

IB Maths: AA HL

Basic Limits & Continuity

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.7 Basic Limits & Continuity
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Basic Limits & Continuity

Question 1

For each of the following, either show that the limit converges and find its value, or else explain why the limit diverges:

a)

$$\lim_{x \rightarrow 4} \frac{1}{x^2 - 9}$$

[2 marks]

b)

$$\lim_{x \rightarrow 3} \frac{1}{x^2 - 9}$$

[2 marks]

c)

$$\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 9}$$

[3 marks]

Question 2

a)

Evaluate the limit

$$\lim_{x \rightarrow -\infty} \left(13 - \frac{619}{x^2} \right)$$

justifying your answer by clear mathematical reasoning.

[2 marks]

b)
Show that the limit

$$\lim_{x \rightarrow +\infty} \frac{3x^2 - 5x + 7}{x^2}$$

converges, and find its value. Be sure to show clear algebraic working.

[3 marks]

Question 3

A student has attempted to evaluate the limit

$$\lim_{x \rightarrow +\infty} (x^3 - x)$$

as follows:

$$\lim_{x \rightarrow +\infty} (x^3 - x) = (+\infty)^3 - (+\infty) = (+\infty) - (+\infty) = 0$$

a)
Explain what is wrong with the student's work.

[2 marks]

b)
Determine the correct evaluation of the limit, justifying your answer by clear mathematical reasoning.

[2 marks]

c)
Use technology to help you sketch the graph of $y = x^3 - x$, and show that the graph confirms your answer to part (b).

[2 marks]

Question 4

Consider the function defined by

$$f(x) = \frac{1}{x^2}$$

a)

Evaluate the limits

(i)

$$\lim_{x \rightarrow 0^-} f(x)$$

(ii)

$$\lim_{x \rightarrow 0^+} f(x)$$

[3 marks]

b)

Evaluate the limits

(i)

$$\lim_{x \rightarrow -\infty} f(x)$$

(ii)

$$\lim_{x \rightarrow +\infty} f(x)$$

[3 marks]

c)

Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = f(x)$.

[2 marks]

d)

Use technology to help you sketch the graph of $y = f(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 5

Consider the function g defined by

$$g(x) = \frac{1}{x-5}$$

a)
Evaluate the limits

(i)
 $\lim_{x \rightarrow 5^-} g(x)$

(ii)
 $\lim_{x \rightarrow 5^+} g(x)$

[3 marks]

b)
Evaluate the limits

(i)
 $\lim_{x \rightarrow -\infty} g(x)$

(ii)
 $\lim_{x \rightarrow +\infty} g(x)$

[3 marks]

c)
Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = g(x)$.

[2 marks]

d)
Use technology to help you sketch the graph of $y = g(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 6

a)

The function f is a piecewise function defined by

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ x+3, & x > 2 \end{cases}$$

Explain why f is not continuous at $x=2$.

[3 marks]

b)

A function g is defined for all $x \in \mathbb{R}$, and it is differentiable at all points $x \in \mathbb{R}$.

Explain why g is continuous at $x=7$.

[2 marks]