Question 1

(a) State the conditions that must be satisfied to be able to model a random variable X with a binomial distribution $B(n, p)$.

[4]

(b) A fair spinner has 8 sectors labelled with the numbers 1 through 8. For each of the following cases, state with a reason whether or not a binomial distribution would be appropriate for modelling the specified random variable.

(i) The random variable S is the number of the sector that the spinner lands on when it is spun.

(ii) The random variable W is the number of times the spinner is spun until it lands on '7' for the first time.

(iii) The random variable Y is the number of times the spinner lands on a prime number when it is spun twelve times.

(iv) On the first spin, it is a 'win' if the spinner lands on an even number. On subsequent spins it is a 'win' if the spinner lands *either* on the same number as the previous spin *or* on a factor of the number from the previous spin. The random variable L is the number of wins when the spinner is spun ten times.

[4]

(a)
- A trial has two outcomes (success or failure)
- An experiment has a finite number of trials (n)
- The probability of success is constant (p)
- The trials are independent of each other.

If these four conditions are met then the number of successes, X, follows a binomial distribution $B(n,p)$.

(a) State the conditions that must be satisfied to be able to model a random variable X with a binomial distribution $B(n, p)$.

[4]

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[4]

(b)(i) S is not binomial as there are more than two outcomes.

(ii) W is not binomial as there is not a fixed number of trials.

(iii) Y is binomial with $B(12, \frac{1}{2})$ as it has 12 independent spins and landing on a prime is a success which has a constant probability of $\frac{1}{2}$.

(iv) L is not binomial as getting a win depends on the previous spin.

EXAM PAPERS PRACTICE

Question 2

A fair coin is tossed 20 times and the number of times it lands heads up is recorded.

$n=20$

(a) Find the expected number of times that the coin will land heads up. $p=\frac{1}{2}$

$E(X)$

[1]

(b) Find the probability that the coin lands heads up 15 times.

[2]

(a) Let X be the number of times the coin lands heads up when tossed 20 times.

$X \sim B(20, \frac{1}{2})$

$E(X) = np$ (from formula booklet)

$E(X) = (20)(\frac{1}{2})$

$\boxed{E(X) = 10}$

A fair coin is tossed 20 times and the number of times it lands heads up is recorded.

(a) Find the expected number of times that the coin will land heads up.

[1]

(b) Find the probability that the coin lands heads up 15 times.

[2]

(b) $X \sim B(20, \frac{1}{2})$

Using the calculator (Probability Distribution)

$P(X=15) = 0.0147857...$

$\boxed{P(X=15) = 0.0148 \text{ (4dp)}}$

Question 3

On any given day during a normal five-day working week, there is a 60% chance that Yussuf catches a taxi to work.

$n=5$ $p=0.6$

(a) Find E(X), the expected number of times Yussuf will catch a taxi to work during a normal five-day working week.

[1]

(b) Find the probability that, during a normal five-day working week, Yussuf never catches a taxi.

[2]

(c) Find the probability that, during a normal five-day working week, Yussuf catches a taxi once at the most.

[2]

(a) Let X be the number of days in a week that Yussuf catches a taxi.

$X \sim B(5, 0.6)$

$E(X) = np$ (from formula booklet)

$E(X) = (5)(0.6)$

$\boxed{E(X) = 3}$

EXAM PAPERS PRACTICE

On any given day during a normal five-day working week, there is a 60% chance that Yussuf catches a taxi to work.

(a) Find $E(X)$, the expected number of times Yussuf will catch a taxi to work during a normal five-day working week.

[1]

(b) Find the probability that, during a normal five-day working week, Yussuf never catches a taxi.

$X=0$

[2]

(c) Find the probability that, during a normal five-day working week, Yussuf catches a taxi once at the most.

[2]

(b) $X \sim B(5, 0.6)$

Using the calculator (Probability Distribution)

$\boxed{P(X=0) = 0.01024}$

On any given day during a normal five-day working week, there is a 60% chance that Yussuf catches a taxi to work.

(a) Find $E(X)$, the expected number of times Yussuf will catch a taxi to work during a normal five-day working week.

[1]

(b) Find the probability that, during a normal five-day working week, Yussuf never catches a taxi.

[2]

(c) Find the probability that, during a normal five-day working week, Yussuf catches a taxi once at the most.

[2]

(c) $X \sim B(5, 0.6)$

At most once means $X \le 1$

Using the calculator (Cumulative Distribution)

$P(X \le 1) = 0.08704$

$\boxed{P(X \le 1) = 0.08704}$

Question 4

A difficult operation to remove a rare form of cancer is found to have a success rate of 78%. Twelve patients are on the waiting list to undergo the operation.

$p=0.78$

$n=12$

(a) Find the probability that all twelve patients' operations are successful.

[2]

(b) Find the probability that all but two patients undergo successful operations.

[2]

(a) Let X be the number of successful operations.

$X \sim B(12, 0.78)$

All successful $\Rightarrow X=12$

Using the calculator (Probability Distribution)

$P(X=12) = 0.0507148...$

$\boxed{P(X=12) = 0.0507 \quad (4dp)}$

A difficult operation to remove a rare form of cancer is found to have a success rate of 78%. Twelve patients are on the waiting list to undergo the operation.

(a) Find the probability that all twelve patients' operations are successful.

[2]

(b) Find the probability that all but two patients undergo successful operations.

[2]

(b) $X \sim B(12, 0.78)$

All but two $\Rightarrow X = 10$

Using the calculator (Probability Distribution)

$P(X=10) = 0.266278...$

$\boxed{P(X=10) = 0.2663 \quad (4dp)}$

Question 5

For a jellyfish population in a certain area of the ocean, there is a 95% chance that any given jellyfish contains microplastic particles in its body. P=0.95

n=40

For a sample size of 40 jellyfish from this population, find the probability of:

(i) exactly 38 jellyfish

(ii) all the jellyfish

having microplastic particles in their bodies.

[4]

Let X be the number of jellyfish in the sample that contain microplastic particles.

$X \sim B(40, 0.95)$

(i) Using the calculator (Probability Distribution)

$P(X=38) = 0.277671...$

$\boxed{P(X=38) = 0.2777 \quad (4dp)}$

(ii) Using the calculator (Probability Distribution)

$P(X=40) = 0.128512 ...$

$\boxed{P(X=40) = 0.1285 \quad (4dp)}$

Question 6

Giovanni is rolling a biased dice, for which the probability of landing on a two is 0.25. He rolls the dice 10 times and records the number of times that it lands on a two. Find the probability that

$p = 0.25$

$n = 10$

(i) the dice lands on a two 4 times.

(ii) the dice lands on a two 4 times by landing on a two 3 times in the first 9 rolls, and then landing on a two on the tenth roll.

[5]

(i) Let X be the number of times the dice lands on a 2 when rolled 10 times.

$X \sim B(10, 0.25)$

Using the calculator (Probability Distribution)

$P(X=4) = 0.145998...$

$\boxed{P(X=4) = 0.1460 \text{ (4dp)}}$

(ii) Let Y be the number of times the dice lands on a 2 when rolled 9 times.

$Y \sim B(9, 0.25)$

[3 two's in the first 9 rolls] and [A two on the 10^{th} roll]

$P(Y=3) \times 0.25$

$0.233596... \times 0.25 = 0.058399...$

$\boxed{0.0584 \text{ (4dp)}}$

Question 7

For cans of a particular brand of soft drink labelled as containing 330 ml, the actual volume of soft drink in a can varies. Although the company's quality control assures that the mean volume of soft drink in the cans remains at 330 ml, it is known from experience that the probability of any particular can of the soft drink containing less than 320 ml is 0.0296.

$n = 24$ $\quad p = 0.0296$

Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans in the pack that contain less than 320 ml of soft drink.

Find the probability that

(i) none of the cans

(ii) exactly two of the cans

(iii) at least two of the cans

contain less than 320 ml of soft drink.

[4]

$L \sim B(24, 0.0296)$

(i) Using the calculator (Probability Distribution)

$P(L=0) = 0.486204...$

$\boxed{P(L=0) = 0.4862 \text{ (4dp)}}$

(ii) Using the calculator (Probability Distribution)

$P(L=2) = 0.124856...$

$\boxed{P(L=2) = 0.1249 \text{ (4dp)}}$

(iii) At least 2 $\Rightarrow L \geq 2$

Using the calculator (Cumulative Distribution)

$P(L \geq 2) = 1 - P(L \leq 1)$

$= 1 - 0.842139...$

$= 0.157861...$

$\boxed{P(L \geq 2) = 0.1579 \text{ (4dp)}}$

EXAM PAPERS PRACTICE

Question 8

The random variable $X \sim \mathrm{B}(40, 0.15)$.

$n=40 \quad p=0.15$

(a) Find:

(i) $\mathrm{P}(3 \leq X < 14)$

(ii) $\mathrm{P}(5 < X < 12)$

[3]

(b) Find $\mathrm{Var}(X)$.

[1]

(c) Find $\mathrm{P}(X \leq 3 \mid X \leq 9)$.

[3]

(a)(i) $3 \leq X < 14$ is $X \leq 13$ without $X \leq 2$

Using the calculator (Cumulative Distribution)

$P(3 \leq X < 14) = P(X \leq 13) - P(X \leq 2)$

$= 0.998596... - 0.048598...$

$= 0.94999...$

$\boxed{P(3 \leq X < 14) = 0.9500 \quad (4dp)}$

(ii) $5 < X < 12$ is $X \leq 11$ without $X \leq 5$

Using the calculator (Cumulative Distribution)

$P(5 < X < 12) = P(X \leq 11) - P(X \leq 5)$

$= 0.988030... - 0.432500...$

$= 0.55553...$

$\boxed{P(5 < X < 12) = 0.5555 \quad (4dp)}$

The random variable $X \sim \mathrm{B}(40, 0.15)$.

(a) Find:

(i) $\mathrm{P}(3 \leq X < 14)$

(ii) $\mathrm{P}(5 < X < 12)$

[3]

(b) Find $\mathrm{Var}(X)$.

[1]

(c) Find $\mathrm{P}(X \leq 3 \mid X \leq 9)$.

[3]

(b) $Var(X) = np(1-p)$ (from formula booklet)

$Var(X) = (40)(0.15)(1-0.15)$

$\boxed{Var(X) = 5.1}$

The random variable $X \sim B(40, 0.15)$.

(a) Find:

(i) $P(3 \leq X < 14)$

(ii) $P(5 < X < 12)$

[3]

(b) Find $\mathrm{Var}(X)$.

[1]

(c) Find $P(X \leq 3 \mid X \leq 9)$.

[3]

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (from formula booklet)

$P(X \leq 3 \mid X \leq 9) = \frac{P(X \leq 3 \cap X \leq 9)}{P(X \leq 9)}$

$= \frac{P(X \leq 3)}{P(X \leq 9)}$ ← $(X \leq 3) \cap (X \leq 9) = (X \leq 3)$

$= \frac{0.130168...}{0.932779...}$ Using the calculator

$= 0.13954...$

$\boxed{P(X \leq 3 \mid X \leq 9) = 0.1395 \quad (4dp)}$

Question 9

Zara is a gymnast. It is known that she has a 20% chance of making a mistake in any given routine.

$p = 0.2$

Zara performs ten routines in a competition.

$n = 10$

(a) (i) Find the expected number of routines in which Zara will make a mistake.

(ii) Find the standard deviation of the number of routines in which Zara makes a mistake.

[3]

(b) Find the probability that Zara makes a mistake in:

(i) none of her routines,

(ii) exactly two of her routines,

(iii) no more than two of her routines.

[6]

(c) Given that Zara makes a mistake in at least 2 of her routines, find the probability that she makes a mistake in exactly 3 of her routines.

[3]

(d) Find the probability that the number of routines in which Zara makes a mistake is less than one standard deviation away from the mean.

[3]

(a) Let X be the number of the 10 routines containing a mistake.

$X \sim B(10, 0.2)$

(i) $E(X) = np$ (from formula booklet)

$E(X) = (10)(0.2)$

$\boxed{\text{Expected number of routines} = 2}$

(ii) $\mathrm{Var}(X) = np(1-p)$ (from formula booklet)

$\mathrm{Var}(X) = (10)(0.2)(1-0.2)$

$\mathrm{Var}(X) = 1.6$

Standard Deviation of $X = \sqrt{\mathrm{Var}(X)}$

Standard Deviation of $X = \sqrt{1.6}$

$= 1.264911...$

$\boxed{\text{Standard Deviation} = 1.26 \quad (3sf)}$

Zara is a gymnast. It is known that she has a 20% chance of making a mistake in any given routine.

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[3]

(d) Find the probability that the number of routines in which Zara makes a mistake is less than one standard deviation away from the mean.

[3]

(b) $X \sim B(10, 0.2)$

(i) Using the calculator (Probability Distribution)

$P(X=0) = 0.107374...$

$\boxed{P(X=0) = 0.1074 \text{ (4dp)}}$

(ii) Using the calculator (Probability Distribution)

$P(X=2) = 0.301989...$

$\boxed{P(X=2) = 0.3020 \text{ (4dp)}}$

(iii) None more than 2 $\Rightarrow X \leq 2$

Using the calculator (Cumulative Distribution)

$P(X \leq 2) = 0.677799...$

$\boxed{P(X \leq 2) = 0.6778 \text{ (4dp)}}$

Zara is a gymnast. It is known that she has a 20% chance of making a mistake in any given routine.

Zara performs ten routines in a competition.

(a) (i) Find the expected number of routines in which Zara will make a mistake.

(ii) Find the standard deviation of the number of routines in which Zara makes a mistake.

[3]

(b) Find the probability that Zara makes a mistake in:

(i) none of her routines,

(ii) exactly two of her routines,

(iii) no more than two of her routines.

[6]

$X \geq 2$

(c) Given that Zara makes a mistake in at least 2 of her routines, find the probability that she makes a mistake in exactly 3 of her routines.

$X = 3$

[3]

(d) Find the probability that the number of routines in which Zara makes a mistake is less than one standard deviation away from the mean.

[3]

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (from formula booklet)

$$P(X=3 \mid X \geq 2) = \frac{P(X=3 \cap X \geq 2)}{P(X \geq 2)}$$

$$= \frac{P(X=3)}{P(X \geq 2)} \longleftarrow (X=3) \cap (X \geq 2) = (X=3)$$

$$= \frac{P(X=3)}{1 - P(X \leq 1)}$$

$$= \frac{0.201326...}{1 - 0.375809...} \quad \text{Using the calculator}$$

$$= 0.32254...$$

$\boxed{P(X=3 \mid X \geq 2) = 0.3225 \text{ (4dp)}}$

Zara is a gymnast. It is known that she has a 20% chance of making a mistake in any given routine.

Zara performs ten routines in a competition.

(a) (i) Find the expected number of routines in which Zara will make a mistake.

(ii) Find the standard deviation of the number of routines in which Zara makes a mistake.

[3]

(b) Find the probability that Zara makes a mistake in:

(i) none of her routines,

(ii) exactly two of her routines,

(iii) no more than two of her routines.

[6]

(c) Given that Zara makes a mistake in at least 2 of her routines, find the probability that she makes a mistake in exactly 3 of her routines.

[3]

(d) Find the probability that the number of routines in which Zara makes a mistake is less than one standard deviation away from the mean.

E(X)

[3]

(d) From (a):

Mean = 2

Standard Deviation = $\sqrt{1.6}$

Less than one standard deviation away from the mean

$2-\sqrt{1.6} < X < 2+\sqrt{1.6}$

$0.735... < X < 3.264...$

X can only be positive integers $\Rightarrow 1 \leq X \leq 3$

$1 \leq X \leq 3$ is $X \leq 3$ without $X = 0$

$P(1 \leq X \leq 3) = P(X \leq 3) - P(X=0)$

$= 0.879126... - 0.107374...$

$= 0.77175...$

$\boxed{0.7718 \text{ (4dp)}}$

Question 10

In the town of Wooster, Ohio, it is known that 90% of the residents prefer the locally produced Woostershire brand sauce when preparing a Caesar salad. The other 10% of residents prefer another well-known brand.

n = 30

30 residents are chosen at random by a pollster. Let the random variable X represent the number of those 30 residents that prefer Woostershire brand sauce.

p = 0.9

(a) Find

(i) $\mathrm{E}(X)$

(ii) $\mathrm{Var}(X)$.

[2]

(b) Find the probability that

(i) 90% or more of the residents chosen prefer Woostershire brand sauce,

(ii) none of the residents chosen prefer the other well-known brand.

[6]

(a) $X \sim B(30, 0.9)$

(i) $E(X) = np$ (from formula booklet)

$E(X) = (30)(0.9)$

$\boxed{E(X) = 27}$

(ii) $Var(X) = np(1-p)$ (from formula booklet)

$Var(X) = (30)(0.9)(1-0.9)$

$\boxed{Var(X) = 2.7}$

In the town of Wooster, Ohio, it is known that 90% of the residents prefer the locally produced Woostershire brand sauce when preparing a Caesar salad. The other 10% of residents prefer another well-known brand.

30 residents are chosen at random by a pollster. Let the random variable X represent the number of those 30 residents that prefer Woostershire brand sauce.

(a) Find

(i) $E(X)$

(ii) $Var(X)$.

[2]

(b) Find the probability that

(i) 90% or more of the residents chosen prefer Woostershire brand sauce,

(ii) none of the residents chosen prefer the other well-known brand.

[6]

(b) (i) 90% of 30 = 27

90% or more ⇒ $X \geq 27$

Using the calculator (Cumulative Distribution)

$P(X \geq 27) = 1 - P(X \leq 26)$

$= 1 - 0.352560...$

$= 0.647439...$

$\boxed{P(X \geq 27) = 0.6474 \ (4dp)}$

(ii) None prefer other brand ⇒ All prefer Woostershire

$\therefore X = 30$

Using the calculator (Probability Distribution)

$P(X = 30) = 0.042391...$

$\boxed{P(X = 30) = 0.0424 \ (4dp)}$