Name: ______________________________________

Question Paper

Date:

Time:

Total marks available: 407

Total marks achieved: ______

Subject: Board Specific Topic Questions for students studying AS level IAL Further Mathematics XFM01 and the A level IAL Further Mathematics YFM01. However students studying other boards may find this useful.

Topic: Decision 1

Sub Topic: Algorithms on graphs

Questions

Q1.
Pupils from ten schools are visiting a museum on the same day. The museum needs to allocate each school to a tour group. The maximum size of each tour group is 42 pupils. A group may include pupils from more than one school. Pupils from each school must be kept in the same tour group. The numbers of pupils visiting from each school are given below.

8 17 9 14 18 12 22 10 15 7

(a) Calculate a lower bound for the number of tour groups required. You must make your method clear.

(2)

(b) Using the above list, apply the first-fit bin packing algorithm to allocate the pupils visiting from each school to tour groups.

(2)

The above list of numbers is to be sorted into descending order.
(c) Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Using your sorted list from (c), apply the first-fit decreasing bin packing algorithm to obtain a second allocation of pupils to tour groups.

(2)

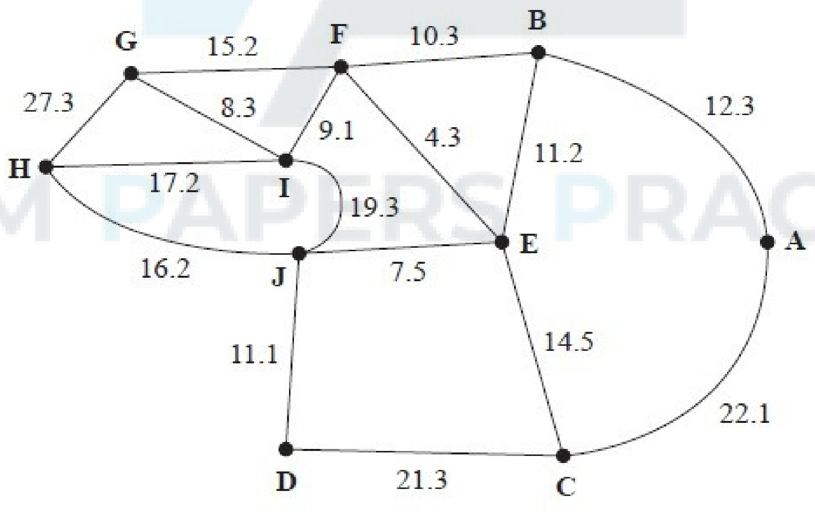


Figure 2

[The total weight of the network is 227.2]

Figure 2 represents the corridors in the museum. The number on each arc is the length, in metres, of the corresponding corridor. Sally is a tour guide in the museum and she must travel along each corridor at least once during each tour. Sally wishes to minimise the length of her route. She must start and finish at the museum's entrance at A.
(e) Use an appropriate algorithm to find the corridors that Sally will need to traverse twice. You should make your method and working clear.

(4)

(f) Write down a possible shortest route, giving its length.

(2)

Sally is now allowed to start at H and finish her route at a different vertex. A route of minimum length that includes each corridor at least once needs to be found.
(g) State the finishing vertex of Sally's new route and calculate the difference in length between this new route and the route found in (f).

(2)

(Total for question = 18 marks)

Q2.

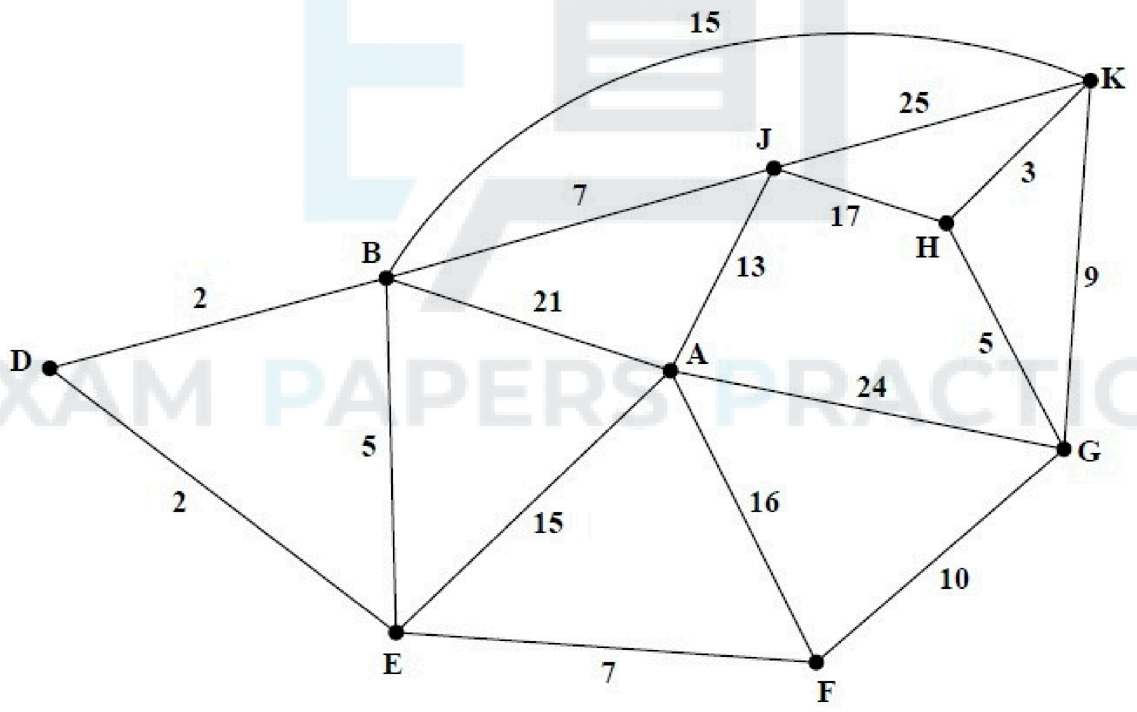


Figure 1

[The total weight of the network is 196]

Figure 1 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Oliver wishes to travel by road from A to K as quickly as possible.
(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to K. State the quickest route.

On a particular day Oliver must travel from B to K via A.

(b) Find a route of minimal time from B to K that includes A, and state its length.

(2)

Oliver needs to travel along each road to check that it is in good repair. He wishes to minimise the total time required to traverse the network.
(c) Use the route inspection algorithm to find the shortest time needed. You must state all combinations of edges that Oliver could repeat, making your method and working clear.

(7)

(Total for question = 15 marks)

Q3.

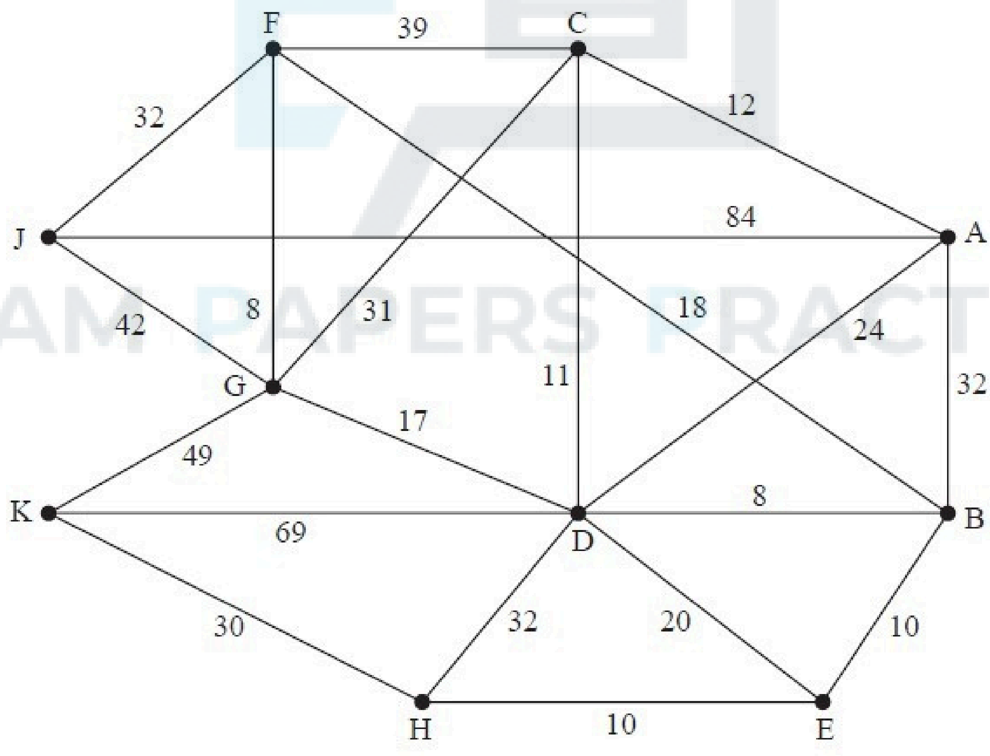


Figure 4

Figure 4 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. Tamasi, who lives at A, needs to collect a caravan. Tamasi can collect a caravan from either J or K.
Tamasi decides to use Dijkstra's algorithm once to find the shortest routes between A and J and

between A and K. (a) State, with a reason, which vertex should be chosen as the starting vertex for the algorithm. **(2)**

(b) Use Dijkstra's algorithm to find the shortest routes from A to J and from A to K. You should state the routes and their
corresponding lengths.

(7)

Tamasi's brother lives at F. He needs to visit Tamasi at A and then visit their mother who lives at H.

(c) Find a route of minimal length that goes from F to H via A.

(1)

(Total for question = 10 marks)

Q4.

(a) Explain what is meant by the term 'path'.

(2)

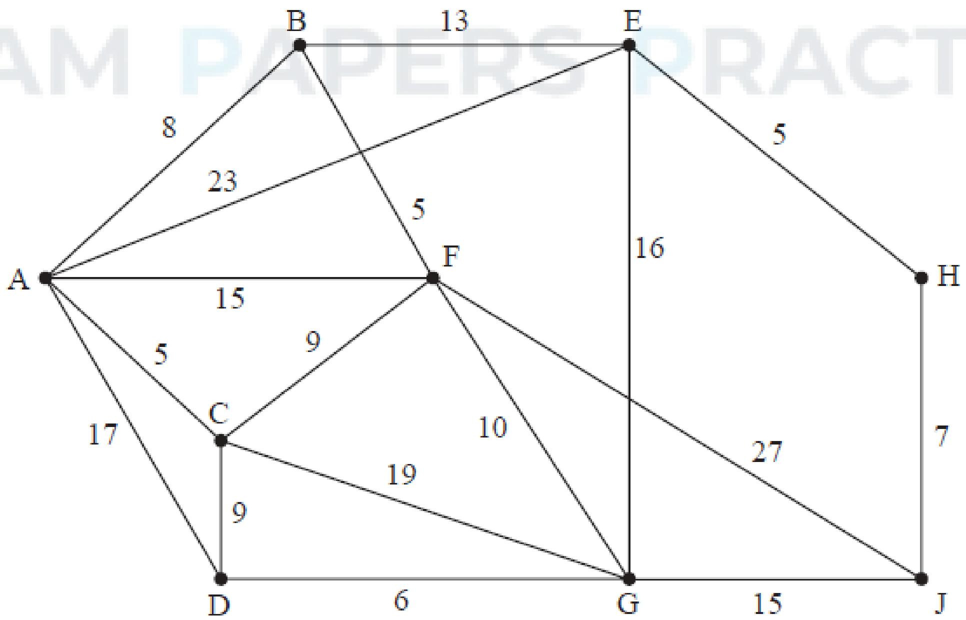


Figure 1

Figure 1 represents a network of roads. The number on each arc represents the length, in km, of the corresponding road. Piatrice wishes to travel from A to J.

(b) Use Dijkstra's algorithm to find the shortest path Piatrice could take from A to J.

State your path and its length.

(6)

Piatrice needs to return from J to A via G.

(c) Find the shortest path Piatrice could take from J to A via G and state its length.

(2)

(Total for question = 10 marks)

Q5.

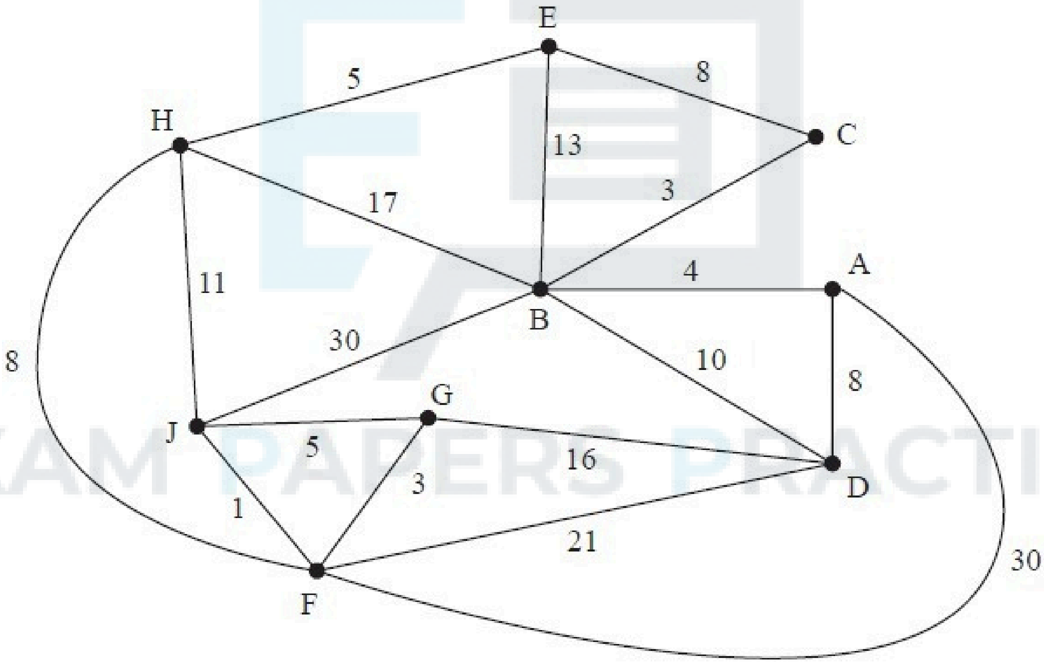


Figure 1

[*The total weight of the network is* 193]

Figure 1 represents a network of roads. The number on each edge represents the length, in miles, of the corresponding road. Jan wishes to travel from A to J. She wishes to minimise the distance she travels.

(a) Use Dijkstra's algorithm to find the shortest path from A to J. Obtain the shortest path and state its length.

(6)

On Monday, Jan needs to travel from her gym at J to her home at H via her office at A.

(b) State the shortest path from J to H via A and its length.

(2)

On Tuesday, Jan needs to check each road. She must travel along each road at least once. Jan must start and finish at A.
(c) Use the route inspection algorithm to find the length of the shortest inspection route. State the roads that should be repeated. You should make your method and working clear.

(5)

On Wednesday, Jan decides to start her inspection route at G but can finish her route at a different node. The inspection route must still traverse each road at least once.
(d) Determine where the route should finish so that the length of the inspection route is minimised. You must give reasons for your answer and state the length of the route.

(3)

(Total for question = 16 marks)

Q6.

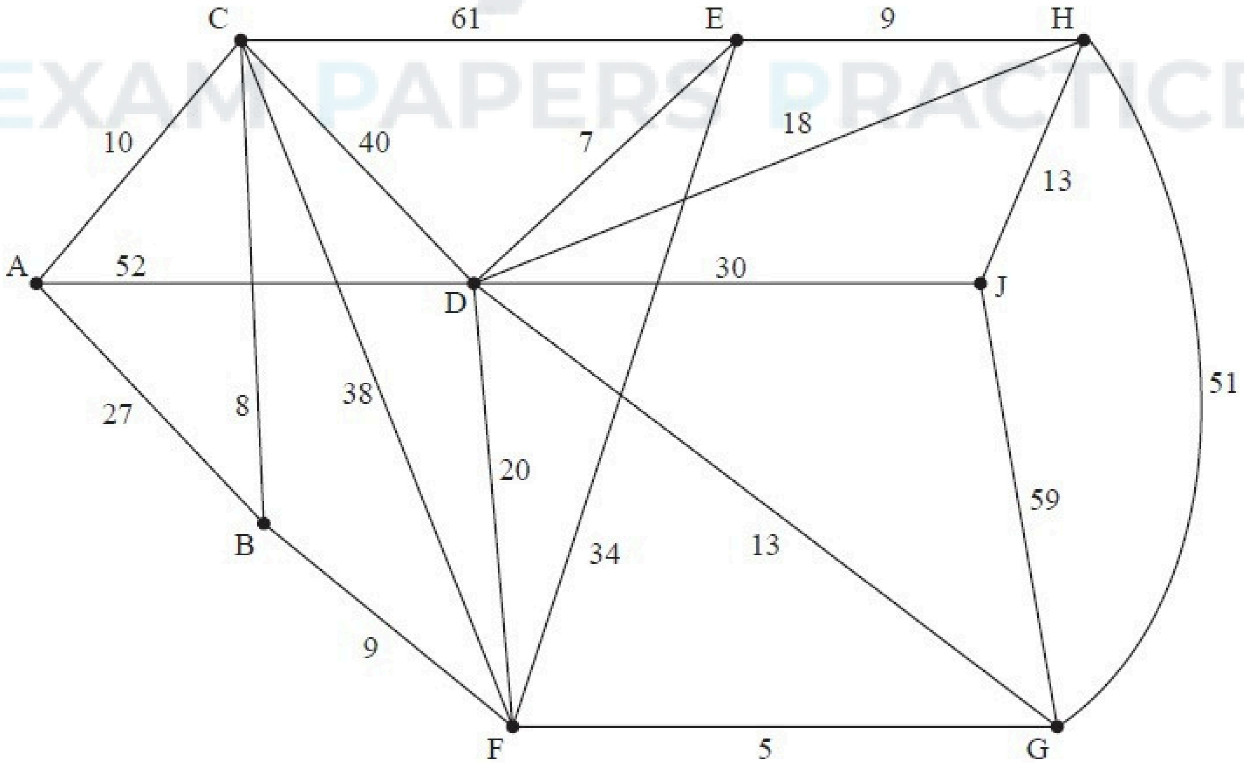


Figure 2

Figure 2 models a network of tracks between nine ranger stations, A, B, C, D, E, F, G, H and J, in a forest. The number on each edge gives the time, in minutes, to travel along the corresponding track. The forest ranger wishes to travel from A to J as quickly as possible.
(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J.
State the quickest route.

(6)

(b) Hence determine the weight of the minimum spanning tree for the network given in Figure 2. Give a reason for your answer.
You do not need to find the minimum spanning tree.

(2)

(Total for question = 8 marks)

Q7.

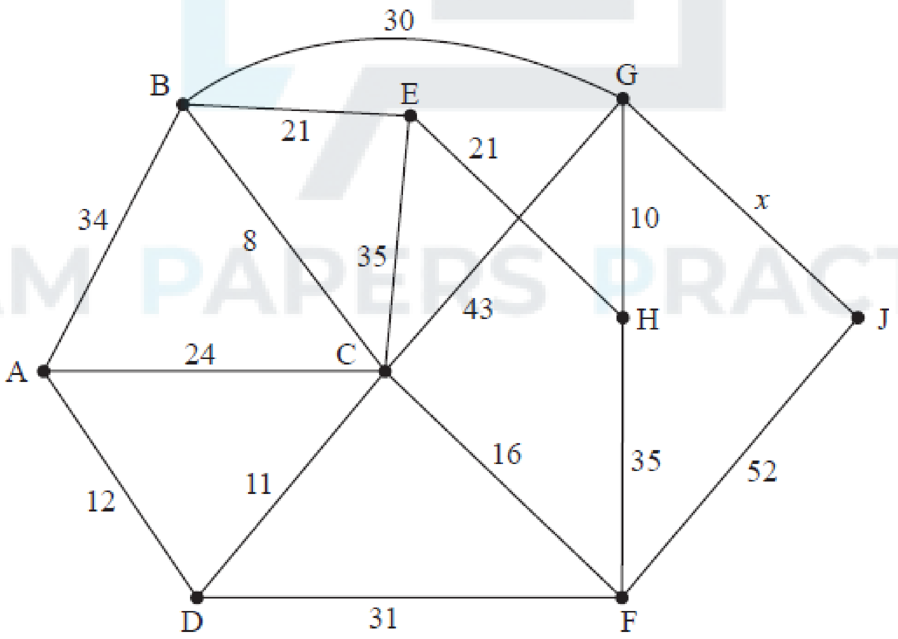


Figure 4

[The total weight of the network is 383 + x]

Figure 4 models a network of roads. The number on each edge gives the time, in minutes, to travel along the corresponding road. The vertices, A, B, C, D, E, F, G, H and J represent nine towns. Ezra wishes to travel from A to H as fast as possible.
The time taken to travel between towns G and J is unknown and is denoted by x minutes.

Dijkstra's algorithm is to be used to find the fastest time to travel from A to H. On Diagram 1 in the answer book the "Order of labelling" and "Final value" at A and J, and the "Working values" at J, have already been completed.
(a) Use Dijkstra's algorithm to find the fastest time to travel from A to H. State the quickest route.

(6)

Ezra needs to travel along each road to check it is in good repair. He wishes to minimise the total time required to traverse the network. Ezra plans to start and finish his inspection route at A. It is given that his route will take at least 440 minutes. (b) Use the route inspection algorithm and the completed Diagram 1 to find the range of possible values of x.

(6)

(c) Write down a possible route for Ezra.

(1)

A new direct road from D to H is under construction and will take 25 minutes to travel along.
Ezra will include this new road in a minimum length inspection route starting and finishing at A.
It is given that this inspection route takes exactly 488 minutes.
(d) Determine the value of x. You must give reasons for your answer.

(2)

(Total for question = 15 marks)

Q8.

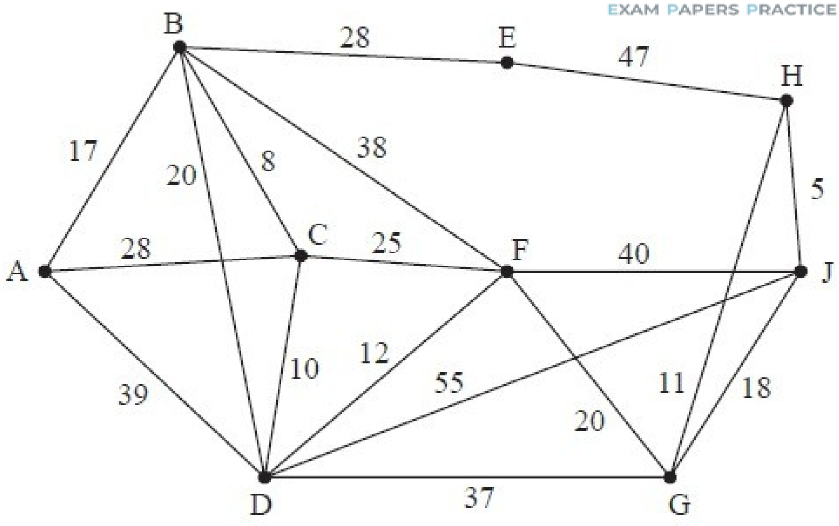


Figure 2

[The total weight of the network is458]

Figure 2 represents a network of roads between nine towns, A, B, C, D, E, F, G, H and J. The number on each edge represents the length, in kilometres, of the corresponding road.
(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.
(ii) State the length of the shortest path from A to J.

(6)

The roads between the towns must be inspected. Claude must travel along each road at least once. Claude will start the inspection route at A and finish at J. Claude wishes to minimise the length of the inspection route.
(b) By considering the pairings of all relevant nodes, find the length of Claude's route. State the arcs that will need to be traversed twice.

(5)

If Claude does **not** start the inspection route at A and finish at J, a shorter inspection route is possible.
(c) Determine the two towns at which Claude should start and finish so that the route has minimum length. Give a reason for your answer and state the length of this route.

(3)

(Total for question = 14 marks)

Q9.

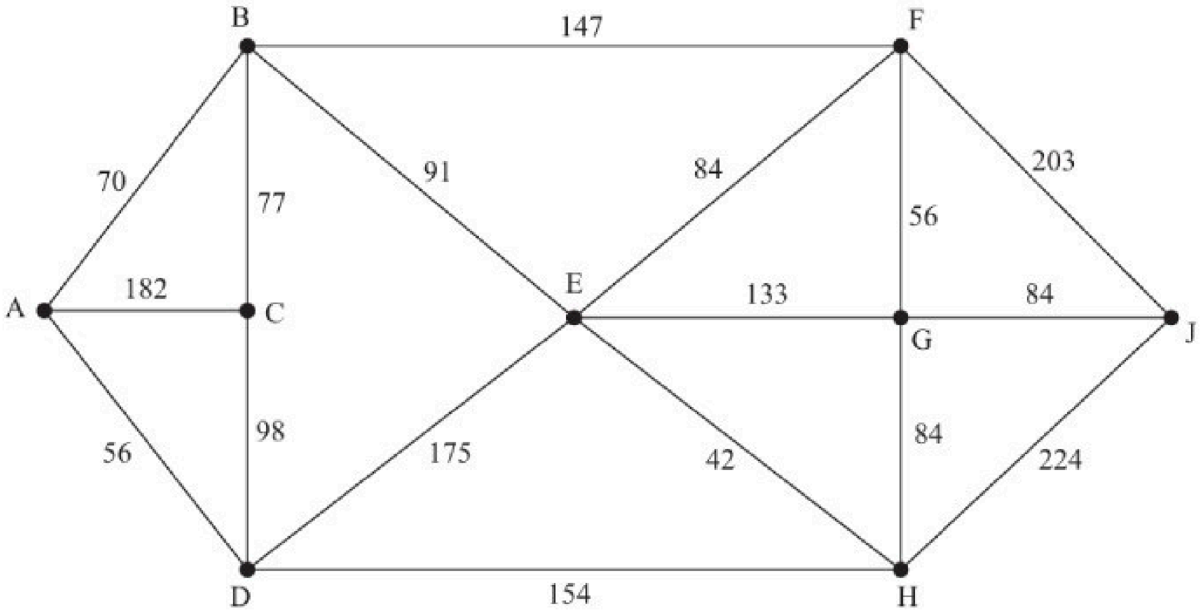


Figure 1
[The total weight of the network is 1960 m]

The network shown in Figure 1 represents the paths between nine attractions in a theme park.
The number on each edge is the length, in metres, of the corresponding path.
(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.
(ii) State the length of the shortest path from A to J in metres.

(6)

Sarita needs to inspect the paths between the attractions. She must travel along each path at least once.
Sarita decides to start and finish her inspection route at A. She wishes to minimise the length of her route.
(b) By considering the pairings of all relevant nodes, find the length of Sarita's route.

(4)

Sarita now decides to start her inspection route at A, but finish at a different attraction.
She must still minimise the length of her route and travel along each path at least once.
(c) (i) Determine where Sarita should finish her route. You must justify your answer.
(ii) Calculate the difference between the lengths of the two inspection routes.

(3)

(Total for question = 13 marks)

Q10.

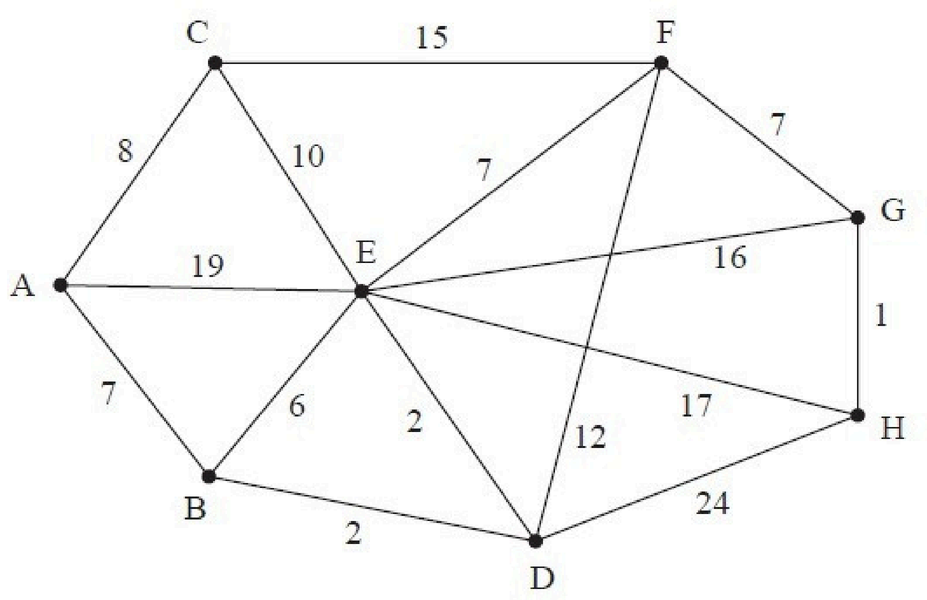


Figure 5

Figure 5 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F, G and H, represent eight towns. Bronwen needs to visit each town. She will start and finish at A and wishes to minimise the total distance travelled.

(a) By applying Dijkstra's algorithm, starting at A, complete the table of least distances in the answer book.

(6)

(b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Bronwen's route. Write down the route that gives this upper bound.

(2)

A reduced network is formed by deleting A and all arcs that are directly joined to A.

(c) (i) Use Prim's algorithm, starting at C, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.

(ii) Hence, calculate a lower bound for the length of Bronwen's route.

(4)

(d) Using only the results from (b) and (c), write down the smallest interval that you can be confident contains the length of Bronwen's optimal route.

(2)

(Total for question = 14 marks)

Q11.

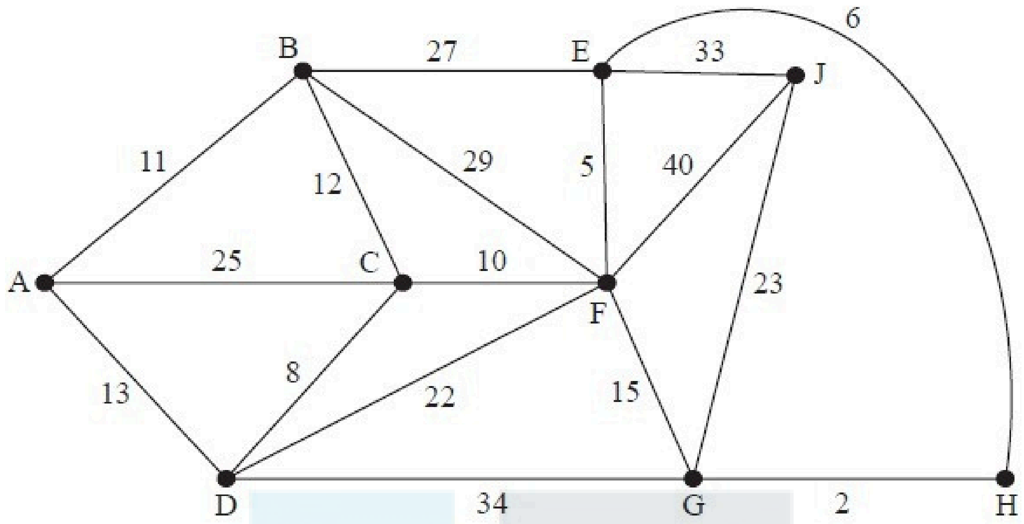


Figure 3

[The total weight of the network is 315]

Figure 3 represents a network of roads between nine parks, A, B, C, D, E, F, G, H and J. The number on each edge represents the length, in miles, of the corresponding road.
(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.
(ii) State the length of the shortest path from A to J.

(6)

The roads between the parks need to be inspected. Robin must travel along each road at least once. Robin wishes to minimise the length of the inspection route. Robin will start the inspection route at C and finish at E.
(b) By considering the pairings of all relevant nodes, find the length of Robin's route.

(4)

(c) State the number of times Robin will pass through G.

(1)

It is now decided to start and finish the inspection route at A. Robin must still minimise the length of the route and travel along each road at least once.
(d) Calculate the difference between the lengths of the two inspection routes.

(1)

(e) State the edges that need to be traversed twice in the route that starts and finishes at A, but do not need to be traversed twice in the route that starts at C and finishes at E.

(1)

(Total for question = 13 marks)

Q12.

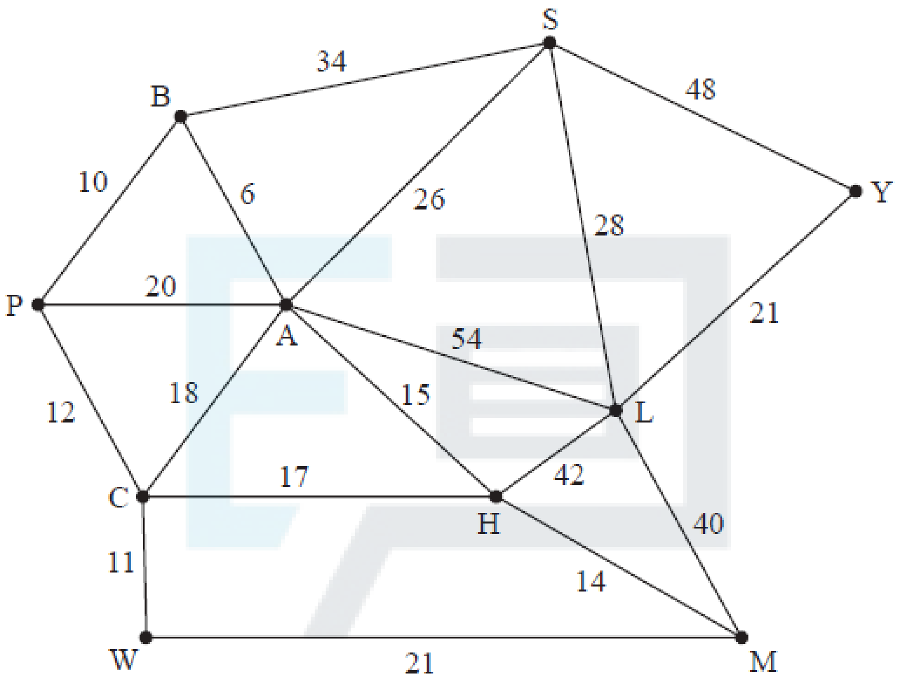


Figure 2

The network in Figure 2 shows the distances, in miles, between ten towns, A, B, C, H, L, M, P, S, W and Y.

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

The table shows the shortest distances, in miles, between the ten towns.

(b) Use Prim's algorithm on the table, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the arcs of your tree.

(3)

(c) State the weight of the minimum spanning tree found in (b).

(1)

Sharon needs to visit all of the towns, starting and finishing in the same town, and wishes to minimise the total distance she travels.
(d) Use your answer to (c) to calculate an initial upper bound for the length of Sharon's route.

(1)

(e) Use the nearest neighbour algorithm on the table, starting at W, to find an upper bound for the length of Sharon's route. Write down the route which gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at Y, an upper bound of length 212 miles was found.
(f) State the best upper bound that can be obtained by using this information and your answers from (d) and (e). Give the reason for your answer.

(1)

(g) By deleting W and all of its arcs, find a lower bound for the length of Sharon's route.

(2)

Sharon decides to take the route found in (e).

(h) Interpret this route in terms of the actual towns visited.

(1)

(Total for question = 14 marks)

Q13.

	A	B	C	D	E	F	G	H
A	–	38	37	x	37	42	41	27
B	38	–	26	32	33	38	37	34
C	37	26	–	39	38	39	30	39
D	x	32	39	–	37	36	29	36
E	37	33	38	37	–	32	33	30
F	42	38	39	36	32	–	31	28
G	41	37	30	29	33	31	–	33
H	27	34	39	36	30	28	33	–

The network represented by the table shows the least distances, in km, between eight museums, A, B, C, D, E, F, G and H.
A tourist wants to visit each museum at least once, starting and finishing at A. The tourist wishes to minimise the total distance travelled. The shortest distance between A and D is x km where $32 \leq x \leq 35$
(a) Using Prim's algorithm, starting at A, obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

(b) Use your answer to (a) to determine an initial upper bound for the length of the tourist's route.

(1)

(c) Starting at A, use the nearest neighbour algorithm to find another upper bound for the length of the tourist's route. Write down the route that gives this upper bound.

(2)

The nearest neighbour algorithm starting at E gives a route of

E – H – A – D – G – C – B – F – E

(d) State which of these two nearest neighbour routes gives the better upper bound. Give reasons for your answer.

(2)

Starting by deleting A, and all of its arcs, a lower bound of 235 km for the length of the route is found.
(e) Determine the smallest interval that must contain the optimal length of the tourist's route. You must make your method and working clear.

(4)

(Total for question = 12 marks)

Q14.

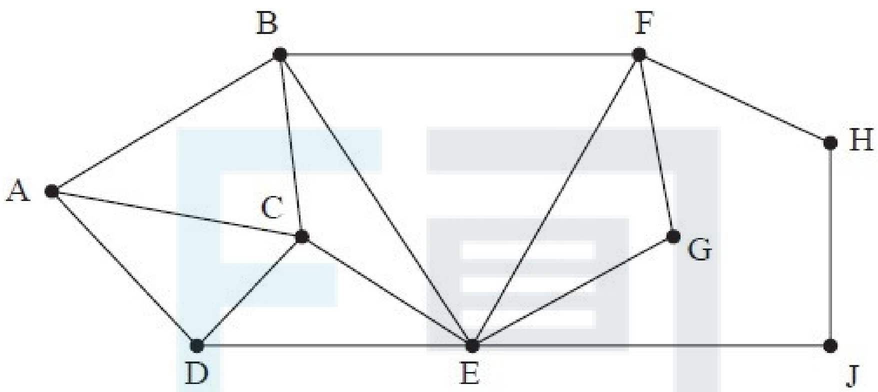


Figure 1

Figure 1 shows a graph, T.

(a) Write down an example of a path from A to J on T.

(1)

(b) State, with a reason, whether A – B – C – D – E – G – F – H – J is an example of a tour on T.

(1)

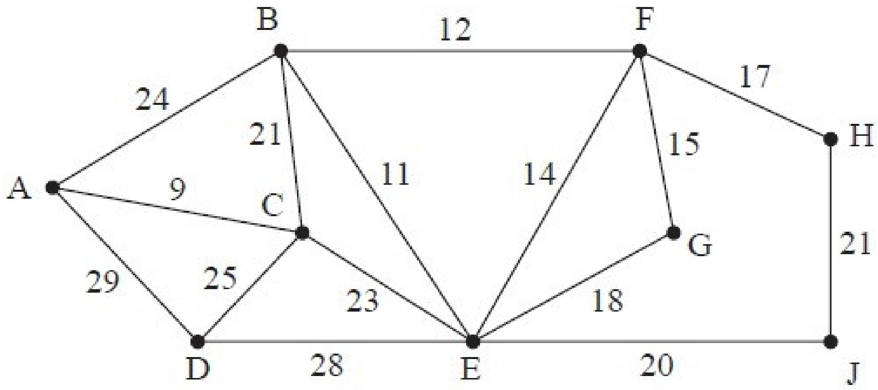


Figure 2

The numbers on the 15 arcs in Figure 2 represent the distances, in km, between nine vertices, A, B, C, D, E, F, G, H and J, in a network.

(c) Use Kruskal's algorithm to find the minimum spanning tree for the network.

You should list the arcs in the order in which you consider them. In each case, state whether or not you are adding the arc to the minimum spanning tree.

(3)

(d) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

(e) State the weight of the minimum spanning tree.

(1)

(Total for question = 7 marks)

Q15.

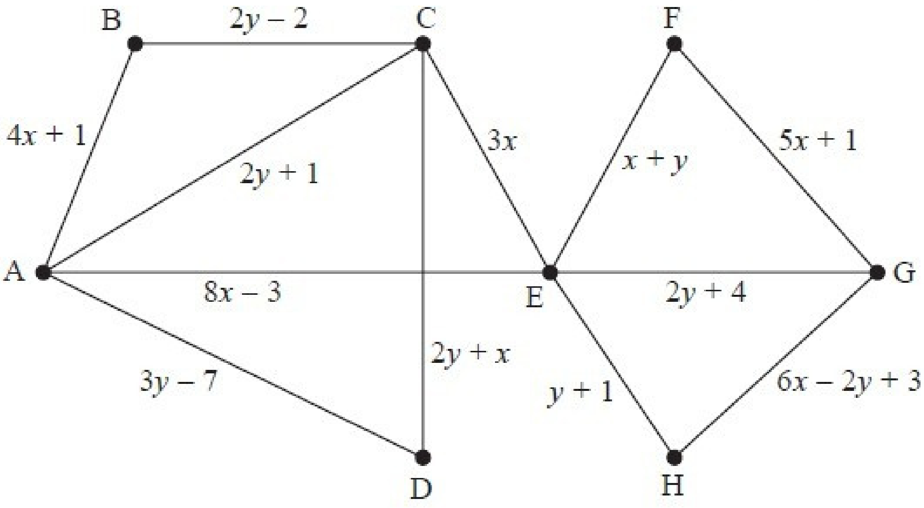


Figure 5

Figure 5 shows a weighted graph that contains 12 arcs and 8 vertices.
It is given that

- no two arcs have the same weight
- x and y are positive integers
- arc CD is not in the minimum spanning tree for the graph

(a) Explain why $y < x + 7$

(2)

It is also given that when Prim's algorithm, starting at A, is applied to the weighted graph, AB is the first arc selected.
(b) Show that $y > 2x$ and write down and simplify two further constraints on the values of x and y.

(3)

(c) Represent these four constraints on Diagram 1 in the answer book.

(4)

(d) Using Diagram 1 **only**, write down the possible pairs of values that x and y can take in the form (x, y).

(2)

The minimum spanning tree for the weighted graph in Figure 5 has total weight 73
Six of the seven arcs in the minimum spanning tree are AB, AD, BC, CE, EF and GH.
(e) Determine the value of x and the value of y. You must make your method and working clear.

(4)

(Total for question = 15 marks)

Q16.
(a) Explain the difference between the classical and the practical travelling salesperson problems.

(2)

The table below shows the distances, in km, between seven museums, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

Fran must visit each museum. She will start and finish at A and wishes to minimise the total distance travelled.
(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Fran's route. Make your method clear.

(2)

Starting at D, a second upper bound of 203 km was found.
(c) State whether this is a better upper bound than the answer to (b), giving a reason for your answer.

(1)

A reduced network is formed by deleting G and all the arcs that are directly joined to G.
(d) (i) Use Prim's algorithm, starting at A, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.
(ii) Hence calculate a lower bound for the length of Fran's route.

(4)

By deleting A, a second lower bound was found to be 188 km.

(e) State whether this is a better lower bound than the answer to (d)(ii), giving a reason for your answer.

(1)

(f) Using only the results from (c) and (e), write down the smallest interval that you can be confident contains the length of Fran's optimal route.

(2)

(Total for question = 12 marks)

Q17.

	A	B	C	D	E	F	G	H
A	–	34	29	35	28	30	37	38
B	34	–	32	28	39	40	32	39
C	29	32	–	27	33	39	34	31
D	35	28	27	–	35	38	41	36
E	28	39	33	35	–	36	33	40
F	30	40	39	38	36	–	34	39
G	37	32	34	41	33	34	–	35
H	38	39	31	36	40	39	35	–

Table 1

Table 1 represents a network that shows the travel times, in minutes, between eight towns, A, B, C, D, E, F, G and H.

(a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the edges of your tree.

(3)

(b) State the weight of the minimum spanning tree.

(1)

	A	B	C	D	E	F	G	H
J	33	37	41	35	x	40	28	42

Table 2

Table 2 shows the travel times, in minutes, between town J and towns A, B, C, D, E, F, G and H.

The journey time between towns E and J is x minutes where x > 28
A salesperson needs to visit all of the **nine** towns, starting and finishing at J.
The salesperson wishes to minimise the total time spent travelling.
(c) Starting at J, use the nearest neighbour algorithm to find an upper bound for the duration of the salesperson's route. Write down the route that gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at E, an upper bound of 291 minutes for the salesperson's route was found.
(d) State the best upper bound that can be obtained by using this information and your answer to (c). Give the reason for your answer.

(1)

Starting by deleting J and all of its arcs, a lower bound of 264 minutes for the duration of the salesperson's route was found.
(e) Determine the value of x. You must make your method and working clear.

(3)

(Total for question = 10 marks)

Q18.

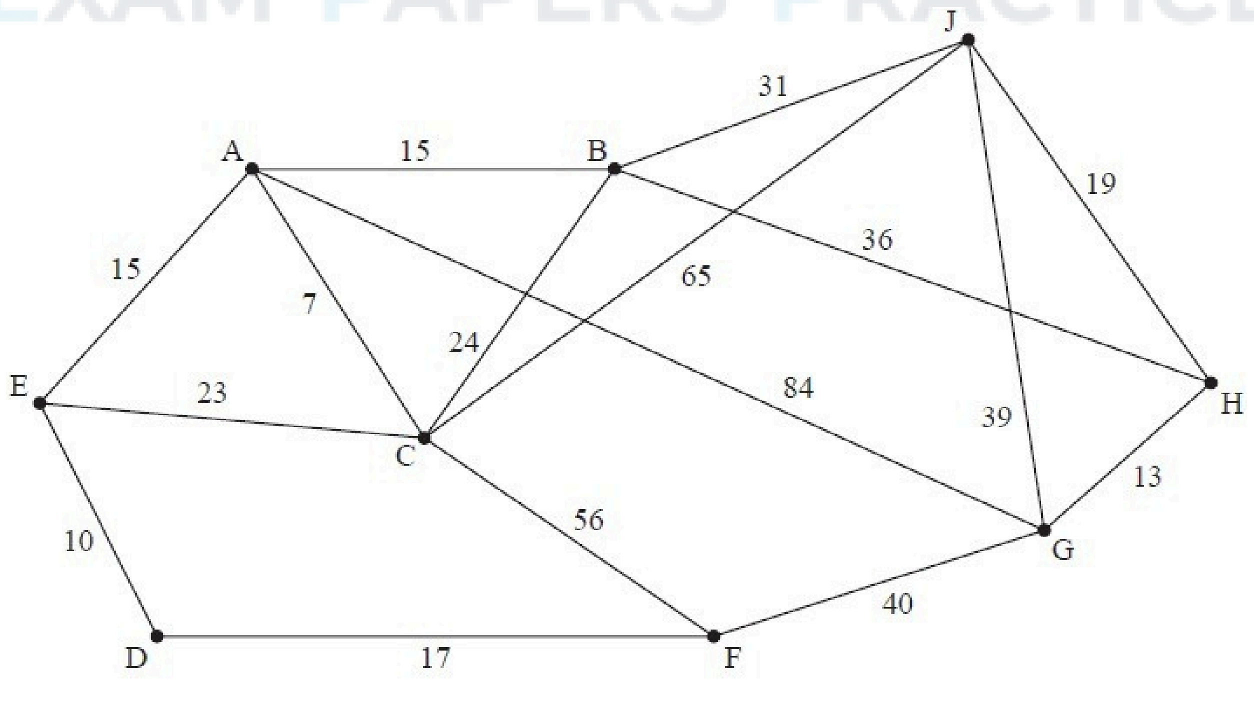


Figure 3

[The total weight of the network is 494]

Direct roads between nine factories, A, B, C, D, E, F, G, H and J, are represented in Figure 3.

The number on each arc represents the lengths, in kilometres, of the corresponding road.

The table below shows the shortest distances, in kilometres, between the nine factories.

	A	B	C	D	E	F	G	H	J
A	-	15	7	25	15	42	64	51	46
B	15	-	22	40	30	57	49	36	31
C	7	22	-	32	22	49	71	58	53
D	25	40	32	-	10	17	57	70	71
E	15	30	22	10	-	27	67	66	61
F	42	57	49	17	27	-	40	53	72
G	64	49	71	57	67	40	-	13	32
H	51	36	58	70	66	53	13	-	19
J	46	31	53	71	61	72	32	19	-

Table of shortest distances

(a) Starting at A, use Prim's algorithm to find a minimum spanning tree for the table of shortest distances. You must state the order in which you select the arcs of your tree.

(3)

(b) State the weight of the minimum spanning tree.

(1)

A route is needed that minimises the total distance to traverse each road at least once. The route must start at E and finish at F.

(c) Determine the length of this route. You must give a reason for your answer.

(2)

It is now decided to start the route at C and finish the route at A. The route must include every road at least once and must still minimise the total distance travelled.

(d) By considering the pairings of all relevant nodes, find the roads that need to be traversed twice.

(4)

Naoko needs to visit all nine factories, starting and finishing at the same factory, and wishes to minimise the total distance travelled.

(e) Starting at B, use the nearest neighbour algorithm on the table of shortest distances to find an upper bound for the length of Naoko's route. Write down the cycle, obtained from the table of shortest distances, which gives this upper bound.

(2)

(f) By deleting C and all of its arcs, use the values in the table of shortest distances to find a lower bound for the length of Naoko's route.

(2)

(Total for question = 14 marks)

Q19.

The table below represents a complete network that shows the least costs of travelling between eight cities, A, B, C, D, E, F, G and H.

	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

Srinjoy must visit each city at least once. He will start and finish at A and wishes to minimise his total cost.

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. You must list the arcs that form the tree in the order in which you select them.

(3)

(b) State the weight of the minimum spanning tree.

(1)

(c) Use your answer to (b) to help you calculate an initial upper bound for the total cost of Srinjoy's route.

(1)

(d) Show that there are two nearest neighbour routes that start from A. You must make the routes and their corresponding costs clear.

(4)

(e) State the best upper bound that can be obtained by using your answers to (c) and (d).

(1)

(f) Starting by deleting A and all of its arcs, find a lower bound for the total cost of Srinjoy's route. You must make your method and working clear.

(3)

(g) Use your results to write down the smallest interval that must contain the optimal cost of Srinjoy's route.

(2)

(Total for question = 15 marks)

Q20.

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-

(a) Explain the difference between the **classical** Travelling Salesman Problem and the **practical** Travelling Salesman Problem.

(2)

The table shows the shortest distances, in miles, between ten towns, A, B, C, D, E, F, G, H, J and K.

Kenzo must visit each town at least once, starting and finishing at A. Kenzo wishes to minimise the total distance travelled.
(b) Use Prim's algorithm, starting at A, to obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

(c) Use your answer to part (b) to determine an initial upper bound for the length of Kenzo's route.

(1)

(d) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Kenzo's route. Write down the route that gives this upper bound.

(3)

Using the answer to part (d), and given that the length of the nearest neighbour route starting at G is 145 miles,
(e) state which of these two nearest neighbour routes gives the better upper bound. Give a reason for your answer.

(1)

(f) By deleting A and all of its arcs, obtain a lower bound for the length of Kenzo's route.

(2)

(g) State the smallest interval that must contain the optimal length of Kenzo's route.

(1)

(Total for question = 13 marks)

Q21.

Kruskal's algorithm finds a minimum spanning tree for a connected graph with n vertices.

(a) Explain the terms

(i) connected graph,

(ii) tree,

(iii) spanning tree.

(3)

(b) Write down, in terms of n, the number of arcs in the minimum spanning tree.

(1) The table below shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	17	–	19	30	–	–
B	17	–	21	23	–	–	–
C	–	21	–	27	29	31	22
D	19	23	27	–	–	40	–
E	30	–	29	–	–	33	25
F	–	–	31	40	33	–	39
G	–	–	22	–	25	39	–

(c) Complete the drawing of the network on Diagram 1 in the answer book by adding the necessary arcs from vertex C together with their weights.

(2)

(d) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(e) State the weight of the minimum spanning tree.

(1)

(Total for question = 10 marks)

Q22.

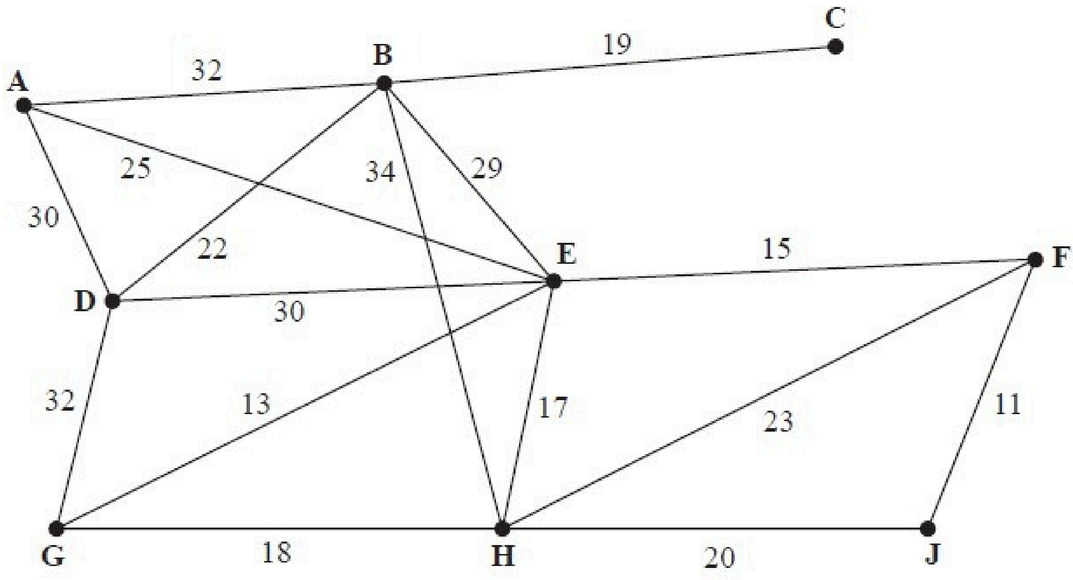


Figure 1

(a) Define the terms

(i) tree,

(ii) minimum spanning tree.

(3)

(b) Use Kruskal's algorithm to find the minimum spanning tree for the network shown in Figure 1. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in the minimum spanning tree.

(3)

(c) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book and state the weight of the minimum spanning tree.

(2)

(Total for question = 8 marks)

Q23.
The table below shows the distances, in metres, between six vertices, A, B, C, D, E and F, in a network.

	A	B	C	D	E	F
A	–	18	23	17	28	19
B	18	–	20	11	–	24
C	23	20	–	–	25	13
D	17	11	–	–	–	22
E	28	–	25	–	–	–
F	19	24	13	22	–	–

(a) Draw the weighted network using the vertices given in Diagram 1 in the answer book.

(2)

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree.

(3)

(c) Draw the minimum spanning tree on Diagram 2 in the answer book and state its total weight.

(2)

(Total for question = 7 marks)

Q24.

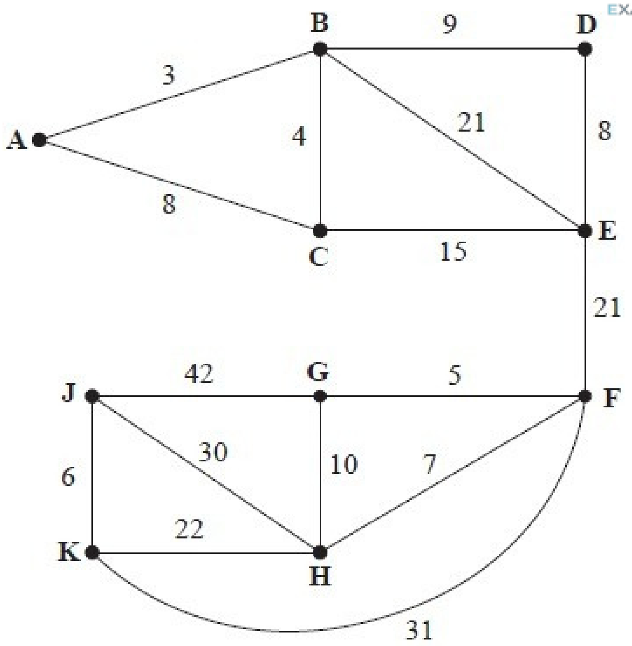


Figure 1

Figure 1 represents a network of roads between ten villages, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding road. The local council needs to find the shortest route from A to J.

(a) Use Dijkstra's algorithm to find the shortest route from A to J. State the route and its length.
(6)

During the winter, the council needs to ensure that all ten villages are accessible by road even if there is heavy snow. The council wishes to minimise the total length of road it needs to keep clear.

(b) Use Prim's algorithm, starting at A, to find a minimum connector for the five villages A, B, C, D and E. You must clearly state the order in which you select the edges of your minimum connector.

 (2)

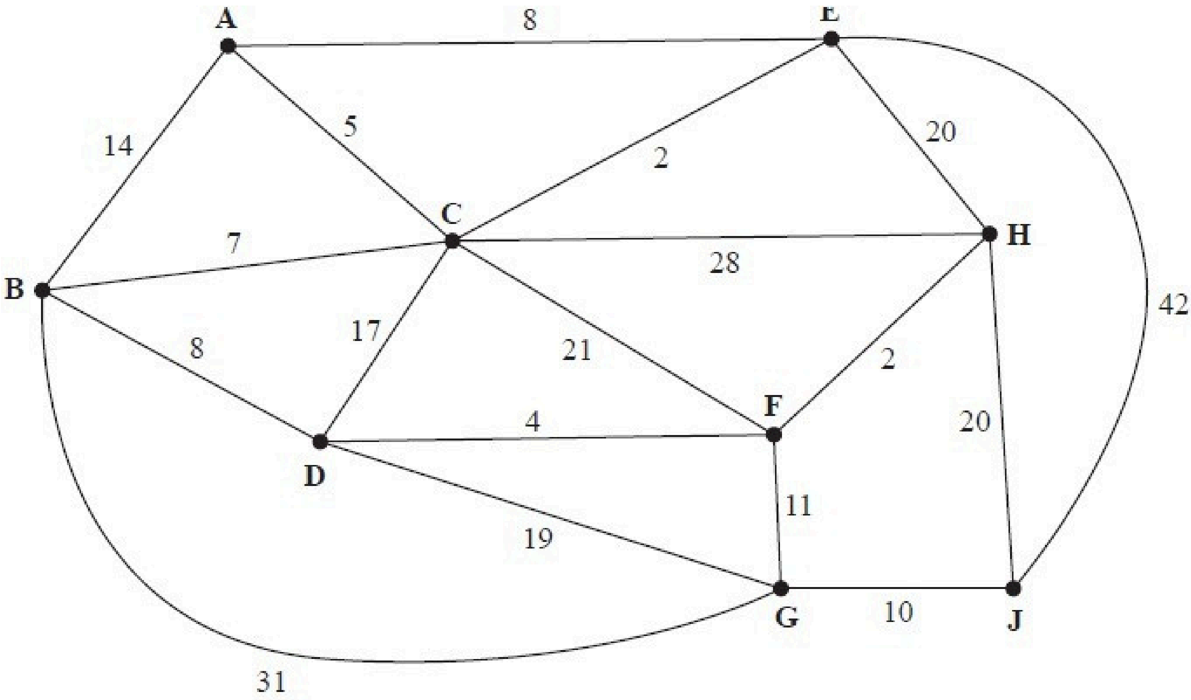
(c) Use Kruskal's algorithm to find a minimum connector for the five villages F, G, H, J and K. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in your minimum connector.

(2) (d) Calculate the total length of road that the council must keep clear of snow to ensure that all ten villages are accessible.

(1)

(Total for question = 11 marks)

Q25.



[*The total weight of the network is 269*]

Figure 3

Figure 3 models a network of roads. The number on each edge gives the time taken, in minutes, to travel along the corresponding road.
(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J. State the quickest route.

(6)

Alan needs to travel along all the roads to check that they are in good repair. He wishes to complete his route as quickly as possible and will start at his home, H, and finish at his workplace, D.
(b) By considering the pairings of all relevant nodes, find the arcs that will need to be traversed twice in Alan's inspection route from H to D. You must make your method and working clear.

(5)

For Alan's inspection route from H to D

(c) (i) state the number of times vertex C will appear,

(ii) state the number of times vertex D will appear.

(2)

(d) Determine whether it would be quicker for Alan to start and finish his inspection route at H, instead of starting at H and finishing at D. You must explain your reasoning and show all your working.

(2)

(Total for question = 15 marks)

Q26.

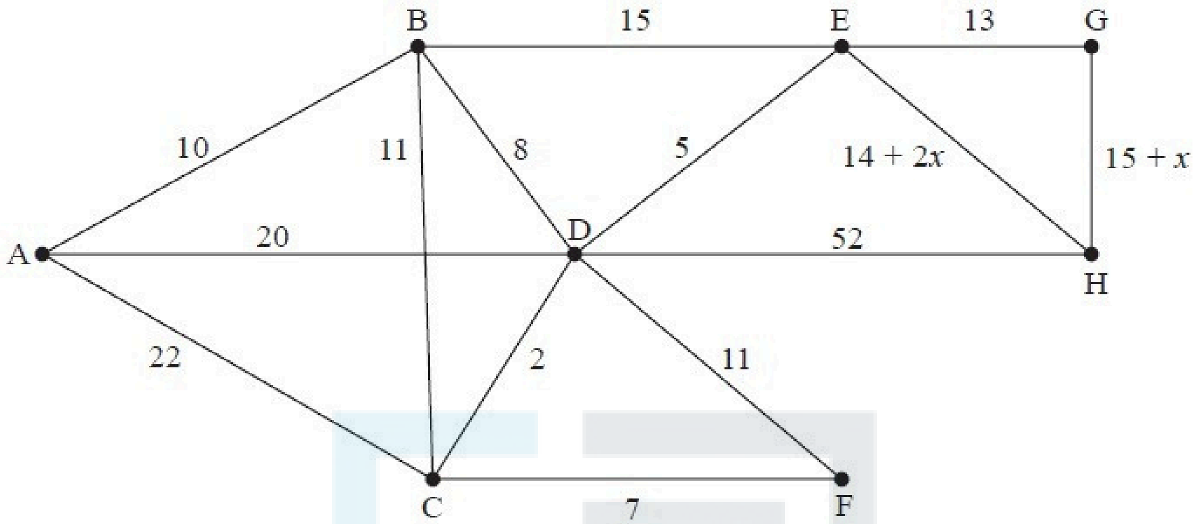


Figure 3

[*The total weight of the network is* 205 + 3*x*]

Figure 3 represents a network of roads. The number on each arc represents the time taken, in minutes, to drive along the corresponding road. Malcolm wishes to minimise the time spent driving from his home at A to his office at H. The delays from roadworks on two of the roads leading in to H vary daily, and so the time taken to drive along these roads is expressed in terms of x, where x is fixed for any given day and x > 0 (a) Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary.

(7)

On Monday, Malcolm needs to check each road. He must travel along each road at least once. He must start and finish at H and minimise the total time taken for his inspection route.
Malcolm finds that his minimum duration inspection route requires him to traverse exactly four roads twice and the total time it takes to complete his inspection route is 307 minutes.
(b) Calculate the minimum time taken for Malcolm to travel from A to H on Monday. You must make your method and working clear.

(4)

(Total for question = 11 marks)

Q27.

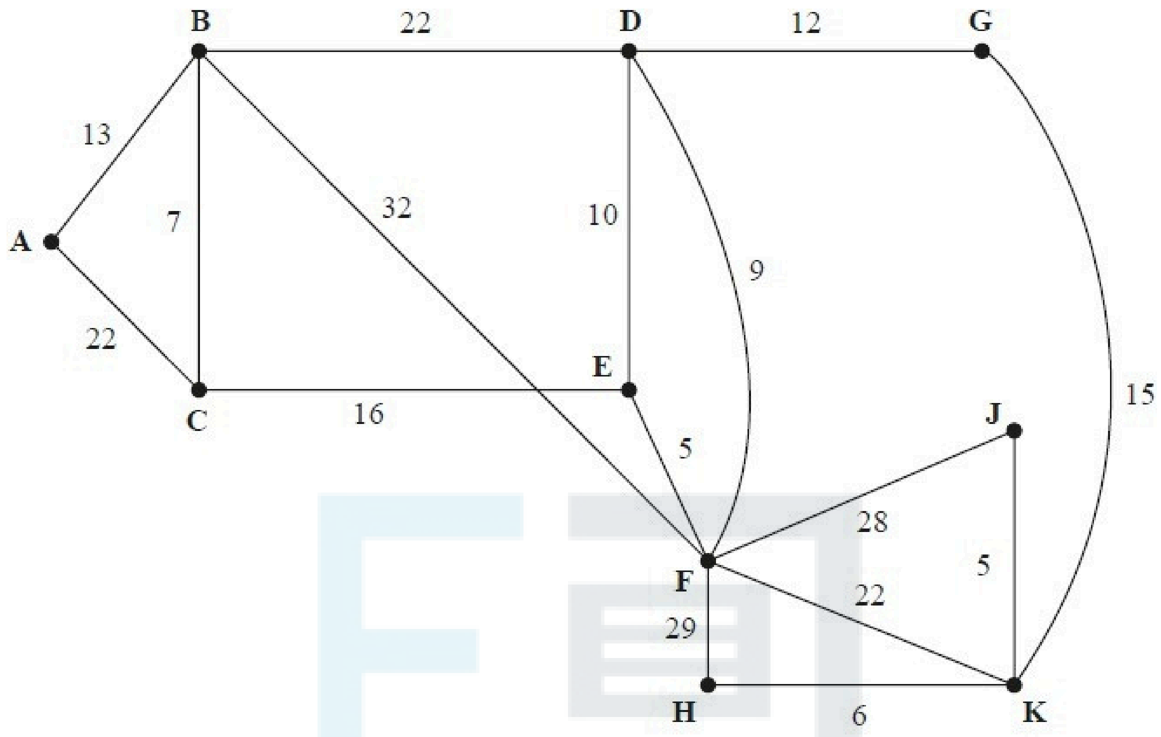


Figure 1

[The total weight of the network is 253]

Figure 1 represents a network of roads between 10 cities, A, B, C, D, E, F, G, H, J and K.
The number on each edge represents the length, in miles, of the corresponding road.
One day, Mabintou wishes to travel from A to H. She wishes to minimise the distance she travels.
(a) Use Dijkstra's algorithm to find the shortest path from A to H. State your path and its length.
(6)

On another day, Mabintou wishes to travel from F to K via A.

(b) Find a route of minimum length from F to K via A and state its length.

(2)

The roads between the cities need to be inspected. James must travel along each road at least once. He wishes to minimise the length of his inspection route. James will start his inspection route at A and finish at J.
(c) By considering the pairings of all relevant nodes, find the length of James' route. State the arcs that will need to be traversed twice. You must make your method and working clear.

(6)

(d) State the number of times that James will pass through F.

(1)

It is now decided to start the inspection route at D. James must minimise the length of his route. He must travel along each road at least once but may finish at any vertex.

(e) State the vertex where the new inspection route will finish.

(1)

(f) Calculate the difference between the lengths of the two inspection routes.

(1)

(Total for question = 17 marks)

Q28.

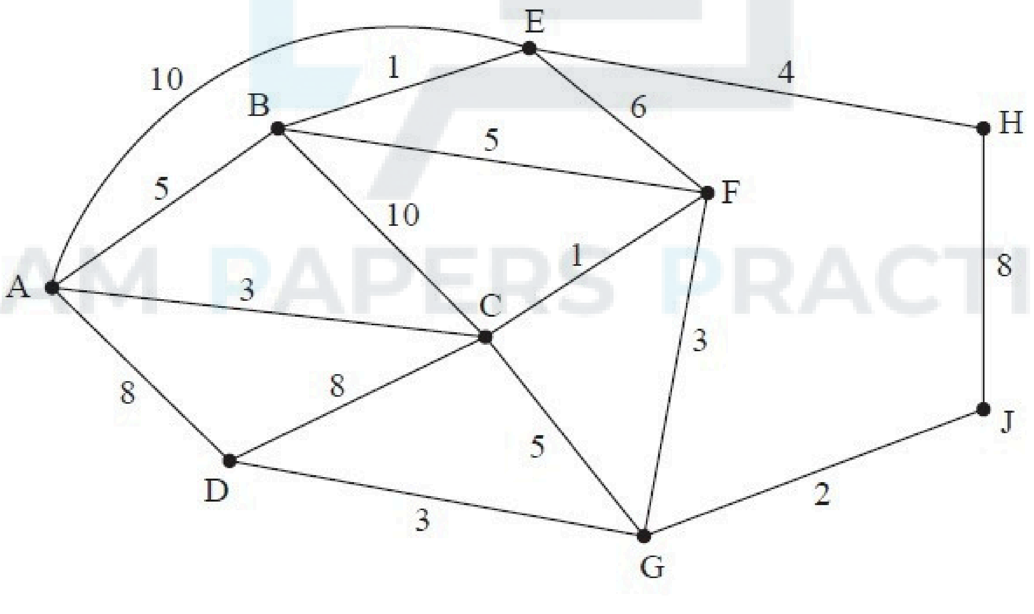


Figure 4

[The total weight of the network is 82]

Figure 4 represents a network of 16 roads in a city. The number on each arc represents the time taken, in minutes, to travel along the corresponding road.

Chan needs to check that the roads are in good repair. He must travel along each road at least once. Chan will start and finish at his office at G and must minimise the total time taken for his inspection route.

For this inspection route,
(a) find the time taken and state a possible route. You must make your method and reasoning clear.

(3)

Chan wonders if he can reduce his travel time by starting from his home at B, travelling along each road at least once and finishing at his office at G.
(b) By considering the pairings of all relevant nodes, find any arcs that would need to be traversed twice in the minimum inspection route from B to G. You must make your method clear, showing your working.

(5)

(c) Determine which of the two routes ending at G is quicker, the one starting at G or the one starting at B. You must justify your answer.

(2)

(Total for question = 10 marks)

Q29.

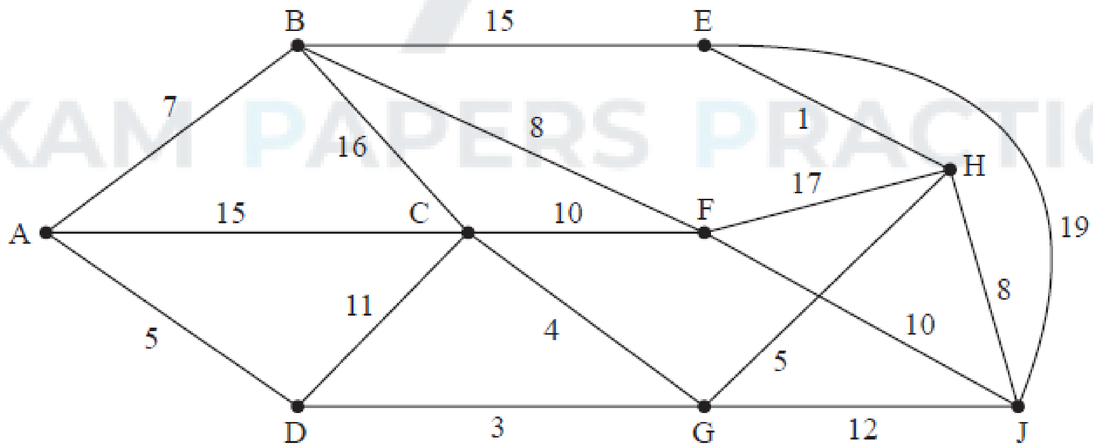


Figure 3

[*The total weight of the network is* 166]

Figure 3 models a network of cycle lanes that must be inspected. The number on each arc represents the length, in km, of the corresponding cycle lane. Lance needs to cycle along each lane at least once and wishes to minimise the length of his inspection route.
He must start and finish at A.
(a) Use an appropriate algorithm to find the length of the route. State the cycle lanes that Lance

will need to traverse twice. You should make your method and working clear.

(6)

(b) State the number of times that vertex C appears in Lance's route.

(1)

It is now decided that the inspection route may finish at any vertex. Lance will still start at A and must cycle along each lane at least once.
(c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of this new minimum route.

(3)

(Total for question = 10 marks)

Q30.

The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.
(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(1)

(Total for question = 8 marks)

Q31.

	A	B	C	D	E	F
A	–	73	56	27	38	48
B	73	–	58	59	43	34
C	56	58	–	46	38	42
D	27	59	46	–	25	32
E	38	43	38	25	–	21
F	48	34	42	32	21	–

The table above shows the least distances, in km, between six cities, A, B, C, D, E and F. Mohsen needs to visit each city, starting and finishing at A, and wishes to minimise the total distance he will travel.

(a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Mohsen's route. You must state your route and its length.

(3)

(b) Starting by deleting A and all of its arcs, find a lower bound for the length of Mohsen's route.

(3)

(c) Use your answers from (a) and (b) to write down the smallest interval that you can be confident contains the optimal length of the route.

(2)

(Total for question = 8 marks)

Q32.
The table below shows the distances, in km, between six data collection points, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

Ferhana must visit each data collection point. She will start and finish at A and wishes to minimise the total distance she travels.
(a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the distance Ferhana must travel. Make your method clear.

(2)

(b) Starting by deleting B, and all of its arcs, find a lower bound for the distance Ferhana must travel. Make your calculation clear.

(3)

(Total for question = 5 marks)

Q33.

The table below shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–

Mei must visit each town at least once. She will start and finish at A and wishes her route to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree in the answer book, use the shortcut method to find an upper bound below 520 km for Mei's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Mei's route.

(2)

(c) Starting by deleting E, and all of its arcs, find a lower bound for the length of Mei's route. Make your method clear.

(3)

(Total for question = 7 marks)

Q34.

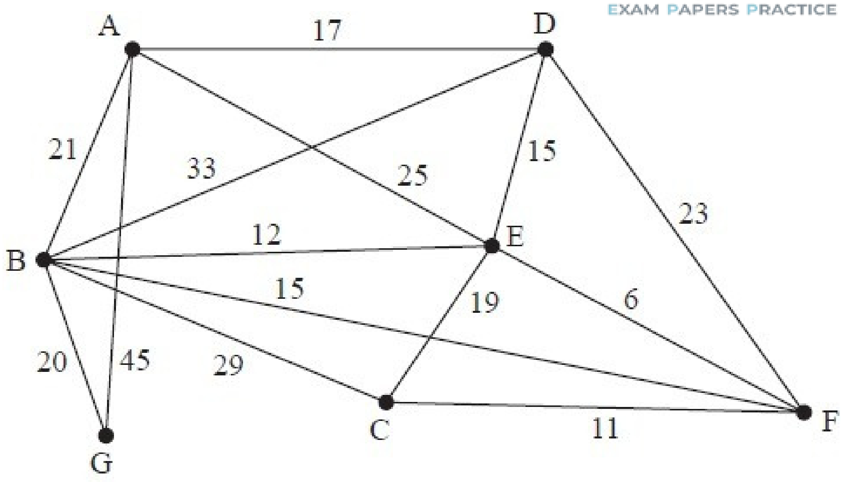


Figure 3

[The total weight of the network is 291]

Figure 3 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F and G, represent seven towns. Derek needs to visit each town. He will start and finish at A and wishes to minimise the total distance travelled.
(a) By inspection, complete the two copies of the table of least distances in the answer book.

(2)

(b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Derek's route. Write down
the route that gives this upper bound.

(2)

(c) Interpret the route found in (b) in terms of the towns actually visited.

(1)

(d) Starting by deleting A and all of its arcs, find a lower bound for the route length.

(3)

Clive needs to travel along the roads to check that they are in good repair. He wishes to minimise the total distance travelled and must start at A and finish at G.
(e) By considering the pairings of all relevant nodes, find the length of Clive's route. State the edges that need to be
traversed twice. You must make your method and working clear.

(5)

(Total for question = 13 marks)

Q35.

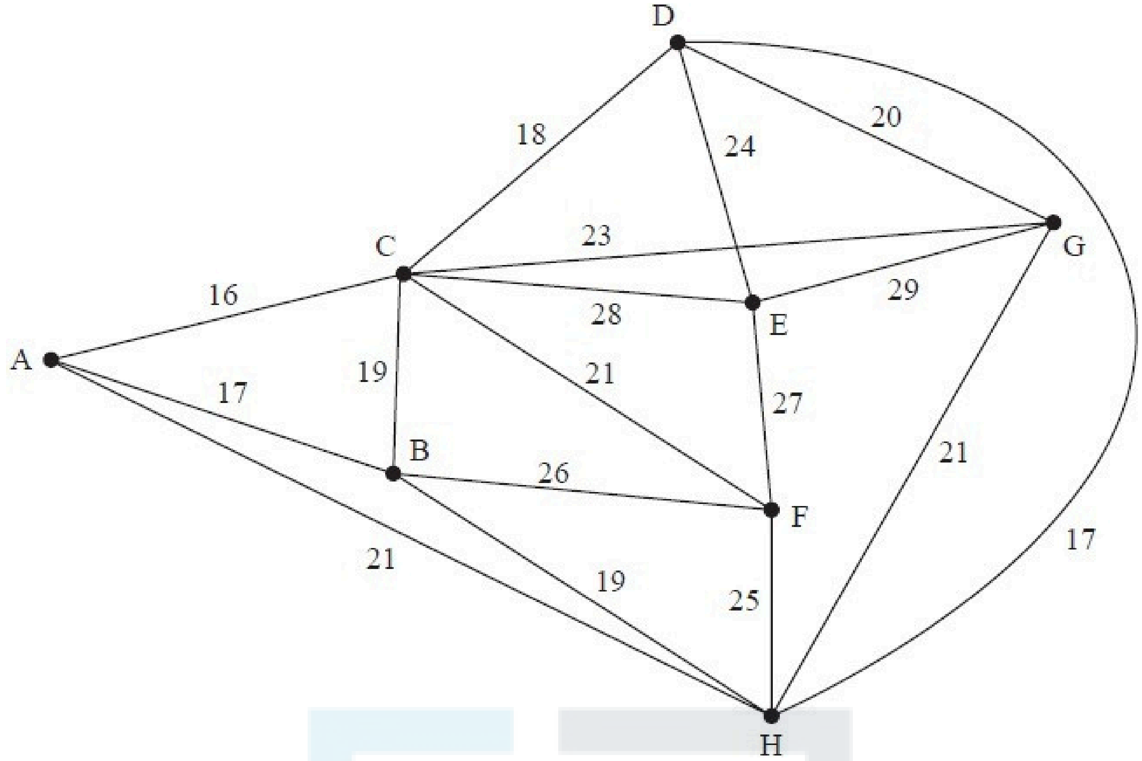


Figure 3

(a) Explain why it is impossible to draw a graph with eight vertices in which the vertex orders are 1, 2, 2, 3, 3, 4, 4 and 6

(1)

Figure 3 shows the network T. The numbers on the arcs represent the distances, in km, between the eight vertices, A, B, C, D, E, F, G and H.

(b) Determine whether or not A – C – D – E – C – B – F is an example of a path on T. You must justify your answer.

(2)

(c) Use Prim's algorithm, starting at A, to find the minimum spanning tree for T. You must clearly state the order in which you select the arcs of the tree.

(3)

(d) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

The weight of arc CF is now increased to a value of x. The minimum spanning tree for T is unique and includes the same arcs as those found in (c).

(e) Write down the smallest interval that must contain x.

(Total for question = 9 marks)

(2)