Name: _____

Mark Scheme

Date:

Time:

Total marks available: **407**

Total marks achieved: _____

Subject: Board Specific Topic Questions for students studying AS level IAL Further Mathematics XFM01 and the A level IAL Further Mathematics YFM01. However students studying other boards may find this useful.

Topic: Decision 1

Sub Topic: Algorithms on graphs

Questions

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\frac{132}{42} = 3.14...$ so lower bound is 4	M1 A1 (2)
(b)	Group 1: **8** **17** **9** 7 Group 2: **14** **18** 10 Group 3: **12** 22 Group 4: 15	M1 A1 (2)
(c)	e.g. middle right 8 17 9 14 18 12 22 10 15 7 17 14 18 22 15 **12** 8 9 10 7 22 **18** 17 14 15 **12** **10** 8 9 7 22 **18** 17 15 **14** **12** **10** **9** 8 7 22 **18** 17 **15** **14** **12** **10** **9** 8 7 e.g. middle left 8 17 9 14 18 12 22 10 15 7 22 **18** 8 17 9 14 12 10 15 7 22 **18** 17 15 **14** 8 9 12 10 7 22 **18** **17** 15 **14** **12** 8 9 10 7 22 **18** **17** 15 **14** **12** 10 9 8 7 22 **18** **17** 15 **14** **12** 10 **9** **8** 7	M1 A1 A1ft A1 (4)
(d)	Group 1: **22** **18** Group 2: **17** **15** 10 Group 3: **14** **12** 9 7 Group 4: 8	M1 A1 (2)
(e)	B(E)C + G(I)H = (11.2 + 14.5) + (8.3 + 17.2) = 51.2* B(F)G + C(EJ)H = (10.3 + 15.2) + (14.5 + 7. 5 + 16.2) = 63.7 B(EJ)H + C(EF)G = (11.2 + 7.5 + 16.2) + (14.5 + 4. 3 + 15.2) = 68.9 Repeat arcs: BE, CE, GI, HI	M1 A1 A1 A1 (4)
(f)	Route e.g. ABEBFECEJIFGIGHIHJDCA Length = 227. 2 + 51.2 = 278.4 (m)	B1 B1ft (2)
(g)	Finishing vertex: C Reduction in lengths: 51.2 – (10.3 + 15.2) = 25.7 (m)	B1 B1 (2)
		18 marks

Notes for Question
PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE FOR ASCENDING IN PART (c) **a1M1:** Attempt to find the lower bound $(132 \pm 22)/42$ (a value of 3.14 (or better) seen with no working can imply this mark) **a1A1:** CSO - correct calculation seen **or** 3.14 followed by 4 – accept 3.1 if correct calculation seen. An answer of 4 with no working scores M0A0 **b1M1:** First six items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the values in bold) **b1A1:** CSO (so no additional/repeated values) **c1M1:** Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then M1 only** **c1A1:** First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot values of 18 and 10 for the second pass or (if middle left) a pivot value of 14

c2A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark
c3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including if middle right a fourth pass with the 15 and 7 **used** as pivots **or** if middle left a fifth pass with the 8 used as a pivot

Sorting list into ascending order in (c)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in **(c)**
- If the list is not reversed in **(c)** but stated in ascending or descending order in **(d)** then remove the last two A marks earned in **(c)**. If the candidate says that the list needs reversing in **(c)** but does not actually show the reversed list in **(c)** then remove the last A mark earned
- **Note that if sorting into ascending order then a 'sort complete' statement is required – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed**

Middle right ascending
(requires sort complete statement– see above)

8 17 9 14 18 12 22 10 15 7
8 9 10 7 **12** 17 14 18 22 15
8 9 7 **10** **12** 17 14 15 **18** 22
8 7 **9** **10** **12** **14** 17 15 **18** 22
7 8 **9** **10** **12** **14** **15** 17 **18** 22

Middle left ascending
(requires sort complete statement – see above)

8 17 9 14 18 12 22 10 15 7
8 17 9 14 12 10 15 7 **18** 22
8 9 12 10 7 **14** 17 15 **18** 22
8 9 10 7 **12** **14** 15 **17** **18** 22
8 7 **9** 10 **12** **14** 15 **17** **18** 22
7 **8** **9** 10 **12** **14** 15 **17** **18** 22

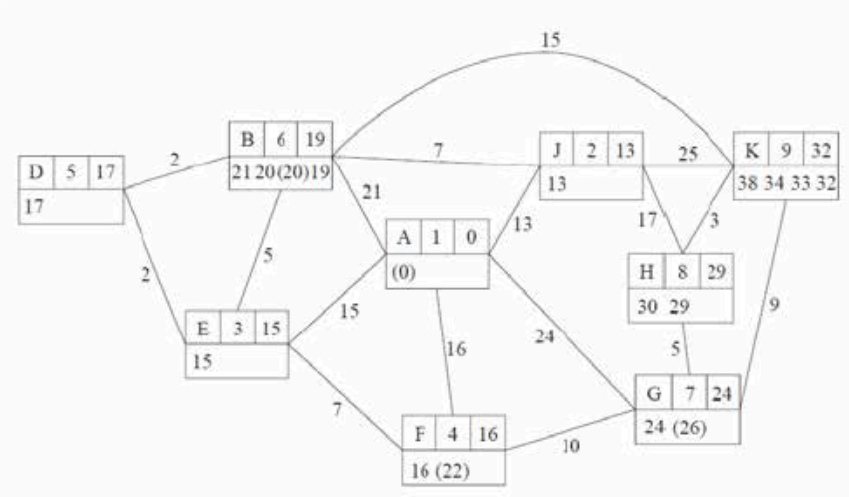
d1M1: First six items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the values in bold)
d1A1: CSO (so no additional/repeated values)

e1M1: Correct three pairings of the correct four odd nodes (B, C, G and H)
e1A1: Any one row correct including pairings **and** totals
e2A1: All three rows correct including pairings **and** totals
e3A1: CAO correct arcs clearly stated: BE, CE, GI and HI – must be these arcs and not e.g. BEC, GIH, or BC via E, etc.

f1B1: Any correct route (checks: 21 vertices, starting and ending at A, BE, CE, GI and HI appearing twice, A(2), B(2), C(2), D(1), E(3), F(2), G(2), H(2), I(3), J(2))
f2B1ft: For 227.2 + their smallest repeat out of a choice of at least **two** totals seen in **(e)** – this mark is dependent on M1 in (e)

g1B1: CAO (C)
g2B1: CAO (25.7) – note that the correct answer can come from incorrect working e.g. 11.2 + 14.5 = 25.7 is B0 (just adding BE and EC together) so this answer need to be checked carefully – correct method is 51.2 – (10.3 + 15.2) (subtracting BF and FG from 51.2) but give bod on a correct answer of 25.7 with no working

Q2.

Question	Scheme	Marks
(a)		M1 A1 (JEFD) A1 (BG) A1ft (HK)
	Quickest route: A – G – H – K	A1
	Shortest time: 32 (mins)	A1ft
		(6)
(b)	Route from B to K via A: B – D – E – A – G – H – K Length: 51 (mins)	B1 B1ft
		(2)
(c)	A(ED)B + F(G)H = 19 + 15 = 34 AF + B(K)H = 16 + 18 = 34 A(G)H + B(DE)F = 29 + 11 = 40 Arcs AF, BK, KH or AE, ED, DB, FG, GH will be traversed twice Route length = 196 + 34 = 230 (mins)	M1 A1ft A1ft A1ft A1A1 A1
		(7)

Notes:

(a)

M1: A larger value replaced by a smaller value at least once in the working values at either B or H or K

A1: All values in J, E, F and D correct and the working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value at F of 22

A1: All values in B and G correct and the working values in the correct order. Penalise order of labelling only once per question (B and G must be labelled in that order and B must be labelled after J, E, F, D). Condone an additional working value of 20 at B and an additional working value of 26 at G

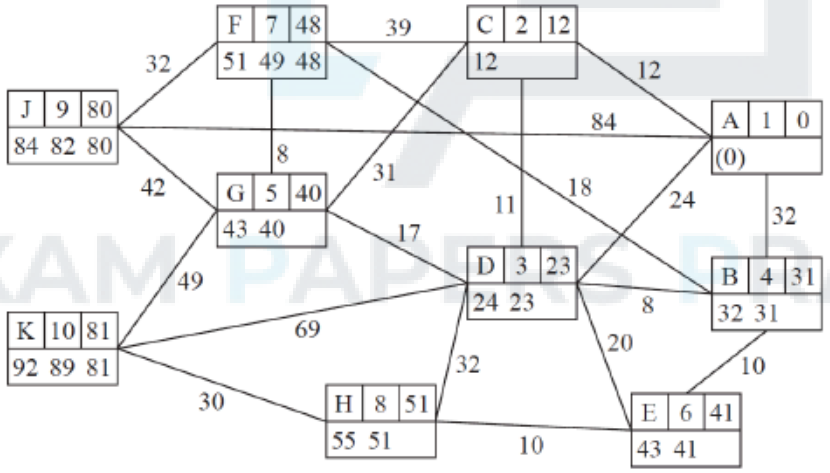
A1ft: All values in H and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (H and K must be labelled in that order and H labelled after all other nodes (excluding K))

A1: CAO (AGHK)

A1ft: Follow through on their final value at K – if their answer is not 32 follow through their final value at K (condone lack of units)

(b)	
B1:	CAO (BDEAGHK)
B1ft:	51 or their final value at B + their final value at K (condone lack of units)
(c)	
M1:	Three distinct pairings of the correct four odd nodes
A1ft:	One row correct including pairing **and** total (the ft on the first three A marks in (c) is for using their final values at B, F and H from (a) for the lengths of AB, AF and AH only)
A1ft:	Two rows correct including pairing **and** totals
A1ft:	All three rows correct including pairing **and** totals
A1:	CAO one combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)
A1:	CAO both combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)
A1:	CAO (230)

Q3.

Question Number	Scheme	Marks
(a)	Start at A e.g. we would be able to find the shortest distance from A to every other vertex e.g. A appears in both required routes	B1 B1dep **(2)**
(b)		M1 A1 (ACDBG) A1 (EFH) A1ft (JK)
	Shortest path to J: A C D G F J Length: 80 (miles) Shortest path to K: A C D B E H K Length: 81 (miles)	A1 A1 A1ft **(7)**
(c)	Route from F to H via A: F G D C A C D B E H	B1 **(1)**
		10 marks
Notes for Question		

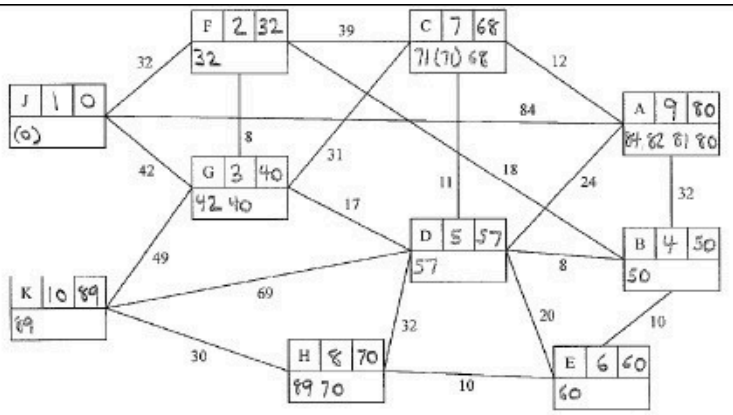
a1B1: CAO (A) – if more than (vertex) A stated then B0
a2B1dep: Correct reason for starting at A (dependent on first B mark) – either need to expicitly mention that A appears on both routes **or** if starting at A then the shortest route to all other vertices (or just to vertices J and K) can be found

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 51 49 48 in that order (so 51 48 49 is incorrect)
It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

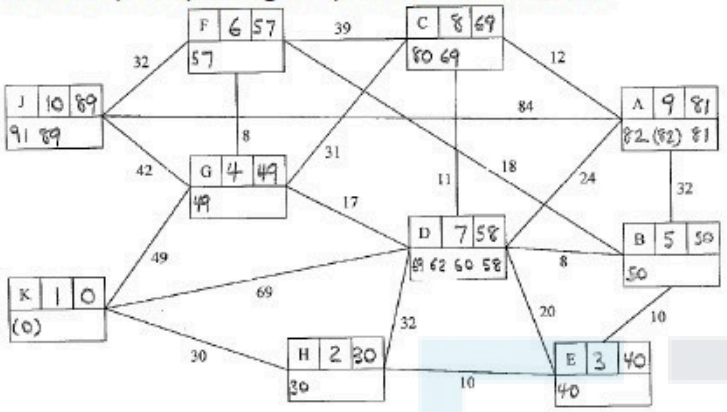
b1M1: Any larger working value replaced by any smaller working value at any two nodes except A and C (for example, **if** correct at B, 32 is replaced by 31 which is a larger value being replaced by a smaller value at one node – as this is a method mark the values do not need to be correct)
b1A1: All values at A, C, D, B and G correct and the working values in the correct order (including order of labelling so nodes must be labelled in the order A, then C, then D, then B, then G). Condone lack of a zero as a working value at A
b2A1: All values at E, F and H correct and the working values in the correct order. Penalise order of labelling only once per question (so E, F and H must be labelled in that order and E must be labelled after A, C, D, B and G)
b3A1ft: All values in J and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are A, F and G). For example, **if** correct then the order of labelling of nodes A, F and G is 1, 7 and 5 respectively so the working values at J should come from A, G and F in that order. The first working value at J should be 0 (the Final value at A) + 84 (the weight of the arc AJ), the second working value at J should be 40 (the Final value at G) + 42 (the weight of the arc GJ) and the final working value at J should be 48 (the Final value at F) + 32 (the weight of the arc FJ). Repeat the process for K (which will have working values from D, G and H with the order of these nodes determined by the candidate's order of labelling at D, G and H)
b4A1: One correct route (either ACDGFJ or ACDBEHK) – allow if reversed (e.g. JFGDCA) and allow if stated in terms of arcs (e.g. AC, CD, DG, GF, FJ)
b5A1: Both routes correct (as for **b4A1** – routes can be reversed and accept in terms of arcs)
b6A1ft: Both lengths correct following through **their** Final values at J and K. Condone correct answers or correct answers following through their diagram even if not explicitly clear which value refers to which path

c1B1: Correct answer only (FGDCACDBEH or FG, GD, DC, CA, AC, CD, DB, BE, EH) – if stated in terms of arcs then arcs AC and CD must appear twice in their route

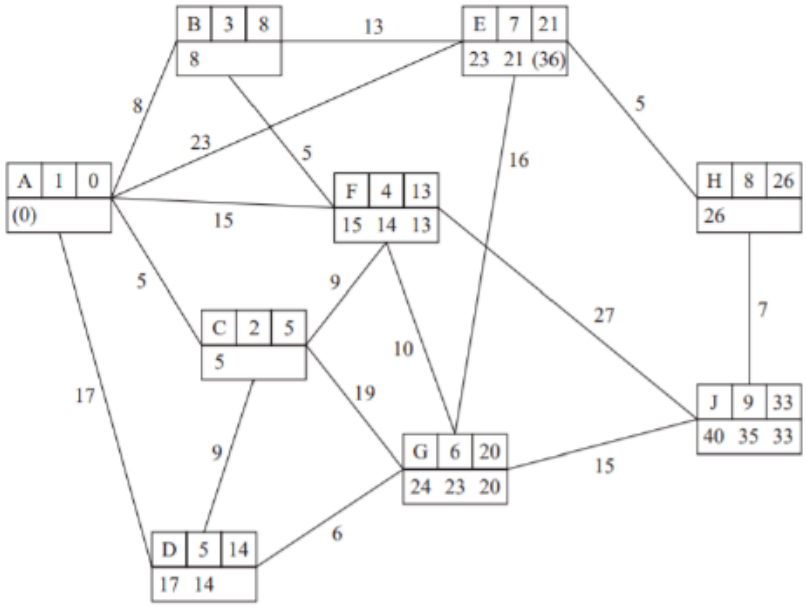
If Dijkstra is completed twice (from J **and** K) then full marks can be awarded. If the candidate uses **just** J or K as the starting vertex in **(b)** then this is not a misread. The candidate can score if starting at J: M1 (as above), A1 (for correct values at J, F, G, B, D), A0, A0, A1 (for route JFGDCA), A1 (for length 80), A0 so 4 out of 7 max.



If starting at K: M1 (as above), A1 (for correct values at K, H, E, G, B), A0, A0, A1 (for route KHEBDCA), A1 (for length 81), A0 so 4 out of 7 max.



Q4.

Question Number	Scheme	Marks
(a)	A path is a (i) finite sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once	B2, 1, 0 **(2)**
(b)	 Shortest path: A B E H J Length: 33 (km)	M1 A1 (ACBFD) A1 (GE) A1ft (HJ) A1 A1ft **(6)**
(c)	Shortest path from J to A via G: J G D C A Length: 20 + 15 = 35 (km)	B1 B1ft **(2)**
		10 marks

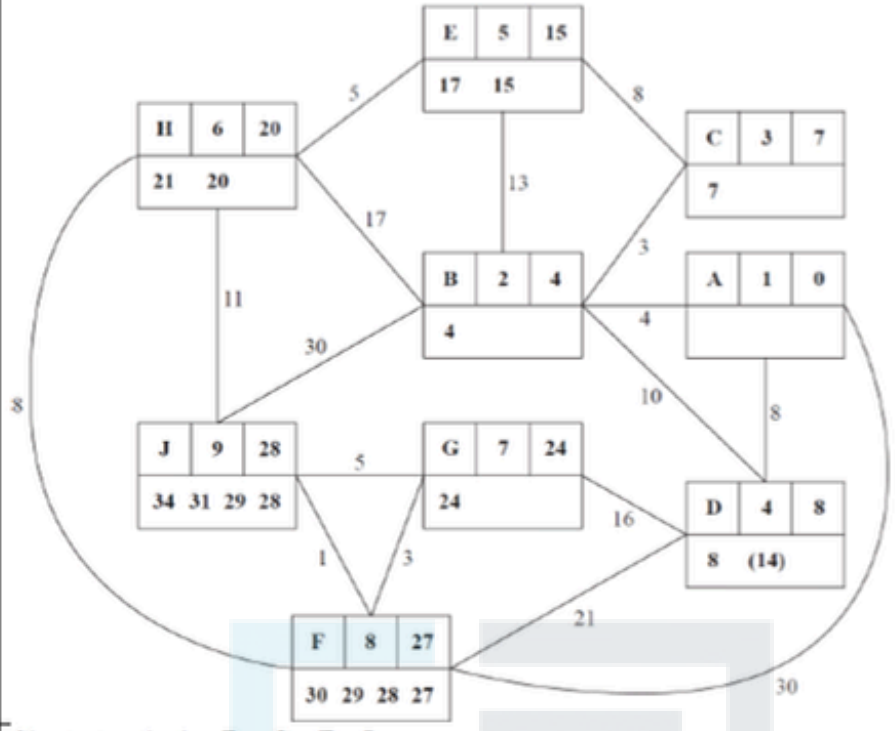
Question Number	Scheme	Marks
	Notes for Question	

a1B1: One of the three points made clearly ('finite, edges', 'end vertex of one edge is the start vertex of the next', 'no vertex appears more than once')
a2B1: All three points made clearly. Candidates who state that a path is a walk in which no vertex appears more than once can score B1B0 only

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 15 14 13 in that order (so 15 13 14 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

b1M1: Working values - a larger value replaced by a smaller value for at least two of the five activities D, E, F, G, J
b1A1: All values at A, C, B, F and D correct and the working values in the correct order
b2A1: All values at G and E correct and the working values in the correct order
b3A1ft: All values in H and J correct on the follow through and the working values in the correct order
b4A1: cao (A B E H J only)
b5A1ft: Follow through on their final value at J **only** (condone lack of units)
c1B1: cao (J G D C A only)
c2B1ft: 35 or follow through their final value at G + 15

Q5.

Question number	Scheme	Marks
(a)		M1 A1 (BCDE) A1 (HG) A1ft (FJ)
	Shortest path: A – D – G – F – J	A1
	Length: 28 (miles)	A1ft (6)
(b)	Shortest path: J – F – G – D – A – B – C – E – H	B1
	Length: 28 + 20 = 48 (miles)	B1ft (2)
(c)	A(BC)E + FG = 15 + 3 = 18* A(DG)F + E(HF)G = 27 + 16 = 43 A(D)G + E(H)F = 24 + 13 = 37	M1 A1 A1
	Repeat arcs: AB, BC, CE, FG	A1
	Length: 193 + 18 = 211 (miles)	A1ft (5)
(d)	EF (13) is the shortest link between two odd nodes excluding G Repeat EF (13) since this is the shortest path excluding G	M1
	We finish at A	A1
	Length of route = 193 + 13 = 206 (miles)	A1 (3)
		(16 marks)

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 21 20 in that order (so 20 21 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**

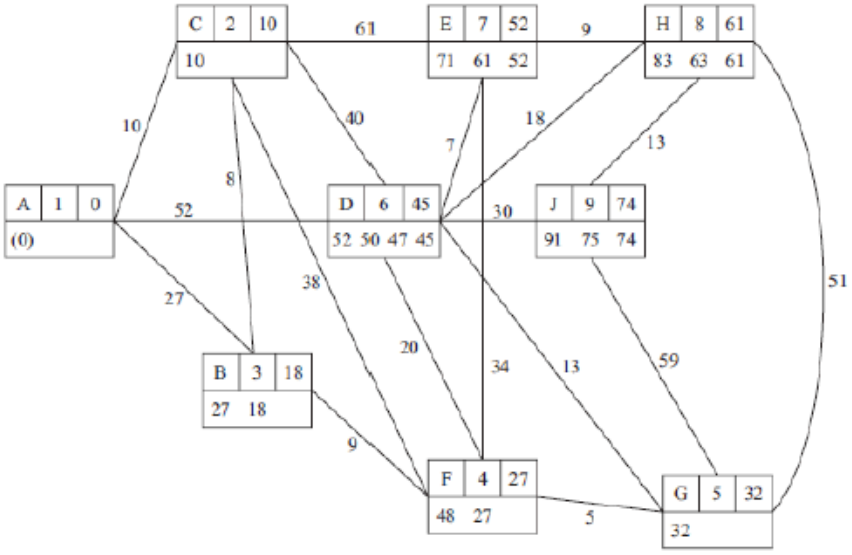
a1M1: A larger value replaced by a smaller value at least twice in the working values at either E, F, H or J
a1A1: All values at B, C, D and E correct and the working values in the correct order
a2A1: All values at H and G correct and the working values in the correct order
a3A1ft: All values in F and J correct on the follow through and the working values in the correct order. To follow through F check that the working values at F follow from the candidate's final values for the nodes that are directly attached to F (which are A, D, H, G (and J)). For example, **if** correct then the order of labelling of nodes A, D, H and G are 1, 4, 6 and 7 respectively so the working values at F should come from A, D, H and G in that order. The first working value at F should be 30 (from A), the second working value at F should be their 29 (the Final value at D) + 21 (the weight of the arc DF), the third working value at F should be their 20 (the Final value at H) + 8 (the weight of the arc HF) and the fourth working value at F should be their 24 (the Final value at G) + 3 (the weight of the arc FG). Repeat the process for J (which will have working values from B, H, G and F with the order of these nodes determined by the candidate's order of labelling at B, H, G and F)
a4A1: CAO (ADGFJ or AD, DG, GF, FJ but **not** JFGDA or equivalent from J to A)
a5A1ft: Follow through their final value at J **only** – if their answer is 28 but this is not the Final Value at J then A0

b1B1: CAO for the route (JFGDABCEH or JF, FG, GD, DA, AB, BC, CE, EH)
b2B1ft: 48 or follow through their final value at J + their final value at H

c1M1: Three distinct pairings of the nodes A, E, F and G
c1A1: Any two rows correct including pairings **and** totals
c2A1: All three rows correct including pairings **and** totals
c3A1: CAO - correct arcs clearly stated and not just in their working as AB, BC, CE and FG (allow BA, CB, etc.) – must be these arcs. Do not accept ABCE or AE via B and C
c4A1ft: Correct answer of 211 **or** follow through 193 + their least total from a choice of three

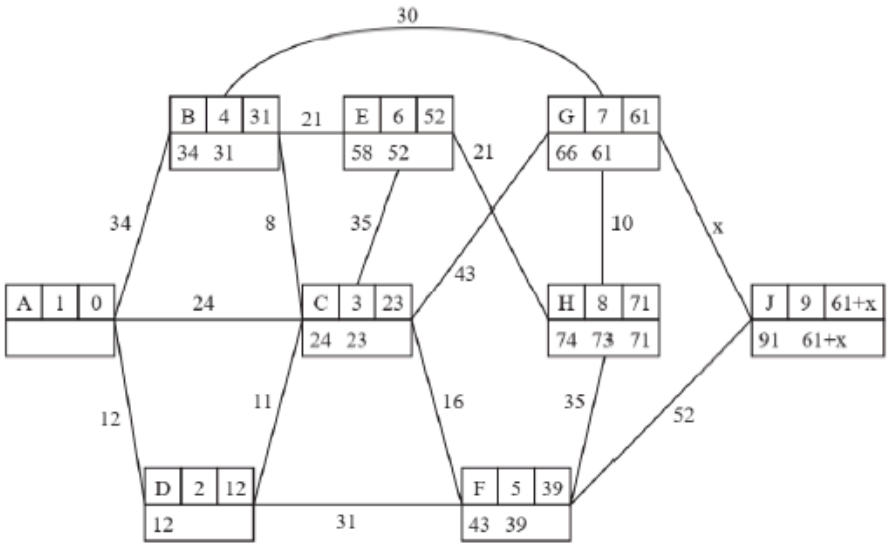
d1M1: Identifies the need to repeat one path of the three (AE, AF, EF) which does not include G (maybe implicit) or listing of only these possible repeats – **this mark is dependent on either scoring the M mark in (c) or stating all three possible paths**. If stating more than these three paths (AE, AF, EF) then it must be clear from later working that they are only considering these three. **As a minimum stating just one of these three paths (or any combination of these three paths with no others) can score this mark (so, for example, just stating AE and AF scores this mark) provided that they do not further imply that a path including G should be repeated (as this would indicate that mentioning one (or more) of these paths is for the purpose of not repeating it)**
d1A1: Identifies EF as the least **and** A as the finishing point. They have to explicitly state that **EF** is the **least** path that **does not include G**
d2A1: CAO (206)

Q6.

Question Number	Scheme	Marks
(a)		M1 A1 (ACBFG) A1 (DE) A1ft (HJ)
	Shortest time to travel from A to J is 74 (mins)	A1ft
	Quickest route is ACBFGDEHJ	A1 (6)
(b)	As the quickest route found in (a) is a path through **all** vertices in the network the weight of this path is equal to the weight of the MST for the network	M1
	Weight of MST is 74 (mins)	A1ft (2)
		8 marks

	Notes
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 71 61 52 in that order (so 71 52 61 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling	
a1M1	A larger value replaced by a smaller value at least twice in the working values at either B, D, E, F, H, J
a1A1	All values at A, C, B, F and G correct and the working values in the correct order
a2A1	All values at D and E correct and the working values in the correct order
a3A1ft	All values in H and J correct on the follow through and the working values in the correct order. To follow through H check that the working values at H follow from the candidate's final values for the nodes that are directly attached to H (which are D, E and G (and J)). For example, **if** correct then the order of labelling of nodes D, E and G are 6, 7 and 5 respectively so the working values at H should come from G, D and E in that order. The first working value at H should be their 32 (the Final value at G) + 51 (the weight of the arc GH), the second working value at H should be their 45 (the Final value at D) + 18 (the weight of the arc DH) and the third working value should be their 52 (the Final value at E) + 9 (the weight of the arc EH) . Repeat the process for J (which will have working values from D, G and H with the order of these nodes determined by the candidate's order of labelling at D, G and H)
a4A1ft	Follow through their final value at J **only** – if answer is 74 but this is not the Final Value at J then A0
a5A1	CAO (ACBFGDEHJ)
b1M1	A statement that the route found in (a) is the MST, either stated directly, or an indication that the route has visited all 9 nodes.
b1A1ft	Follow through their answer to (a) provided that the route from (a) passed through all nine nodes in the network (a weight of 74 with no reason scores M0)

Q7.

(a)	 Fastest time: 71 (minutes) Quickest route: ADCBGH	M1 A1 (DCB) A1 (FE) A1ft (GH) A1ft A1 (6)
(b)	AD + EH = 12 + 21 = 33*	M1 A1
	A(DCB)E + D(CBG)H = 52 + 59 = 111	A1
	A(DCBG)H + D(CB)E = 71 + 40 = 111	A1
	$(383+x)+33\ldots440 \Rightarrow x\ldots\ldots$	depM1
	$24 \leqslant x < 30$	A1 (6)
(c)	e.g. ABCEBGCDADFJGHEHFCA	B1 (1)
(d)	If a direct road DH opens then only A and E are odd therefore the shortest inspection route is (383 + x) + 25 (DH) + 52 (AE)	M1
	$460 + x = 488$ therefore $x = 28$	A1 (2)
		15 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 74 73 71 in that order (so 74 71 73 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**

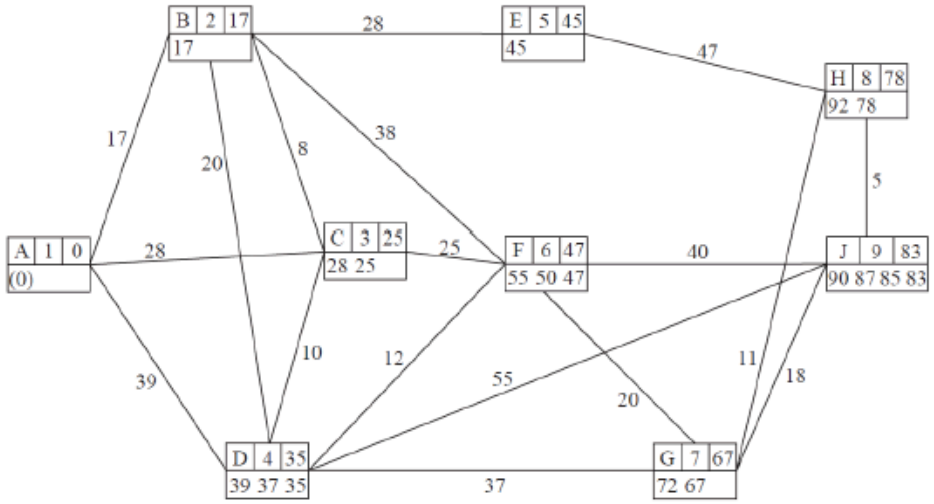
a1M1: A larger value replaced by a smaller value at least twice in the working values at either B, C, E, F, G, H
a1A1: All values at D, C and B correct and the working values in the correct order
a2A1: All values at F and E correct and the working values in the correct order
a3A1ft: All values in G and H correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are C and B). For example, **if** correct then the order of labelling of nodes C and B are 3 and 4 respectively so the working values at G should come from C and B in that order. The first working value at G should be their 23 (the Final value at C) + 43 (the weight of the arc CG), the second working value at G should be their 31 (the Final value at B) + 30 (the weight of the arc BG). Repeat the process for H (which will have working values from F, E and G with the order of these nodes determined by the candidate's order of labelling at F, E and G)
a4A1ft: Follow through their final value at H **only** – if answer is 71 but this is not the Final Value at H then A0
a5A1: CAO (ADCBGH)

b1M1: Three distinct pairings of the nodes A, D, E and H
b1A1: Any one row correct including pairing **and** total
b2A1: Any two rows correct including pairings **and** totals
b3A1: All three rows correct including pairings **and** totals
b2dM1: $(383 + x)$ + (their least pairing total) with any inequality sign or equal to 440 – dependent on first M mark in (b). Give bod if not all totals are shown (so if they only give two totals then they should be using the least of these two) but they must have shown all three distinct pairings of the four odd nodes
b4A1: cao $(24 \leqslant x < 30)$ – condone $24 \leqslant x \leqslant 29$

c1B1: cao (check: starting and finishing at A, 19 nodes, AD and EH repeated in route, with A(3), B(2), C(3), D(2), E(2), F(2), G(2), H(2), J(1)) – can be given in terms of arcs

d1M1: $(383 + x) + 25$ + their 52 (where 'their 52' must be the length of their shortest path from A to E in either (a) **or** (b) **or** they state/imply the shortest path from A to E is 52) - a correct value of 28 with no working can imply this mark only
d1A1: cao (28) from correct working **and** correct reasoning that A and E are the only odd nodes **or** that we only need to pair A and E (as a minimum accept mention of A and E only but ignore any mention of the new direct road from D to H)

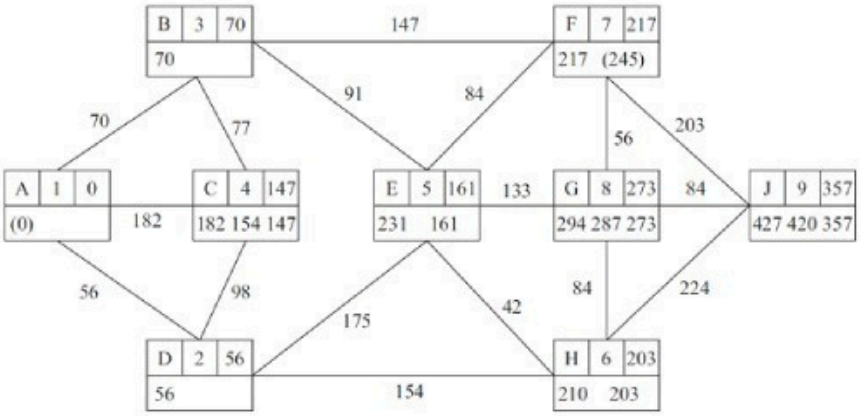
Q8.

(a)(i)		M1 A1 (ABCDE) A1 (FG) A1ft (HJ)
	Shortest path from A to J is ABCDFGHJ	A1
(a)(ii)	Length of shortest path from A to J is 83 (km)	A1ft (6)
(b)	B(CD)F + HJ = 30 + 5 = 35* B(CDFG)H + F(GH)J = 61 + 36 = 97 B(CDFGH)J + F(G)H = 66 + 31 = 97	M1 A1 A1
	Repeat arcs: BC, CD, DF and HJ	A1
	Route length is 458 + 35 = 493 (km)	A1ft (5)
(c)	The shortest path between any of the pairs of the four odd nodes (A, B, F and H) is AB (17)	M1
	So start at F and finish at H (or vice-versa)	A1
	Length of route is 458 + 17 = 475 (km)	B1 (3)
		14 marks
	Special Case for Part B – using A B F H – Mark as Misread	
(b)	AB + F(G)H = 17 + 31 = 48* A(BCD)F + B(CDFG)H = 47 + 61 = 108 A(BCDFG)H + B(CD)F = 78 + 30 = 108	M1 A1 A1
	Repeat arcs: AB, FG and GH	A1
	Route length is 458 + 48 = 506 (km)	A1ft (5)
	Remove final two A marks earned in this section so max 3/5	

	Notes for Question	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at D the working values must be 39 37 35 in that order (so 39 35 37 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**	
a1M1	A larger value replaced by a smaller value in the working values of at least two of the nodes C, D, F, G, H, J	
a1A1	All values at A, B, C, D and E correct and the working values in the correct order	
a2A1	All values at F and G correct and the working values in the correct order	
a3A1ft	All values in H and J correct on the follow through and the working values in the correct order. To follow through H check that the working values at H follow from the candidate's final values for the nodes that are directly attached to H (which are E and G). For example, if correct then the order of labelling of nodes E and G are 5 and 7 respectively so the working values at H should come from E and G in that order. The first working value at H should be their 45 (the Final value at E) + 47 (the weight of the arc EH), the second working value at H should be their 67 (the Final value at G) + 11 (the weight of the arc GH). Repeat the process for J (which will have working values from D, F, G and H with the order of these nodes determined by the candidate's order of labelling at D, F, G and H)	
a4A1	CAO (ABCDFGHJ)	
a5A1ft	Follow through their final value at J **only** – if answer is 83 but this is not the Final Value at J then A0	
b1M1	Three distinct pairings of the nodes B, F, H and J with one row correct (including total)	
b1A1	Any two rows correct including pairings **and** totals	
b2A1	All three rows correct including pairings **and** totals	
b3A1	CAO - correct arcs clearly stated and not just in their working as BC, CD, DF and HJ – must be these arcs. Do not accept BF or B(CD)F or BF via C and D	
b4A1ft	Correct route length (493) or follow through their least repeat + 458	
c1M1	Indicates the need to find the shortest path between any pair of the correct four odd nodes (A, B, F, H)	
c1A1	CAO (F, H)	
c1B1	CAO (475)	

	Special Case for Part B – using A B F H – Mark as Misread	
b1M1	Three distinct pairings of the nodes A, B, F and H with one row correct (including total)	
b1A1	Any two rows correct including pairings **and** totals	
b2A1	All three rows correct including pairings **and** totals	
b3A1	CAO - correct arcs clearly stated and not just in their working as AB, FG and GH – must be these arcs. Do not accept FH or F(G)H or FH via G	
b4A1ft	Correct route length (506) or follow through their least repeat + 458	
	Remove final two A marks earned in this section so max 3/5	

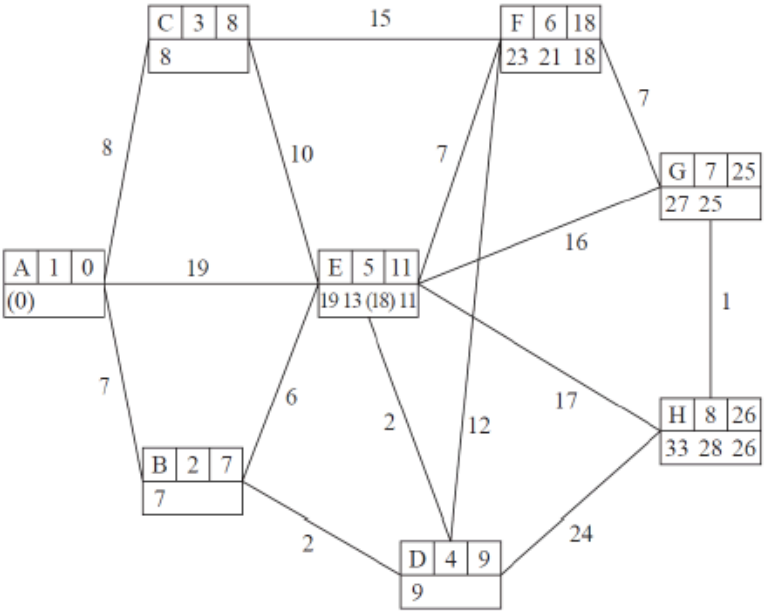
Q9.

Question Number	Scheme	Marks
(a) (i)		M1 A1 (ADBCE) A1 (HF) A1ft (GJ)
	Shortest path from A to J is ABFGJ	A1
(a) (ii)	Shortest length is 357 (metres)	A1ft **(6)**
(b)	Consider all pairings of A, C, E and J	M1
	AC + EJ 147 + 210 = 357 AE + CJ 161 + 364 = 525 AJ + CE 357 + 168 = 525	A1ft A1ft
	Length of route = 1 960 + 357 = 2 317 (metres)	A1 ft **(4)**
(c)(i)	Consider arcs between odd nodes not using A – shortest CE so repeat. Finish at J	M1 A1
(c)(ii)	Difference in length their "357" – 168 = 189 metres or their "2317" – 2128 = 189 (metres)	A1ft **(3)**
		13 Marks

Notes for Question	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 182 154 147 in that order (so 182 147 154 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**
ai1M1	A larger value replaced by a smaller value at least twice in the working values at either C, E, G, H, J
ai1A1	All values at A, D, B, C and E correct and the working values in the correct order
ai2A1	All values at H and F correct and the working values in the correct order
ai3A1ft	All values in G and J correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H (and J)). For example, **if** correct then the order of labelling of nodes E, F and H are 5, 7 and 6 respectively so the working values at H should come from E, H and F in that order. The first working value at G should be their 161 (the Final value at E) + 133 (the weight of the arc EG), the second working value at G should be their 203 (the Final value at H) + 84 (the weight of the arc HG) and the third working value should be their 217 (the Final value at F) + 56 (the weight of the arc FG) . Repeat the process for J (which will have working values from F, G and H with the order of these nodes determined by the candidate's order of labelling at F, G and H)

ai4A1	CAO (accept route written in arcs)
aii1A1ft	Follow through their final value at J **only** – if answer is 357 but the final value at J is not 357 then A0
b1M1	The correct three pairings of the correct four nodes (A, C, E and J)
b1A1ft	Two rows correct including pairings **and** totals. Ft for AC, AE and AJ only using their values from (a) but allow recovery if correct totals seen here
b2A1ft	All three rows correct including pairings **and** totals. Ft for AC, AE and AJ only using their values from (a) but allow recovery if correct totals seen here
b3A1ft	1960 + their smallest from a choice of three totals
ci1M1	Considers arcs between odd nodes not including A (e.g. listing possible arcs to be repeated CE CJ and EJ) (A statement that CE is the shortest implies this mark)
ci1A1	Clear indication that CE is shortest (e.g. a statement or tick or starring of CE, not just using 168) (we do not need to see all values explicitly listed here) and therefore finish at J
cii1A1ft	**Dependent on M mark in (b)** 189 or their "357" – 168

Q10.

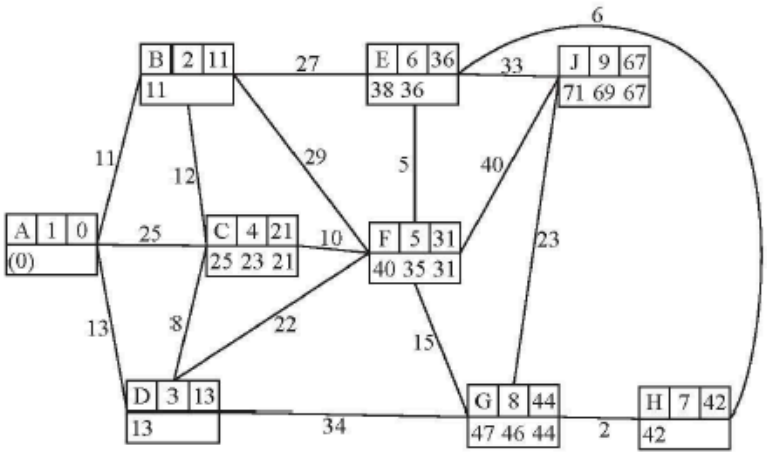
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td> 
</td><td>M1
A1 (ABCD)
A1 (EF)
A1ft (GH)

dM1

A1 (6)</td></tr>
<tr><td></td><td>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th><th>G</th><th>H</th></tr>
<tr><td>A</td><td>-</td><td>7</td><td>8</td><td>9</td><td>11</td><td>18</td><td>25</td><td>26</td></tr>
<tr><td>B</td><td>7</td><td>-</td><td>14</td><td>2</td><td>4</td><td>11</td><td>18</td><td>19</td></tr>
<tr><td>C</td><td>8</td><td>14</td><td>-</td><td>12</td><td>10</td><td>15</td><td>22</td><td>23</td></tr>
<tr><td>D</td><td>9</td><td>2</td><td>12</td><td>-</td><td>2</td><td>9</td><td>16</td><td>17</td></tr>
<tr><td>E</td><td>11</td><td>4</td><td>10</td><td>2</td><td>-</td><td>7</td><td>14</td><td>15</td></tr>
<tr><td>F</td><td>18</td><td>11</td><td>15</td><td>9</td><td>7</td><td>-</td><td>7</td><td>8</td></tr>
<tr><td>G</td><td>25</td><td>18</td><td>22</td><td>16</td><td>14</td><td>7</td><td>-</td><td>1</td></tr>
<tr><td>H</td><td>26</td><td>19</td><td>23</td><td>17</td><td>15</td><td>8</td><td>1</td><td>-</td></tr>
</table>
</td><td></td></tr>
<tr><td>(b)</td><td>NNA: A – B – D – E – F – G – H – C – A</td><td>B1</td></tr>
<tr><td></td><td>7 + 2 + 2 + 7 + 7 + 1 + 23 + 8 = 57 (km)</td><td>B1 (2)</td></tr>
<tr><td>(c)(i)</td><td>Prim (starting at C): CE, DE, BD, EF, FG, GH</td><td>M1 A1</td></tr>
<tr><td></td><td>RMST weight = 10 + 2 + 2 + 7 + 7 + 1 = 29</td><td></td></tr>
<tr><td>(c)(ii)</td><td>29 + 7(AB) + 8(AC) = 44 (km)</td><td>M1 A1 (4)</td></tr>
<tr><td>(d)</td><td>44 ⩽ optimal distance ⩽ 57</td><td>B2, 1, 0 (2)</td></tr>
<tr><td></td><td></td><td>14 marks</td></tr>
</table>

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 23 21 18 in that order (so 23 18 21 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling
a1M1: Any larger working value replaced by any smaller working value at at **least two nodes** except A, B, C and D **a1A1:** All values at A, B, C and D correct and the working values in the correct order. Condone lack of a zero as a working value at A **a2A1:** All values at E and F correct and the working values in the correct order. Penalise order of labelling only once per question (so E and F must be labelled in that order and E must be labelled after D) **a3A1ft:** All values at G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H). For example, if correct then the order of labelling of nodes E, F and H are 5, 6 and 8 respectively so the working values at G should come from E and F in that order. The first working value at G should be 18 (the Final value at F) + 7 (the weight of the arc FG) and the second working value at G should be 11 (the Final value at E) + 16 (the weight of the arc EG). Repeat the process for H (which will have working values from D, E and G with the order of these nodes determined by the candidate's order of labelling at D, E and G) **a2M1:** Correct entries in the table following through their final values – dependent on the previous M mark (need only fill in either the A row or A column) **a4A1:** cao **b1B1:** Correct nearest neighbour route starting and finishing at A (A – B – D – E – F – G – H – C – A) **b2B1:** cao (57) on length of route **ci1M1:** First three arcs (CE, DE, BD) **or** all 7 nodes {C, E, D, B, F, G, H} correctly chosen in order. If any explicit rejections seen then M1only in (c)(i). Order of nodes may be seen at the top of the matrix/table {4, 1, 3, 2, 5, 6, 7}. Award M0 for a correct tree with no working. Award M1 only for the first three arcs (oe) selected correctly if starting at a different node than C. If correct values circled in the table but no indication of order of selection then M0 **ci1A1:** cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen). If AB and AC added explicitly in (c)(i) then A0 but can score both marks in (c)(ii) **cii2M1:** Weight of RMST + 7 + 8 (two smallest arcs incident to A) with $19 \leqslant$ RMST $\leqslant 39$ (if clearly not six arcs in RMST then M0). Give bod if 15 is added to the total of six values circled in the table provided those six values sum to a value between 19 and 39 inclusive **cii2A1:** cao (44) – if correct RMST stated in (c)(i) followed by 44 (with no additional working) then award M1A1 in (c)(ii). This mark is dependent on Prim's algorithm being used to find the RMST (allow this mark if rejections seen in (c)(i) when applying Prim). So in (c) M1A0M1A1 is possible e.g. if only stating the node (instead of the arc) selection in order when applying Prim. If the correct six values are circled in the table and added to 15 to give 44 but the order of arc/node selection is not stated (so no indication that Prim has been applied) then A0 (as the qu. says, 'Hence…') **d1B1:** **Any** indication of an interval from their answer to **(c)(ii)** to their answer to **(b)** with one value correct (e.g. $44 \sim 57$ scores B1 but $57 \sim 44$ or $57 \leqslant$ optimal distance $\leqslant 44$ scores B0). **If correct route seen in (b) but the upper bound not stated in (b) allow recovery in part (d) it stated here (but still withold the second mark in (b)).** **d2B1:** cao ($44 \leqslant$ optimal distance $\leqslant 57$) including correct inequalities (allow $44 <$ optimal distance $\leqslant 57$) – allow interval notation e.g., [44, 57] or (44, 57]

Q11.

Question Number	Scheme	Marks
(a)(i)		M1 A1 (ABDC) A1 (FE) A1ft (HGJ)
	Shortest path from A to J: ADCFEHGJ	A1
(a)(ii)	Length of shortest path from A to J: 67 (miles)	A1ft **(6)**
(b)	AC + EJ = A(D)C + E(HG)J = 21 + 31 = 52	M1 A1
	AE + CJ = A(DCF)E + C(FEHG)J = 36 + 46 = 82	
	AJ + CE = A(DCFEHG)J + C(F)E = 67 + 15 = 82	A1
	Route length is 315 + 52 = 367 (miles)	A1ft **(4)**
(c)	Pass through G a total of 3 times	B1 **(1)**
(d)	Difference in inspection routes is 67 – 52 = 15 (miles)	B1ft **(1)**
(e)	Arcs CF and EF do not need to be repeated	B1 **(1)**
		13 marks
	Notes for Question	
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 40 35 31 in that order (so 40 31 35 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**		

a1M1: A larger value replaced by a smaller value at least twice in the working values at either C, E, F, G or J
a1A1: All values at A, B, D and C correct and the working values in the correct order
a2A1: All values at F and E correct and the working values in the correct order
a3A1ft: All values in H, G and J correct on the follow through and the working values in the correct order.
To follow through G say check that the working value(s) at G follow from the candidate's final values for the nodes that are directly attached to G (which are D, F, H and J). For example, **if** correct then the order of labelling of nodes D, F and H are 3, 5 and 7 respectively so the working values at G should come from D, F and H in that order. The first working value at G should be their 23 (the Final value at D) + 34 (the weight of the arc DG), the second working value at G should be their 31 (the Final value at F) + 15 (the weight of the arc FG) and the third working value at G should be their 42 (the Final value at H) + 2 (the weight of arc GH). Repeat this exact process for H and J for the follow through for this mark
a4A1: Correct shortest path from A to J (ADCFEHGJ) **only** – not from J to A
a5A1: Follow through their final value at J **only** (condone lack of units) – if their answer is 67 but this is not their Final Value at J then **A0**

Condone for the final two marks in (a) the 'shortest path' and 'length of shortest path' written on the wrong lines

b1M1: Three distinct pairings of nodes A, C, E and J
b1A1: Any one row correct including pairing **and** total
b2A1: All three rows correct including pairings **and** totals
b3A1ft: Correct route length (367) from the correct pairing or follow through 315 + their least total from a choice of three

c1B1: CAO (3)

d1B1ft: Correct answer of 15 either from correct or no incorrect working or follow through (their Final value at J from (a) – their least repeat from (b)) – **this mark is dependent on having scored both M marks in (a) and (b)**

e1B1: CAO (CF, EF only)

Q12.

(a)	Kruskal: AB(6), BP(10), CW(11), CP(12), HM(14), AH(15), reject CH(17), reject AC(18), reject AP(20), reject MW(21), LY(21), AS(26), LS(28) (not BS, LM, HL, SY, AL)	M1 A1 A1 **(3)**
(b)	Prim: AB, BP, CP, CW, AH, HM, AS, LS, LY	M1 A1 A1 **(3)**
(c)	143 (miles)	B1 **(1)**
(d)	286 (miles)	B1ft **(1)**
(e)	NNA starting at W: W – C – P – B – A – H – M – L – Y – S – W 11 + 12 + 10 + 6 + 15 + 14 + 40 + 21 + 48 + 55 = 232	M1 A1 **(2)**
(f)	The best upper bound is the one starting at Y as 212 is less than both 232 and 286	B1 **(1)**
(g)	(143 – 11) + 11 + 21 = 164 (miles)	M1 A1 **(2)**
(h)	WCPBAHMLYSACW	B1 **(1)**
		14 marks

Notes for Question
a1M1: First four arcs (AB, BP, CW, CP) correctly chosen and at least one rejection seen at some point **a1A1:** All arcs in tree selected correctly and in the correct order (AB, BP, CW, CP, HM, AH, LY, AS, LS) – no other arcs in MST **a2A1:** cso including all rejections correct and at the correct time – **note that LY can be accepted before MW is rejected**. We do not need to see the explicit rejection of arcs BS to AL but if these are explicitly rejected then they must be in the correct order. Note that a list of all the arcs in the correct order followed by a list of the arcs in the MST can score full marks **b1M1:** First three arcs correctly chosen in order (AB, BP, CP,…) **or** first four nodes {A, B, P, C, …} correctly chosen in order. If any **explicit** rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 2, 4, - , - , -, 3, -, -, -} so do check carefully for this. Starting at any other node can score M1 only for first three arcs chosen correctly **b1A1:** First six arcs correctly chosen in order (AB, BP, CP, CW, AH, HM,…) **or** all ten nodes {A, B, P, C, W, H, M, S, L, Y} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 2, 4, 6, 9, 7, 3, 8, 5, 10} (**no** missing numbers) **b2A1:** cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) **c1B1:** cao (143) – this mark can be awarded if seen in (b) (although if answered in (c) too then mark according to the answer given in (c)) **d1B1ft:** Follow through double their answer from (c) **e1M1:** Nearest neighbour route starting at W – must have at least W – C – P – B – A – H – … allow if stated in terms of arcs **e1A1:** CAO on length (232) **and** route (must return to W and can be stated in terms of arcs) **f1B1:** An indication that 212 is the minimum (of 212 and the answers to (d) and (e)) – this mark is dependent on the correct values in (d) and (e) so accept an answer of the form 'the one starting at Y (or the route with weight 212) as it is the least' – we do not need to see explicit mention of the values in (d) and (e) provided they are correct in (d) and (e) **g1M1:** (weight of their MST from (c) **or** (b) **or** 132 **only**) – 11 + 11(WC) + 21(MW) (oe so may not see the –11 + 11). A correct answer of 164 can imply this (and the next) mark **g1A1:** 164 **h1B1:** cao – either the route must be written out in full (in terms of nodes or arcs) **or** they must make it absolutely clear that the route begins exactly as in (e) (which must therefore be correct) but after S, towns A and C are visited before (returning to) W. Just stating that A, C, W are visited twice (or similar) is B0

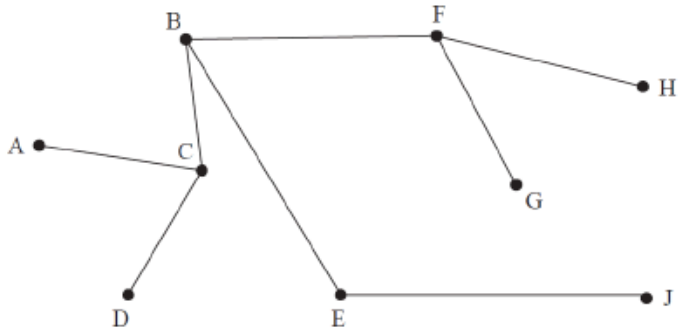
Q13.

Question Number	Scheme	Marks
(a)	Prim: AH, FH, EH, FG, DG, CG, BC	M1 A1 A1 **(3)**
(b)	Initial upper bound 2(201) = 402 (km)	B1ft **(1)**
(c)	A – H – F – G – D – B – C – E – A 27 + 28 + 31 + 29 + 32 + 26 + 38 + 37 = 248	M1 A1 **(2)**
(d)	Nearest neighbour starting at E has a length of $212 + x$ As $x \leqslant 35 \Rightarrow$ the NN route starting at E is at most 247 (km) and therefore the NN starting at E gives the better upper bound as it is less than the one starting at A (which was 248 (km))	M1 A1 **(2)**
(e)	Lower bound is given by $(201-27)+27+x=235$	M1
	$x = 34$	A1
	$235 \leqslant$ optimal length $\leqslant 246$	M1 A1 **(4)**
		12 marks

Notes for Question
a1M1: Prim's – first three arcs correctly chosen in order (AH, FH, EH, …) **or** first four nodes {A, H, F, E, …} correctly chosen in order. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, -, - , 4 , 3, -, 2} so check there too. Starting at any other node apart from A can score M1 only for first three arcs chosen correctly **a1A1:** First five arcs correctly chosen in order (AH, FH, EH, FG, DG, …) **or** all eight nodes {A, H, F, E, G, D, C, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 7, 6, 4, 3, 5, 2} (**no** missing numbers) **a2A1:** CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) **b1B1ft:** Follow through double the stated length of their MST **c1M1:** NN starting at A – must have at least A – H – F – G – D –… allow if stated in terms of arcs **c2A1:** CAO on length (248) **and** route (must return to A but can be stated in terms of arcs) **d1M1:** Calculating the correct length of the NN route starting at E ($212 + x$) **and** attempting to use the range of values for x to determining the better upper bound (implied by 247 seen or [244, 247]). **d1A1:** Correct best upper bound stated (the one starting at E) together with a correct comparison of **248** (possibly implicit - if this value is not explicitly stated in (d) then 248 must have been seen in (c)) with 247 (or an indication of 'at most' 247). **For those who obtained an answer of 211 in (c) and say that $212 + x$ is always bigger (without using the given interval for x to find the UB) then no marks in this part** **e1M1:** Correct method for calculating x (which is the weight of MST from (a)/(b) – 27 + two smallest arcs incident to A (the 27 and x) equal to 235). If using the doubled value from (b) then **M0**. If not using the weight of the MST from (a) then they must be using either 174 **or** $26 + 30 + 29 + 31 + 28 + 30$ **or** explicitly using the correct six arcs only (BC, CG, GD, GF, FH, HE) so not just circled in one of the tables. The correct value of x (with either no working or no incorrect working) clearly stated can imply this mark **e1A1:** CAO for x (34) – **as a minimum** must have seen the calculation $201 + x = 235$ to award this mark **e2M1:** Any indication of an interval from 235 to either 246, 247 or 248 (this mark is **not** dependent on the previous M mark)

e2A1: CAO (condone $235 <$ optimal length $\leqslant 246$ and allow equivalent interval notation e.g. (235, 246] or [235, 246]) – this mark is dependent on all previous marks in (e) (so must have found that x equals 34). The correct interval (with no others) with no supporting working scores **M0A0M1A0** The minimum requirement for full marks is: $201 + x = 235 \Rightarrow x = 34 \therefore [235, 246]$ For those who simply state $x = 34$ (only) followed by the correct interval they score **M1A0M1A0**

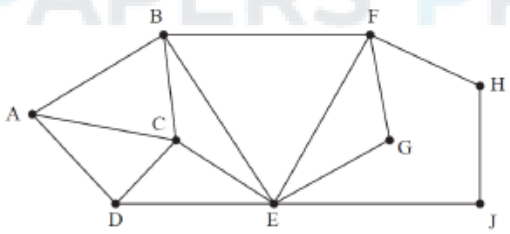
Q14.

Question Number	Scheme	Marks
(a)	e.g. A – B – F – H – J	B1 (1)
(b)	A – B – C – D – E – G – F – H – J is not an example of a tour on T as although it contains every vertex it does not return to A	B1 (1)
(c)	Kruskal: AC(9), BE(11), BF(12), not EF(14), FG(15), FH(17), not EG(18), EJ(20), $\left\{\begin{matrix}\text{not HJ (21)}\\ \text{BC (21)}\end{matrix}\right\}$, not CE(23), not AB(24), CD(25) (not DE, AD)	M1 A1 A1 (3)
(d)		B1 (1)
(e)	130 (km)	B1 (1)
		7 marks

Notes for Question

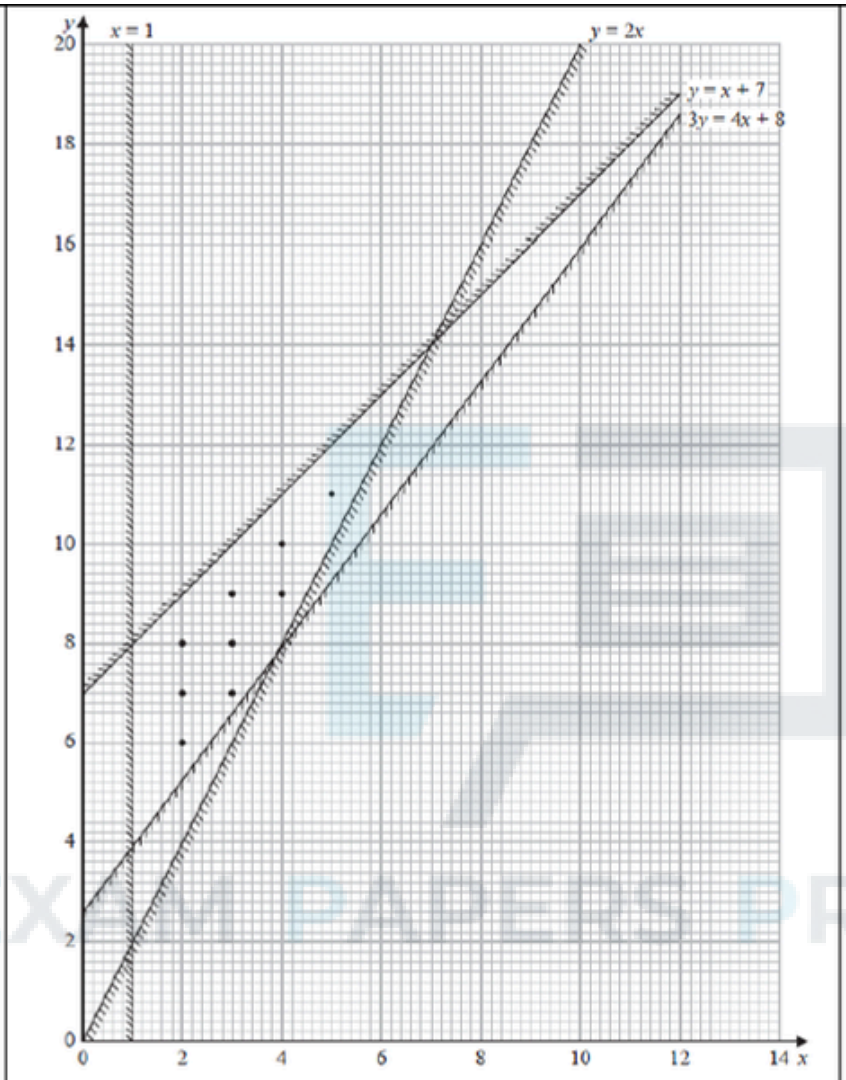
a1B1: Any correct example of a path (so no vertex appearing more than once) from A to J
b1B1: No with a correct reason i.e. must mention that a tour must begin and start at the same vertex, e.g. the route given does not finish at A scores B1
c1M1: Kruskal's algorithm - first four arcs (AC, BE, BF, FG) correctly chosen **and** at least one rejection seen at some point (the rejection need not be correct or at the correct time)
c1A1: All arcs in tree selected correctly in the correct order (AC, BE, BF, FG, FH, EJ, BC, CD) with no additional arcs included in MST
c2A1: cso - including all rejections correct and at the correct time (do not need to see DE and/or AD rejected but if they are rejected then must be after CD has been selected). Note that BC can be included before HJ rejected
d1B1: cao
e1B1: cao (130) – no units required/ignore units even if incorrect

For reference:



Q15.

Question Number	Scheme	Marks
(a)	If CD is not in the tree then AD must be as these are the only two arcs incident to D or the weight of CD is greater than the weight of AD	M1
	$2y+x>3y-7 \Rightarrow y<x+7$ *	A1 (2)
(b)	$4x+1<2y+1 \Rightarrow y>2x$ *	B1
	$4x+1<8x-3 \Rightarrow x>1$	B1
	$4x+1<3y-7 \Rightarrow 3y>4x+8$	B1 (3)
(c)		



B1
B1
B1
B1

(4)

(d)	$(2,6),(2,7),(2,8)$ $(3,7),(3,8),(3,9)$ $(4,9),(4,10)$ $(5,11)$	B1ft B1 (2)
(e)	Arcs in the tree are given as AB, AD, BC, CE, EF and GH – the one remaining arc in the MST is EH (because the weight of arcs EG and FG > weight of arc EH)	M1
	Total weight of the MST is therefore $14x+5y-4$	A1
	$14x+5y-4=73$ and testing integer value points inside FR	M1dep

	$x=3$ and $y=7$	A1 (4)
		15 marks

Notes for Question
a1M1: Explaining that if CD is not in the tree than AD must be e.g. 'the MST must contain D so if CD is not in the tree then AD is'. Must explicitly mention arc AD for this mark, so as a minimum accept, 'AD must be in the MST' **a1A1:** Correct reasoning and derivation of the given result ($2y+x>3y-7 \Rightarrow y<x+7$) – as the answer is given we must see at least $2y+x>3y-7$ or $3y-7<2y+x$ before the required answer **SC (Special Case) in (a):** $2y+x>3y-7 \Rightarrow y<x+7$ without any explanation given (or if explanation is incorrect) can score M1A0 **b1B1:** CAO – **must see at least** $4x+1<2y+1$ before the given answer of $y>2x$ and not just arc AB < arc AC or $4x<2y$ **b2B1:** CAO ($x>1$) – but allow equivalents, e.g., $x-1>0$, $1<x$, $4x>4$, etc. but must be two terms only **b3B1:** CAO ($3y>4x+8$) – but allow **exact** equivalents e.g. $y>\frac{4}{3}x+\frac{8}{3}$, $4x-3y<-8$, $4x+8-3y<0$, or equivalent but must be three terms only In (c), the lines can be drawn as either dashed or non-dashed lines (or a combination of the two). The lines must be long enough to define the correct feasible region and pass through one small square of the points stated below: $y=2x$ must pass within one small square of (0, 0) and (7, 14) $y=x+7$ must pass within one small square of (0, 7) and (7, 14) $x=1$ must pass within one small square of (1, 0) and (1, 10) $3y=4x+8$ must pass within one small square of $(1,4)$ and $(7,12)$ **c1B1:** Any one line correctly drawn (ignore any shading) **c2B1:** Any two lines correctly drawn (ignore any shading) **c3B1:** Any three lines correctly drawn (ignore any shading) **c4B1:** All four lines correctly drawn and shading which implies the correct region (but region need not be labelled)

d1B1ft: At least 4 pairs of integer coordinates correctly stated for points inside their region. This mark is dependent on scoring at least the first two marks in (c) (so must have drawn at least two lines correctly) and the candidate must have drawn exactly four lines. The region must not be infinite but need not ncessarily be bounded by all four lines. If the candidate's region does not contain 4 integer coordinates then B0. Note that integer points on the lines that define the boundary of the region are not counted as being inside the region (regardless of if the candidate has strict inequalities or not) **d2B1:** All 9 coordinates correct (and no others) – **dependent on all four lines correctly drawn in (c)** **e1M1:** States that the remaining arc (in the MST) is one of either EH, EG or FG (and no others). Only one of these three arcs need to be stated for this mark. Allow this mark for either the expression $(4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+X$ or the equation

$(4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+X=73$ where $X=y+1$ or $2y+4$ or $5x+1$ (or equivalent equations/expressions). Their expression/equation need not be simplified but for reference (if correct) they are $14x+4y-5+X$ and $14x+4y-5+X=73$ (with the expression for X as before). Note that stating $14x+4y-5$ (or equivalent) and then separately stating one of the expressions for X would imply this mark

e2A1: A correct expression for the weight of the MST either simplified $(14x+5y-4)$ or not $((4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+(y+1))$. This mark can be implied if a correct equation is seen, e.g. $14x + 5y = 77$ (or equivalent and again need not be simplified). **If more than one equation or expression seen then they must clearly select the correct expression/equation for this mark** (so stating more than one expression for the weight of the MST is A0)

e1M1dep: This mark is dependent on the first M mark in this part and the first B mark in (d). Setting their linear expression (in x and y) for the weight of the MST equal to 73 and then substituting into this equation at least one integer pair of values of x and y from (d). This mark can also be awarded for substituting at least one integer pair of values of x and y into their linear expression. The correct answers can imply this mark

e1A1: Correct answers only ($x = 3$ **and** $y = 7$) from correct working – do not accept any other answers stated as well but accept as a coordinate (3, 7) – **must have drawn the correct four lines in (c) but need not have stated all nine correct coordinates in (d)**. As a minimum for full marks in (e) the candidate must have stated a correct expression (e.g. $14x + 5y - 4$ or equivalent) or equation (e.g. $14x + 5y = 77$ or equivalent) before then stating the correct answer

The correct answer with no method or working scores no marks in (e)

Q16.

Question Number	Scheme	Marks
(a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited exactly once	B2, 1, 0 (2)
(b)	NNA starting at A: A – B – D – F – C – G – E – A 25 + 24 + 35 + 27 + 29 + 31 + 35 = 206 (km)	B1 B1 (2)
(c)	The better upper bound is the one starting at D as it is smaller	B1dep (1)
(d)(i)	Prim (starting at A): AB, BD, BE, EF, CF	M1 A1
	RMST weight = 25 + 24 + 27 + 28 + 27 = 131	
(d)(ii)	131 + 29 (CG) + 31 (EG) = 191 (km)	M1 A1 (4)
(e)	The better lower bound is the one found by deleting G as this is the larger of the two	B1dep (1)
(f)	191 ≤ optimal distance ≤ 203	B1ft B1 dep (2)
		12 marks

Notes for Question

a1B1: Understands the difference is connected to the number of times each vertex may be visited – condone 'point' (oe) for vertex (must refer to both problems in their answer but not necessarily by name)
a2B1: Correctly identifies which is classical (each node visited 'exactly once' or 'once') and which is practical (each node visited 'at least once' but B0 for 'more than once' oe – it must be clear that for the practical case that a node may be visited more than once but not necessarily more than once). Must use correct language (e.g. vertex or node) but condone singular/plural confusion e.g. vertex for vertices, or poor spelling (in this part a mark of B0B1 is not possible)

b1B1: Correct nearest neighbour route starting at A (must return to A) – possibly stated in terms of arcs e.g. AB, BD, DF, CF, CG, EG, EA
b2B1: CAO (206) on length of route

c1B1dep: CAO dependent on the correct UB in (b) – allow 'yes it is' (as question asks, 'state whether this (an upper bound of 203) is a better upper bound than the answer to (b)') **and** with some indication that this value is smaller than the one in (b) e.g. '203 < 206 so yes it is' scores B1

di1M1: Must be using Prim's algorithm not NNA. First three arcs (or all 6 nodes / or numbers across the top of the matrix) selected correctly. First three arcs are AB, BD, BE, first six nodes are A, B, D, E, F, C and so numbers across the matrix would be 1, 2, 6, 3, 4, 5. Award M1 only for a correct tree with either no working or if starting at a different node than A
di1A1: CAO (order of arc selection clear) – in terms of arcs only for this mark - AB, BD, BE, EF, CF – condone those that state AB, BD, BE, EF, CF, CG, EG or AB, BD, BE, EF, CF, EG, CG (these candidates are most likely adding on the two smallest arcs incident to G for the next part of the question)

dii2M1: Adding two least weighted arcs (CG(29) + EG(31)) to the length of their answer from d(i) (where 100 ≤ d(i) ≤ 160) - condone if parts (d)(i) and (d)(ii) are combined together as a single part (d)
dii2A1: CAO (191)

e1B1dep: CAO dependent on the correct LB in (d)(ii) – allow 'no it isn't' (as question asks, 'state whether this (a lower bound of 188) is a better lower bound than the answer to (d)(ii)') and with some indication that this value is smaller than the one in (d)(ii) e.g. '188 < 191 so no it isn't' scores B1

If the candidate's answer to (b) is less than 188 then no marks can be awarded in (f)

f1B1ft: Their numbers correctly used, accept any inequalities or any indication of an interval from

their largest of the two values (188 or d(ii)) to their smallest of the two values (203 or (b))

e.g. condone for B1 only 203 – 191 = 12

f2B1dep: This mark is dependent on the previous B mark - CAO including correct inequalities (accept either 191 ≤ optimal distance ≤ 203 or 191 < optimal distance ≤ 203) or equivalent notation e.g. [191, 203] or (191, 203]

Q17.

(a)	Prim: AE, AC, CD; BD, AF; CH, BG	M1 A1 A1 **(3)**
(b)	Weight of MST is 205 (minutes)	B1 **(1)**
(c)	J – G – B – D – C – A – E – F – H – J 28 + 32 + 28 + 27 + 29 + 28 + 36 + 39 + 42 = 289 (minutes)	M1 A1 **(2)**
(d)	The best upper bound is the one starting at J as 289 is less than 291	B1 **(1)**
(e)	The two smallest arcs incident to J are 28 and the min(x, 33) but 28 + 33 + 205 $\neq$ 264	B1
	$205 + 28 + x = 264$	M1
	$x = 31$	A1 **(3)**
		10 marks

	Notes for Question	
a1M1:	Prim's – first three arcs correctly chosen in order (AE, AC, CD, …) or first four nodes {A, E, C, D, …} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 3, 4 , 2 , -, -, -}. Starting at any other node can score M1 only for first three arcs chosen correctly	
a1A1:	First five arcs correctly chosen in order (AE, AC, CD, BD, AF, …) or all eight nodes {A, E, C, D, B, F, H, G} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 5, 3, 4, 2, 6, 8, 7} (no missing numbers)	
a2A1:	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)	
b1B1:	CAO for weight of MST (205) – no units required	
c1M1:	Nearest neighbour starting at J with first five nodes correct (J – G – B – D – C –) Accept arcs JG GB BD DC	
c1A1:	Correct nearest neighbour route (must return to J) (may be listed as arcs JG GB BD DC CA AE EF FH HJ) and correct length (289)	
d1B1:	Accept any wording indicating that the answer from (c) is smaller than 291 and therefore the better upper bound An indication that 289 is the minimum (of 291 and 289) – this mark is dependent on the correct value in (c) so accept an answer of the form 'the one starting at J (or the route with weight 289) as it is the least'	
e1B1:	Correct justification that the two smallest arcs incident to J are 28 and x - may calculate that the two smallest arcs must be 28 and 31 and then state that as no other arc has length 31, this must be the value of x	
e1M1:	Forming the equation: weight of MST from (b) + 28 + x = 264	
e1A1:	CAO for x	

Q18.

Question Number	Scheme	Marks
(a)	Prim: AC, AB, AE; DE, DF, BJ; HJ, GH **or** AC, AE, DE; AB, DF, BJ; HJ, GH	M1 A1 A1 (3)
(b)	127 (km)	B1 (1)
(c)	If starting at E and finishing at F then the shortest path between C and H needs to be traversed twice. So the length of the route is 58 + 494 = 552 (km)	M1 A1 (2)
(d)	Route must start at C and finish at A therefore need to consider pairings of the nodes A, E, F and H	M1
	AE + FH = 15 + 53 = 68 AF + EH = 42 + 66 = 108 AH + EF = 51 + 27 = 78	A1 A1
	Repeat roads: AE, FG, GH	A1 (4)
ALT	**Special Case – considers C, E, F and H – mark as misread removing final 2 A marks earned in this section**	M1
	CE + FH = 22 + 53 = 75 CF + EH = 49 + 66 = 115 CH + EF = 58 + 27 = 85	A1 A1
	Repeat roads: CA, AE, FG, GH	A1 (4)
(e)	B – A – C – E – D – F – G – H – J – B 15 + 7 + 22 + 10 + 17 + 40 + 13 + 19 + 31 = 174 (km)	M1 A1 (2)
(f)	(127 – 7) + 7 + 22 = 149 (km)	M1 A1 (2)
		14 marks

	Notes
a1M1:	Prim's – first three arcs correctly chosen in order (AC, AB, AE, … or AC, AE, DE) **or** first four nodes {A, C, B, E, … or A, C, E, D,…} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 3, 2, - , 4 , -, -, -, - or 1, -, 2, 4, 3, -, -, -, -}. Starting at any other node can score M1 only for first three arcs chosen correctly
a1A1:	First six arcs correctly chosen in order {AC, AB, AE, DE, DF, BJ, …} **or** all nine nodes {A, C, B, E, D, F, J, H, G} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 3, 2, 5, 4, 6, 9, 8, 7} (**no** missing numbers). **Or** the alternative six arcs {AC, AE, DE, AB, DF, BJ,…}, nine nodes {A, C, E, D, B, F, J, H, G} or numbers {1, 5, 2, 4, 3, 6, 9, 8, 7}
a2A1:	CSO – all arcs correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)
b1B1:	CAO (127)
c1M1:	Indication that the shortest path between C and H needs to be traversed twice (correct answer can imply this mark)
c1A1:	CAO (552)
d1M1:	The correct three pairings of the correct four nodes (A, E, F and H)
d1A1:	Two rows correct including pairings **and** totals
d2A1:	All three rows correct including pairings **and** totals
d3A1:	CAO (AE, FG, GH only but in any order) – must be stated as edges so **A0** for AE, FH or AE F(G)H
	Special Case – considers C, E, F and H – mark as misread removing last two A marks earned in this section, so max 2/4
d1M1:	The correct three pairings of the four nodes (C, E, F and H)
d1A1:	Two rows correct including pairings **and** totals
d2A1:	All three rows correct including pairings **and** totals
d3A1:	CAO (CA, AE, FG, GH only but in any order) – must be stated as edges so **A0** for CE, FH or C(A)E F(G)H
e1M1:	Nearest neighbour starting at B with first five nodes correct (B – A – C – E – D –) Accept arcs so BA, AC, CE, ED,
e1A1:	Correct nearest neighbour route (must return to B) and correct length (174) Accept arcs so BA, AC, CE, ED, DF, FG, GH, HJ, JB
f1M1:	(weight of their MST from (b) **or** 127 **only**) – 7 + 7(AC) + 22(CE or CB) (oe so may not see the –7 + 7). A correct answer of 149 with no working can imply this (and the next) mark. If (b) is incorrect a value of their "127" from (b) plus 22 scores M1 (Note new calculation to find RMST 120 + 7 + 22 = 149 may be seen)
f1A1:	CAO (149)

Q19.

Question Number	Scheme	Marks
(a)	Prim: AE, EG, CE; DG, CF; DH, BF	M1 A1 A1 (3)
(b)	Weight of MST = 197	B1 (1)
(c)	Initial upper bound = 2(197) = 394	B1ft (1)
(d)	A – E – G – D – H – B – F – C – A 23 + 24 + 26 + 33 + 38 + 34 + 32 + 38 = 248	M1 A1
	A – E – G – D – H – F – C – B – A	A1
	23 + 24 + 26 + 33 + 38 + 32 + 35 + 36 = 247	A1 (4)
(e)	247	B1ft (1)
(f)	Weight of RMST is 174	B1ft
	Lower bound = 174 + 23 + 35 = 232	M1 A1 (3)
(g)	232 □ optimal value □ 247	M1 A1 (2)
		15 marks

	Notes for Question	
a1M1: Prim's – first three arcs correctly chosen in order (AE, EG, CE, …) **or** first four nodes {A, E, G, C, …} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 4, - , 2 , -, 3, -}. Starting at any other node can score M1 only for first three arcs chosen correctly **a1A1:** First five arcs correctly chosen in order (AE, EG, CE, DG, CF, …) **or** all eight nodes {A, E, G, C, D, F, H, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 4, 5, 2, 6, 3, 7} **(no** missing numbers) **a2A1:** cso – all arcs correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) **b1B1:** cao (197 – ignore units) should come from 23 + 24 + 25 + 26 + 32 + 33 + 34 **c1B1ft:** Follow through double their answer to **(b)** **Mark (d) and (e) together** **d1M1:** Nearest neighbour starting at A with first five nodes correct (A – E – G – D – H –) **d1A1:** One correct route (must return to A) **d2A1:** One correct value **or** both correct routes **d3A1:** Both correct values (do not isw if values doubled) **and** both correct routes (must both return to A) **SC in (d) correct Hamiltonian paths and corresponding weights (AEGDHBFC (210) and AEGDHFCB (211)) scores M1A1A0A0** **e1B1ft:** Follow through their least weight route from **(d)** – must have or imply two Hamiltonian cycles in **(d)** or **(e)** **f1B1ft:** Either 174 **or** 24 + 25 + 26 + 32 + 33 + 34 **or** 197 – 23 **or** the weight of their MST from **(b)** – 23 **f1M1:** Weight of RMST + 23 + 35 (two smallest arcs incident to A) with 151£ RMST £ 197 (if clearly not six arcs in RMST then M0) **f1A1:** cao (232) – if correct answer with no working then awarded B0M1A1 – as a minimum for full marks accept 174 + 23 + 35 = 232 but 174 + 58 = 232 scores B1M1A0 **g1M1:** Any indication of an interval from their answer to **(f)** to their answer to **(e)** with one value correct **g1A1:** cao (either 232£ optimal value £ 247 or 232< optimal value £ 247)	

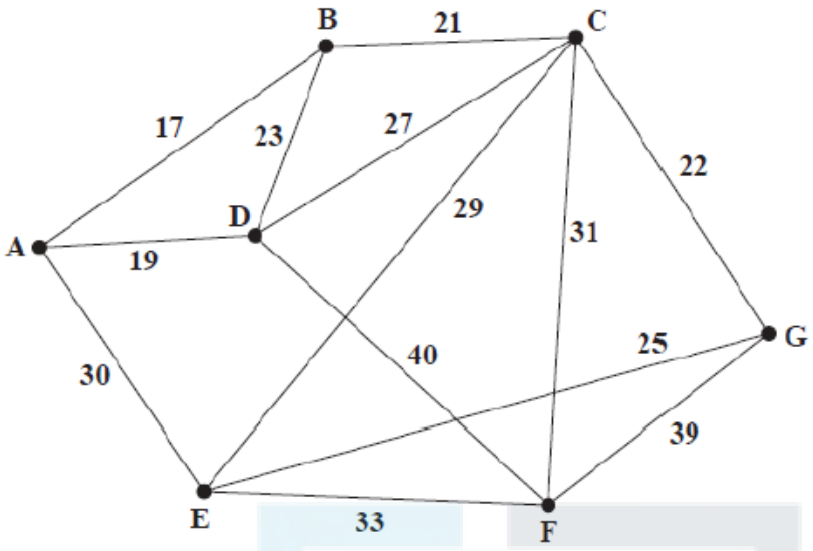
Q20.

Question Number	Scheme	Marks
(a)	**Classical** problem must visit **every vertex exactly once** and **returns to start** but **practical** problem must visit **every vertex at least once** and **returns to start**	B1 B1
		(2)
(b)	Either AB, BC, CK, HK, JK, CD, GJ, FG, EG or AB, BC, CK, HK, JK, GJ, FG, EG, CD	M1 A1 A1
		(3)
(c)	$89 \times 2 = 178$ (miles)	B1
		(1)
(d)	ABCKHJGFEDA 140 (miles)	M1 A1 A1
		(3)
(e)	Route from A as it is the smaller value or 140 because $140 < 145$	B1ft
		(1)
(f)	Lower Bound = 89 – 16 + (16 + 19) = 108 (miles)	M1 A1
		(2)
(g)	$108 \leqslant L \leqslant 140$	B1ft
		(1)
		13 marks

Notes for Question		
a1B1	General idea that Classical visits vertices once but practical is at least once. Any mention of travelling along every arc is B0	
a2B1	Need to see all the words in bold (accept equivalent technical language e.g. node instead of vertex) – must include return to start	
b1M1	Prim's – first three arcs correctly chosen in order (AB, BC, CK, …) **or** first four nodes {A, B, C, K, … } correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 2, 3, - , - , -, -, -, -, 4 }. Starting at any other node can score M1 only for first three arcs chosen correctly	
b1A1	First six arcs correctly chosen in order {AB, BC, CK, HK, JK, CD, …} or {AB, BC, CK, HK, JK, GJ, …}**or** all 10 nodes {A, B, C, K, H, J, D, G, F, E} or {A, B, C, K, H, J, G, F, E, D}correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 2, 3, 7, 10, 9, 8, 5, 6, 4} or {1, 2, 3, 10, 9, 8, 7, 5, 6, 4} (**no** missing numbers).	
b2A1	CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)	
c1B1	CAO	

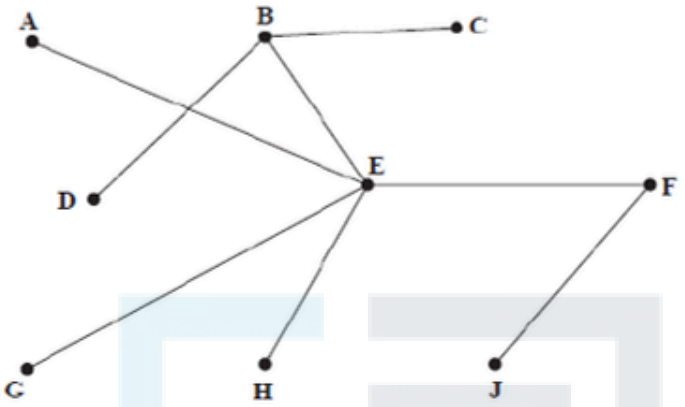
d1M1	NN route attempted – first seven nodes correct ABCKHJG (accept arcs for this mark AB, BC, CK, KH, HJ, JG or complete numbering on table {1, 2, 3, 10, 9, 8, 7, 5, 6, 4} which is clearly a separate calculation from part (b))	
d1A1	CAO – route correct including return to A (accept arcs AB, BC, CK, KH, HJ, JG, GF, FE, ED, DA) (condone if arcs stated alphabetically instead of in route order)	
d2A1	CAO – distance 140	
e1B1ft	States A or 140 **and** reason correct (smaller value) but ft their answer to (d) dependent on M1 in (d)	
f1M1	Their "89" – 16 + two shortest from A (a new calculation to find the RMST may be seen e.g. new table with A deleted and a new MST calculation, with the correct 8 arcs circled BC CD CK EG FG GJ JK KH plus two shortest from A)	
f1A1	CAO – an answer of 108 implies both of these marks	
g1B1ft	FT their answers to (e) and (f) but dep on B1 in (e) and M1 in (f) and their (f) < their (e) (accept $108 < L \leqslant 140$ or interval notation [108, 140] or (108, 140])	

Q21.

Question	Scheme	Marks
(a)	e.g. accept (i) Every pair of nodes connected by a path (ii) Connected graph with no cycles (iii) All nodes connected	B1 B1 B1
		(3)
(b)	$n - 1$	B1
		(1)
(c)		M1 A1
		(2)
(d)	Kruskal: AB, AD, BC, CG, reject BD, EG, reject CD, reject CE, reject AE, CF	M1 A1 A1
		(3)
(e)	135 (km)	B1
		(1)
		(10 marks)

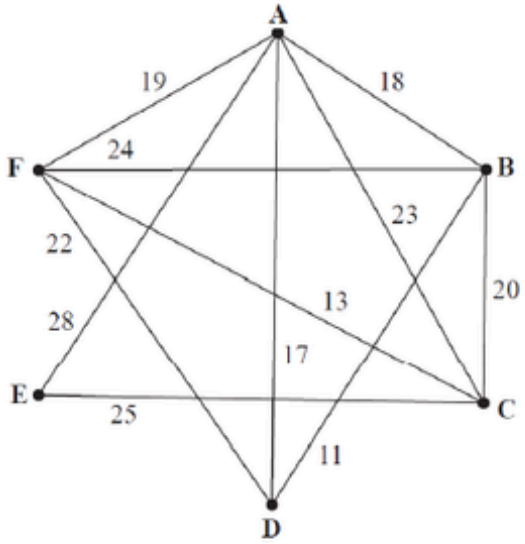
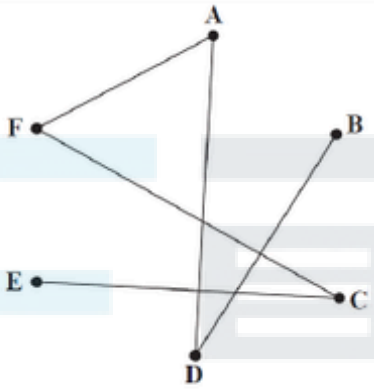
Notes:
(a) **In (a), all technical language used must be correct – for example, do not accept 'point' for node, etc** **(i)B1: every pair** and **path** (or clear definition of path) – no bod - not describing complete graph **(ii)B1: connected** and **no cycles** (not 'loops', 'circles', etc. unless 'cycle' seen as well) **(iii)B1: all nodes** connected (accept definition of minimum spanning tree)
(b) **B1:** cao
(c) **M1:** Either all five arcs correct (ignore weights) or at least three arcs correct (including weights) **A1:** cso (arcs **and** weights) – no additional arcs
(d) **M1:** Kruskal's – first three arcs (AB, AD, BC,… or weights 17, 19, 21, …) chosen correctly **and at least one rejection seen at some point**. **For M1 only**: follow through from their diagram from (c) **A1:** All six arcs (AB, AD, BC, CG, EG, CF or weights 17, 19, 21, 22, 25, 31) chosen correctly and no additional arcs (no follow through from an incorrect network in (c)) **A1:** cso All selections and rejections correct (in correct order and at the correct time) – do not accept weights or a contradiction between arcs and their weights (e.g. AB (16))
B1: cao (ignore lack of units)

Q22.

Question Number	Scheme	Marks
(a)(i)	A tree is a connected graph with no cycles	B1
(ii)	A minimum spanning tree is a tree that contains all vertices The total length of its arcs is as small as possible	B1 B1 (3)
(b)	Kruskal: FJ(11), EG(13), EF(15), EH(17), not GH (18), BC (19), not HJ (20), BD (22), not FH(23), AE (25), BE (29) (not AD, DE, DG, AB, BH)	M1 A1 A1 (3)
(c)		B1
	(Weight of the tree is) 151	B1 (2)
		8 marks

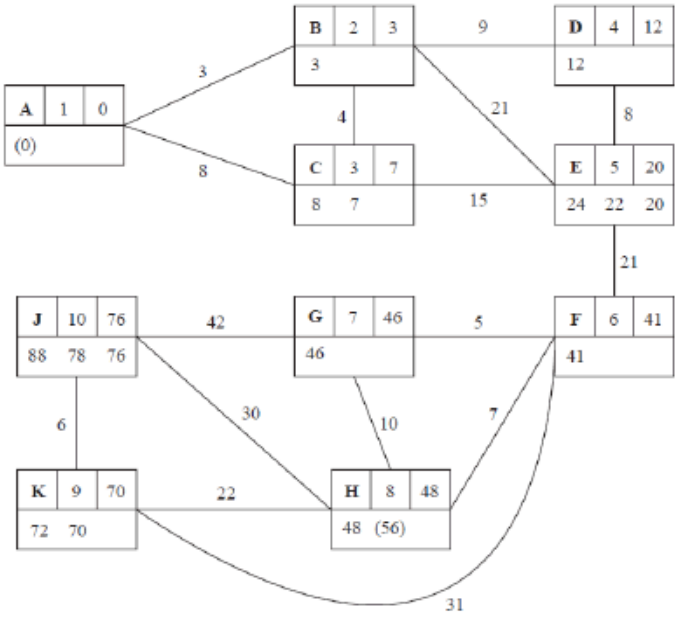
Notes for Question
ai1B1: **Connected + no cycle(s)** (must contain these two points – do not allow 'circle', 'loop' etc. for cycle(s)) – if not using the word 'connected' then allow 'a graph that connects the vertices/nodes' (condone issues with plural or singular e.g. cycle for cycles) **aii2B1**: Contains **all vertices/nodes** (must be clear that all vertices (or nodes) are in a MST) **aii3B1**: Total **length** of arcs is **minimised** (must contain the three points regarding weight/length, arcs/edges and minimised/smallest (oe)) **b1M1**: Kruskal's algorithm - first four arcs (FJ, EG, EF, EH) correctly chosen **and** at least one rejection seen at some point **b1A1**: All arcs in tree selected correctly in the correct order (FJ, EG, EF, EH, BC, BD, AE, BE) with no additional arcs included in MST **b2A1**: CSO including all rejections correct and at the correct time (do not need to see AD, DE, DG, AB, BH rejected but if they are they must be rejected correctly (i.e. in this order) but note that AD, DE have the same weight as do DG and AB so they could appear in either order) Note that stating all the arcs in order (e.g. FJ, EG, EF, EH, GH, BC, HJ, BD, FH, AE, BE, AD, DE, DG, AB, BH) and then stating only those in the tree in the correct order is fine for all three marks in this part **c1B1**: CAO (tree) **c2B1**: CAO (151)

Q23.

Question Number	Scheme	Marks
(a)		M1 A1 (2)
(b)	Kruskal: BD(11), CF(13), AD(17), reject AB(18), AF(19), reject BC(20), reject DF(22), reject AC(23), reject BF(24), CE(25) (not AE)	M1 A1 A1 (3)
(c)		B1
	Weight of MST = 85 (metres)	B1 (2)
		7 marks

Notes for Question
a1M1: At least 8 correct arcs with corresponding correct values or all 11 correct arcs **a1A1:** CSO (11 arcs only + correct values) – give bod **b1M1:** Kruskal's: first three arcs (BD, CF, AD) correctly chosen and at least one rejection seen at some point **b1A1:** All arcs in tree selected correctly and in the correct order (BD, CF, AD, AF, CE) – no other arcs in MST **b2A1:** CSO including all rejections correct and at the correct time – AE need not be considered but if AE is considered then it must be rejected after CE has been added to the MST **c1B1:** CAO (tree) **c2B1:** CAO (85)

Q24.

Question Number	Scheme	Marks
(a)	 Route: ABDEFHKJ Length: 76 (km)	M1 A1 (ABCDE) A1 (FGH) A1ft (KJ) A1 A1ft (6)
(b)	Prim: AB, BC; BD, DE	M1; A1 (2)
(c)	Kruskal: FG, JK, FH, not GH, HK, (not HJ), (not FK), (not GJ)	M1; A1 (2)
(d)	Total length: 85 (km)	B1 (1)
		11 marks

Notes for Question

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 24 22 20 in that order (so 24 20 22 is incorrect)
It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either C or E or J or K
a1A1: All values in A, B, C, D and E correct and the working values in the correct order at C and E (including order of labelling). Condone lack of 0 in A's working value
a2A1: All values in F, G and H correct and the working values in the correct order. Penalise order of labelling only once per question (F, G and H must be labelled in that order and F must be labelled after A, B, C, D and E). Note that an additional working value of 56 at H after the 48 is not an error so 48 56 is fine, however, any other number or 56 48 in this order is incorrect and scores A0 in this part
a3A1ft: All values in K and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through K check that the working value at K follows from the candidate's final values from their feeds into K (which will come from nodes F, H and possibly even J (in the order in which the candidate has labelled them)) and that the final value, and order of

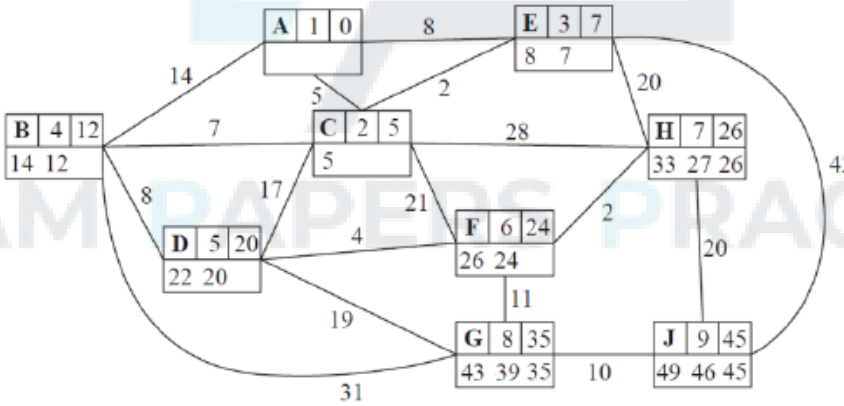
labelling, follows through correctly. Repeat this process for J (which will possibly have working values from G, H and K with the order of these values determined by the candidate's order of labelling at G, H and K)
a4A1: CAO - correct route (ABDEFHKJ) not from J to A
a5A1ft: Follow through on their final value at J only (so if 76 given as the answer and the final value at J is not 76 then A0)

b1M1: First two arcs (AB, BC) chosen correctly in order, or first three nodes (ABC) chosen correctly in order. If any rejections seen at any point, or just a list of **all** the arcs in order, or only a list of weights then M0 (condone for M1only those who find the MST for the entire network)
b1A1: CSO (must be considering arcs so must be AB, BC, BD, DE or BA, BC, etc.) – do not isw if candidates continue and find the MST for the entire network

c1M1: First two arcs (FG, JK) chosen correctly in order **and at least one rejection seen at some point** – no marks in this part if candidates apply Kruskal to the entire network or if only a list of weights given
c1A1: CSO – all selections and rejections correct in the correct order and at the correct time. Note that stating all the arcs in order (e.g. GF, JK, FH, GH, KH, JH, FK, GJ) and then stating only those in the tree in the corret order is fine for both marks in this part

d1B1: CAO (85)

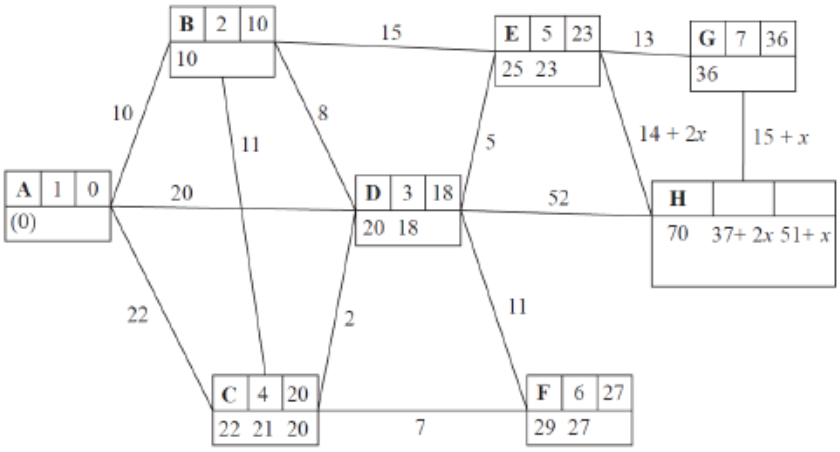
Q25.

Question Number	Scheme	Marks
(a)	 Shortest time: 45 (minutes) Quickest route: A C B D F G J	M1 A1 (CEBD) A1 (FH) A1ft (GJ) A1ft A1 (6)
(b)	A(CB)D + HJ = 20 + 20 = 40 A(CBDF)H + D(FG)J = 26 + 25 = 51 A(CBDFG)J + D(F)H = 45 + 6 = 51 Repeated arcs: AC, BC, BD, HJ	M1 A1 A1 A1 A1 (5)
(c)(i) (ii)	Vertex C: 4 times Vertex D: 3 times	B1 B1 (2)
(d)	H to H requires the consideration of the shortest path from A to J only (as these are the only two odd nodes) 314 > 309 or 45 > 40 so quicker to start at H and finish at D	B1 B1 (2)
		15 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at J the working values must be 49 46 45 in that order (so 49 45 46 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling** **a1M1**: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A or C **a1A1**: All values at C, E, B and D correct and the working values in the correct order (including order of labelling) **a2A1**: All values at F and H correct and the working values in the correct order. Penalise order of labelling only once per question (F and H must be labelled in that order and F must be labelled after C, E, B and D) **a3A1ft:** All values in G and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will mostly likely come from nodes B, D and F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from E, H and G with the order of these values determined by the candidate's order of labelling at E, H and G)

Question Number	Scheme	Marks
	a4A1ft: Follow through on their final value at J **only** (condone lack of units) - so if 45 given as the answer and the final value at J is not 45 then A0 **a5A1**: CAO - correct route (from either A to J or J to A) – ACBDFGJ or JGFDBCA **b1M1**: Correct three distinct pairings of the correct four odd nodes A, D, H and J **b1A1**: **Any** row correct including pairing **and** total **b2A1**: **Any** two rows correct including pairings **and** totals **b3A1**: **All** three rows correct including pairings **and** totals **b4A1**: CAO correct edges clearly stated and not just in their working as AC, BC, BD, HJ. Must be these arcs and not AD, ACBD or AD via B and C **ciB1**: CAO (Vertex C: 4) **ciiB1**: CAO (Vertex D: 3) **d1B1**: Correct reasoning (that to travel from H to H) only the shortest path between A and J needs traversing twice – as a minimum must mention either 'A to J only' or refer to A and J being the only odd nodes (e.g. odd nodes: A and J is fine but not 'A and J are odd nodes') **d2B1**: Either 'it will be slower' or 'it will be quicker from H to D' (if saying 'quicker' then it must be clear that they are talking about H to D) + correct numerical argument (not just stating the values 314 and 309 (or 45 and 40) and saying 'slower' - there must be some comparison of these two values)	

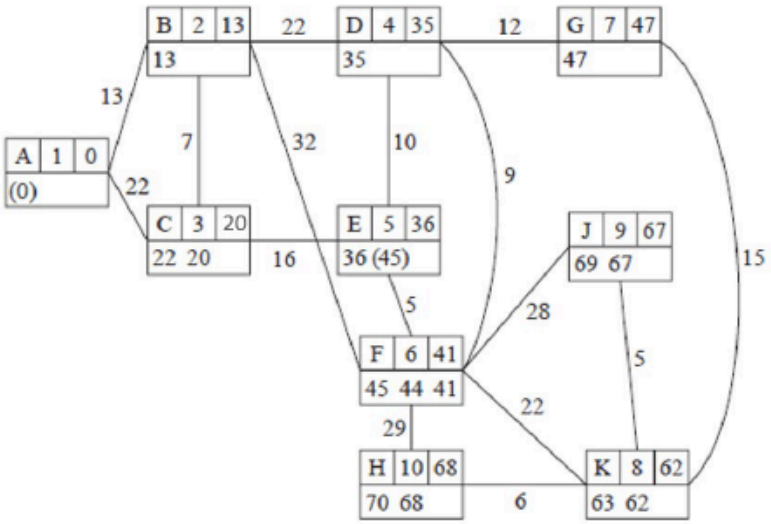
Q26.

Question Number	Scheme	Marks
(a)		M1 A1 A1 A1ft
	Shortest path from A to H via DH: ABDH length: 70	A1
	Shortest path from A to H via EH: ABDEH length: $37 + 2x$	A1
	Shortest path from A to H via GH: ABDEGH length: $51 + x$	A1 (7)
(b)	A to H are the only two odd nodes in the network so repeat arcs in path ABDEH	M1
	$3x + 205 + 37 + 2x = 307$	M1
	$x = 13$	A1
	Time taken is 63 (minutes)	A1 (4)
		11 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 22 21 20 in that order (22 20 21 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling
a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A, B, G or H (or once with at least two working values seen at H) **a1A1:** All values in A, B, D and C correct and the working values in the correct order at D and C (including order of labelling) **a2A1:** All values E and F correct and the working values in the correct order. Penalise order of labelling only once per question **a3A1ft:** All values in G and H correct on the follow through and the working values in the correct order (the order at H must be correct but give bod). Penalise order of labelling only once per question. Ignore permanent label and final value at H only. Allow unsimplified expressions in x for the working values at H **a4A1:** ABDH and 70 **a5A1:** ABDEH and $37 + 2x$ **a6A1:** ABDEGH and $51 + x$ If A0A0A0 for the final three marks in **(a)** then award A1A0A0 for all 3 routes stated correctly or all 3 correct values stated explicitly (so not just left in the working values at H)

b1M1: Indication of repeating arcs in a path from A to H. As a minimum: stating A and H as the odd nodes for the network (not just stating A and H) **or** stating a route from A to H with 5 nodes only **or** stating the need to repeat a path/route from A to H - this mark is for making their method clear **b2M1:** $3x + 205 +$ (one of their paths involving x) $= 307$ – this mark is for making their working clear **b1A1:** CAO ($x = 13$) – this mark is dependent on the second M mark only **b2A1:** CAO (63) – this mark is dependent on the second M mark only **SC** If M0M0 then **both** correct answers of $x = 13$ and 63 score M0M0A0A1 only (so treating the final mark as a B mark)

Q27.

Question Number	Scheme	Marks
(a)	 Shortest path: A B D G K H Length: 68 (miles)	M1 A1 (ABCDE) A1 (FGK) A1ft (JH) A1 A1ft (6)
(b)	Route from F to K via A: F E C B A B D G K Length: 41 + 62 = 103 (miles)	B1 B1ft (2)
(c)	AJ + CE = 67 + 16 = 83 AC + EJ = 20 + 32 = 52 AE + CJ = 36 + 48 = 84 Repeated arcs: AB, BC, EF, FK, JK Length: 253 + 52 = 305 (miles)	M1 A1 A1 A1 A1 A1ft (6)
(d)	Vertex F: 4 times	B1 (1)
(e)	(Start at D and therefore) finish at D	B1 (1)
(f)	Difference = 305 – (253 + 16) = 36 (miles)	B1 (1)
		17 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 45 44 41 in that order (so 45 41 44 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**
a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either C, F, K, J, or H **a1A1**: All values at A, B, C, D and E correct and the working values in the correct order (including order of labelling) – if a working value of 45 appears at E then it must appear after the 36 so therefore 45 36 at E (in this order) is A0 **a2A1**: All values at F, G and K correct and the working values in the correct order (F, G and K must be labelled in that order and F must be labelled after A, B, C, D and E) **a3A1ft**: All values in J and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working value at J follows from the candidate's final values from their feeds into J (which will mostly likely come from nodes F and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from F and K with the order of these values determined by the candidate's order of labelling at F and K) **a4A1**: CAO (ABDGKH or AB, BD, DG, GK, KH) **a5A1ft**: Follow through on their final value at H **only** (condone lack of units) so if answer given as 68 but final value at H is not 68 then A0 **b1B1**: CAO (FECBABDGK or FE, EC, CB, BA, AB, BD, DG, GK) **b2B1ft**: Follow through their final value at F + their final value at K **or** 103 **c1M1**: Correct three distinct pairings of the correct four odd nodes of A, C, E and J **c1A1**: **Any** row correct including pairing **and** total **c2A1**: **Any** two rows correct including pairings **and** totals **c3A1**: **All** three rows correct including pairings **and** totals **c4A1**: CAO correct edges clearly stated and not just in their working as AB, BC, EF, FK and JK – must be these arcs **c5A1ft**: Follow through their value of their smallest pairing total + 253 **d1B1**: CAO (4 only) **e1B1**: CAO (D) **f1B1**: CAO (36)

Q28.

Question Number	Scheme	Marks
(a)	Pair the odd nodes: C, D **or** repeated arcs are CF, FG, DG	B1
	Time = 82 + 7 = 89	B1
	e.g. route GDGJHEADCABEFBCFCGFG	B1 (3)
(b)	BC + DG = B(F)C + DG = 6 + 3 = 9*	M1 A1
	BD + CG = B(FG)D + C(F)G = 11 + 4 = 15	A1
	BG + CD = B(F)G + C(FG)D = 8 + 7 = 15	A1
	Repeat arcs: BF, CF, DG	A1 (5)
(c)	Route starting from G is quicker	B1
	e.g. difference = (82 + 9) – 89 = 2 or 9 – 7 = 2	B1 (2)
		10 marks

Notes for Question
a1B1: cao (correctly stating the two odd nodes **or** correct repeated arcs stated) – so must either state that C and D are odd **or** state the arcs CF, FG, DG only, but B0 if only stating C and D or CD (without mention of 'odd') **a2B1:** cao (89) **a3B1:** Correct route: checks – starts and finishes at G, 20 nodes, CF, FG and DG repeated, A(2), B(2), C(3), D(2), E(2), F(3), G(4), H(1), J(1) **b1M1:** Correct three distinct pairings of the correct four odd nodes B, C, D and G **b1A1:** Any one row correct including pairing **and** total **b2A1:** Any two rows correct including pairings **and** totals **b3A1:** All three rows correct including pairings **and** totals **b4A1:** cao - correct arcs clearly stated and not just in their working as BF, CF and DG – must be these arcs. Do not accept BC, BFC or BC via F **c1B1:** cao (oe e.g. B to G is slower) – dependent on the correct repeats arcs (possibly implied) in (a) and (b) **or** clearly implied in (c) (e.g. correct values compared in this part) – must be clear that it is the route starting at G which is quicker **c2B1:** cao (difference of 2 **or** comparing 89 and 91 **or** comparing 7 with 9)

Q29.

Question Number	Scheme	Marks
(a)	A(DG)C + D(GH)E = 12 + 9 = 21	M1 A1
	AD + C(GH)E = 5 + 10 = 15*	A1
	A(DGH)E + C(G)D = 14 + 7 = 21	A1
	Repeated arcs: AD, CG, GH, EH	A1
	Length of route: 166 + 15 = 181 (km)	A1ft (6)
(b)	Vertex C: 3 times	B1 (1)
(c)	CD (7) is the shortest path between two odd nodes excluding A	M1
	Repeat CGD (7) since this is the shortest path excluding A The route finishes at E	A1
	Length of route = 166 + 7 = 173 (km)	A1 (3)
		10 marks

	Notes for Question	
a1M1: Three distinct pairings of the correct four odd nodes (A, C, D, E) **a1A1:** One row correct including pairings **and** totals **a2A1:** Two rows correct including pairings **and** totals **a3A1:** All three rows correct including pairings **and** totals **a4A1:** The smallest repeat **arcs** (accept AD, CG, GH, EH only) **a5A1ft:** Correct answer of 181 **or** 166 + their least **b1B1:** cao (3) **c1M1:** Identifies the need to repeat one path of the three (DE, CE, CD) which does not include A (this maybe implicit) **or** listing of only these three possible repeats. This mark is dependent on either scoring the M mark in **(a)** or stating all three possible paths in this part. As a minimum accept the stating of one of these three paths **c1A1:** Identifies C(G)D as the least **and** E as the finishing point. They have to explicitly state that C(G)D is the least path of those that do not include A (this can be done by stating that CD is the least of CD, CE, DE only (so with no others) **or** stating that CD is the least of those that don't include A but not for just 'CD is the least') **c2A1:** cao (173)		

Q30.

Question	Scheme	Marks
(a)	E.g. if use CD as shortcut get 807 or if use CF + AD get 793	M1 A1
		(2)
(b)	A F E D B C A	B1
	82 113 98 130 110 217 = 750	B1
		(2)
(c)	length of RMST = 439	B1
	439 + 82 + 113 = 634	M1 A1
		(3)
(d)	634 < optimal ≤ 750	B1ft
		(1)
		(8 marks)

Notes:

(a)

M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated.

A1: cao – shortcut and length must be consistent.

(Examples shortcuts: CD = 807, CF + AD = 793, CF + BD = 664, AD + EF + FC = 715, DF FC = 785 etc.)

(b)

B1: cao

B1: cao

(c)

B1: cao

M1: Adding two least weighted arcs to their RMST length

A1: cao

(d)

B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)

Q31.

Question Number	Scheme	Marks
(a)	NNA: A – D – E – F – B – C – A 27+25+21+34+58+56 = 221 (km)	M1 A1 A1 (3)
(b)	RMST weight = 118 (km)	B1
	118 + 27 + 38 = 183 (km)	M1 A1 (3)
(c)	183 ⩽ length ⩽ 221	M1 A1 (2)
		8 marks

Notes for Question

a1M1: Nearest neighbour A – D – E – F – B or accept 1 5 6 2 3 4 across the top of the table
a1A1: Route correctly stated, must return to A, accept link back to A
a2A1: Length correctly stated. Do not ISW if candidates then go on to double the route length

b1B1: CAO for RMST weight (either 118 or 34 + 21 + 25 + 38) – maybe implied by later working
b1M1: Adding 27 + 38 (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs
b1A1: CAO - if 183 seen without working then award all 3 marks in **(b)**

c1M1: Their answers from **(a)** and **(b)** correctly used, accept any inequalities or any indication of an interval from their 183 to their 221 (so 183 – 221 can score this mark). Please note that UB > LB for this mark
c1A1: CAO (no follow through on their values) including correct inequalities or equivalent set notation (but condone 183 < length ⩽ 221)

Q32.

Question Number	Scheme	Marks
(a)	Nearest neighbour: A – B – F – D – E – C – A 35 31 44 39 53 42 = 244 (km)	M1 A1 (2)
(b)	MST with B removed: AC, CD, DF, DE gives a RMST weight of 172 (km)	B1
	172 + 31 + 35 = 238 (km)	M1 A1 (3) **5 marks**

Notes for Question

a1M1: Nearest neighbour A – B – F – D – E – C – (condone lack of return to start) **or** correct route length of 244. Accept AB, BF, FD, DE, EC but do not accept weights only. Accept 1 2 6 4 5 3 across the top of the table
a1A1: CAO both route (either in terms of vertices (ABFDECA) or arcs (AB, BF, FD, DE, EC, CA) but not weights) **and** length correct (244) do not ISW if this value is then doubled to 488

b1B1: CAO for RMST weight (either 172 or 42 + 47 + 44 + 39) – maybe implied by later working
b1M1: Adding 31 + 35 (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs
b1A1: CAO - if 238 seen without working then award B0M1A1

Q33.

Question Number	Scheme	Marks
(a)	e.g. add CD and remove AD, BA and BC gives 516 (km) e.g. add EF and remove EB, BA and AF gives 509 (km)	M1 A1 (2)
(b)	NNA: A – B – E – F – D – C – A	B1
	57 66 69 78 71 76 = 417 (km)	B1 (2)
(c)	Length of RMST = 248	B1
	248 + 66 + 69 = 383 (km)	M1 A1 (3)
		7 marks

Notes for Question

a1M1: Must clearly start with 2(length of given MST) and add and subtract at least one arc (to give a network of weight < 628) – graph must be connected and Eulerian

a1A1: CAO – shortcut(s) and length must be consistent (with length stated < 520). The shortcuts must be clearly stated (that is the arcs added and subtracted) and network must be connected and Eulerian

b1B1: CAO (must return to A) – must be stated in terms of either the nodes or arcs (e.g. AB, BE, EF,…) but not just the weights of the arcs

b2B1: CAO (417)

c1B1: Correct length of RMST (248) – maybe implied by later working

c1M1: Adding the two correct least weighted arcs (66 and 69) to their RMST length (231 ⩽ length ⩽ 265)

– give bod but their RMST must only contain 4 arcs – this mark can be implied by the correct value for the lower bound

c1A1: CAO (383) – if correct answer with no working then award B0M1A1

Q34.

Question Number	Scheme	Marks
(a)		B2, 1, 0 (2)

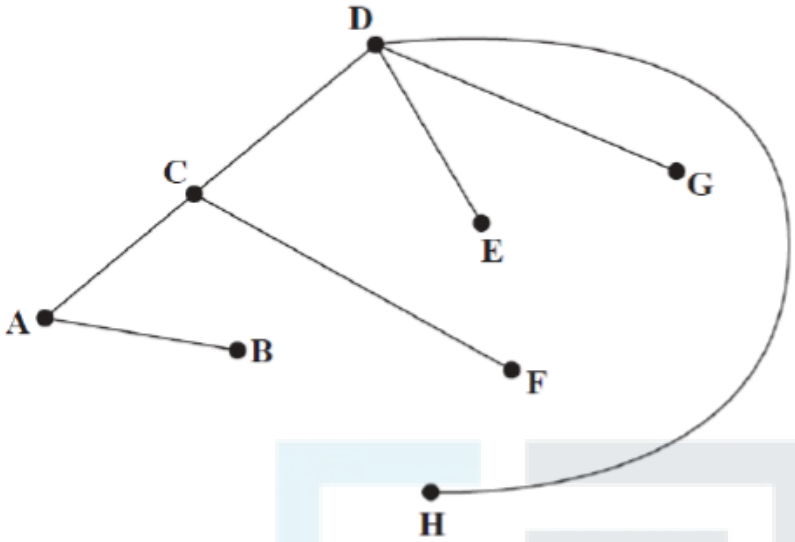
	A	B	C	D	E	F	G
A	-	21	**42**	17	25	31	41
B	21	-	26	27	12	15	20
C	**42**	26	-	**32**	17	11	46
D	17	27	**32**	-	15	**21**	47
E	25	12	17	15	-	6	32
F	31	15	11	**21**	6	-	35
G	41	20	46	47	32	35	-

(b)	NNA starting at A: A – D – E – F – C – B – G – A 17 + 15 + 6 + 11 + 26 + 20 + 41 = 136 (km)	M1 A1 (2)
(c)	A – D – E – F – C – F – B – G – B – A	B1 (1)
(d)	RMST weight = 12 + 6 + 11 + 15 + 20 = 64	B1
	64 + 17 (AD) + 21 (AB) = 102 (km)	M1 A1 (3)
(e)	AC + EG = 42 + 32 = 74	M1
	AE + CG = 25 + 46 = 71	A1
	AG + CE = 41 + 17 = 58*	A1
	Repeat arcs: AB, BG, CF, EF	A1
	Length: 291 + 58 = 349 (km)	A1ft (5)
		13 marks

Notes for Question
Condone lack of, or incorrect units throughout this question **a1B1:** At least two of the six values correct (in either table) – these are the bold values. The two values can be the same (for example, 42 in both cells AC and CA would score this mark) **a2B1:** Fully correct (the six bold values in both tables) **b1M1:** Nearest neighbour route starting at A – must have at least A – D – E – F – C – B – … Allow if stated in terms of arcs (AD, DE, EF, FC, CB,…) rather than nodes (note that arcs AD and DA are equivalent and so therefore AD, ED, EF, FC, CB,… is acceptable) **b1A1:** CAO on length (136) **and** route (must return to A and can be stated in terms of arcs rather than nodes) **c1B1:** CAO (ADEFCFBGBA or in terms of arcs, e.g., AD, DE, EF, FC, CF, FB, BG, GB, BA) **d1B1:** CAO on the weight of the RMST (using arcs BE, EF, CF, DE, BG) – accept either 64 **or** 12 + 6 + 11 + 15 + 20 – could be seen or implied by later working as part of the lower bound calculation **d1M1:** Adding correct two least weighted arcs from A (17(AD) + 21(AB)) to their RMST length where their RMST length x is $58 \leq x \leq 70$ **d1A1:** CAO (102). If this answer is stated with no working then award B0M1A1 **e1M1:** Correct three distinct pairings of the correct four odd nodes (A, C, E and G) **e1A1:** Any two rows correct including pairings **and** totals **e2A1:** All three rows correct including pairings **and** totals

e3A1: CAO correct edges clearly stated and not just in their working as AB, BG, CF and EF – must be these arcs. Do not accept AG, ABG or AG via B (and similarly for CE) **e4A1ft:** Follow through their value of their smallest pairing total + 291

Q35.

Question number	Scheme	Marks
(a)	E.g. You cannot have a graph with an odd number of odd vertices E.g. $\frac{1+2+2+3+3+4+4+6}{2}=12.5$ which is not an integer and so therefore not possible to have a graph with the given vertex orders	B1 (1)
(b)	As vertex C appears more than once A – C – D – E – C – B – F is not an example of a path on T	B2, 1, 0 (2)
(c)	AC, AB, CD; DH, DG; CF, DE	M1; A1; A1 (3)
(d)		B1 (1)
(e)	$21 < x < 25$	B2, 1, 0 (2)
		(9 marks)

Notes for Question

a1B1: CAO – common examples that score B1:

- Cannot have (a graph with an) odd number of odd vertices
- Cannot have a graph with three odd vertices
- The sum of the degrees/order (of the vertices) is 25 which is not even therefore not possible (but not just for obtaining 25 and saying 'impossible'). The 25 must be linked either in words to the 'sum of the degrees/order' or explicitly showing 1+ 2 + 2 + 3 + 3 + 4 + 4 + 6 = 25 so just '25 is not even' scores B0
- The sum of the degrees/order (of the vertices) is 25 which is odd therefore not possible (with equivalent justification of the 25 as in the previous bullet-point)
- $\frac{1+2+2+3+3+4+4+6}{2}=12.5$ which is not an integer so therefore impossible. They do not have to explain that they are using the result that $\sum$ vertex degrees = 2(no of arcs) but they must explain why a value of 12.5 leads to the required graph not being possible. A value of 12.5 with no working (or explanation) scores B0

In (a) do not condone clearly incorrect technical language e.g. using 'arc' when it should be 'vertex'

b1B1: No + attempt at a reason which includes **either** the mention of a cycle/circle/loop etc. **or** the repeating of a vertex/node/point etc. is sufficient for this mark (condone incorrect technical language) – give bod (but 'no because there is a repeated arc' **only** scores B0 unless we also see mention of a repeated vertex (oe) as well)
b2DB1: No + correct reason (dependent on first B mark in (b)) – no bod – must refer to C appearing twice (**not** just that a vertex is repeated) or that it contains the cycle C – D – E – C (**not** just that it contains a cycle). **All technical language must be correct if used for this mark and do not isw any incorrect reasoning**

The minimum acceptable answer for both marks in this part is, 'it is not a path as C appears twice'

c1M1: Prim's – first three arcs correctly chosen in order (AC, AB, CD) **or** first four nodes (A, C, B, D) correctly chosen in order. **If any explicit rejections seen at some point then M1 (max) only.** Order of nodes may be seen at the top of a matrix/table {1, 3, 2, 4, -, -, -, -}. However, do not accept a list of weights only (as the weights in the network are not unique)
c1A1: First five arcs correctly chosen in order (AC, AB, CD, DH, DG) **or**
all eight nodes {A, C, B, D, H, G, F, E} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 3, 2, 4, 8, 7, 6, 5} (**no** missing numbers). However, do not accept a list of weights only (as the weights in the network are not unique)
c2A1: CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional incorrect arcs). They must be considering arcs for this final mark (do not accept a list of the weights of each arc, nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

Misread in (c): Starting at a node other than A scores **M1 only – must** have the first three arcs (or four nodes) correct (and in the correct order) – condone any rejections seen for this mark

d1B1: CAO (ignore weights on arcs even if incorrect)

e1B1: $x < 25$ or $x \geqslant 25$ or $x < 24$ or $x \geqslant 24$ or equivalent notation e.g. (… , 25)
e2B1: CAO ($21 < x < 25$) or equvialent notation (e.g. (21, 25))