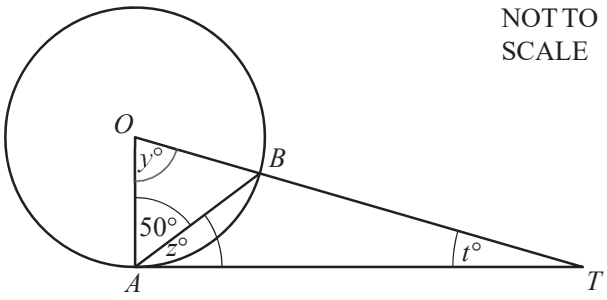
(a) angle BDE, [1]

The angle BDE is **58** degrees.

(b) angle AND. [2]

the angle AND is 77 degrees.

Question 16



TA is a tangent at *A* to the circle, centre *O*.
Angle *OAB* = 50°.

Find the value of

(a) *y*, [1]

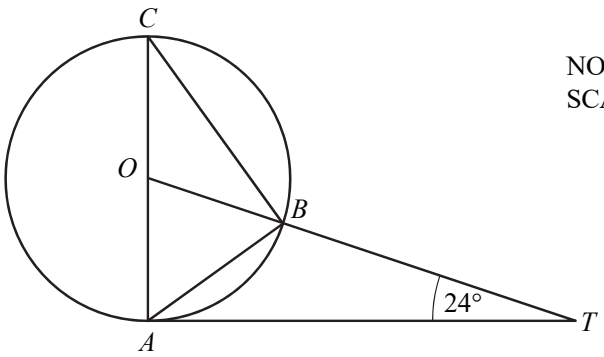
$y = 80$

(b) *z*, [1]

$z = 40$

(c) *t*. [1]

$t = 10$



NOT TO SCALE

A, *B* and *C* are points on a circle, centre *O*.
TA is a tangent to the circle at *A* and *OBT* is a straight line.
AC is a diameter and angle *OTA* = 24°.

Calculate

(a) angle *AOT*, [2]

$\mathrm{AOT} = 66°$

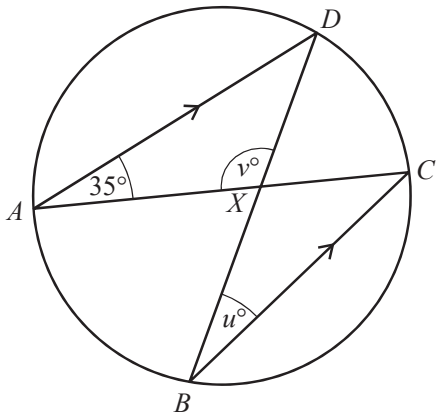
(b) angle *ACB*, [1]

$ACB = 33°$

(c) angle *ABT*. [2]

$ABT = 123°$

(a)



NOT TO SCALE

A, B, C and D are points on the circle.
AD is parallel to BC.
The chords AC and BD intersect at X.

[3]

Find the value of u and the value of v.

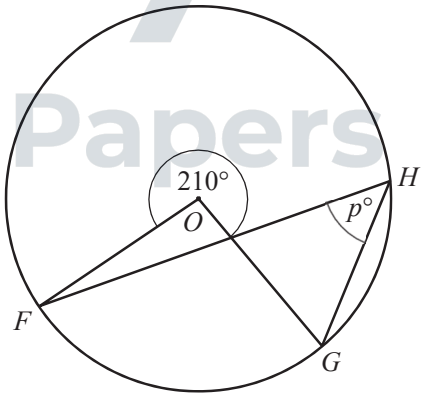
- From parallel lines (interior alternate angles):

$A\hat{D}B = u° = 35°$

- Angles in a $\triangle$ add up to 180°

$V = 180° - (2 \times 35°) = 110°$

(b)



NOT TO SCALE

F, G and H are points on the circle, centre O.

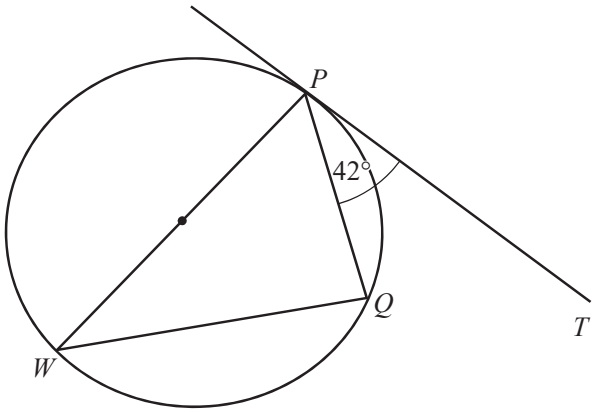
[2]

Find the value of p.

$*p = \frac{FOG}{2}$ $\Rightarrow p = \frac{150°}{2}$

$\cdot F\hat{O}G = 360° - 210°$ $\Rightarrow p = 75°$

$\Rightarrow F\hat{O}G = 150°$



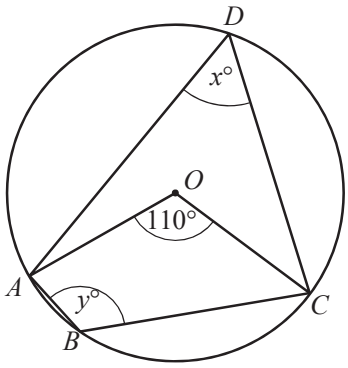
NOT TO SCALE

In the diagram, PT is a tangent to the circle at P.
PW is a diameter and angle $TPQ = 42°$.

[2]

Find angle PWQ.

$$180 - 48° - 90° = 42°$$



NOT TO SCALE

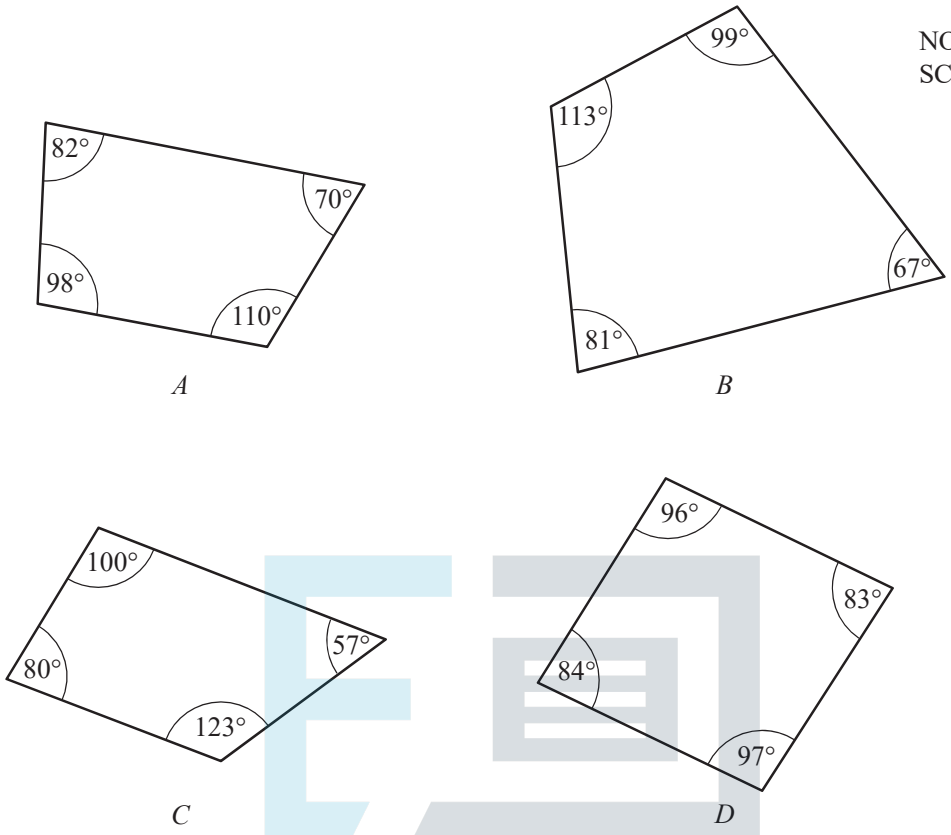
A, B, C and D lie on the circle, centre O.

Find the value of x and the value of y. [2]

$x = 55$

$y = 125$

Question 21



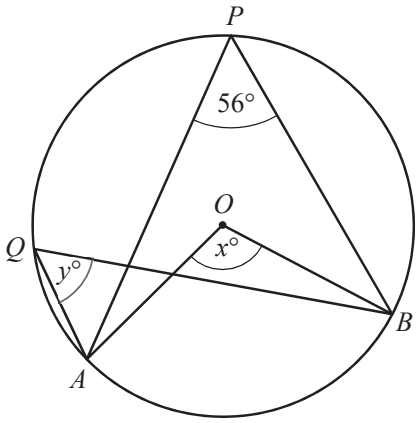
The diagram shows four quadrilaterals A, B, C and D.

Which one of these could be a cyclic quadrilateral? [1]

In cyclic quadrilateral, opposite angles are supplementary.

Thus property holds only in option (B)

Hence, correct option is (B)



NOT TO SCALE

A, *B*, *P* and *Q* lie on the circle, centre *O*.
Angle *APB* = 56°.

Find the value of

(a) *x*,

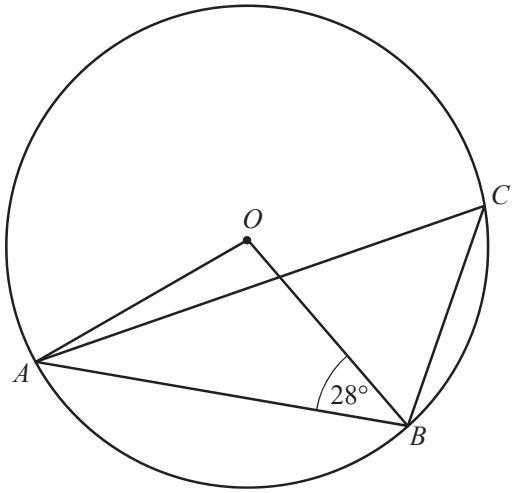
$x = 2 \times 56 = 112.$

[1]

(b) *y*.

$y = 56.$

[1]



NOT TO SCALE

In the diagram, A, B and C lie on the circumference of a circle, centre O.

Work out the size of angle ACB.

Give a reason for each step of your working. [4]

$OA = OB$ radius of the circle

Therefore

$\triangle OAB$ is isosceles triangle

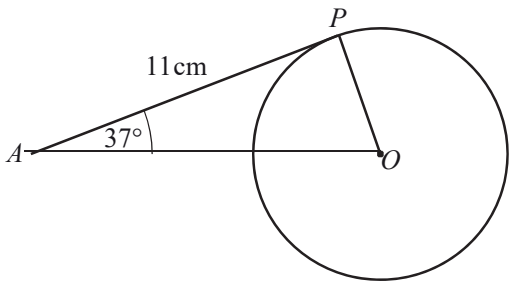
$\angle OAB = \angle OBA = 28°$ (isosceles triangles base angles are equal)

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\angle AOB = 180° - 56° = 124°$

$\angle ACB = 124°/2 = 62°$

$\angle ACB = 62°$



NOT TO SCALE

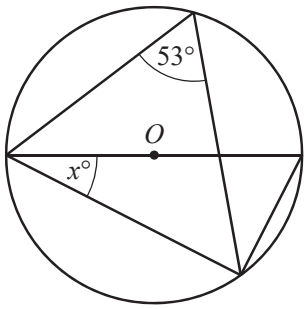
In the diagram, AP is a tangent to the circle at P.
O is the centre of the circle, angle $PAO = 37°$ and $AP = 11$ cm.

(a) Write down the size of angle OPA.

angle $OPA = 90°$ [1]

(b) Work out the radius of the circle.

25 cm [2]



NOT TO SCALE

The diagram shows a circle, centre O.

Find the value of x.

$x = 53^\circ$

[2]