EXAM PAPERS PRACTICE

Algebraic Fractions

Model Answers

EXAM PAPERS PRACTICE

Question 1

Write as a single fraction in its simplest form.

$$\frac{5}{x-3}+\frac{3}{x+7}+\frac{1}{2}$$ [4]

$\frac{5}{x-3}+\frac{3}{x+7}+\frac{1}{2}=\frac{5\cdot2\cdot(x+7)+3\cdot2\cdot(x-3)+(x-3)\cdot(x+7)}{2\cdot(x-3)\cdot(x+7)}$

Now, simplify the numerator:

$10(x+7)+6(x-3)+\left(x^2-3x+7x-21\right)$

$=10x+70+6x-18+x^2+4x-21$

$=x^2+20x+31$

So, the combined fraction is:

$\frac{x^2+20x+31}{2\cdot(x-3)\cdot(x+7)}$

This expression is in its simplest form.

Question 2

Write as a single fraction in its simplest form.

$$\frac{x+1}{x}-\frac{y-1}{y}$$ [3]

$\frac{x+1}{x}-\frac{y-1}{y}=\frac{(x+1)y-(y-1)x}{xy}$

Now, simplify the numerator:

$(x+1)y-(y-1)x$

$=xy+y-yx+x$

$=x+y$

So, the combined fraction is:

$\frac{x+y}{xy}$

This expression is in its simplest form.

EXAM PAPERS PRACTICE

Question 3

Write as a single fraction in its simplest form.

(a) $\frac{x^2 - 3x}{x^2 - 9}$ [3]

To simplify the given expression $\frac{x^2-3x}{x^2-9}$, factor both the numerator and the denominator:

$\frac{x^2-3x}{x^2-9} = \frac{x(x-3)}{(x-3)(x+3)}$

Now, cancel out the common factor $(x - 3)$ in the numerator and denominator:

$\frac{x(x-3)}{(x-3)(x+3)} = \frac{x}{x+3}$

So, the simplified form of $\frac{x^2-3x}{x^2-9}$ is $\frac{x}{x+3}$.

(b) $\frac{3}{x-4} + \frac{2}{2x+5}$

1. Find a common denominator, which is the product of the denominators $(x - 4)$ and $(2x + 5)$:$(x - 4)(2x + 5)$.
2. Express each fraction with this common denominator:

[3]

$\frac{3}{x-4} \cdot \frac{2x+5}{2x+5} + \frac{2}{2x+5} \cdot \frac{x-4}{x-4}$

This simplifies to:

$\frac{3(2x+5)}{(x-4)(2x+5)} + \frac{2(x-4)}{(x-4)(2x+5)}$

1. Combine the numerators over the common denominator:

$\frac{6x+15+2x-8}{(x-4)(2x+5)}$

1. Combine like terms in the numerator:

$\frac{8x+7}{(x-4)(2x+5)}$

So, the sum $\frac{3}{x-4} + \frac{2}{2x+5}$ as a single fraction in its simplest form is $\frac{8x+7}{(x-4)(2x+5)}$.

Question 4

Simplify. $\frac{x^3y + 2xy^3}{x^2y^2}$ [2]

$\frac{x^3y+2xy^3}{x^2y^2} = \frac{xy(x^2+2y^2)}{x^2y^2}$

Now, cancel the common factor xy from the numerator and denominator:

$\frac{xy(x^2+2y^2)}{x^2y^2} = \frac{x^2+2y^2}{xy}$

So, the simplified form of $\frac{x^3y+2xy^3}{x^2y^2}$ is $\frac{x^2+2y^2}{xy}$.

EXAM PAPERS PRACTICE

Question 5

Write as a single fraction.

$$1 - \frac{2}{p} - \frac{3}{t}$$

[2]

To write the expression $1 - \frac{2}{p} - \frac{3}{t}$ as a single fraction, find a common denominator, which is pt.

Now, express each term with this common denominator:

$1 - \frac{2}{p} - \frac{3}{t} = \frac{pt}{pt} - \frac{2t}{pt} - \frac{3p}{pt}$

Combine the numerators over the common denominator:

$\frac{pt-2t-3p}{pt}$

So, the expression $1 - \frac{2}{p} - \frac{3}{t}$ written as a single fraction is $\frac{pt-2t-3p}{pt}$.

Question 6

Simplify.

$$\frac{42np - 7n}{12pt - 2t + 18mp - 3m}$$

[4]

To simplify the expression $\frac{42np-7n}{12pt-2t+18mp-3m}$, factor out the common factor in the numerator and denominator.

Factor out $7n$ from the numerator:

$7n(6p - 1)$

Factor out $2t - 3m$ from the denominator:

$(2t - 3m)(6p - 1)$

Now, cancel the common factor $(6p - 1)$ from the numerator and denominator:

$\frac{7n}{2t-3m}$

So, the simplified form of $\frac{42np-7n}{12pt-2t+18mp-3m}$ is $\frac{7n}{2t-3m}$.

EXAM PAPERS PRACTICE

Question 7

Simplify. [4]

$$\frac{4+10w}{8-50w^2}$$

To simplify the expression $\frac{4+10w}{8-50w^2}$, factor both the numerator and the denominator:

$4 + 10w = 2(2 + 5w)$

$8 - 50w^2 = 2\left(4 - 25w^2\right)$

Now, rewrite the fraction with the factored forms:

$\frac{2(2+5w)}{2(4-25w^2)}$

Cancel the common factor of 2 in the numerator and denominator:

$\frac{2+5w}{4-25w^2}$

So, the simplified form of $\frac{4+10w}{8-50w^2}$ is $\frac{2+5w}{4-25w^2}$.

Question 8

Write as a single fraction in its simplest form. [3]

$$3 - \frac{t+2}{t-1}$$

To write the expression $3 - \frac{t+2}{t-1}$ as a single fraction in its simplest form, find a common denominator, which is $(t - 1)$.

Now, express each term with this common denominator:

$3 - \frac{t+2}{t-1} = \frac{3(t-1)}{t-1} - \frac{t+2}{t-1}$

Combine the numerators over the common denominator:

$\frac{3(t-1)-(t+2)}{t-1}$

Now, distribute 3 in the numerator:

$\frac{3t-3-t-2}{t-1}$

Combine like terms in the numerator:

$\frac{2t-5}{t-1}$

So, the expression $3 - \frac{t+2}{t-1}$ written as a single fraction in its simplest form is $\frac{2t-5}{t-1}$.

Question 9

Write as a single fraction, in its simplest form.

$$\frac{1-x}{x} - \frac{2+x}{1-2x}$$

[4]

$\frac{(1-x)(1-2x)}{x(1-2x)} - \frac{(2+x)x}{x(1-2x)}$

Combine the numerators:

$\frac{(1-x)(1-2x)-(2+x)x}{x(1-2x)}$

Now, distribute and simplify the numerator:

$(1 - 2x - x + 2x^2) - (2x + x^2)$

Combine like terms in the numerator:

$1 - 2x - x + 2x^2 - 2x - x^2$

Combine like terms:

$2x^2 - 6x + 1$

So, the combined fraction is:

$\frac{2x^2-6x+1}{x(1-2x)}$

This expression is in its simplest form.

EXAM PAPERS PRACTICE

Question 10

(a) Write $\frac{1}{y} - \frac{2}{x}$ as a single fraction in its lowest terms. [2]

To write $\frac{1}{y} - \frac{2}{x}$ as a single fraction in its lowest terms, find a common denominator. In this case, the common denominator is xy.
Now, express each fraction with this common denominator:
$\frac{x}{xy} - \frac{2y}{xy}$
Combine the numerators:
$\frac{x-2y}{xy}$
So, $\frac{1}{y} - \frac{2}{x}$ as a single fraction in its lowest terms is $\frac{x-2y}{xy}$.

(b) Write $\frac{x^2+x}{3x+3}$ in its lowest terms. [3]

To simplify the fraction $\frac{x^2+x}{3x+3}$, factor out the common factor in the numerator: $\frac{x(x+1)}{3(x+1)}$
Now, cancel out the common factor $x+1$ in the numerator and denominator: $\frac{x}{3}$
So, the simplified form of $\frac{x^2+x}{3x+3}$ is $\frac{x}{3}$.
0𝔹

Question 11

Write as a single fraction in its simplest form

$$\frac{x}{3} + \frac{x-1}{2}.$$

[2]

To write the expression $\frac{x}{3} + \frac{x-1}{2}$ as a single fraction in its simplest form, find a common denominator, which is the product of 3 and 2, i.e., 6 .
Now, express each fraction with this common denominator:
$\frac{x}{3} \cdot \frac{2}{2} + \frac{x-1}{2} \cdot \frac{3}{3} = \frac{2x}{6} + \frac{3(x-1)}{6}$
Combine the numerators over the common denominator:
$\frac{2x+3(x-1)}{6}$
Distribute and simplify the numerator:
$\frac{2x+3x-3}{6} = \frac{5x-3}{6}$
So, the expression $\frac{x}{3} + \frac{x-1}{2}$ as a single fraction in its simplest form is $\frac{5x-3}{6}$.

EXAM PAPERS PRACTICE

Question 12

Write as a single fraction in its simplest form

$$\frac{4}{2x+3} - \frac{2}{x-3}$$ [3]

To write the expression $\frac{4}{2x+3} - \frac{2}{x-3}$ as a single fraction in its simplest form, find a common denominator, which is $(2x+3)(x-3)$.
Now, express each fraction with this common denominator:
$\frac{4}{2x+3} \cdot \frac{x-3}{x-3} - \frac{2}{x-3} \cdot \frac{2x+3}{2x+3}$
Combine the numerators over the common denominator:
$\frac{4(x-3)-2(2x+3)}{(2x+3)(x-3)}$
Distribute and simplify the numerator:
$\frac{4x-12-4x-6}{(2x+3)(x-3)} = \frac{-18}{(2x+3)(x-3)}$
So, the expression $\frac{4}{2x+3} - \frac{2}{x-3}$ as a single fraction in its simplest form is $\frac{-18}{(2x+3)(x-3)}$.

Question 13

Simplify $\frac{x}{3} + \frac{5x}{9} - \frac{5x}{18}$. [2]

$\frac{x}{3} \cdot \frac{6}{6} + \frac{5x}{9} \cdot \frac{2}{2} - \frac{5x}{18} \cdot \frac{1}{1}$
Combine the numerators over the common denominator:
$\frac{6x+10x-5x}{18}$
Combine like terms in the numerator:
$\frac{11x}{18}$
So, the simplified form of $\frac{x}{3} + \frac{5x}{9} - \frac{5x}{18}$ is $\frac{11x}{18}$.

EXAM PAPERS PRACTICE

Question 14

Write as a fraction in its simplest form

$$\frac{x-3}{4}+\frac{4}{x-3}.$$ [3]

$\frac{(x-3)^2}{4(x-3)}+\frac{16}{4(x-3)}$

Combine the numerators over the common denominator:

$\frac{(x-3)^2+16}{4(x-3)}$

Now, expand and simplify the numerator:

$\frac{x^2-6x+9+16}{4(x-3)}=\frac{x^2-6x+25}{4(x-3)}$

So, the expression $\frac{x-3}{4}+\frac{4}{x-3}$ as a single fraction in its simplest form is $\frac{x^2-6x+25}{4(x-3)}$.

Question 15

Write as a single fraction in its simplest form

$$\frac{5}{x}-\frac{4}{x+1}.$$ [2]

$\frac{5(x+1)}{x(x+1)}-\frac{4x}{x(x+1)}$

Combine the numerators over the common denominator:

$\frac{5(x+1)-4x}{x(x+1)}$

Now, distribute and simplify the numerator:

$\frac{5x+5-4x}{x(x+1)}$

Combine like terms in the numerator:

$\frac{x+5}{x(x+1)}$

So, the expression $\frac{5}{x}-\frac{4}{x+1}$ as a single fraction in its simplest form is $\frac{x+5}{x(x+1)}$.

EXAM PAPERS PRACTICE

Question 16

Simplify $\frac{x+2}{x} - \frac{x}{x+2}$. Write your answer as a fraction in its simplest form.

To simplify the expression $\frac{x+2}{x} - \frac{x}{x+2}$, find a common denominator. In this case, the common denominator is $x(x+2)$.
Now, express each fraction with this common denominator:
$\frac{(x+2)^2}{x(x+2)} - \frac{x^2}{x(x+2)}$
Combine the numerators over the common denominator:
$\frac{(x+2)^2 - x^2}{x(x+2)}$
Now, expand and simplify the numerator:
$\frac{x^2+4x+4-x^2}{x(x+2)}$
Combine like terms in the numerator:
$\frac{4x+4}{x(x+2)}$
Factor out the common factor in the numerator:
$\frac{4(x+1)}{x(x+2)}$
So, the simplified form of $\frac{x+2}{x} - \frac{x}{x+2}$ is $\frac{4(x+1)}{x(x+2)}$.

Question 17

14 (a) Write $\frac{3}{x} - \frac{2}{x+1}$ as a single fraction in its simplest form.

To write the expression $\frac{3}{x} - \frac{2}{x+1}$ as a single fraction in its simplest form, find a common denominator. In this case, the common denominator is $x(x+1)$.
Now, express each fraction with this common denominator:
$\frac{3(x+1)}{x(x+1)} - \frac{2x}{x(x+1)}$ [3]
Combine the numerators over the common denominator:
$\frac{3(x+1)-2x}{x(x+1)}$
Now, distribute and simplify the numerator:
$\frac{3x+3-2x}{x(x+1)}$
Combine like terms in the numerator:
$\frac{x+3}{x(x+1)}$
So, the expression $\frac{3}{x} - \frac{2}{x+1}$ as a single fraction in its simplest form is $\frac{x+3}{x(x+1)}$.

(b) Solve the equation $\frac{3}{x} - \frac{2}{x+1} = 0$.

To solve the equation $\frac{3}{x} - \frac{2}{x+1} = 0$, follow these steps:
1. Find a common denominator, which is $x(x+1)$.
2. Express each fraction with this common denominator:
$\frac{3(x+1)}{x(x+1)} - \frac{2x}{x(x+1)} = \frac{3(x+1)-2x}{x(x+1)}$
1. Combine the numerators: [1]
$\frac{3x+3-2x}{x(x+1)} = \frac{x+3}{x(x+1)}$
1. Set the numerator equal to zero:
$x + 3 = 0$
1. Solve for x :
$x = -3$
So, the solution to the equation $\frac{3}{x} - \frac{2}{x+1} = 0$ is $x = -3$.

EXAM PAPERS PRACTICE

Question 18

Work out as a single fraction

$$\frac{2}{x-3} - \frac{1}{x+4}.$$ [3]

To work out the expression $\frac{2}{x-3} - \frac{1}{x+4}$ as a single fraction, find a common denominator. In this case, the common denominator is $(x-3)(x+4)$.

Now, express each fraction with this common denominator:

$\frac{2(x+4)}{(x-3)(x+4)} - \frac{1(x-3)}{(x-3)(x+4)}$

Combine the numerators over the common denominator:

$\frac{2(x+4)-(x-3)}{(x-3)(x+4)}$

Now, distribute and simplify the numerator:

$\frac{2x+8-x+3}{(x-3)(x+4)}$

Combine like terms in the numerator:

$\frac{x+11}{(x-3)(x+4)}$

So, the expression $\frac{2}{x-3} - \frac{1}{x+4}$ as a single fraction is $\frac{x+11}{(x-3)(x+4)}$.

Question 19

Write $2x - \frac{10x}{5-x}$ as a single fraction. [2]

To write the expression $2x - \frac{10x}{5-x}$ as a single fraction, find a common denominator, which is $(5-x)$.

Now, express each term with this common denominator:

$2x \cdot \frac{5-x}{5-x} - \frac{10x}{5-x} \cdot \frac{5}{5}$

Combine the numerators over the common denominator:

$\frac{2x(5-x)-10x \cdot 5}{5-x}$

Distribute and simplify the numerator:

$\frac{10x-2x^2-50x}{5-x}$

Combine like terms in the numerator:

$\frac{-2x^2-40x}{5-x}$

Factor out a common factor of $-2x$:

$\frac{-2x(x+20)}{5-x}$

So, the expression $2x - \frac{10x}{5-x}$ as a single fraction is $\frac{-2x(x+20)}{5-x}$.

Question 20

Write as a single fraction in its simplest form.

$$\frac{x+3}{x-3} - \frac{x-1}{x+1}$$ [4]

To write the expression $\frac{x+3}{x-3} - \frac{x-1}{x+1}$ as a single fraction in its simplest form, find a common denominator, which is $(x-3)(x+1)$.
Now, express each fraction with this common denominator:
$\frac{(x+3)(x+1)}{(x-3)(x+1)} - \frac{(x-1)(x-3)}{(x-3)(x+1)}$
Combine the numerators over the common denominator:
$\frac{(x+3)(x+1)-(x-1)(x-3)}{(x-3)(x+1)}$
Now, distribute and simplify the numerator:
$\frac{x^2+4x+3-(x^2-4x+3)}{(x-3)(x+1)}$
Combine like terms in the numerator:
$\frac{x^2+4x+3-x^2+4x-3}{(x-3)(x+1)}$
Combine like terms:
$\frac{8x}{(x-3)(x+1)}$
So, the expression $\frac{x+3}{x-3} - \frac{x-1}{x+1}$ as a single fraction in its simplest form is $\frac{8x}{(x-3)(x+1)}$.

Question 21

Write the following as a single fraction in its simplest form. [3]

$$\frac{x+2}{3} - \frac{2x-1}{4} + 1$$

To combine the given expression $\frac{x+2}{3} - \frac{2x-1}{4} + 1$ into a single fraction, find a common denominator. In this case, the common denominator is 12 (the least common multiple of 3 and 4).
Now, express each term with this common denominator:
$\frac{(x+2)\cdot 4}{3\cdot 4} - \frac{(2x-1)\cdot 3}{4\cdot 3} + 1$
Combine the numerators over the common denominator:
$\frac{4(x+2)-3(2x-1)}{12} + 1$
Distribute and simplify the numerator:
$\frac{4x+8-6x+3}{12} + 1$
Combine like terms in the numerator:
$\frac{-2x+11}{12} + 1$
To add the fraction and the whole number, express 1 as a fraction with the common denominator 12 :
$\frac{-2x+11}{12} + \frac{12}{12}$
Combine the fractions:
$\frac{-2x+11+12}{12}$
Combine like terms in the numerator:
$\frac{-2x+23}{12}$
So, the expression $\frac{x+2}{3} - \frac{2x-1}{4} + 1$ as a single fraction in its simplest form is $\frac{2x+23}{12}$.

Question 22

Simplify the following.

$$\frac{h^2 - h - 20}{h^2 - 25}$$ [4]

To simplify the expression $\frac{h^2-h-20}{h^2-25}$, factor both the numerator and the denominator:

$\frac{(h-5)(h+4)}{(h-5)(h+5)}$

Now, cancel out the common factor $(h - 5)$ in the numerator and denominator:

$\frac{h+4}{h+5}$

So, the simplified form of $\frac{h^2-h-20}{h^2-25}$ is $\frac{h+4}{h+5}$.

Question 23

Simplify fully.

$$\frac{x^2 - x - 20}{x^3 - 10x^2 + 25x}$$ [5]

To simplify the expression $\frac{x^2-x-20}{x^3-10x^2+25x}$, factor both the numerator and the denominator:

$\frac{(x-5)(x+4)}{x(x-5)(x-5)}$

Now, cancel out the common factor $(x - 5)$ in the numerator and denominator:

$\frac{x+4}{x(x-5)}$

So, the simplified form of $\frac{x^2-x-20}{x^3-10x^2+25x}$ is $\frac{x+4}{x(x-5)}$.

Question 24

Write as a single fraction in its simplest form.

$$\frac{3}{x+10} - \frac{1}{x+4}$$ [3]

To write the expression $\frac{3}{x+10} - \frac{1}{x+4}$ as a single fraction in its simplest form, find a common denominator, which is $(x+10)(x+4)$.
Now, express each fraction with this common denominator:
$\frac{3(x+4)}{(x+10)(x+4)} - \frac{1(x+10)}{(x+10)(x+4)}$
Combine the numerators over the common denominator:
$\frac{3(x+4)-(x+10)}{(x+10)(x+4)}$
Distribute and simplify the numerator:
$\frac{3x+12-x-10}{(x+10)(x+4)}$
Combine like terms in the numerator:
$\frac{2x+2}{(x+10)(x+4)}$
Factor out the common factor in the numerator:
$\frac{2(x+1)}{(x+10)(x+4)}$
So, the expression $\frac{3}{x+10} - \frac{1}{x+4}$ as a single fraction in its simplest form is $\frac{2(x+1)}{(x+10)(x+4)}$.

Question 25

15 Write the following as a single fraction in its simplest form.

$$\frac{x+1}{x+5} - \frac{x}{x+1}$$ [4]

To write the expression $\frac{x+1}{x+5} - \frac{x}{x+1}$ as a single fraction in its simplest form, find a common denominator, which is $(x+5)(x+1)$.
Now, express each fraction with this common denominator:
$\frac{(x+1)^2}{(x+5)(x+1)} - \frac{x(x+5)}{(x+5)(x+1)}$
Combine the numerators over the common denominator:
$\frac{(x+1)^2-x(x+5)}{(x+5)(x+1)}$
Now, distribute and simplify the numerator:
$\frac{x^2+2x+1-x^2-5x}{(x+5)(x+1)}$
Combine like terms in the numerator:
$\frac{-3x+1}{(x+5)(x+1)}$
So, the expression $\frac{x+1}{x+5} - \frac{x}{x+1}$ as a single fraction in its simplest form is $\frac{-3x+1}{(x+5)(x+1)}$.

Question 26

Write $\frac{2}{x-2}+\frac{3}{x+2}$ as a single fraction.

Give your answer in its simplest form. [3]

To combine the fractions $\frac{2}{x-2}+\frac{3}{x+2}$ into a single fraction, find a common denominator, which is $(x-2)(x+2)$.

Now, express each fraction with this common denominator:

$\frac{2(x+2)}{(x-2)(x+2)}+\frac{3(x-2)}{(x-2)(x+2)}$

Combine the numerators over the common denominator:

$\frac{2(x+2)+3(x-2)}{(x-2)(x+2)}$

Distribute and simplify the numerator:

$\frac{2x+4+3x-6}{(x-2)(x+2)}$

Combine like terms in the numerator:

$\frac{5x-2}{(x-2)(x+2)}$

So, the expression $\frac{2}{x-2}+\frac{3}{x+2}$ as a single fraction in its simplest form is $\frac{5x-2}{(x-2)(x+2)}$.

Question 27

Write as a single fraction in its simplest form.

$$\frac{2}{x}+\frac{1}{2x}+\frac{1}{2}$$ [2]

$\frac{2}{x}+\frac{1}{2x}+\frac{1}{2}=\frac{2\cdot2}{2\cdot x}+\frac{1}{2\cdot x}+\frac{x}{2\cdot x}$

Now, combine the numerators over the common denominator:

$\frac{4+1+x}{2x}=\frac{x+5}{2x}$

So, $\frac{2}{x}+\frac{1}{2x}+\frac{1}{2}$ can be written as a single fraction in its simplest form as $\frac{x+5}{2x}$.

Question 28

Simplify this fraction.

$$\frac{x^2-5x+6}{x^2-4}$$ [4]

To simplify the given fraction $\frac{x^2-5x+6}{x^2-4}$, you can factor both the numerator and the denominator, and then cancel out common factors.
First, factor the numerator and denominator:
Numerator: x^2-5x+6 factors as $(x-2)(x-3)$.
Denominator: x^2-4 factors as $(x-2)(x+2)$.
Now, rewrite the fraction with the factored forms:
$\frac{(x-2)(x-3)}{(x-2)(x+2)}$
Cancel out the common factor $(x-2)$ from the numerator and denominator:
$\frac{(x-2)(x-3)}{(x-2)(x+2)}$
The simplified fraction is:
$\frac{x-3}{x+2}$
So, $\frac{x^2-5x+6}{x^2-4}$ simplifies to $\frac{x-3}{x+2}$.

Question 29

Write as a single fraction, in its simplest form.

$$\frac{3}{x+2}-\frac{2}{x-1}$$ [3]

$\frac{3}{x+2}-\frac{2}{x-1}=\frac{3\cdot(x-1)}{(x+2)(x-1)}-\frac{2\cdot(x+2)}{(x+2)(x-1)}$
Now, combine the numerators over the common denominator:
$\frac{3(x-1)-2(x+2)}{(x+2)(x-1)}$
Distribute and simplify the numerator:
$\frac{3x-3-2x-4}{(x+2)(x-1)}=\frac{x-7}{(x+2)(x-1)}$
So, $\frac{3}{x+2}-\frac{2}{x-1}$ can be written as a single fraction in its simplest form as $\frac{x-7}{(x+2)(x-1)}$.

Question 30

Write as a single fraction in its simplest form.

$$\frac{2x-1}{3}-\frac{2}{x+1}$$ [3]

$\frac{2x-1}{3}-\frac{2}{x+1}=\frac{(2x-1)\cdot(x+1)}{3\cdot(x+1)}-\frac{2\cdot3}{3\cdot(x+1)}$

Now, combine the numerators over the common denominator:

$\frac{(2x-1)(x+1)-2\cdot3}{3\cdot(x+1)}$

Expand and simplify the numerator:

$\frac{2x^2+2x-x-1-6}{3\cdot(x+1)}=\frac{2x^2+x-7}{3\cdot(x+1)}$

So, $\frac{2x-1}{3}-\frac{2}{x+1}$ can be written as a single fraction in its simplest form as $\frac{2x^2+x-7}{3\cdot(x+1)}$.

Question 31

Simplify. $\dfrac{x^2-16}{x^2-3x-4}$ [4]

To simplify the given fraction $\frac{x^2-16}{x^2-3x-4}$, you can factor the numerator and denominator, and then cancel out common factors.

First, factor the numerator and denominator:

Numerator: x^2-16 factors as $(x+4)(x-4)$.

Denominator: x^2-3x-4 factors as $(x-4)(x+1)$.

Now, rewrite the fraction with the factored forms:

$\frac{(x+4)(x-4)}{(x-4)(x+1)}$

Cancel out the common factor $(x-4)$ from the numerator and denominator:

$\frac{(x+4)(x-4)}{(x-4)(x+1)}$

The simplified fraction is:

$\frac{x-4}{x+1}$

So, $\frac{x^2-16}{x^2-3x-4}$ simplifies to $\frac{x-4}{x+1}$.

Question 32

Write as a single fraction in its simplest form.

$$\frac{3}{x+2} - \frac{4}{2x-5} \qquad [3]$$

$\frac{3}{x+2} - \frac{4}{2x-5} = \frac{3(2x-5)}{(x+2)(2x-5)} - \frac{4(x+2)}{(x+2)(2x-5)}$

Now, combine the numerators over the common denominator:

$\frac{3(2x-5)-4(x+2)}{(x+2)(2x-5)}$

Distribute and simplify the numerator:

$\frac{6x-15-4x-8}{(x+2)(2x-5)} = \frac{2x-23}{(x+2)(2x-5)}$

So, $\frac{3}{x+2} - \frac{4}{2x-5}$ can be written as a single fraction in its simplest form as $\frac{2x-23}{(x+2)(2x-5)}$.

Question 33

(a) Write as a single fraction in its simplest form.

$$\frac{3}{2x-1} - \frac{1}{x+2}$$ [3]

$\frac{3}{2x-1} - \frac{1}{x+2} = \frac{3(x+2)}{(2x-1)(x+2)} - \frac{(2x-1)}{(2x-1)(x+2)}$

Now, combine the numerators over the common denominator:

$\frac{3(x+2)-(2x-1)}{(2x-1)(x+2)}$

Distribute and simplify the numerator:

$\frac{3x+6-2x+1}{(2x-1)(x+2)} = \frac{x+7}{(2x-1)(x+2)}$

So, $\frac{3}{2x-1} - \frac{1}{x+2}$ can be written as a single fraction in its simplest form as $\frac{x+7}{(2x-1)(x+2)}$.

(b) Simplify.

$$\frac{4x^2-16x}{2x^2+6x-56}$$ [4]

To simplify the given fraction $\frac{4x^2-16x}{2x^2+6x-56}$, you can first factor the numerator and denominator and then cancel out common factors.

Factor the numerator and denominator:

Numerator: $4x^2 - 16x$ factors as $4x(x-4)$.

Denominator: $2x^2 + 6x - 56$ factors as $2(x-4)(x+7)$.

Now, rewrite the fraction with the factored forms:

$\frac{4x(x-4)}{2(x-4)(x+7)}$

Cancel out the common factor $(x-4)$ from the numerator and denominator:

$\frac{4x(x-4)}{2\cdot(x-4)(x+7)}$

Simplify further:

$\frac{4x}{2(x+7)}$

Now, simplify the fraction by dividing both the numerator and denominator by 2 :

$\frac{2x}{x+7}$

So, $\frac{4x^2-16x}{2x^2+6x-56}$ simplifies to $\frac{2x}{x+7}$.

Question 34

Write as a single fraction, in its simplest form. [4]

$$\frac{3}{2x} + \frac{2x}{3} + 3 + 2x$$

$\frac{3}{2x} + \frac{2x}{3} + 3 + 2x = \frac{3 \cdot 3}{2 \cdot 3x} + \frac{2x \cdot 2x}{3 \cdot 2x} + \frac{3 \cdot 2x}{3 \cdot 2x} + \frac{2x \cdot 6x}{1 \cdot 6x}$

Now, combine the numerators over the common denominator:

$\frac{9+4x^2+6x+12x^2}{6x}$

Combine like terms in the numerator:

$\frac{16x^2+6x+9}{6x}$

Factor the numerator if possible:

$\frac{(4x+3)(4x+3)}{6x}$

Now, simplify the fraction by canceling common factors in the numerator and denominator:

$\frac{(4x+3)}{2}$

So, $\frac{3}{2x} + \frac{2x}{3} + 3 + 2x$ can be written as a single fraction in its simplest form as $\frac{4x+3}{2}$.

Question 35

Write as a single fraction in its simplest form.

$$\frac{2}{x} - \frac{2}{x+1}$$ [3]

$\frac{2}{x} - \frac{2}{x+1} = \frac{2 \cdot (x+1)}{x \cdot (x+1)} - \frac{2 \cdot x}{x \cdot (x+1)}$

Combine the numerators over the common denominator:

$\frac{2(x+1)-2x}{x \cdot (x+1)}$

Distribute and simplify the numerator:

$\frac{2x+2-2x}{x \cdot (x+1)}$

Combine like terms in the numerator:

$\frac{2}{x \cdot (x+1)}$

So, $\frac{2}{x} - \frac{2}{x+1}$ can be written as a single fraction in its simplest form as $\frac{2}{x \cdot (x+1)}$.

□

Question 36

Solve the equation.

$$\frac{3}{2x} + \frac{1}{x+1} = 0$$ [3]

$\frac{3}{2x} + \frac{1}{x+1} = \frac{3\cdot(x+1)}{2x\cdot(x+1)} + \frac{1\cdot 2x}{(x+1)\cdot 2x}$

Combine the numerators over the common denominator:

$\frac{3(x+1)+2x}{2x\cdot(x+1)}$

Distribute and simplify the numerator:

$\frac{3x+3+2x}{2x\cdot(x+1)}$

Combine like terms in the numerator:

$\frac{5x+3}{2x\cdot(x+1)}$

Now, set the numerator equal to zero and solve for x :

$5x + 3 = 0$

Subtract 3 from both sides:

$5x = -3$

Divide by 5 :

$x = -\frac{3}{5}$

So, the solution to the equation $\frac{3}{2x} + \frac{1}{x+1} = 0$ is $x = -\frac{3}{5}$.

Question 37

19 Simplify.

$$\frac{x^2 + 6x - 7}{3x + 21}$$ [4]

To simplify the given fraction $\frac{x^2 6x-7}{3x+21}$, you can factor the numerator and look for common factors in the numerator and denominator:

Factor the numerator:

$x^2 + 6x - 7 = (x + 7)(x - 1)$

Now, rewrite the fraction with the factored numerator:

$\frac{(x+7)(x-1)}{3x+21}$

Now, check for common factors. Both the numerator and denominator have a common factor of 7:

$\frac{7(x+7)(x-1)}{7\cdot(3x+3)}$

Cancel out the common factor:

$\frac{(x+7)(x-1)}{3x+3}$

Now, simplify further by factoring out a common factor of 3 in the denominator:

$\frac{(x+7)(x-1)}{3(x+1)}$

So, $\frac{x^2+6x-7}{3x+21}$ simplifies to $\frac{(x+7)(x-1)}{3(x+1)}$.

Question 38

(a) Factorise $x^2 + x - 30$. [2]

To factorize the quadratic expression $x^2 + x - 30$, we are looking for two numbers whose product is the product of the coefficient of x^2 term (1) and the constant term (−30) and whose sum is the coefficient of the x term (1).
The numbers that satisfy these conditions are 6 and -5 because $6 \times (-5) = -30$ and $6 + (-5) = 1$.
Now, split the x term using these numbers:
$x^2 + x - 30 = x^2 + 6x - 5x - 30$
Now, group the terms:
$x^2 + 6x - 5x - 30 = (x^2 + 6x) + (-5x - 30)$
Factor out the common factor from each group:
$x(x + 6) - 5(x + 6)$
Now, factor out the common factor of $(x + 6)$:
$(x + 6)(x - 5)$
So, the factorization of $x^2 + x - 30$ is $(x + 6)(x - 5)$.

[1]

(b) Simplify $\dfrac{(x-5)(x+4)}{x^2+x-30}$.

To simplify the expression $\frac{(x-5)(x+4)}{x^2+x-30}$, you can use the factorization of the quadratic denominator $x^2 + x - 30$ obtained in the previous response, which is $(x + 6)(x - 5)$.
Now, substitute this factorization into the expression:
$\frac{(x-5)(x+4)}{x^2+x-30} = \frac{(x-5)(x+4)}{(x+6)(x-5)}$
Now, cancel out the common factor $(x - 5)$ in the numerator and denominator:
$\frac{(x-5)(x+4)}{(x+6)(x-5)}$
The simplified expression is:
$\frac{x+4}{x+6}$
So, $\frac{(x-5)(x+4)}{x^2+x-30}$ simplifies to $\frac{x+4}{x+6}$.

[3]

Question 39

22 Write as a single fraction in its simplest form.

$$\frac{2}{x+3} + \frac{3}{x+2}$$

$\frac{2}{x+3} + \frac{3}{x+2} = \frac{2\cdot(x+2)}{(x+3)(x+2)} + \frac{3\cdot(x+3)}{(x+3)(x+2)}$
Combine the numerators over the common denominator:
$\frac{2(x+2)+3(x+3)}{(x+3)(x+2)}$
Distribute and simplify the numerator:
$\frac{2x+4+3x+9}{(x+3)(x+2)}$
Combine like terms in the numerator:
$\frac{5x+13}{(x+3)(x+2)}$
So, $\frac{2}{x+3} + \frac{3}{x+2}$ can be written as a single fraction in its simplest form as $\frac{5x+13}{(x+3)(x+2)}$.