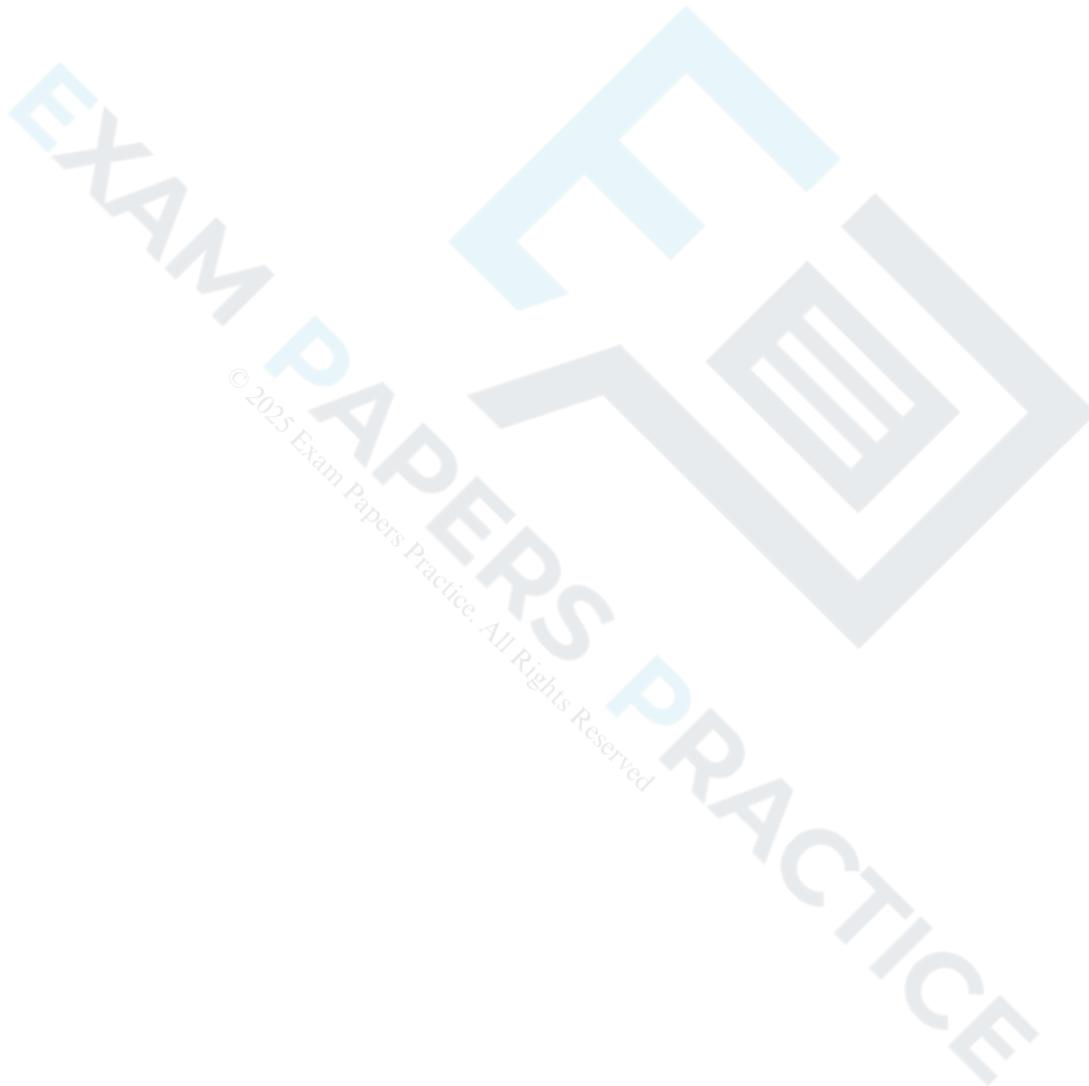




Question	Answer/Indicative content	Marks	Part marks and guidance	
	$y = 12, -12$	A1	<p>No extras</p> <p><b>Examiner's Comments</b></p> <p>Many candidates realised what needed to be done in this unstructured question and a large proportion secured the first five marks by correctly differentiating the equation of the curve and equating this to 8, the gradient of the line. A relatively common error at this stage was to equate to the negative reciprocal of the gradient, showing confusion regarding parallel and perpendicular gradients. The resulting disguised quadratic proved far more difficult than usual as many candidates did not recognise this out of context, as it is more usually seen as a question in its own right. Of the candidates who did realise the need to make a substitution, many did not multiply by <math>x^2</math> and incorrectly substituted <math>y</math> for <math>x^2</math> and <math>y^2</math> for <math>x^4</math>; they secured no more marks. Those who proceeded correctly usually factorised the simple resulting quadratic and remembered to take the square root to find <math>x</math>, although it was quite common to omit the <math>-3</math>. Thereafter, the vast majority of successful candidates found the corresponding value(s) of <math>y</math> correctly, although a number erroneously substituted into the line rather than the curve. This question proved</p>	<p>If curve equated to line and before differentiating:</p> <p>First four marks <b>B1 M1 A1 B1</b> available as main scheme            Then <b>M0</b> for equating as this not been explicitly done            Allow the <b>M1</b> for the substitution  <b>DM1</b> for quadratic as main scheme (dependent on a correct substitution)  <b>A0</b> for the 9 (as follows wrong working)  <b>DM1</b> for square rooting (dependent on a correct substitution)  <b>A0</b> for the co-ordinates (as follows wrong working).            Max mark 7/10</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					appropriately discriminating with less than a quarter of candidates scoring full marks.	
			<b>Total</b>	<b>10</b>		



Question	Answer/Indicative content	Marks	Part marks and guidance	
2	$(3k - 1)^2 - 4 \times k \times -4$  $= 9k^2 + 10k + 1$  $9k^2 + 10k + 1 < 0$  $(9k + 1)(k + 1) < 0$  $-1, -\frac{1}{9}$  $-1 < k < -\frac{1}{9}$	*M1  A1  M1  DM1  A1  M1  A1	<p>Attempts <math>b^2 - 4ac</math> or an equation or inequality involving <math>b^2</math> and <math>4ac</math>. Must involve <math>k^2</math> in first term (but no <math>x</math> anywhere). If <math>b^2 - 4ac</math> not stated, must be clear attempt.</p> <p>Correct discriminant, simplified to 3 terms</p> <p>States discriminant <math>&lt; 0</math> or <math>b^2 &lt; 4ac</math>.</p> <p>Correct method to find roots of a three term quadratic</p> <p>Both values of <math>k</math> correct</p> <p>Chooses "inside region" of inequality</p> <p>Allow  <math>"k &lt; -\frac{1}{9} \text{ and } k &gt; -1"</math>            etc. must be strict inequalities for A mark</p> <p><b>Examiner's Comments</b></p> <p>This unstructured question proved to be very demanding. Most candidates recognised the need to find the discriminant and the majority realised that this needed to be less than zero. Given that both terms involved algebraic manipulation, determining the discriminant proved challenging to a large number of candidates. Similarly, the solution of the resulting quadratic inequality proved</p>	<p>Must be working with the discriminant explicitly and not only as part of the quadratic formula. Allow <math>\sqrt{b^2 - 4ac}</math> for first M1 A1</p> <p>Can be awarded at any stage. Doesn't need first M1. No square root here</p> <p>Allow correct region for their inequality</p> <p>Do not allow  <math>"k &lt; -\frac{1}{9} \text{ or } k &gt; -1"</math>;</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>challenging, with added difficulty seeming to result from the fact that both roots were negative; a significant number thought that <math>-\frac{1}{9}</math> was less than <math>-1</math>, showing these roots in the wrong positions on the x-axis and getting the inequality the wrong way round when their intention was to choose the inside region. The best candidates handled all these obstacles well and produced short fluent solutions gaining all seven marks (as achieved by around one-third of candidates); some candidates were unable to start the question at all, instead trying to solve the equation using the quadratic formula.</p>
			<b>Total</b>	<b>7</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
3	i	$\frac{p-7}{-4-^{-}2}$ or $\frac{7-p}{-2-^{-}4}$	M1	uses $\frac{y_2 - y_1}{x_2 - x_1}$ (at least 3 out of 4 correct)	<b>Alternative method:</b> Equation of line through one of the given points with gradient 4 <b>M1</b> Substitutes <b>other</b> point into their equation <b>M1</b>
	i	$\frac{p-7}{-4-^{-}2} = 4$ or $\frac{7-p}{-2-^{-}4} = 4$ $p = -1$	A1  A1	Correct, unsimplified equation  <b>Examiner's Comments</b> <input type="checkbox"/>  There was a variety of approaches to this question, many of which worked well, with errors mostly being seen in the subtraction of negative numbers. The most successful method was to find the equation of the line through the given point and then substitute $x$ for the other point. Also very successful was the informal method of counting up or down in 4s. The gradient method needed more care with the negative numbers and was by far the method most prone to error, both in substitution and subsequent calculation.	Obtains $p = -1$ (Accept $y = -1$ ) <b>A1</b> <b>Note:</b> Other "informal" methods can score full marks provided <b>www</b>
	ii	$\frac{-2+6}{2} = m, \quad \frac{7+q}{2} = 5$	M1	Correct method (may be implied by one correct coordinate)	Use the same marking principle for candidates who add / subtract half the difference to an end point or use similar triangles or other valid "informal" methods
	ii	$m = 2$	A1		

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$q = 3$	A1	<u>Examiner's Comments</u> <input type="checkbox"/> Again informal methods were often more successful than formal ones; use of the mid-point formula was more successful in finding $m$ than $q$ .	
	iii	$\sqrt{(-2-d)^2 + (7-3)^2}$	*M1	Correct method to find line length / square of line length using Pythagoras' theorem (at least 3 out of 4 correct)	<b>SC: B1</b> for each value of $d$ found or "spotted" from correct working  <b>Note:</b> Other "informal" methods can score full marks provided <b>www</b>
	iii	$d^2 + 4d + 20 = 52$	B1	$(2\sqrt{13})^2 = 52$ or $2\sqrt{13} = \sqrt{52}$	
	iii	$d^2 + 4d - 32 = 0$ $(d+8)(d-4) = 0$	DM1	Correct method to solve 3 term quadratic, must involve their "52"	
	iii	$d = -8$ or $4$	A1	<u>Examiner's Comments</u> In this question candidates again used a variety of methods, with many spotting that the difference between $d$ and $-2$ had to be $\sqrt{36}$ ; commonly the negative root was missed here leading to a score of 3 out of 4. Most candidates approached this more formally, using Pythagoras' theorem and again weak algebra and difficulties with negative values led to the loss of marks; some struggled to square $2\sqrt{13}$ accurately.	
		<b>Total</b>	<b>10</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
4	i	$2x^2 - 3x - 5 = \frac{-10x - 11}{2}$	*M1	Substitute for $x/y$ or attempt to get an equation in 1 variable only	or $10x + 2(2x^2 - 3x - 5) + 11 = 0$
	i	$4x^2 + 4x + 1 = 0$	A1	Obtain correct 3 term quadratic – could be a multiple e.g. $2x^2 + 2x + 0.5 = 0$	If $x$ is eliminated, expect $k(8y^2 + 48y + 72) = 0$
	i	$(2x + 1)(2x + 1) = 0$	DM1	Correct method to solve resulting 3 term quadratic	
	i	$x = -\frac{1}{2}$	A1		SC If DM0 and $x = -\frac{1}{2}$ spotted
	i	$y = -3$	A1	<b>Examiner's Comments</b>  Almost all candidates recognised the need to eliminate a variable and chose to eliminate $y$ . There were errors in finding the quadratic, but most then went on to factorise correctly and find the values of both variables; forgetting to find $y$ is now comparatively rare. A large number of candidates, however, found the substitution of $x = -\frac{1}{2}$ to find $y$ difficult and many lost this mark.	<b>B1</b> for $x$ value, <b>B1</b> for $y$ value <b>B1</b> justifying only one root

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	Line is a tangent to the curve	B1✓	<p>Must be consistent with their answers to their quadratic in (i).  <b>1 repeated root</b> – indicates one point.            Accept tangent, meet at, intersect, touch etc. but do not accept cross  <b>2 roots</b> – indicates meet at two points  <b>0 roots</b> – indicates do not meet. Do not accept “do not cross”</p> <p><u>Examiner's Comments</u></p> <p>One acceptable response was that one root implied that the line was a tangent to the curve. The question did not specify that a geometrical comment was required and so “meeting at one point” was another acceptable response. Candidates who made an error in part (i) were rewarded for a consistent conclusion relating to their roots. Use of the word “cross” is unhelpful; for example, in the case where there are no solutions saying “they do not cross” does not exclude the possibility that they touch. A number of candidates were using stock phrases irrespective of their answer to (i), such as “they are perpendicular” or “it just touches the x-axis” or stating the line was a tangent when they had found two different roots; these of course gained no credit.</p>	Follow through from their solution to (i)
		<b>Total</b>	<b>6</b>		

Question			Answer/Indicative content	Marks	Part marks and guidance	
5		i	$n = 0$	B1	Allow $3^0$  <u>Examiner's Comments</u>  Only a tiny number of candidates failed to secure the mark for this simple recall of index notation; $\frac{1}{3}$ and 1 were occasionally seen.	
		ii	$\frac{1}{t^3} = 64$ (or $4^3$ )	M1	or $t^3 = \frac{1}{64}$ or $64t^3 = 1$ or $\left(\frac{1}{t}\right)^3 = 64$	Allow embedded
		ii	$t = \frac{1}{4}$	A1	$4^{-1}$ is <b>A0</b> $t = \pm \frac{1}{4}$ is <b>A0</b>  <u>Examiner's Comments</u>  Most candidates knew how to deal with the negative index and rewrote the equation as $\frac{1}{t^3} = 64$ or equivalent. Thereafter, however, a significant number could not proceed further, with $-4$ being a common wrong answer.	$4^{-1}$ <b>www</b> alone implies <b>M1 A0</b>
		iii	$2p^2 = 8$	M1	or $8p^6 = 8^3$ . Allow $2p^{\frac{6}{3}} = 8$ for <b>M1</b>	If not 512, evidence of $8 \times 8 \times 8$ needed.
		iii	$p = 2$	A1	<b>www</b>	<b>SC Spotted B1</b> for 2, <b>B1</b> for $-2$ , <b>B1</b> for justifying exactly 2 solutions

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	or $p = -2$	A1	www  <u>Examiner's Comments</u>  Although a large majority of candidates realised the need to find a cube root, many applied this only to the $p^6$ term and not to the 8. Those that were successful often omitted the negative solution thus surrendering the final mark.	SC $8p^2 = 8, p = \pm 1$ B1
			Total	6		

Question		Answer/Indicative content	Marks	Part marks and guidance	
6	i	$\frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times -2}}{2 \times 1}$	M1	Valid attempt to use quadratic formula	No marks for attempting to factorise
	i	$= \frac{6 \pm \sqrt{44}}{2}$	A1		
	i	$= 3 \pm \sqrt{11}$	A1	Both roots correct and simplified	
		OR:			
	i	$(x-3)^2 - 9 - 2 = 0$ $x-3 = \pm\sqrt{11}$	M1 A1	Correct method to complete square	Must get to $(x-3)$ and $\pm$ stage for the M mark, constants combined correctly gets A1
	i	$x = 3 \pm \sqrt{11}$	A1	Rearranged to correct form cao	
				<u>Examiner's Comments</u>  Almost all candidates recognised from the wording of the question that factorisation was not appropriate and most opted to use the quadratic formula. Most were successful in the initial substitution but a significant number failed to deal accurately with the negative value of $c$ . Many had difficulty simplifying the resulting surd. Some only divided the first term by 2; others erroneously divided $\sqrt{44}$ by 2 to get $\sqrt{22}$ . Most of the candidates who attempted to complete the square were successful, although a number failed to find two roots. Overall, about 70% of candidates secured full marks.	
	ii	$\frac{dy}{dx} = 2x - 6$	B1		

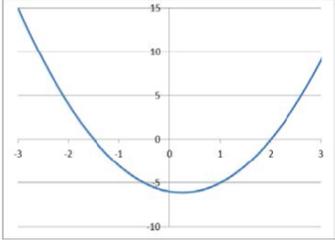
Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	= - 16	B1	<p>www</p> <p><u>Examiner's Comments</u></p> <p>This was generally more successful than part (i). Almost all candidates correctly differentiated the expression and most accurately substituted the given value of <math>x</math> to get <math>-16</math>; 85% of candidates gained both marks. The most common error was to use 5 instead of <math>-5</math>.</p>	
		<b>Total</b>	<b>5</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance		
7	i	$y = -x^3 - 3x^2 + 4x - kx + k$	M1	Attempt to multiply out brackets	Must have $\pm x^3$ and 5 or 6 terms
	i		A1	Can be unsimplified	
	i	$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$	M1	Attempt to differentiate their expansion	If using product rule:
	i		A1	(M0 if signs have changed throughout)	Clear attempt at correct rule M1*
	i	When $x = -3, \frac{dy}{dx} = 0$	M1*	Sets $\frac{dy}{dx} = 0$	Differentiates both parts correctly A1
	i		DM1*		Expand brackets of both parts *DM1
	i			Substitutes $x = -3$ into their	
	i	$-27 + 18 + 4 - k = 0$		$\frac{dy}{dx} = 0$	Then as main scheme

Question		Answer/Indicative content	Marks	Part marks and guidance	
	i	$k = -5$	A1	<p><b>www</b></p> <p><b>Examiner's Comments</b></p> <p>More than half of candidates secured all seven marks available for this question and many clear, compact solutions were seen. Others also scored highly producing solutions marred only by arithmetical error. The conceptual problems that arose came from difficulties in differentiating <math>kx</math> or differentiating <math>k</math> to give 1; most knew to set their derivative to zero and substitute <math>x = -3</math>. Only a few candidates substituted into either the original expression or its expanded form without any attempt at differentiation.</p>	
	ii	$\frac{d^2y}{dx^2} = -6x - 6$	M1	Evaluates second derivative at $x = -3$ or other fully correct method	<p><b>Alternate valid methods include:</b></p> <ol style="list-style-type: none"> <li>1) Evaluating gradient at either side of <math>-3</math></li> <li>2) Evaluating <math>y</math> at either side of <math>-3</math></li> <li>3) Finding other turning point and stating "negative cubic so min before max"</li> </ol>

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	When $x = -3$ , $\frac{d^2y}{dx^2}$ is positive so min point	A1	No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in $k$ value)  <b>Examiner's Comments</b>  Most candidates found the second derivative and considered the sign at $x = -3$ ; only a few equated to zero in error. As his result was independent of $k$ , this was by far the easiest route to success; candidates considering signs or using other methods rarely made any progress.	
	iii	$-3x^2 - 6x + 9 = 9$	M1	Sets their gradient function from (i) (or from a restart) to 9	Allow first M even if $k$ not found but look out for correct answer from wrong working.  <u>Alternative Methods:</u>  <b>Note:</b> Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent  If curve equated to line before differentiating:  M0 A0, can get M1M1 but A0 ww  Maximum mark 2/5
	iii	$3x(x + 2) = 0$ $x = 0$ or $x = -2$	A1	Correct $x$ -values	
	iii	When $x = 0$ , $y = -9$ for line $y = -5$ for curve	M1	One of their $x$ -values substituted into both curve and line / substituted into one and verified to be on the other	
	iii	When $x = -2$ , $y = -27$ for line $y = -27$ for curve	M1	Conclusion that $x = -2$ is the correct value or Second $x$ -value substituted into both curve and line / verified as above	
	iii	$x = -2$ , $y = -27$	A1	$x = -2$ , $y = -27$ <b>www (Check <math>k</math> correct)</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii			<p><b>Examiner's Comments</b></p> <p>This proved, appropriately, the most challenging question on the paper with only about half of candidates making any progress. Even high scoring candidates rarely produced a complete solution as it was necessary to identify a point here both the gradient of the line was equal to the gradient of the curve and such that the point was on both the line and the curve. Most commonly, candidates equated their derivative to 9 and tried to solve the resulting quadratic. Often the solution <math>x = 0</math> was ignored or omitted; others substituted their solution(s) into the line only, sometimes offering both solutions. Many tried to equate the line and curve but were unable to solve the resulting cubic equation. A few who equated using the original form of the equation noticed that <math>x - 1</math> was a common factor and the resulting quadratic had a repeated root, thus implying tangency but this approach was rarely rigorously explained.</p>	<p><b>Alternative method</b></p> <p>Attempt to solve equations of curve and tangent simultaneously <b>and</b> uses valid method to establish at least one root of the resulting cubic (<math>x^3 + 3x^2 - 4 = 0</math> oe) <b>M1</b>            All roots found <b>A1</b>            Either            1) States <math>x = -2</math> is repeated root so tangent <b>M2</b>            (If double root found but not explicitly stated that repeated root implies tangent then <b>M0</b> but <b>B1</b> if <math>(-2, -27)</math> found)            Or            2) Substitutes one <math>x</math> value into their gradient function to determine if equal to gradient of the line <b>M1</b>            Substitutes other <math>x</math> value into their gradient function to determine if equal to gradient of the line or conclusion that <math>-2</math> is the correct one <b>M1</b>  <math>x = -2, y = -27</math> <b>A1 www</b></p> <p><b>SC Trial and Improvement</b></p> <p>Finds at least one value at which the gradient of the curve is 9 <b>B1</b>            Verifies on both line and curve <b>B1 2/5</b></p>
		<b>Total</b>	<b>14</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
8	i $(2x + 3)(x - 2) = 0$  $x = -\frac{3}{2}, x = 2$	M1	Correct method to find roots	
	i 	A1	Correct roots	
	i	B1	Reasonably symmetrical positive quadratic curve, must cross x axis	
	i	B1	y intercept (0, -6) only	Indicated on graph or clearly stated, but there must be a curve

Question		Answer/Indicative content	Marks	Part marks and guidance	
	i		B1	<p>Good curve, with correct roots indicated and min point in 4th quadrant (not on axis)</p> <p><b>Examiner's Comments</b></p> <p>This graph proved much more accessible than the one in Q5, with around four-fifths of candidates securing four or five marks. Almost all realised it was a positive quadratic and many were successful in factorising to obtain the two <math>x</math>-intercepts, although there were some sign errors. Most realised the <math>y</math>-intercept was <math>(0, -6)</math> but a significant number used this as the turning point rather than using the <math>x</math>-intercepts to locate the turning point. Some candidates who were successful in part (ii) had the wisdom to return and correct an error in this part.</p>	Only allow final B1 if curve is clearly intended to be a quadratic symmetrical about min point in 4th quadrant
	ii	$\frac{dy}{dx} = 4x - 1 = 0$	M1	Attempt to find $x$ coordinate of vertex by differentiating and equating / comparing to zero, completing the square, finding the midpoint of their roots oe	SC Award B1 (FT) for $x < 0$ if clearly from their graph
	ii	Vertex when $x = \frac{1}{4}$	A1	cao	NB Look for solution to 9ii done in the space below 9i graph

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$x < \frac{1}{4}$	A1FT	$x <$ their vertex, allow $\leq$  <b>Examiner's Comments</b>  Many candidates did not understand the term "decreasing function" and instead gave the region between their roots found in part (i); indeed nearly half of candidates failed to score at all in this part. Those that approached by differentiation were usually more successful in finding the minimum point than those who completed the square.	
	iii	$2x^2 - x - 6 = 4$	M1	Set quadratic expression equal to 4	
	iii	$2x^2 - x - 10 = 0$			
	iii	$(2x - 5)(x + 2) = 0$	M1	Correct method to solve <b>resulting</b> three term quadratic	<b>Not</b> $2x^2 - x - 6 = 0$ with no use of 4
	iii	$x = \frac{5}{2}, x = -2$	A1	Must have both solutions – no mark for one spotted root	

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii	Distance $PQ = 4\frac{1}{2}$	B1FT	FT from their $x$ values found from their resulting quadratic, provided $y = 4$  <b>Examiner's Comments</b>  Around three quarters of candidates secured the first three marks of this question, equating to 4, simplifying and solving to find the $x$ values of $P$ and $Q$ . Again, factorisation was both the most appropriate and most frequent approach. Rather than then subtracting these values, many saw the word "distance", used the "distance formula" and re-substituted to find $y$ (not always getting 4) and then used Pythagoras' theorem. This led to a lot of unnecessary arithmetical difficulty.	Allow $\frac{9}{2}$ oe, but do not accept unsimplified expressions like $\sqrt{\frac{81}{4}}$
		<b>Total</b>	<b>12</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
9	i	$3(x^2 + 3x) + 10$  $= 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4} + 10$  $= 3\left(x + \frac{3}{2}\right)^2 + \frac{13}{4}$	B1  M1  A1	$\left(x + \frac{3}{2}\right)^2$  $10 - 3p^2$ or $\frac{10}{3} - p^2$  Allow $p = \frac{3}{2}, q = \frac{13}{4}$ <b>A1 www</b>	If $p, q$ found correctly, then ISW slips in format.  $3(x + 1.5)^2 - 3.25$ <b>B1 M0 A0</b> $3(x + 1.5) + 3.25$ <b>B1 M1 A1 (BOD)</b> $3(x + 1.5x)^2 + 3.25$ <b>B0 M1 A0</b> $3(x^2 + 1.5)^2 + 3.25$ <b>B0 M1 A0</b> $3(x - 1.5)^2 + 3.25$ <b>B0 M1 A1 (BOD)</b> $3 \times (x + 1.5)^2 + 3.25$ <b>B0M1A0</b>
	i			<b>Examiner's Comments</b>  The fact that the first digit was given in this "completing the square" question appeared to ease the difficulty somewhat, but this is still an area which many candidates find difficult with less than two-thirds achieving full marks. Identifying the value of $p$ was usually very well done; the problems usually occurred in the calculation of $q$ , with both arithmetic problems, particularly with the squaring, and structural misunderstanding when the candidates failed to multiply by 3.	
	ii	$\left(-\frac{3}{2}, \frac{13}{4}\right)$	B1	FT i.e. – their $p$	If restarted e.g. by differentiation:

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii		B1	FT i.e. their $q$  <b>Examiner's Comments</b>  This question provided a follow through from the previous part which enabled many candidates with poor arithmetic to earn credit for their understanding of the relationship between the format and the graph. Many secured both marks as a result. Those who re-started by differentiation were usually less successful, again due to the difficulties with the fraction work.	Correct method to find $x$ value of minimum point <b>M1</b> Fully correct answer <b>www</b> <b>A1</b>
	iii	$9^2 - (4 \times 3 \times 10)$	M1	Uses $b^2 - 4ac$	Use of $\sqrt{b^2 - 4ac}$ is <b>M0</b> unless recovered
	iii	$= -39$	A1	Ignore $> 0, < 0$ etc. <b>ISW</b> comments about number of roots  <b>Examiner's Comments</b>  Most candidates are familiar with the term discriminant and only a few erroneously used $\sqrt{b^2 - 4ac}$ . Around one in ten candidates substituted correctly but then made arithmetical errors. Commonly seen was $9^2 = 49$ and the subtraction $81 - 120$ often resulted in 39 or $\pm 49$ or $\pm 41$ .	
		<b>Total</b>	<b>7</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
10	i	$f(x) = 6x^2 + 2x$ $f'(x) = -12x^{-3} + 2$	M1	$kx^{-3}$ obtained by differentiation	<b>ISW</b> incorrect simplification after correct expression
	i		A1	$-12x^{-3}$	
	i		B1	$2x$ correctly differentiated to 2  <b>Examiner's Comments</b>  This was very well done, with over 90% of candidates securing all three marks despite the added difficulty of negative powers of $x$ . Even candidates whose overall total was very low recognised and performed the routine of differentiation efficiently. Where errors did occur, these were usually in converting the original expression.	
	ii	$f''(x) = 36x^{-4}$	M1	Attempt to differentiate <b>their (i)</b> i.e. at least one term "correct"	Allow constant differentiated to zero
	ii		A1	Fully correct <b>cao</b> No follow through for <b>A</b> mark  <b>Examiner's Comments</b>  Again, this was very well done, with almost all candidates recognising the notation and differentiating again, usually successfully.	<b>ISW</b> incorrect simplification after correct expression
		<b>Total</b>	<b>5</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
11	$k = x^3$ $8k^2 + 7k - 1 = 0$ $(8k - 1)(k + 1) = 0$ $k = \frac{1}{8}, k = -1$ $x = \frac{1}{2}, x = -1$	<p>M1*</p> <p>DM1*</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Use a substitution to obtain a quadratic or factorise into 2 brackets each containing <math>x^3</math></p> <p>Correct method to solve a quadratic</p> <p>Both values of <math>k</math> correct</p> <p>Attempt to cube root at least one value to obtain <math>x</math></p> <p>Both values of <math>x</math> correct and no other values</p> <p><b>Examiner's Comments</b></p> <p>This disguised quadratic was approached well by the vast majority of candidates, with about two-thirds achieving all five marks. It was very rare to see candidates leap straight to the quadratic formula with no attempt to find the cube roots at the end, which has been a problem with similar questions in the past. Most candidates opted to perform a substitution and then to factorise, although those who opted for a two-bracket approach using <math>x^3</math> were also often successful. Some candidates who used the quadratic formula failed to deal accurately with <math>a = 8</math>, but most earned the first three marks with apparent ease. Thereafter a small number of candidates opted to cube rather than find cube roots, but the main loss of credit was due to lack of accuracy at the end.</p>	<p>No marks if whole equation cube rooted etc.</p> <p>No marks if straight to formula with no evidence of substitution at start and no cube rooting / cubing at end</p> <p><b>Spotted solutions:</b></p> <p>If M0 DMO or M1 DM0 SC B1 <math>x = -1</math> <b>www</b></p> <p>SC B1 <math>x = \frac{1}{2}</math> <b>www</b></p> <p>(Can then get 5/5 if both found <b>www</b> and exactly two solutions justified)</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>The assertion “you can’t cube root a negative number” was seen regularly; <math>\sqrt[3]{\frac{1}{8}} = \pm \frac{1}{2}</math> was less common but not rare. This confusion between cube and square roots clearly needs addressing as it let down an otherwise well-answered question where only the very lowest-scoring candidates made no progress at all.</p>
			<b>Total</b>	<b>5</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
12	i	$4\sqrt{45}$	M1	or $4\sqrt{5}\sqrt{3} \times \sqrt{3}$ (not just $4\sqrt{5 \times 3} \times \sqrt{3}$ ) or $\sqrt{720}$ or $\sqrt{240} \times \sqrt{3}$ or better	For method mark, makes a correct start to manipulate the expression i.e. at least combines two parts correctly or splits one part correctly
	i	$=12\sqrt{5}$	A1	Correctly simplified answer  <b>Examiner's Comments</b>  Most candidates were successful with this easy starter, but a significant minority found it quite challenging. Most earned at least a method mark for correct surd manipulation of some kind, but the accuracy was more of a problem, with some arithmetic errors and also conceptual ones such as $4 \times 3\sqrt{5} = 7\sqrt{5}$ .	
	ii	$\frac{20\sqrt{5}}{5} = 4\sqrt{5}$	B1	<b>cao</b> , do not allow unsimplified, do not allow if clearly from wrong working  <b>Examiner's Comments</b>  Around 85% of candidates were successful in rationalising the denominator. Where no credit was earned, this was usually due to a lack of understanding rather than arithmetical error with a significant minority appearing not to know how to rationalise the given expression. Simply rewriting it as $20(\sqrt{5})^{-1}$ was quite common.	

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii	$5\sqrt{5}$	B1	<p>cao <b>www</b>, do not allow unsimplified, do not allow if clearly from wrong working</p> <p><b>Examiner's Comments</b></p> <p>This was generally less successful than parts (i) and (ii), with just under three-quarters of candidates earning the mark. Many of those who did not give the answer in the required form did at least understand the notation as <math>(\sqrt{5})^3</math> was often seen, but then simplified to <math>3\sqrt{5}</math>.</p>	
		<b>Total</b>	<b>4</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
13	i	$2\sqrt{3}$	B1	cao  <b>Examiner's Comments</b>  Most candidates recognised the need to rationalise the denominator and did so efficiently.  There were occasional numerical errors such as and a small minority appeared not to know how to start, usually giving the answer as.	Do not accept $\frac{6\sqrt{3}}{3}$
	ii	$10\sqrt{3} - 18\sqrt{3}$	B1	$\sqrt{27} = 3\sqrt{3}$ so i, not just $\sqrt{9}\sqrt{3}$  <b>Examiner's Comments</b>  Again, most candidates were successful with this part. A relatively common error was to write with many of the weaker candidates it was unclear where incorrect answers had come from.	
	ii	$-8\sqrt{3}$	B1		
	iii	$3^{\frac{5}{2}} = 3^2 \times 3^{\frac{1}{2}}$	B1	Separate $\sqrt{3}$ from $3^{\frac{5}{2}}$  <b>Examiner's Comments</b>  This was generally less successful than parts (i) and (ii), with only about three quarters of candidates earning both marks. was a commonly seen wrong answer.	Allow only $3 \times 3 \times 3^{\frac{1}{2}}$ , $3^2 \times \sqrt{3}$ , $3 \times 3 \times \sqrt{3}$ , or $\sqrt{81}\sqrt{3}$ , $3\sqrt{9}\sqrt{3}$ for first mark
	iii	$9\sqrt{3}$	B1		
		<b>Total</b>	<b>5</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
14	i	y coordinate of the centre is -5	B1	Correct y value	Alt
	i	Radius = 5	B1	Correct radius	Shows only meets x axis at one point B1
	i	Centre is five units below x axis and radius is five, so just touches the x-axis	B1	Correct explanation based on the above — allow clear diagram <b>www</b>  <b>Examiner's Comments</b>  The simplest, and most common, approach to show that the circle did not go above the x axis was to identify the centre and radius from the equation and state/show on a diagram that the circle just touched the axis at a single point. The majority of candidates showed clear solutions to this effect. Some, however, tested just a single point (usually $y = 1$ ) and showed this was not a point on the circle, which was of course insufficient.	Correct y value for the centre B1 Correct explanation B1 <b>www</b>
	ii	$CP^2 = (6 - 2)^2 + (k + 5)^2$  $CP^2 < 25 \Rightarrow 16 + k^2 + 10k + 25 < 25$	M1	Attempt to find $CP$ or $CP^2$	Alternative Puts $x = 6$ to into equation of circle M1
	ii	$k^2 + 10k + 16 < 0$	A1	Correct three term quadratic expression*	Correct three term quadratic equation*, could be in terms of $y$ A1
	ii	$(k + 2)(k + 8) < 0$	A1	$k = -2$ and $k = -8$ found	$k = -2$ and $k = -8$ found (allow $y$ ) A1
	ii	$-8 < k < -2$	M1	Chooses "inside region" for their roots of their quadratic	Then as main scheme

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii		A1	<p>Must be strict inequalities for the A mark</p> <p>* Or <math>(k + 5)^2 &lt; 9</math></p> <p><b>Examiner's Comments</b></p> <p>Most candidates took the correct approach to this, substituting <math>x = 6</math> and then solving the quadratic and finding the values of <math>k</math> that corresponded to the points on the circumference. A large number of candidates then stopped and failed to identify the correct range of values being any value between these. Those who carried on were usually correct, but it was fairly common to not give the answer as the strict inequalities required for the point to be inside the circle. There were some neat alternative solutions using Pythagoras' theorem to find the values of <math>k</math> from a good sketch.</p>	<p>* Or <math>(k + 5)^2 = 9</math></p> <p><b>SC</b></p> <p>Trial and improvement</p> <p><b>B2</b> if final answer correct (<b>B1</b> if inequalities are not strict)</p> <p>Can only get 5/5 if fully explained</p>
	iii	$(2y - 2)^2 + (y + 5)^2 = 25$ $5y^2 + 2y + 4 = 0$ $b^2 - 4ac = 4 - 4 \times 5 \times 4 = -76$ $< 0$ , so line and circle do not meet	M1*	Attempts to eliminate $x$ or $y$ from equation of circle	If $y$ eliminated: $5x^2 + 4x + 16 = 0$
	iii		A1	Correct three term quadratic obtained	$b^2 - 4ac = 16 - 4 \times 5 \times 16 = -304$
	iii		M1dep*	Correct method to establish quadratic has no roots e.g. considers value of $b^2 - 4ac$ , tries to find roots from quadratic formula	<b>No marks for purely graphical attempts</b>

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii		A1	Correct clear conclusion <b>www AG</b>  <b>Examiner's Comments</b>  There were a large number of fully correct solutions to the request to prove that the line and circle do not meet. Most performed the easier substitution for $x$ , but $(2y)^2 = 2y^2$ was a fairly common error. Many were able to use the discriminant, or the quadratic formula, to explain their reasoning clearly. Only a few claimed that the line and circle did not meet because the quadratic could not be factorised. Some of the weaker candidates again resorted to testing a single point (sometimes the centre) or drawing a poor diagram.	
			<b>Total</b>	<b>12</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
15	i	$y = 6x^3 + 4x^{-\frac{1}{2}} + 5x$	B1	$\frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$ soi	
	i	$\frac{dy}{dx} = 18x^2 - 2x^{-\frac{3}{2}} + 5$	M1	Attempt to differentiate, any term correct	
	i		A1	Two correct terms	
	i		A1	Fully correct, no "+c"	
				<b>Examiner's Comments</b> <input type="checkbox"/> <input type="checkbox"/> This differentiation was extremely well done, with around four in five candidates securing all the available marks; the ability to recognise and deal with a fractional negative term was much better than in some previous sessions. This term remained the main cause of error, although some errors were made with the first term. The inclusion of a constant when differentiating is now very rare indeed.	
	ii	$\frac{d^2y}{dx^2} = 36x + 3x^{-\frac{5}{2}}$	M1	Attempt to differentiate their $\frac{dy}{dx}$	Any term still involving x correct — follow through from their expression for the M mark only

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii		A1	cao www in either part  Examiner's Comments <input type="checkbox"/> <input type="checkbox"/>  The need to differentiate again was apparent to most candidates, and again the standard of dealing with the fractional negative term was very high. Some candidates made arithmetical errors here and a few failed to  simplify $\frac{6}{2}$ .	
<b>Total</b>				<b>6</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
16	$k = x^2$ $4k^2 + 3k - 1 = 0$  $(4k - 1)(k + 1) = 0$  $k = \frac{1}{4}, k = -1$  $x = \pm \sqrt{\frac{1}{4}}$	M1*  M1dep*  A1  M1	Substitute for $x^2$  Attempt to solve resulting quadratic  Correct values of k soi  Attempt to square root	<p>No marks if whole equation square rooted etc.</p> <p>No marks if straight to formula with no evidence of substitution at start and no square rooting/squaring at end.</p> <p>If factorising into two brackets:  <math>(4x^2 - 1)(x^2 + 1) = 0</math> M1 A1  <math>(2x + 1)(2x - 1)(x^2 + 1) = 0</math>            M1 A1 A1 as before</p> <p>Spotted solutions:            If M0 DM0 or M1 DM0</p> <p>SR B1 <math>x = \frac{1}{2}</math> www</p> <p>SC B1 <math>x = -\frac{1}{2}</math> www</p> <p>(Can then get 5/5 if both found www and exactly two solutions justified)</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
			$x = \pm \frac{1}{2}$	A1	Final answers correct, no extras  <u><b>Examiner's Comments</b></u>  This disguised quadratic was well approached by the vast majority of candidates, with just over half achieving all 5 marks. This continued an improving trend over the last few sessions, with fewer candidates going straight to the quadratic formula with no attempt to square root at the end, which has been a problem with similar questions in the past. The most common approach was to perform a substitution and then to factorise, although those who opted for a two-bracket approach using $x^2$ were also often successful. Fewer candidates than in previous sessions used the quadratic formula; those that did were usually successful. Completing the square was rarely seen. Only a small number of candidates opted to square rather than square root, but the main loss of credit was due to lack of accuracy at the end. Although most candidates correctly dismissed any roots from $x^2 = -1$ , some did not. More common was the absence of the negative square root
			<b>Total</b>	<b>5</b>	

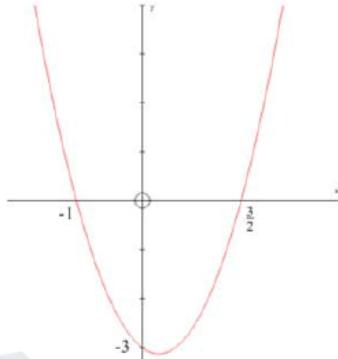
Question	Answer/Indicative content	Marks	Part marks and guidance	
17	$5x^2 + 10x + 2 = 5(x^2 + 2x) + 2$ $= 5[(x + 1)^2 - 1] + 2$ $= 5(x + 1)^2 - 3$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p><math>p = 5</math></p> <p><math>q = 1</math></p> <p><math>2 - 5</math> “their <math>q^2</math>” or <math>\frac{2}{5}</math> – “their <math>q^2</math>”  <b>Must be evidence of squaring <math>r = -3</math></b></p> <p><b>Examiner's Comments</b></p> <p>This “completing the square” question was tackled well by the majority of candidates, many securing all four marks. In keeping with previous sessions, almost all earned the first two marks, seeing that <math>p</math> was 5 and <math>q</math> was 1, although <math>q = 5</math> was seen relatively often among weaker candidates. Failure to multiply by five when working out the constant was the most common error amongst candidates who did not achieve full marks. Those who took out 5 as a factor of the full expression often made errors with the resulting fractions.</p>	<p>If <math>p</math>, <math>q</math> and <math>r</math> found correctly, then ISW slips in format.  <math>5(x + 1)^2 + 3</math> B1 B1 M0 A0  <math>5(x + 1) - 3</math> B1 B1 M1 A1  <b>(BOD)</b>  <math>5(x + 1 x)^2 - 3</math> B1 B0 M1 A0  <math>5(x^2 + 1)^2 - 3</math> B1 B0 M1 A0  <math>5(x - 1)^2 - 3</math> B1 B0 M1 A0  <math>5 x (x + 1)^2 - 3</math> B0 B1 M1 A0</p>

Question	Answer/Indicative content	Marks	Part marks and guidance
			Total



Question		Answer/Indicative content	Marks	Part marks and guidance	
18	i	$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1	Attempt to differentiate, at least two non-zero terms correct	These Ms may be awarded in either order
	i		A1	Fully correct	
	i	When $x = 4$ , $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$	
	i	$\frac{dy}{dx} = 0$ gives $a = 13$	M1	Sets their $\frac{dy}{dx}$ to 0. Must be seen	
	i		A1	<b>Examiner's Comments</b>  Differentiating and setting to zero and substituting $x = 4$ was the obvious strategy and, although the arithmetic proved troublesome for some, many candidates were able to secure full marks for this part.	
	ii	$\frac{d^2y}{dx^2} = 12x - 26$	M1	Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign	<b>Alternate valid methods include:</b> 1) Evaluating gradient at either side of $4(x > \frac{1}{3})$ e.g. at 3, -16 at 5, 28 2) Evaluating $y = -46$ at 4  and either side of  $4(x > \frac{1}{3})$ e.g. (3, -37), (5, -33)

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	When $x = 4$ , $\frac{d^2y}{dx^2} > 0$ so minimum	A1	<b>www</b>  <b>Examiner's Comments</b>  Considering the sign of the second derivative was by far the most common approach for this part and was generally successful. Some candidates equated their second derivative to zero, a confusion that has been common for many sessions.	If using alternatives, working must be fully correct to obtain the A mark
	iii	$6x^2 - 26x + 8 = 0$	M1	Sets their derivative to zero	Could be $(6x - 2)(x - 4) = 0$  or $(3x - 1)(2x - 8) = 0$
	iii	$(3x - 1)(x - 4) = 0$	M1	Correct method to solve quadratic ( <b>appx 1</b> )	
	iii	$x = \frac{1}{3}$	A1	oe  <b>Examiner's Comments</b>  A small minority omitted this question, but most candidates were comfortable in returning to their expression in (i), equating to zero and finding the other root. An alternative method not seen before by several markers was to equate the second derivative to the negative of the value found in (ii); this is perfectly valid for cubics and was usually successful.	
		<b>Total</b>	<b>10</b>		

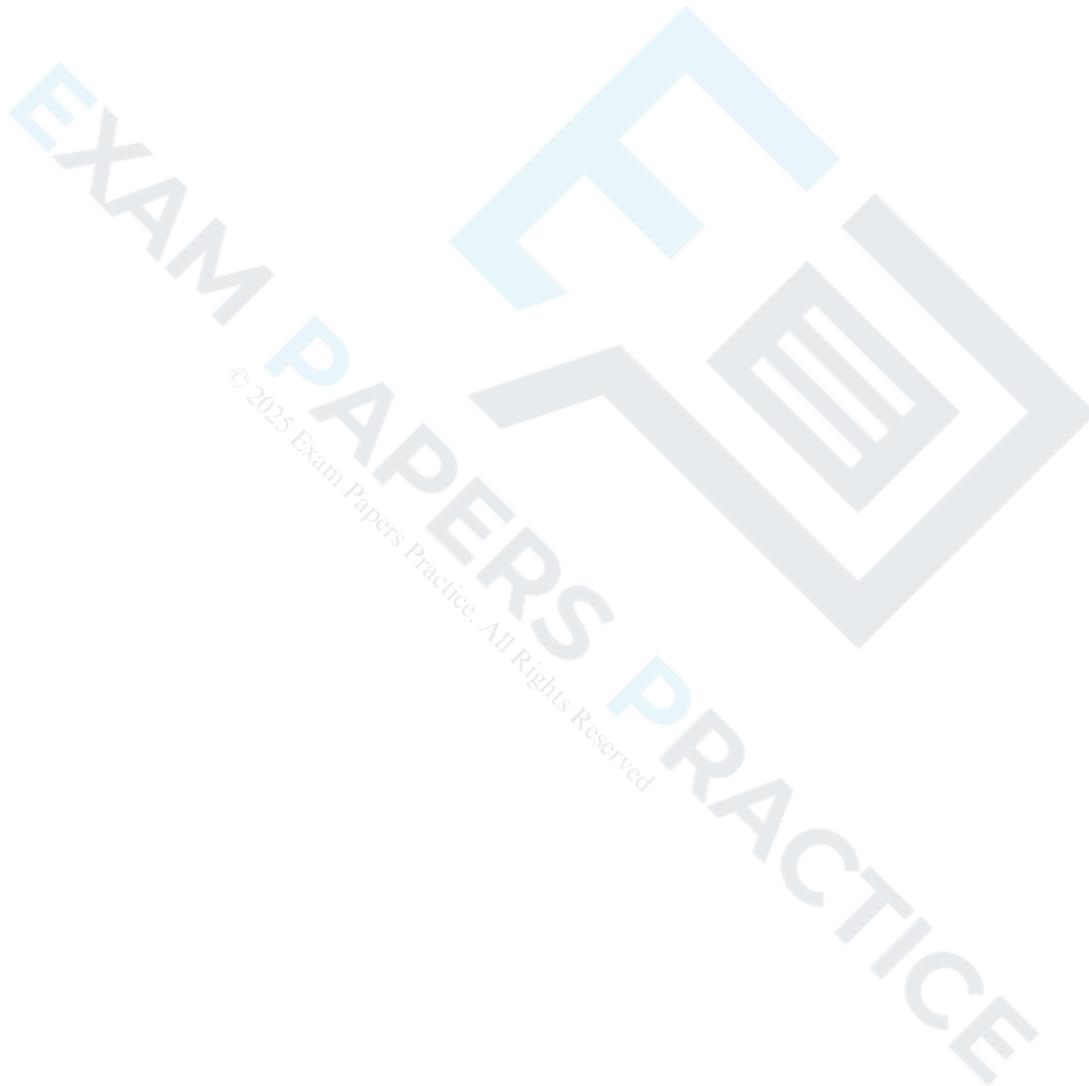
Question		Answer/Indicative content	Marks	Part marks and guidance
19	i	$(2x - 3)(x + 1) = 0$	M1	Correct method to find roots – see appendix 1
	i	$x = \frac{3}{2}, x = -1$	A1	Correct roots
	i		A1ft	Good curve: <ul style="list-style-type: none"> <li>• Correct shape, symmetrical positive quadratic</li> <li>• Minimum point in the correct quadrant for their roots (ft)</li> <li>• their x intercepts correctly labelled (ft)</li> </ul>
	i		B1	y intercept at (0, -3). Must have a graph.  <b>Examiner's Comments</b>  Most candidates recognised this as a quadratic and provided an appropriate sketch, although there was a tendency for some to become steep/vertical extremely quickly rather indicate increasing gradient. The points of intersection on the x-axis were usually accurate with the occasional sign swaps. Although the y-intercept was usually correctly identified as -3, it was very common to see this as vertex of the graph which lost an accuracy mark; candidates were expected to indicate the vertex would be in the correct quadrant for their roots

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$x < -1, x > \frac{3}{2}$	M1	Chooses the “outside region”	If restarted, fully correct method for solving a quadratic inequality including choosing “outside region” needed for M1
	ii		A1ft	Follow through x-values in (i). Allow “ $x < -1, x > \frac{3}{2}$ ”, “ $x < -1$ or $x > \frac{3}{2}$ ” but do not allow “ $x < -1$ and $x > \frac{3}{2}$ ”  <b>Examiner's Comments</b>  Most candidates used their answer to part (i) and chose the correct outside region, although choosing the inside region was a frequently seen error. The notation used to describe the region was usually correct; incorrect language such as joining the two sections with the word ‘and’ lost the accuracy mark.	NB e.g. $-1 > x > \frac{3}{2}$ scores <b>M1A0</b> Must be strict inequalities for A mark
	iii	$b^2 - 4ac = 1^2 - 4 \times 2 \times -(3 + k)$	M1	Rearrangement and use of $b^2 - 4ac < 0$ , must involve 3 and $k$ in constant term (not $3k$ )	Alt for first two marks: <b>M1</b> Attempt to find turning point and form inequality $k < y_{min}$
	iii	$25 + 8k < 0$	A1	$p + 8k < 0$ oe found, any constant $p$ . $p$ need not be simplified	A1 turning point correct  $(\frac{1}{4}, -\frac{25}{8})$

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii	$k < -\frac{25}{8}$	A1	<p>Correct final answer</p> <p><b>Examiner's Comments</b></p> <p>This proved demanding for many candidates. Although some secured all three marks, many earned no credit as they either put the discriminant equal to zero or, as was frequently seen, to <math>k</math>, making no attempt to rearrange the given equation. Accuracy marks were often lost as candidates failed to deal with the minus signs both in the discriminant and in the expression for <math>c</math>. A few candidates found the turning point of their graph either by differentiation or by completing the square but these approaches were far less common.</p>	<p>If M0 (either scheme) SC B1</p> <p>seen <math>k = -\frac{25}{8}</math> or <math>k &gt; -\frac{25}{8}</math></p>
		<b>Total</b>	<b>9</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
20	$x^2 - (5 - 2x)^2 = 3$ $3x^2 - 20x + 28 = 0$ $(3x - 14)(x - 2) = 0$ $x = \frac{14}{3}, x = 2$ $y = -\frac{13}{3}, y = 1$	<p>M1*</p> <p>A1</p> <p>M1dep</p> <p>A1</p> <p>A1</p>	<p>Substitute for <math>x/y</math> or valid attempt to eliminate one of the variables</p> <p>Three term quadratic in solvable form</p> <p>Correct method to solve three term quadratic – see appendix 1</p> <p>Both <math>x</math> values correct</p> <p>Both <math>y</math> values correct. Allow 1 A mark for one correct pair of <math>x</math> and <math>y</math> from correct factorisation.</p> <p><b>Examiner's Comments</b></p> <p>The vast majority of candidates opted to substitute for <math>y</math> and so form a quadratic in <math>x</math> as the first step in solving this pair of simultaneous equations. Sign errors meant that not all candidates obtained the correct quadratic and even those who did find it difficult to factorise. Attempts to use the formula were also hampered by the relatively large number 28 and so many candidates got no further. Those who did succeed usually remembered to substitute to find <math>y</math>, but sign errors were again quite common in this part. Nonetheless, a significant proportion of candidates produced full, clear and accurate solutions.</p>	<p>If <math>y</math> eliminated:</p> $3y^2 + 10y - 13 = 0$ $(3y + 13)(y - 1) = 0$ <p><b>Spotted solutions: If M*0</b></p> <p>SC B1 <math>x = 2, y = 1</math> <b>www</b></p> <p>SC B1 <math>x = \frac{14}{3}, y = -\frac{13}{3}</math> <b>www</b></p> <p>Must show on both line and curve (Can then get 5/5 if both found <b>www</b> and exactly two solutions justified)</p>

Question	Answer/Indicative content	Marks	Part marks and guidance
			Total
		5	



Question		Answer/Indicative content	Marks	Part marks and guidance	
21		$k = x^{\frac{1}{3}}$	M1*	Use a substitution to obtain a quadratic, or factorise into 2 brackets each containing $x^{\frac{1}{3}}$	No marks if whole equation cubed / rooted etc. No marks if straight to quadratic formula with no evidence of substitution at start and no cube rooting / cubing at end.
		$k^2 - k - 6 = 0$ $(k - 3)(k + 2) = 0$	M1dep	Attempt to solve resulting three-term quadratic – see guidance in appendix 1	
		$k = 3, k = -2$	A1	Correct values of $k$	Spotted solutions: If M0 DMO or M1 DM0
		$x = 3^3, x = -2^3$	M1	Attempt to cube at least one value	SC B1 $x = 27$ <b>www</b>
		$x = 27, x = -8$	A1	Final answers correct ISW	SC B1 $x = -8$ <b>www</b> (Can then get 5/5 if both found <b>www</b> and exactly two solutions justified)
<b>Examiner's Comments</b>					
			<p>This disguised quadratic was well approached by the vast majority of candidates. The most common approach was to perform a substitution and then to factorise, although some candidates did make their choice of substitution clear, which made it difficult to award partial credit in the cases where errors then occurred. As the resulting quadratic was simple, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and the number who tried to cube root, rather than to cube, their solutions was comparatively large.</p>		
<b>Total</b>			<b>5</b>		

Question			Answer/Indicative content	Marks	Part marks and guidance
22		i	$5^8$	B1	cao  <b>Examiner's Comments</b>  Almost all candidates secured this easy mark, but the error of $(5^2)^4 = 5^6$ was quite common.
		ii	$5^{-\frac{1}{4}}$	M1	Fourth root $\equiv \frac{1}{4}$ so i
		ii		A1	cao www  <b>Examiner's Comments</b>  Again, most candidates were able to gain both marks dealing with both the fractional and negative elements of the index.
		iii	$5^{\frac{9}{2}}$	M1	$(5^{\frac{3}{2}})^3$ or $5^3 \times 5^{\frac{3}{2}}$ or other correct product of two simplified powers of 5
		iii		A1	oe cao www  <b>Examiner's Comments</b>  This part of the question proved rather more demanding with a minority of candidates securing both marks. Those who recognised that $\sqrt{5} = 5^{\frac{1}{2}}$ were usually able to go on and complete the question successfully; those who tried to multiply out were less successful.
			<b>Total</b>	<b>5</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
23		$\frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{8\sqrt{3}+8}{3-1}$ $4\sqrt{3}+4$	<p>M1</p> <p>A1</p> <p>A1</p> <p><b>Examiner's Comments</b></p> <p>Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all three marks. Errors were sometimes seen both in evaluating the numerator and the denominator, and occasionally in performing the final division</p>	<p>Multiply top and bottom by <math>\sqrt{3}+1</math> or <math>-\sqrt{3}-1</math> evidence of multiplying out needed</p> <p>Either numerator or denominator correct</p> <p>Final answer cao</p> <p>Alternative: M1 Correct method to solve simultaneous equations formed from equating expression to <math>a\sqrt{3}+b</math> A1 Either <math>a</math> or <math>b</math> correct A1 Both correct</p>	
		<b>Total</b>	<b>3</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance
24	$y = 4x^2 + ax^{-1} + 5$	B1	$ax^{-1}$ soi
	$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct
		A1	Fully correct
	At stationary point, $8x - ax^{-2} = 0$	M1	Sets their derivative to 0
	$a = 8x^3$ oe	A1	Obtains expression for $a$ in terms of $x$ , or $x$ in terms of $a$ <b>www</b>
	When $a = 8x^3$ , $y = 32$ $32 = 4x^2 + 8x^2 + 5$	M1	Substitutes their expression and 32 into equation of the curve to form single variable equation
	$x = \frac{3}{2}$ oe	A1	Obtains correct value for $x$ . Allow $x = \frac{\sqrt{27}}{\sqrt{12}}$ .
			Ignore $-\frac{3}{2}$ given as well.
	$a = 27$	A1	Obtains correct value for $a$ . Ignore $-27$ given as well.
	OR		
	$y = 4x^2 + ax^{-1} + 5$	B1	$ax^{-1}$ soi
	$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct
	A1	Fully correct	
$32 = 4x^2 + ax^{-1} + 5$	M1	Substitutes 32 into equation of the curve to find expression for $a$	
$a = 27x - 4x^3$	A1	Obtains expression for $a$ in terms of $x$ <b>www</b>	

$x = \frac{\sqrt[3]{a}}{2}$  oe,  $a = 18x$  oe also fine

or expression for  $a$  e.g.  
 $a^{\frac{2}{3}} = 9$

Question	Answer/Indicative content	Marks	Part marks and guidance
	At stationary point, $8x - ax^{-2} = 0$ $8x - (27x - 4x^3)x^{-2} = 0$  $x = \frac{3}{2}$ <b>oe</b>          $a = 27$	M1          A1          A1	Sets derivate to zero and forms single variable equation  Obtains correct value for x.  Allow $x = \sqrt{\frac{27}{12}}$ .    Ignore $-\frac{3}{2}$ given as well.  Obtains correct value for a. Ignore $-27$ given as well.

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p><b>Examiner's Comments</b></p> <p>Many candidates obtained at least the first four marks for this demanding final question, by correctly differentiating and setting equal to zero; the most common errors at this stage were to equate to 32 or to leave the constant term 5 in the derivative. Thereafter, a significant proportion candidates went on to secure at least 7 of the 8 marks by finding an expression for a and correctly substituting this and 32 into the equation of the curve, or other equivalent methods. Some did not spot this way forward and others lost marks due to incorrect simplification of algebra or arithmetical slips. Many did not spot the factor of 3 in</p> $x^2 = \frac{27}{12}$ <p>and so were then unable to finish the question. Occasionally candidates appear to consider a to be a variable passing through the point (x, 32); often these attempts were unclear and involved the (often unrealised) creation of functions of the form xy and the subsequent attempts at implicit differentiation were incorrect.</p>
			<b>Total</b>	<b>8</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
25	i	Centre of circle (4, 3)	B1	Correct centre	Or $r^2 = 4^2 + 3^2 + 20$ so  ISW after $\sqrt{45}$  <b>Examiner's Comments</b>  This proved to be a very successfully answered question, with around nine in ten candidates securing all three marks.
	i	$(x-4)^2 - 16 + (y-3)^2 - 9 - 20 = 0$	M1	$(x \pm 4)^2 - 4^2$ and $(y \pm 3)^2 - 3^2$ seen (or implied by correct answer)	
	i	$r^2 = 45$		$\sqrt{45}$ or better <b>www</b>	
	i	$r = \sqrt{45}$	A1		
	ii	At A, $y = 0$ so $x^2 - 8x - 20 = 0$ $(x-10)(x+2) = 0$	M1	Valid method to find A e.g. put $y = 0$ and attempt to solve quadratic (allow slips) or Pythagoras' theorem	Alternative for finding gradient: M1 Attempt at implicit differentiation as evidenced by $2y \frac{dy}{dx}$ term           <b>A1</b> $2x + 2y \frac{dy}{dx} - 8 - 6 \frac{dy}{dx} = 0$  and substitution of (10, 0) to obtain 2.
	ii	A = (10, 0)	A1	Correct answer found	
	ii	Gradient of radius = $\frac{3-0}{4-10} = -\frac{1}{2}$	M1	Attempts to find gradient of radius (3 out of 4 terms correct for their centre, their A)	
	ii	Gradient of tangent = 2	B1		
	ii	$y - 0 = 2(x - 10)$	M1	Equation of line through their A, any non-zero gradient	

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$y = 2x - 20$	A1	Correct answer in any three-term form	<b>Examiner's Comments</b>  Just over half of candidates obtained full marks in this part, with errors appearing at all stages. Some put $x$ rather than $y$ equal to 0 when trying to find A and the alternative method of using Pythagoras' theorem often led to slips. There were a significant number of problems finding the gradient and errors such as $-\frac{3}{6} = -\frac{1}{3}$ were commonly seen.
	iii	$A' = (-2, 6)$	B1	Finds the opposite end of the diameter	<b>Examiner's Comments</b>  Many candidates did not realise that the point required for the parallel line was the opposite end of the diameter. Most did use the same gradient as in (ii), but some used the negative reciprocal. An interesting method sometimes seen was consideration of translation of the original line.
	iii	$y - 6 = 2(x + 2)$	M1	Line through their A' parallel to their line in (ii)	
	iii	$y = 2x + 10$	A1	Correct answer in any three-term form	
	iv	$OC = \sqrt{3^2 + 4^2} = 5$	M1	Attempts to find the distance from O to their centre and subtract from their radius	ISW incorrect simplification

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv	$(0 <) r < \sqrt{45} - 5$	A1	Correct inequality, condone $\leq$	<b>Examiner's Comments</b>  This proved very demanding, with many candidates unable to start; those who drew a diagram were generally more successful but less than a quarter of candidates secured both marks. Even amongst those who found the maximum length of the radius to be $\sqrt{45} - 5$ , it was quite rare to see the correct inequality.
			<b>Total</b>	<b>14</b>		

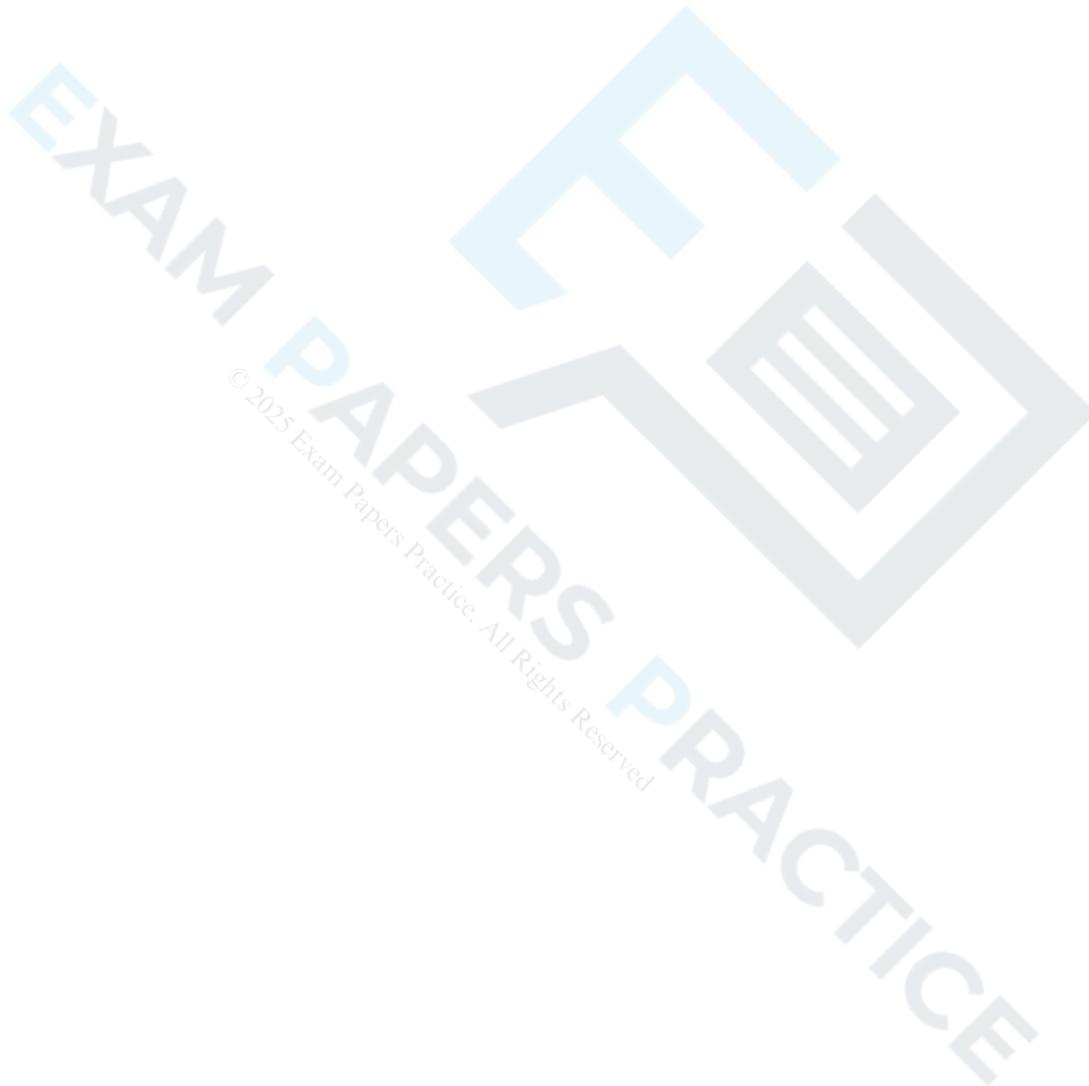
Question		Answer/Indicative content	Marks	Part marks and guidance	
26		$x^2 + (2 - 2k)x + 11 + k = 0$	M1*	Attempt to rearrange to a three-term quadratic	Each Ms depend on the previous M  -2 > k > 5 scores <b>M1A0</b>  Allow "k < -2 or k > 5" for A1 <b>Do not allow "k &lt; -2 and k &gt; 5"</b>  <b>Examiner's Comments</b>  Although this question proved demanding to many candidates, there were a large number of neat solutions. Most candidates understood the nature of the question and many gained at least three of the four method marks available. Accuracy was the main barrier to complete success as a large number either failed to rearrange the given equation correctly or to substitute correctly into the discriminant. Repeated sign errors often resulted in apparently correct critical values for k that did not receive credit as they were from wrong working.
		$(2 - 2k)^2 - 4(11 + k)$	M1dep*	Uses $b^2 - 4ac$ , involving k and not involving x	
		$4k^2 - 12k - 40 > 0$	A1	Correct simplified inequality obtained <b>www</b>	
		$k^2 - 3k - 10 > 0$			
		$(k - 5)(k + 2)$	M1dep*	Correct method to find roots of 3-term quadratic	
		$k < -2, k > 5$	M1dep*	$b^2 - 4ac > 0$ and chooses "outside region"	
			A1	Fully correct, strict inequalities.	
		<b>Total</b>	<b>7</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
27	i	$-2(x^2 - 6x - 2)$	B1	or $a = -2$	$-2(x - 3)^2 - 22$ B1 B1 M0 A0
	i	$= -2[(x - 3)^2 - 2 - 9]$	B1	$b = -3$	$-2(x - 3) + 22$ 4/4 (BOD)
	i		M1	$4 + 2b^2$	$-2(x - 3x)^2 + 22$ B1 B0 M1 A0
	i	$= -2(x - 3)^2 + 22$	A1	$c = 22$ If $a$ , $b$ and $c$ found correctly, then ISW slips in format.  <b>If signs of all terms changed at start, can only score SC B1 for fully correct working to obtain <math>2(x - 3)^2 - 22</math></b>  If done correctly and then signs changed at end, do not ISW, award B1B1M1A0	$-2(x^2 - 3)^2 + 22$ B1 B0 M1 A0 $-2(x + 3)^2 + 22$ B1 B0 M1 A0 $-2x(x - 3)^2 + 22$ B0 B1 M1 A0 $-2(x^2 - 3) + 22$ B1 B0 M1 A0  <b>Examiner's Comments</b>  The negative coefficient of $x^2$ generally did not daunt candidates and there were many clear and accurate solutions, aided by the integer arithmetic. There were the usual errors when trying to find the constant term and these were exacerbated by the need to multiply two negative numbers together. Some candidates however, chose to change all the signs to make the question easier; this approach earned a maximum of one mark in this part, with the possibility of follow through marks in part (i). Others treated the expression as an equation to achieve the same effect; at this level it is expected that candidates should know the difference.
	ii	(3, 22)	B1ft	Allow follow through “– their $b$ ”	May restart.

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii		B1ft	Allow follow through "their c"	Follow through marks are for their final answer to (i)  <b>Examiner's Comments</b>  This part was sometimes omitted with candidates apparently not seeing connection between the parts. Others found the coordinates by differentiation and substitution but most used (i) and so were allowed follow-through marks had they made errors in the previous part.
			<b>Total</b>	<b>6</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
28	i	$(2^{-2})^3$ or $2^{15} \div 2^{21}$	B1	Valid attempt to simplify	Correct use of either index law  $\left(\frac{1}{2}\right)^6$ oe is B1  <b>Examiner's Comments</b>  This simple index question was very well done, with around 90% securing both marks.
	i	$2^{-6}$	B1	Correct answer. Accept $p = -6$ .	
	ii	$5 \times (2^2)^{\frac{2}{3}} + 3 \times (2^4)^{\frac{1}{3}}$	M1	Attempts to express both terms or a combined term as a power of 2	e.g. Both $4 = 2^2$ and $16 = 2^4$ <b>soi</b>  If M0  <b>Examiner's Comments</b>  Although there were a significant number of excellent solutions, this question proved much more demanding than expected with less than a third of candidates securing all three marks. Many reached $5 \times 2^{\frac{4}{3}} + 3 \times 2^{\frac{4}{3}}$ but then went no further, or even "simplified" this to $10^{\frac{4}{3}} + 6^{\frac{4}{3}}$ . Many of those who did obtain $8 \times 2^{\frac{4}{3}}$ appeared not to realise 8 was a power of 2. Some of those who did then made errors adding the powers, either through incorrect addition or multiplying so that $5 \times 2^3 \times 2^{\frac{4}{3}} = 2^4$ .
	ii	$= 5 \times 2^{\frac{4}{3}} + 3 \times 2^{\frac{4}{3}}$ or $10 \times 2^{\frac{1}{3}} + 6 \times 2^{\frac{1}{3}}$	B1	Correctly obtains $2^{\frac{4}{3}}$ or $2^{\frac{1}{3}}$ for either term	
	ii	$= 8 \times 2^{\frac{4}{3}}$			
	ii	$= 2^{\frac{13}{3}}$	A1	Correct final answer	

Question	Answer/Indicative content	Marks	Part marks and guidance	
		Total	5	



Question	Answer/Indicative content	Marks	Part marks and guidance	
29	<p>Let <math>y^{\frac{1}{4}} = x</math></p> <p><math>2x^2 - 7x + 3 = 0</math></p> <p><math>(2x - 1)(x - 3) = 0</math></p> <p><math>x = \frac{1}{2}, x = 3</math></p> <p><math>y = \left(\frac{1}{2}\right)^4, y = 3^4</math></p> <p><math>y = \frac{1}{16}, y = 81</math></p> <p><u>Alternative by rearrangement and squaring:</u></p> <p><math>2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0, 7y^{\frac{1}{4}} = 2y^{\frac{1}{2}} + 3</math></p> <p><math>49y^{\frac{1}{2}} = 4y + 12y^{\frac{1}{2}} + 9, 37y^{\frac{1}{2}} = 4y + 9</math></p> <p><math>16y^2 - 1297y + 81 = 0</math></p> <p><math>(16y - 1)(y - 81) = 0</math></p> <p><math>y = \frac{1}{16}, y = 81</math></p> <p>OR methods may be combined:</p> <p>e.g. after <math>37y^{\frac{1}{2}} = 4y + 9</math></p> <p><math>4y - 37y^{\frac{1}{2}} + 9 = 0</math></p>	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>M2*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>M1*</p>	<p>Use a substitution to obtain a quadratic or</p> <p>factorise into two brackets each containing <math>y^{\frac{1}{4}}</math></p> <p>Correct method to solve resulting quadratic</p> <p>Both values correct</p> <p>Attempt to raise to the fourth power</p> <p>Correct final answers</p> <p>Rearrange and square both sides twice</p> <p>Correct quadratic obtained</p> <p>Correct method to solve resulting quadratic</p> <p>Correct final answers</p> <p>Rearrange, square both sides and substitute</p>	<p>No marks if whole equation raised to fourth power etc.</p> <p>No marks if straight to formula with no evidence of substitution at start and no raising to fourth power / fourth rooting at end.</p> <p>No marks if <math>y^{\frac{1}{4}} = x</math> and then <math>2x - 7x^2 + 3 = 0</math>.</p> <p>If M0 DM0 or M1 DM0 SC B1 <math>y = 81</math> <b>www</b></p> <p>SC B1 <math>y = \frac{1}{16}</math> <b>www</b></p> <p>(Can then get 5/5 if both found <b>www</b> and exactly two solutions justified)</p>

Question	Answer/Indicative content	Marks	Part marks and guidance
	$4x^2 - 37x + 9 = 0$ $(4x - 1)(x - 9) = 0$ $x = \frac{1}{4}, x = 9$	M1dep*  A1  M1dep*	Correct method to solve resulting quadratic   Attempt to square

Question		Answer/Indicative content	Marks	Part marks and guidance	
		$y = \left(\frac{1}{4}\right)^2, y = 9^2$	A1	Correct final answers	<p><b>Examiner's Comments</b></p> <p>This disguised quadratic was well approached by the vast majority of candidates, with around three-quarters of candidates achieving all 5 marks. A very small number of candidates factorised into two brackets, but the most common approach was as usual to perform a substitution and then to factorise. There was some confusion with choice of substitution with many incorrectly obtaining <math>2x - 7x^2 + 3</math>; this earned no credit. Again, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and a small number tried to take the fourth root, rather than to raise to the power four. Some only squared, implying an incorrect substitution. A few gave answers like <math>\pm 81</math>, which lost the final accuracy mark, as</p> <p>did poor attempts at <math>\left(\frac{1}{2}\right)^4</math></p> <p>which was variously seen as <math>\frac{1}{32}, \frac{1}{64}</math> or <math>\frac{1}{256}</math>.</p>
		<b>Total</b>	<b>5</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
30	$x^2 + (3x + 4)^2 = 34$  $10x^2 + 24x - 18 = 0$ $5x^2 + 12x - 9 = 0$  $(5x - 3)(x + 3) = 0$  $x = \frac{3}{5}, x = -3$	M1*  A1  M1dep*  A1	Substitute for $x/y$ or valid attempt to eliminate one of the variables  Correct three term quadratic in solvable form  Attempt to solve resulting three term quadratic  Correct $x$ values	If $x$ eliminated:  $10y^2 - 8y + 290 = 0$ $5y^2 - 4y + 145 = 0$  $(5y - 29)(y + 5) = 0$  Award <b>A1 A0</b> for one pair correctly found from correct quadratic

Question		Answer/Indicative content	Marks	Part marks and guidance	
		$y = \frac{29}{5}, y = -5$	A1	Correct y values	<p><b>Spotted solutions:</b> If M0 DM0</p> <p>SC B1 <math>x = \frac{3}{5}, y = \frac{29}{5}</math> <b>www</b>            SC B1 <math>x = -3, y = -5</math> <b>www</b></p> <p>Must show on both line and curve (Can then get 5/5 if both found <b>www</b> and exactly two solutions justified)</p> <p><b>Examiner's Comments</b></p> <p>This familiar question was very well done with many candidates scoring full marks. The vast majority of candidates opted to substitute for y and so form a quadratic in x. There were some errors, for example <math>16 - 34 = 22</math>, but most substitutions were very good and clearly shown. As in most recent sessions, candidates remain more likely to factorise, accurately, rather than depend on the quadratic formula. This usually resulted in the correct values of x, but a significant number of accuracy errors then occurred when substituting for y. Forgetting to work out the second variable was not entirely absent.</p>
		<b>Total</b>	<b>5</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
31	$\frac{3 + \sqrt{20}}{3 + \sqrt{5}} \times \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$ $\frac{-1 + 3\sqrt{5}}{9 - 5}$ $-\frac{1}{4} + \frac{3}{4}\sqrt{5}$	M1  B1  A1  A1	Attempt to rationalise the denominator – must attempt to multiply  $\sqrt{20} = 2\sqrt{5}$ soi  Either numerator or denominator correct and simplified to no more than two terms  Fully correct and fully simplified. Allow $\frac{-1 + 3\sqrt{5}}{4}$ , order reversed etc. <b>Do not ISW</b> if then multiplied by 4 etc.	Alternative: <b>M1</b> Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{5} + b$  <b>B1</b> $\sqrt{20} = 2\sqrt{5}$ soi  <b>A1</b> Either $a$ or $b$ correct <b>A1</b> Both correct  <b>Examiner's Comments</b>  Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all four marks. The conversion from $\sqrt{20}$ to $2\sqrt{5}$ was usually well done; most errors that occurred were seen when expanding and simplifying the numerator. Some candidates obtained the correct answer but then, seemingly unsatisfied with the fractional values of $a$ and $b$ found, multiplied by 4; this lost the final mark.
	<b>Total</b>	<b>4</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance		
32	i	$\int (2x - 5 + 4x^{-2}) dx = x^2 - 5x - 4x^{-1}$	M1	Attempt to rewrite integrand in a suitable form	Attempt to divide all 3 terms by $x^2$ , or attempt to multiply all 3 terms by $x^{-2}$ so
	i	$(4a^2 - 10a - \frac{2}{a}) - (a^2 - 5a - \frac{4}{a}) = 0$	A1	Obtain $2x - 5 + 4x^{-2}$	Allow if third term is written in fractional form
	i	$3a^2 - 5a + \frac{2}{a} = 0$ $3a^3 - 5a^2 + 2 = 0$ AG	M1	Attempt integration of their integrand	Their integrand must be written as a polynomial ie with all terms of the form $kx^n$ , and no brackets At least two terms must increase in power by 1 Allow if the $-5$ disappears
	i		A1	Obtain $x^2 - 5x - 4x^{-1}$	Allow unsimplified (eg $\frac{4}{-1} x^{-1}$ )
	i		M1	Attempt use of limits	Must be $F(2a) - F(a)$ ie subtraction with limits in the correct order Allow if no brackets ie $4a^2 - 10a - \frac{2}{a} - a^2 - 5a - \frac{4}{a}$ Must be in integration attempt, but allow M1 for limits following M0 for integration eg if fraction not dealt with before integrating
i		A1	Equate to 0 and rearrange to obtain $3a^3 - 5a^2 + 2 = 0$	Must be equated to 0 before multiplying through by $a$ At least one extra line of working required between $(4a^2 - 10a - \frac{2}{a}) - (a^2 - 5a - \frac{4}{a}) = 0$ and the final answer <b>AG</b> so look carefully at working	

**Examiner's Comments**

The quality of responses to this question varied considerably. Not knowing how to deal with the rational expression proved to be a stumbling block for many candidates, resulting in flawed integration attempts.

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>These candidates could still gain a mark for attempting to use limits correctly in their integral, and many did gain this mark though some went back to using the original function. Candidates who appreciated the need to first rewrite the integrand usually did so successfully. Dividing each term by <math>x^2</math> tended to be the more popular and successful approach. Some attempted to multiply through by <math>x^2</math> which gave the correct final term, but errors in applying rules of indices sometimes led to the first two terms being wrong.</p> <p>The integration was usually carried out correctly on whatever function they had at this stage, as was the attempted use of limits. Of the candidates who had been successful up to this point, fewer than half were then able to show the given equation correctly. Giving the first term as <math>2a^2</math> rather than <math>(2a)^2</math> was a common error, and substituting values into <math>4x^{-1}</math> caused difficulties for many; those who first rewrote this term in fractional form were usually more successful.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$f(1) = 3 - 5 + 2 = 0$ AG $f(a) = (a - 1)(3a^2 - 2a - 2)$ $a = \frac{2 \pm \sqrt{4+24}}{6} = \frac{2 \pm 2\sqrt{7}}{6} = \frac{1 \pm \sqrt{7}}{3}$ hence $a = \frac{1}{3}(1 + \sqrt{7})$	B1	Confirm $f(1) = 0$ – detail required	<b>Allow working in <math>x</math> not <math>a</math> throughout</b> $3(1)^3 - 5(1)^2 + 2 = 0$ is enough B0 for just $f(1) = 0$ If using division must show '0' on last line If using coefficient matching must show 'R = 0' If using inspection then there must be some indication of no remainder eg expand to show correct cubic
	ii		M1	Attempt full division by $(a - 1)$ , or equiv method	Must be complete method – ie all 3 terms attempted Long division – must subtract lower line (allow one slip) Inspection – expansion must give at least three correct terms of the cubic Coefficient matching – must be valid attempt at all coeffs of quadratic, considering all relevant terms each time
	ii		A1	Obtain $3a^2$ and one other correct term	Could be middle or final term depending on method Must be correctly obtained Coeff matching – allow for $A = 3$ etc
	ii		A1	Obtain fully correct quotient	Could appear as quotient in long division, or as part of a product if using inspection. For coeff matching it must now be explicit not just $A = 3, B = -2, C = -2$

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii		M1	Attempt to solve quadratic	Using the quadratic formula, or completing the square (see guidance sheet) though negative root may be lost at any point M0 if factorising attempt as expected root is a surd Quadratic must come from division attempt, even if this was not good enough for first M1
	ii		A1	Obtain $\frac{1}{3}(1 + \sqrt{7})$ only	<p>Must give the positive root only, so A0 if negative root still present (but condone <math>a = 1</math> also given) Allow aef but must be a simplified surd as per request on question paper (ie simplify <math>\sqrt{28}</math>)</p> <p><u><input type="checkbox"/>Examiner's Comments</u> <input type="checkbox"/></p> <p>The majority of candidates chose to use the factor theorem to confirm that <math>a = 1</math> was a root, and this was invariably correct. Other methods were also used, but some candidates lost an easy mark by failing to address this part of the question. Finding the quotient was done well, although the absence of a linear term in the function caused problems for some, especially those using long division. There was a variety of methods used to find the quotient, including inspection. This method is fine if done successfully but it makes gaining partial credit extremely unlikely as no working is shown. Candidates attempting this method would be well</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>advised to expand their brackets to check for accuracy.</p> <p>Those who had the correct quotient could usually solve the resulting equation correctly, although completing the square was not as successful as using the formula. The surd was invariably simplified correctly, but only a small minority of candidates referred back to the question and appreciated the need to reject the negative solution.</p>
			<b>Total</b>	<b>12</b>	

Question	Answer/Indicative content	Marks	Part marks and guidance	
33	i $(x + 4) - 2x = (2x - 7) - (x + 4)$	M1	Attempt to eliminate $d$ to obtain equation in $x$ only	Equate two expressions for $d$ , both in terms of $x$ Could use $a + (n - 1)d$ twice, and then eliminate $d$ Could use $u_1 + u_2 + u_3 = S_3$ or $u_2 = \frac{1}{2}(u_1 + u_3)$
	i <b>OR</b>			
	i $2x + d = x + 4$ $2x + 2d = 2x - 7$	A1	Obtain correct equation in just $x$	Allow unsimplified equation A0 if brackets missing unless implied by subsequent working or final answer
	i $2x = 15$ $x = 7.5$	A1	Obtain $x = 7.5$	Any equivalent form Allow from no working or T&I
	i			<b>Alt method:</b> B1 – state, or imply, $2x + 2d = 2x - 7$ , to obtain $d = -3.5$ M1 – attempt to find $x$ from second equation in $x$ and $d$ A1 – obtain $x = 7.5$

Question		Answer/Indicative content	Marks	Part marks and guidance	
	i				<p><b>Examiner's Comments</b></p> <p>Many candidates were successful in this part of the question, with the most popular approach being to first find <math>d = -3.5</math> and then use a second equation to find <math>x</math>. This was usually successful, although sign errors proved a pitfall for some. However, a number of candidates made no further progress beyond finding <math>d</math>, often because they did not consider a third equation. The other common method was to find two expressions for <math>d</math> by considering the difference of consecutive terms which could then be equated and solved. This was an elegant and concise method, but a lack of brackets resulted in errors being made. Other, more creative, solutions were also seen including adding the sum of the three terms and equating this to an expression for <math>S_3</math>.</p>
	ii	terms are 16, 12, $9^{12/16} = 0.75$ , $9^{9/12} = 0.75$	B1	List 3 terms	Ignore any additional terms given

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	common ratio of 0.75 so GP	B1	Convincing explanation of why it is a GP	<p>Must show two values of 0.75, or unsimplified fractions</p> <p>Must state, or imply, that ratio has been checked twice, using both pairs of consecutive terms</p> <p>No need to show actual division of terms to justify 0.75, so allow eg arrows from one term to the next with 'x0.75'</p> <p><b>SR B2</b> if 16, 12, 9 never stated explicitly in a list but are so in a convincing method for <math>r = 0.75</math> twice</p>
	ii	$S_{\infty} = \frac{16}{1-0.75} = 64$	M1	Attempt use of $\frac{a}{1-r}$	<p>Must be correct formula</p> <p>Could be implied by method</p> <p>Allow if used with their incorrect <math>a</math> and / or <math>r</math></p> <p>Allow if using <math>a = 8</math>, even if 16 given correctly in list</p>
	ii		A1	Obtain 64	A0 if given as 'approximately 64'

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii				<p><input type="checkbox"/> <b>Examiner's Comments</b> <input type="checkbox"/></p> <p>Virtually all of the candidates gained the first mark for stating the three relevant terms, and most also gained the final two marks for finding the sum to infinity, though a few used <math>\frac{4}{3}</math> as their ratio. It was the second mark that proved to be the most challenging. Candidates had been asked to verify that the terms did form a geometric progression, and were expected to provide a convincing proof that considered the ratio between two pairs of terms, or an equivalent justification. Whilst some candidates did provide this explanation, far too many assumed that it was a geometric progression and simply found the ratio from a single pair of terms.</p>
	iii	$\frac{(2x-7)}{(x+4)} = \frac{(x+4)}{2x}$ $4x^2 - 14x = x^2 + 8x + 16$	M1*	Attempt to eliminate $r$ to obtain equation in $x$ only	Equate two expressions for $r$ , both in terms of $x$ Could use $ar^{n-1}$ twice, and then eliminate $r$ from simultaneous eqns
	iii	OR			

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii	$2xr = x + 4$ $2xr^2 = 2x - 7$  $3x^2 - 22x - 16 = 0$ $(3x + 2)(x - 8) = 0$ $x = -\frac{2}{3}, x = 8$	A1	Obtain $3x^2 - 22x - 16 = 0$	Allow $6x^2 - 44x - 32 = 0$ Allow $3x^3 - 22x^2 - 16x = 0$ , or a multiple of this Allow any equivalent form, as long as no brackets and like terms have been combined Condone $no = 0$ , as long as implied by subsequent work
	iii		M1d*	Attempt to solve quadratic	Dependent on first M1 for valid method to eliminate $r$ See guidance sheet for acceptable methods
	iii		A1	Obtain $x = -\frac{2}{3}$	Allow recurring decimal, but not rounded or truncated Condone $x = 8$ also given Allow from no working or T&I

Question		Answer/Indicative content	Marks	Part marks and guidance
	iii			<p><input type="checkbox"/> <u>Examiner's Comments</u> <input type="checkbox"/></p> <p>This proved to be a challenging question for many candidates. Whilst most were able to make some attempt at it, it was often not enough to gain even the first mark. The most efficient solution was to equate two algebraic expressions for the ratio, and then rearrange them to get a quadratic which could then be solved. Some candidates were able to provide a concise and elegant solution in this way. Some candidates did embark on this method, but then attempted to first simplify their fractions which invariably went wrong. Others started with the generic equations for the <math>n</math>th term of a geometric progression so that when they eliminated <math>r</math> their equation involved the square or square root of a rational expression.</p>
		<b>Total</b>	<b>11</b>	

Question	Answer/Indicative content	Marks	Part marks and guidance	
34	i	M1	Use $\tan x = \frac{\sin x}{\cos x}$ and rearrange to a form not involving fractions	Must be used and not just stated Must multiply all terms by $\cos x$ so $4\cos^2 x - 1$ is M0, but allow M1 for $\cos x(4\cos x - 1)$ even if subsequent errors
	i	M1	Use $\sin^2 x = 1 - \cos^2 x$	Must be used and not just stated Must be used correctly, so M0 for $1 - 2\cos^2 x$ Not dependent on previous M mark, so M0 M1 possible Must be attempting quadratic in $\cos x$ so M0 for $\cos^2 x = 1 - \sin^2 x$
	i	A1	Obtain $6\cos^2 x - \cos x - 2 = 0$ with no errors seen	Must be equation ie = 0 Allow poor notation (eg $\cos$ not $\cos x$ , or $\tan x = \frac{\sin}{\cos}(x)$ ) as long as final answer is correct
			<p><b><u>Examiner's Comments</u></b></p> <p>Most candidates could quote both of the required identities and then attempt to use them. Whilst <math>\sin^2 x = 1 - \cos x</math> was usually used correctly, the use of <math>\tan x</math> caused more problems as candidates were expected to also deal with the fraction to gain the method mark and a number struggled to do so. Some candidates used poor notation, such as omitting the <math>x</math> from their trigonometric ratios, and others spoiled an otherwise correct solution by failing to give an equation as their final answer.</p>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$(3\cos x - 2)(2\cos x + 1) = 0$ $\cos x = \frac{2}{3}, \cos x = -\frac{1}{2}$ $x = 48.2^\circ, 312^\circ, 120^\circ, 240^\circ$	M1	Attempt to solve quadratic in $\cos x$	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, including $x = \cos x$
	ii		M1	Attempt to find $x$ from root(s) of quadratic	Attempt $\cos^{-1}$ of at least one of their roots Allow for just stating $\cos^{-1}$ (their root) inc if $ \cos x  > 1$ Not dependent so M0 M1 possible If going straight from $\cos x = k$ to $x = \dots$ then award M1 only if their angle is consistent with their $k$
	ii		A1	Obtain at least 2 correct angles	Allow 3sf or better Must come from correct solution of quadratic – ie correct factorisation or correct substitution into formula so A0 if two correct roots from eg $(3\cos x + 2)(2\cos x + 1) = 0$ Allow radian equivalents – 0.841, 5.44, $\frac{2\pi}{3}$ or 2.09, $\frac{4\pi}{3}$ or 4.19

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii		A1	Obtain all 4 correct angles, with no extra in given range	Must now be in degrees  <b>SR</b> If no working shown then allow <b>B1</b> for 2 correct angles (poss in rads) or <b>B2</b> for 4 correct angles, no extras  <u>Examiner's Comments</u>  This part of the question was done very well by the majority of the candidates, who were able to identify the fact that the given equation was a quadratic in $\cos x$ and attempt an appropriate method to solve it. The four required roots then usually followed, though some candidates struggled to find the secondary angles with $318.2^\circ$ being a common wrong answer. Others lost marks by discarding the negative root to the quadratic, failing to realise that this would also lead to valid solutions.
		<b>Total</b>	<b>7</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
35	$ar = -3, \frac{a}{1-r} = 4$	B1	State $ar = -3$	Any correct statement, including $a \times r^{(2-1)} = -3$ etc soi
		B1	State $\frac{a}{1-r} = 4$	Any correct statement, not involving $r^\infty$ (unless it becomes 0) soi
	$-\frac{3}{r} = 4(1-r)$	M1*	Attempt to eliminate either $a$ or $r$	Using valid algebra so M0 for eg $a = -3 - r$ Must be using $ar^k$ and $\pm a/(\pm 1 \pm r)$ Award as soon as equation in one variable is seen
	$4r^2 - 4r - 3 (= 0) / a^2 - 4a - 12 (= 0)$	A1	Obtain correct simplified quadratic	Any correct quadratic not involving fractions or brackets ie $4r^2 = 4r + 3$ gets A1
	$(2r-3)(2r+1) = 0 / (a-6)(a+2) = 0$	M1d*	Attempt to solve 3 term quadratic	See Appendix 1 for acceptable methods
	$r = -\frac{1}{2}$	M1**	Identify $r = -\frac{1}{2}$ as only ratio with a minimally acceptable reason	M0 if no, or incorrect, reason given Must have correct quadratic, correct factorisation and correct roots (if stated)  If $r = -\frac{1}{2}$ is not explicitly identified then allow M1 when they use only this value to find $a$ (or later eliminate the other value)  Could accept $r = -\frac{1}{2}$ as $r < 1$ or reject $r = \frac{3}{2}$ as $> 1$ Could reject $a = -2$ as $S_\infty$ is positive Could refer to convergent / divergent series
	$a = 6$	A1	Obtain $a = 6$ only	If solving quadratic in $a$ , then both values of $a$ may be seen initially - A1 can only be awarded when $a = 6$ is given as only solution

Question			Answer/Indicative content	Marks	Part marks and guidance	
			for sum to infinity – $-1 < r < 1$	A1d**	<p>Convincing reason for <math>r = -\frac{1}{2}</math> as the only possible ratio</p> <p><b>Examiner's Comments</b></p> <p>Candidates were able to make a good start to this question, but only the most able could make progress beyond the first five marks. The majority could attempt the two relevant equations and then eliminate one of the variables, usually <math>a</math>. Substituting the equation for the sum to infinity into the equation for the second term usually resulted in the correct quadratic, whereas the fraction involved in doing the substitution the other way around caused problems for some. Nevertheless, many candidates did obtain the correct quadratic which they could then attempt to solve. Candidates then had to select the correct common ratio and also provide some reasoning for this choice. No credit was available for picking <math>r = -0.5</math> with no, or an incorrect, reason. To gain full marks, the reasoning for the selection of <math>r = -0.5</math> had to be convincing and fully complete. It was not sufficient to reject <math>r = 1.5</math> without also explaining why the other was being accepted.</p>	<p>Must refer to <math> r  &lt; 1</math> or <math>-1 &lt; r &lt; 1</math> or in words A0 if additional incorrect statement</p> <p>No credit for answer only unless both <math>r</math> first found</p>
			<b>Total</b>	<b>8</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
36	i	$f(-2) = 12 - 22(-2) + 9(-2)^2 - (-2)^3 = 12 + 44 + 36 + 8$	M1	Attempt $f(-2)$ or equiv	M0 for using $x = 2$ (even if stated to be $f(-2)$ ) Allow slips in evaluation as long as intention is clear At least one of the second or fourth terms must be of the correct sign Allow any other valid method to divide by $(x + 2)$ as long as remainder is attempted (see guidance in part (iii) for acceptable methods)
	i	= 100	A1	Obtain 100  <b>Examiner's Comments</b>  The overwhelming majority of candidates gained both marks in this question, usually by using the remainder theorem. Some candidates decided to use algebraic division instead; a longer and often less successful tactic.	Do not ISW if subsequently given as $-100$ If using division, just seeing 100 on bottom line is fine unless subsequently contradicted by eg $-100$ or $\frac{100}{x+2}$
	ii	$f(3) = 12 - 66 + 81 - 27 = 0$	B1	Attempt $f(3)$ , and show = 0  <b>Examiner's Comments</b>  Virtually all of the candidates gained this mark, with the factor theorem being by far and away the most common method. It was not sufficient to do the division or state the factorisation, candidates also had to demonstrate that there was no remainder in order to gain the mark.	$12 - 22(3) + 9(3)^2 - (3)^3 = 0$ is enough B0 for just stating $f(3) = 0$ If using division must show '0' on last line or make equiv comment such as 'no remainder' If using coefficient matching must show 'R = 0' Just writing $f(x)$ as the product of the linear factor and the correct quadratic factor is not enough evidence - need to show that the expansion would give $f(x)$ Ignore incorrect terminology eg ' $x = 3$ is a factor' or ' $(3 - x)$ is a root'

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii	$f(x) = (3 - x)(x^2 - 6x + 4)$	M1	Attempt complete division by $(3 - x)$ or $(x - 3)$ , or equiv	Must be complete method - ie all 3 terms attempted Allow M1 if dividing $x^3 - 9x^2 + 22x - 12$ by $(3 - x)$ oe Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of quadratic, considering all relevant terms each time Synthetic division - must be using 3 (not $-3$ ) and adding within each column (allow one slip); expect to see $  \begin{array}{r rrrr}  3 & 1 & -9 & 22 & -12 \\  & & 3 & -18 & \\  \hline  & 1 & -6 & 4 &   \end{array}  $
	iii		A1	Obtain $x^2 - 6x + 4$ or $-x^2 + 6x - 4$	Allow A1 even if division is inconsistent eg dividing $f(x)$ by $(x - 3)$ or $-f(x)$ by $(3 - x)$ Must be explicit and not implied ie $A = 1$ etc in coeff matching method or just the bottom line in the synthetic division method is not enough

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii		A1	Obtain $(3 - x)(x^2 - 6x + 4)$ or $(x - 3)(-x^2 + 6x - 4)$  <b>Examiner's Comments</b>  This part of the question was also very well done, with the majority of the candidates gaining all of the three marks available. Algebraic division tended to be the most common method, with most candidates coping admirably with the negative coefficient of $x^3$ and the function being written in order of ascending powers. Some candidates made the division easier by reversing the order of the terms. Others opted to use the negative of either the function or the factor, or both. Care then had to be taken when finally writing the function as the product of a linear factor and a quadratic factor as the reversal of signs was not always taken into consideration. Other methods included coefficient matching and inspection, both of which were usually successful. However, candidates using inspection do need to be careful as no method is shown which makes it unlikely that partial credit can be awarded in the case of an incorrect answer.	Must be written as a product, just stating the quadratic quotient by itself is not enough Must come from a method with consistent signs in the divisor and dividend
	iv	$x = 3$	B1	State $x = 3$	At any point

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iv	$x = 3 \pm \sqrt{5}$	M1	Attempt to find roots of quadratic quotient	Can gain M1 if using an incorrect quotient from (iii), as long as it is a three term quadratic and comes from a division attempt by $(3 - x)$ or $(x - 3)$ See Appendix 1 for acceptable methods
	iv		A1	Obtain $x = 3 \pm \sqrt{5}$  <b>Examiner's Comments</b>  This was also very well done, with well over half of the candidates gaining full marks. Most remembered to include $x = 3$ as a root, and finding the other two roots was also very well done with a variety of methods being used. For candidates using the quadratic formula the method mark will only be awarded if the correct formula is seen, either the generic statement or after substitution, so it is essential that candidates show full details of their method.	Must be in simplified surd form Allow A1 if from $-f(x) = 0$ eg $(x - 3)(x^2 - 6x + 4) = 0$
		<b>Total</b>	<b>9</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
37	i	$\tan x(\sin x - \cos x) = 6 \cos x$ $\tan x \left( \frac{\sin x}{\cos x} - 1 \right) = 6$ $\tan x(\tan x - 1) = 6$	M1	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once	Must be used clearly at least once – either explicitly or by writing eg ‘divide by $\cos x$ ’ at side of solution Allow M1 for any equiv eg $\sin x = \cos x \tan x$ Allow poor notation eg writing just $\tan$ rather than $\tan x$
	i	$\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ AG	A1	Obtain $\tan^2 x - \tan x - 6 = 0$  <b>Examiner's Comments</b>  A variety of methods were seen for this proof, some more efficient than others. Most candidates did get there in the end, but full credit was only given if the correct notation had been used throughout. Candidates must also ensure that each step is clearly and convincingly detailed when a proof has been requested.	Correct equation in given form, including = 0 Correct notation throughout so A0 if eg $\tan$ rather than $\tan x$ seen in solution
	ii	$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3, \tan x = -2$	M1	Attempt to solve quadratic in $\tan x$	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, inc $x = \tan x$
	ii	$x = \tan^{-1}(3), x = \tan^{-1}(-2)$	M1	Attempt to solve $\tan x = k$ at least once	Attempt $\tan^{-1} k$ at least once Not dependent on previous mark so M0M1 possible If going straight from $\tan x = k$ to $x = \dots$ , then award M1 only if their angle is consistent with their $k$

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$x = 71.6^\circ, 252^\circ, 117^\circ, 297^\circ$	A1	Obtain two correct solutions	Allow 3sf or better Must come from a correct method to solve the quadratic (as far as correct factorisation or substitution into formula) Allow radian equivs ie 1.25 / 4.39 / 2.03 / 5.18
	ii		A1	Obtain all 4 correct solutions, and no others in range  <b>Examiner's Comments</b>  This question was generally very well done, and many candidates gained full marks on this question. The most common error was to completely discount the solution resulting from $\tan^{-1}(-2)$ as it resulted in a negative angle rather than appreciating it would still generate other angles within the given range. It was also disappointing to see candidates with a correct method failing to gain full marks due to rounding errors. As in previous questions involving trigonometry, some candidates did not ensure their calculator was in the correct mode before proceeding. Angles given in radians could gain some credit, but candidates did not actually consider which measure they were using so the typical error was $\tan^{-1}(3) = 1.25$ and hence 189.25.	Must now all be in degrees Allow 3sf or better A0 if other incorrect solutions in range $0^\circ - 360^\circ$ (but ignore any outside this range)  <b>SR</b> If no working shown then allow <b>B1</b> for each correct solution (max of <b>B3</b> if in radians, or if extra solns in range).
		<b>Total</b>	<b>6</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
38	i	$u_{20} = 5 + 19 \times 3$	M1	Attempt $u_{20}$	Must be using correct formula, with $a = 5$ and $d = 3$ Could use $u_n = 3n + 2$ Could attempt to list terms
	i	$= 62$	A1	Obtain 62  <b>Examiner's Comments</b>  This question was invariably correct, with most candidates using the formula for the $n$ th term of an AP. Other methods included firstly generating an $n$ th term expression for the sequence, and some just resorted to manually listing the terms.	If listing terms then need to indicate that 62 is the required answer
	ii	$S_{20} = {}^{20}/_2 (10 + 57)$ $S_9 = {}^9/_2 (10 + 24)$	M1	Explicitly attempt either $S_{20}$ or $S_9$	Must be using correct formula with $a = 5$ and $d = 3$ Use of formula must be explicit, so M0 for eg $S_{20} = 670$ with no other evidence Could use $\frac{1}{2}n(a + l)$ , with $l$ obtained from $a + (n - 1)d$ – expect to see ${}^{20}/_2 (5 + 62)$ and $l$ or ${}^9/_2 (5 + 29)$ Could use $\Sigma(3n + 2)$ , with correct formulae for $\Sigma n$ and $\Sigma 1$
	ii	${}^{20}/_2 (10 + 57) - {}^9/_2 (10 + 24)$	M1	Attempt $S_{20} - S_9$ , where both summations have been shown explicitly	Can get M1 if formulae have not yet been evaluated M0 for $S_{20} - S_{10}$ (see below for one exception)

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$= 670 - 153$ $= 517$ AG	A1	Evaluate both summations and hence obtain 517 CWO	<p>AG so detail is required – only award A1 if both unsimplified sums are seen, as well as both evaluated sums</p> <p>SR Allow B1 if only <math>670 - 153 = 517</math> seen</p> <p>Explicitly detailing only one summation will get M1M0A0</p> <p>Allow 3/3 for <math>S_{20} - S_{10} + u_{10}</math> as long as all explicit</p> <p>Allow 3/3 for manually summing terms as long as all terms are shown and are all correct, but no partial credit if wrong</p>
	ii	OR $u_{10} = 5 + 9 \times 3 = 32$	M1	Attempt $u_{10}$	Must be shown explicitly
	ii	$S = {}^{11}/_2 (32 + 62)$	M1	Attempt required sum	Must have $n = 11$ Or $S = {}^{11}/_2 (2 \times 32 + 10 \times 3)$

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	= 517 AG	A1	Obtain 517  <u>Examiner's Comments</u>  The purpose of this part of the question was to assist candidates in finding an appropriate strategy with which to attempt the final part. Because the answer was given, candidates were expected to show full detail of their method and too many solutions did not address this. Most candidates gained at least the first mark for attempting the sum of the first twenty terms, but a number then struggled to make any further progress. Subtracting the sum of the first ten terms was the most common error; some candidates gave up at this point, whereas others made a valiant, but not always convincing, attempt to obtain the given 517. Some candidates listed, and summed, the relevant eleven terms. This approach gained full credit in this part of the question, but was not a method that could then be replicated in part (iii).	Detail reqd – award M0M1A0 if no evidence for $u_{10} = 32$
	iii	$S_{2N} = {}^{2N}/_2 (10 + 3(2N - 1))$	B1	Correct (unsimplified) $S_{2N}$ soi	Or ${}^{2N}/_2 (5 + 5 + 3(2N - 1))$ , or equiv, from $\frac{1}{2}n(a + l)$ Or ${}^3/_2 (2N)(2N + 1) + 2(2N)$ , or equiv, from $\Sigma(3n + 2)$
	iii	$S_{N-1} = {}^{N-1}/_2 (10 + 3(N - 2))$	B1	Correct (unsimplified) $S_{N-1}$ soi Or $S_N - u_N$ soi	Or $N - \frac{1}{2} (5 + 5 + 3(N - 2))$ , or equiv, from $\frac{1}{2}n(a + l)$ Or ${}^3/_2 (N - 1)(N) + 2(N - 1)$ , or equiv, from $\Sigma(3n + 2)$

Question		Answer/Indicative content	Marks	Part marks and guidance	
	iii	$N(6N + 7) - {}^{N-1}/_2 (3N + 4) = 2750$	M1*	Subtract attempt at $S_{N-1}$ from $S_{2N}$ equate to 2750	Expressions could still be unsimplified Must have attempted to use correct formula, with $a = 5$ , $d = 3$ and correct $n$ each time Allow sign errors, resulting from lack of essential brackets M0 for $S_{2N} - S_N$ but M1 for $S_{2N} - S_N + u_N$
	iii	$9N^2 + 13N - 5496 = 0$	A1	Rearrange to obtain $9N^2 + 13N - 5496 (= 0)$	aef not involving brackets and with like terms combined
	iii	$(9N + 229)(N - 24) = 0$	M1d*	Attempt to solve 3 term quadratic	Any valid attempt to solve quadratic (see guidance) to obtain at least the positive root If solving an incorrect quadratic then method <b>must</b> be shown for M1 to be awarded
	iii	$N = 24$	A1	Obtain $N = 24$ only CWO	No need to consider the negative root, but A0 if found but not discarded  Answer only gains full credit
	iii	OR ${}^{N+1}/_2 (2(5 + 3(N - 1)) + 3M) = 2750$	M1*	Attempt sum from $u_N$ to $u_{2N}$	Correct formula, $a = 5 + 3(N - 1)$ , $d = 3$ , and $n = N$ or $N + 1$
	iii		M1d*	Use $n = N + 1$	Use $n = N + 1$ only
	iii		A1	Correct unsimplified sum = 2750	Just equate to 2750, no need to rearrange
	iii	$9N^2 + 13N - 5496 = 0$	A1	Obtain correct quadratic	Or ${}^{N+1}/_2 ((5 + 3(N - 1)) + (5 + 3(2N - 1)))$ from ${}^{1/2}n(a + l)$
	iii	$(9N + 229)(N - 24) = 0$	M1dd*	Attempt to solve 3 term quadratic	Quadratic must have come from sum = 2750

Question			Answer/Indicative content	Marks	Part marks and guidance
		iii	$N = 24$	A1	Obtain $N = 24$ only  <u>Examiner's Comments</u>  This final part of the question proved to be challenging for even the most able candidates, and fully correct solutions were in the minority. It was disappointing that so few candidates made the link between what they had been asked to do in the previous part of the question and what was now required of them. The first mark was available for finding the sum of the first $2N$ terms, and this was gained by just over half of the candidates. To make any further progress candidates now had to consider the sum of the first $N - 1$ terms, and then equate the difference to 2750. Only a minority actually attempted this, with the most common error being to subtract the sum of the first $N$ terms. A lack of care with brackets meant that some candidates could not obtain the correct, simplified, quadratic despite the initial part of the solution being correct. An elegant alternative method that was sometimes seen considered the sum of $N + 1$ terms, starting on the $N$ th term and finishing on the $2N$ th term, and an equally efficient method used the $n$ th term definition of the sequence.
			<b>Total</b>	<b>11</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
39	i	$\log_3 x^2 - \log_3(x + 4)$ $= \log_3 \frac{x^2}{x+4}$	B1*	Obtain $\log_3 x^2 - \log_3(x + 4)$	Allow no base Could be implied if both log steps done together Allow equiv eg $2(\log_3 x - \log_3(x + 4)^{0.5})$
	i		B1d*	Obtain $\log_3 \frac{x^2}{x+4}$ or equiv single term	CWO so B0 if eg $\frac{\log x^2}{\log(x+4)}$ seen in solution No ISW if subsequently  Must now have correct base in final answer - eg $\log_3(\frac{x^2}{x+4})$ incorrectly 'simplified' eg condone if omitted earlier  <b>Examiner's Comments</b>  The majority of candidates were able to produce a fully correct solution to this part of the question. Of the remainder, most were aware of the power law but too often this was not used as the first step or the second term was incorrect at this stage so no fully correct expression was ever seen. Some candidates obtained the correct expression but then incorrectly cancelled within the logarithm, which was penalised. Another relatively common error was for the difference of the two logarithms to result in a fraction with a logarithm appearing in the denominator. Even if this subsequently was written as the required single term, the error in the method was still penalised.

Question	Answer/Indicative content	Marks	Part marks and guidance	
	ii $\frac{x^2}{x+4} = 3^2$ $x^2 = 9(x+4)$ $x^2 - 9x - 36 = 0$ $(x-12)(x+3) = 0 \quad x = 12$	M1*	Attempt correct method to remove logs	Equation must be of format $\log_3 f(x) = 2$ , with $f(x)$ being the result of a legitimate attempt to combine logs (but condone errors such as incorrect simplification of fraction) Allow use of their (i) only if it satisfies the above criteria, so $x^2 - (x+4) = 9$ is M0 whether or not in (i)
	ii	A1	Obtain any correct equation	Not involving logs
	ii	M1d*	Attempt complete method to solve for $x$	Solving a 3 term quadratic - see additional guidance Must attempt at least one value of $x$
	ii	A1	Obtain $x = 12$ as only solution	Must be from a correct solution of a correct quadratic, and A0 if other root (if given) is not $x = -3$ A0 if $x = -3$ still present Not necessary to consider $x = -3$ , and then discard, but A0 if discarded for incorrect reason  <b>NB</b> Despite not being 'hence' allow full credit for other valid attempts, such as combining $\log_3(x+4)$ with $\log_3 9$ on right-hand side before removing logs,  $3x - \frac{2}{3}\log_3(x+4) = 1$ or starting with $\log_3$ SR in (i) $\frac{\log x^2}{\log(x+4)}$ becoming $\log_3 \frac{x^2}{x+4}$  was penalised as an error in notation, but is eligible for full credit in (ii)
				<b>Examiner's Comments</b>

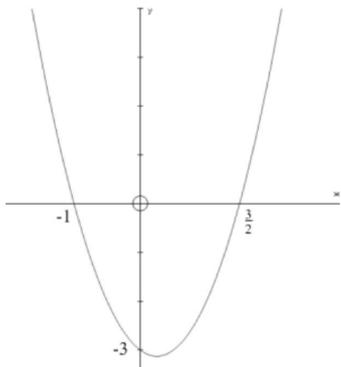
Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>Most candidates who had correctly combined logarithms in the first part of the question could then carry out the correct process to remove the logarithms in this part of the question and solve the ensuing equation with ease. Only the most astute candidates appreciated that <math>-3</math> was not a valid solution to the given equation and thus needed discarding, which meant that three out of four was the modal mark. To gain any credit in this part of the question it was expected that there had been a valid attempt in part (i) to write the two logarithms as a single term.</p>
			<b>Total</b>	<b>6</b>	

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Question		Answer/Indicative content	Marks	Part marks and guidance	
40	a	$\frac{1}{4}x^4 \dots$  $-3x^2 + c$	<b>M1</b> <b>(AO1.1a)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>[3]</b>	Attempt to integrate   Correct integral including +c	At least one power increases by one
	b	$\frac{4}{x^2} = 4x^{-2}$ (i) $x^2$  $-4x^{-1} \dots$  $-x + c$ oe	<b>B1</b> <b>(AO1.1)</b>  <b>M1</b> <b>(AO1.1a)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>[3]</b>	soi  Attempt to integrate a power not a positive integer  Correct integral including +c Penalise omission of +c only once	
	b	(ii) $\int_1^2 \left(\frac{4}{x^2} - 1\right) dx - \int_2^5 \left(\frac{4}{x^2} - 1\right) dx$  Area = $2\frac{4}{5}$ oe	<b>M1</b> <b>(AO3.1a)</b> <b>A1FT</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>[3]</b>	Add absolute areas Correct integrals seen or $[their(ii)(a)]_1^2 - [their(ii)(a)]_2^5$  BC	Both M1 and A1 may be implied by correct answer  SC1 for ""
		<b>Total</b>	<b>9</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
41	a	<p>DR</p> $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 5$ $= \frac{1}{2} + 1 + \frac{7}{2} - 5 = 0$ <p>Since <math>f\left(\frac{1}{2}\right) = 0</math> therefore <math>(2x - 1)</math> is a factor'</p>	<p><b>*M1</b> (AO2.1)</p> <p><b>dep*E1</b> (AO2.1)</p> <p>[2]</p>	<p>Must show an intermediate line of reasoning without brackets or indices</p>	<p>OR</p> <p><b>*M1</b> Attempt to divide <math>f(x)</math> by <math>(2x - 1)</math></p> <p><b>dep*E1</b> State 'No remainder, hence <math>2x - 1</math> is a factor</p>

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	<p>DR</p> <p>Substituting <math>x = \sin \theta</math> into the equation in part (a) gives the equation in part (b)</p> <p>so since <math>x = \frac{1}{2}</math> is a solution in part (a), <math>\sin \theta = \frac{1}{2}</math> is a solution in part (b)</p> <p>Hence <math>\theta = 30</math> or <math>150</math></p> <p>Attempt method for finding quadratic factor in terms of <math>x</math> or <math>\sin \theta</math></p> <p><math>2x^2 + 3x + 5</math></p> <p><math>2x^2 + 3x + 5 = 0</math> has no solutions because</p> <p><math>D = 9 - 4 \times 2 \times 5 &lt; 0</math></p> <p>So there are no more solutions of the given equation</p>	<p>M1 (AO3.1a)</p> <p>E1 (AO3.2a)</p> <p>A1 (AO1.1)</p> <p>M1 (AO1.1a)</p> <p>A1 (AO1.1)</p> <p>M1 (AO2.1)</p> <p>E1 (AO2.4)</p> <p>[7]</p>	<p>Connect the equations given in part (a) and (b)</p> <p>Interpret to give a solution for the equation</p> <p>for both correct with no extras</p> <p>Attempt to obtain quadratic factor by any correct method</p> <p>Attempt to solve the quadratic factor</p> <p>Explicitly use <math>b^2 - 4ac &lt; 0</math> oe</p>	<p>Must be shown</p> <p>Or consider the existence of further solutions, e.g. by calculus</p>	
		<b>Total</b>	<b>9</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
42	a	$x = \frac{3}{2}, x = -1$ 	<b>B1</b> <b>(AO1.1)</b>  <b>B1</b> <b>(AO1.1)</b>  <b>B1</b> <b>(AO1.1)</b>  <b>[3]</b>	BC Correct roots  Good curve: <ul style="list-style-type: none"> <li>• Correct shape, symmetrical positive quadratic</li> <li>• FT Minimum point in the correct quadrant for their roots</li> <li>• FT their x intercepts correctly labelled</li> </ul> y intercept at (0, -3)	Must have a curve
	b	$x \in \left(-1, \frac{3}{2}\right)$	<b>M1</b> <b>(AO1.1)</b>  <b>A1FT</b> <b>(AO1.1)</b>  <b>[2]</b>	Choosing the interval between their x intercepts  This interval identified clearly FT their x values in part (a)	Other clear notation is acceptable

Question		Answer/Indicative content	Marks	Part marks and guidance	
	c	No real roots implies that the discriminant is negative  $b^2 - 4ac = 1^2 - 4 \times 2 \times -(3 + k) < 0$  $25 + 8k < 0$  $k < -\frac{25}{8}$	M1 (AO3.1a)  A1 (AO1.1)  A1 (AO3.2a)  [3]	OR  M1 Attempt to find turning point and use $k < y_{\min}$  A1 Turning point at	
		<b>Total</b>	<b>8</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
43	a	$y^7 + 7xy^6 + 21x^2y^5 + 35x^3y^4$	<b>B2</b> (AO1.1 1.1)  [2]	B1 for three terms correct	
	b	$21x^2y^5 = 35x^3y^4$  $\frac{x}{y} = \frac{3}{5}$ or 0.6	<b>M1</b> (AO3.1a)  <b>A1</b> (AO1.1)  [2]	Equate their terms in $x^2y^5$ and $x^3y^4$	
	c	$P(L=2) = {}_7C_2 \left(\frac{3}{8}\right)^2 \left(\frac{5}{8}\right)^5$ and $P(L=3) = {}_7C_3 \left(\frac{3}{8}\right)^3 \left(\frac{5}{8}\right)^4$  Let $p = \frac{3}{8}$ and $1-p = \frac{5}{8}$ , then $\frac{p}{1-p} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$  so from part (b) $21p^2(1-p)^5 = 35p^3(1-p)^4$ and $P(L=2) = P(L=3)$	<b>B1</b> (AO3.3)  <b>M1</b> (AO3.4)  <b>E1</b> (AO2.1)  [3]	Connect to part (b)  AG	
		<b>Total</b>	<b>7</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance			
44		DR $2(1 - \sin^2 x) = 2 - \sin x$  $2\sin^2 x - \sin x = 0$  $\sin x(2\sin x - 1) = 0$   $\sin x = \frac{1}{2}$ so $x = 30$ or $x = 150$  $\sin x = 0$ so $x = 0$ or $x = 180$	M1(AO3.1a)  A1(AO1.1)  M1(AO1.1a)   A1(AO1.1)  A1(AO1.1)  [5]	Use $\cos^2 x = 1 - \sin^2 x$ and simplify  Obtain $2\sin^2 x - 1\sin x = 0$  Attempt to solve a 2 term quadratic in $\sin x$ and use correct order of operations to obtain $x$	One step of simplification must be seen   Use any valid method Must be seen		
		<b>Total</b>	<b>5</b>				

Question		Answer/Indicative content	Marks	Part marks and guidance	
45	a	$s = 9(6)^2 - 1.5(6)^3 = 0$  E.g. The boomerang is at O E.g. She catches the boomerang	M1(AO1.1)  E1(AO3.4)  [2]		
	b	$v = 18t - \frac{9}{2}t^2$  When $v = 0$ , $t = 0$ or $t = 4$  At $t = 0$ , $s = 0$ , so maximum displacement must be when $t = 4$ giving $s = 48$ m	M1(AO1.1)  M1(AO1.1) A1(AO1.1)  E1(AO3.4)  [4]	Imply deduction that greatest distance is when velocity = 0 and solve	
	c	$t = 5$  $v = -22.5$	B1(AO3.1b)  B1(AO3.4)  [2]		
	d	$a = 18 - 9t$  Acceleration = $-27$ ( $\text{m s}^{-2}$ )	M1(AO1.1)  A1(AO1.1)  [2]		
		<b>Total</b>	<b>10</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
46	a	<p>Resultant force from the tug boats is positive so it is moving east</p> <p>There is zero resultant force in the j direction, so it is not moving north or south</p>	<p>E1(AO2.2a)</p> <p>E1(AO2.2a)</p> <p>[2]</p>	(600i)	
	b	<p><math>350 + 250 - 200 = 100000a</math></p> <p>Obtain <math>0.004 \text{ m s}^{-2}</math></p>	<p>M1(AO3.3)</p> <p>A1(AO1.1)</p> <p>[2]</p>	<p>Use <math>F = ma</math></p> <p>. Allow sign errors and one missing force</p>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	c	$400 = 1.5t + \frac{1}{2}(0.004)t^2$ $0.002t^2 + 1.5t - 400 = 0$ Obtain 209 (seconds)  $v^2 = 1.5^2 + 2(0.004)(400)$ Obtain 2.33 (m s <sup>-1</sup> )	M1(AO3.1b)  A1(AO1.1)  M1(AO3.4)  A1(AO1.1)  M1(AO3.4)  A1(AO1.1)  [6]	Use $s = ut + \frac{1}{2}at^2$  Obtain correct quadratic. Any equivalent form  Use any method to solve their quadratic. If negative root given ( -958.63088 ) this must be clearly discarded. Use $v^2 = u^2 + 2as$ with their $a$ or $v = u + at$ with their $a$ and $t$ . Accept better (2.3345235)	Including BC  Accept better (208.630877) but not 208	
		<b>Total</b>	<b>10</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
47		$x^2 + 8x + (x - 10)^2 = 84$  $2x^2 - 12x + 16 = 0$  $x = 2, x = 4$  $x = 2$ and $y = -8$ $x = 4$ and $y = -6$	<b>M1(AO1.1a)</b>  <b>A1(AO1.1b)</b> <b>A1(AO1.1)</b> <b>A1(AO1.1)</b>  <b>[4]</b>	Substitute the linear equation into the quadratic  Correctly simplified answer BC, but allow by any valid method Values should be paired correctly	<b>OR M1</b> $(y + 10)^2 + 8(y + 10) + y^2 = 84$  $A1 \ 2y^2 + 28y + 96 = 0$ <b>A1</b> $y = -8,$ $y = -6$	
		<b>Total</b>	<b>4</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
48	a	Identify AP with $a = 5000$ and $d = 1500$  $\frac{n}{2}(2(5000) + (n-1)1500)$ $= n(750n + 4250)$	<b>M1(AO3.1b)</b>  <b>A1(AO1.1)</b>  <b>[2]</b>	Identification recognised by an attempt at the sum formula or $n$ th term formula for an AP  Or $750n^2 + 4250n$	
	b	$\frac{5000(1 - (0.9)^n)}{1 - 0.9}$  Obtain $50000(1 - (0.9)^n)$	<b>M1(AO3.1b)</b>  <b>A1(AO3.1b)</b> <b>A1(AO1.1)</b>  <b>[3]</b>	Identification recognised by an attempt at the sum formula with $n$ , $n - 1$ or $n + 1$ or with a positive sign in numerator  Obtain correct unsimplified sum Or $50000 - 50000(0.9)^n$	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	c	Obtain $750n^2 + 4250n - 385000 = 0$  $n = 20$ or $n = -\frac{77}{3}$  State 20 years	<b>M1(AO3.1b)</b>  <b>A1(AO1.1)</b>  <b>A1(AO3.4)</b>  <b>[3]</b>	Equate to 385 000 and solve a 3 term quadratic = 0  BC both required Allow different methods for solving the quadratic	<b>OR</b> <b>M1</b> For writing down and summing the total profit for at least the first four years (may be implied BC) <b>A1</b> For finding that the total is equal to 385 000 for $n = 20$  <b>A1</b> state 20 years	
	d	Firm A's profits continue to grow  Firm B's profits eventually plateau at £50 000 as $(0.9)^n$ tends to 0 with large enough $n$	<b>E1(AO3.4)</b> <b>E1(AO3.2a)</b>  <b>[2]</b>	Some mention is required about the effect of $(0.9)^n$		
		<b>Total</b>	<b>10</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
49	a	$fg(x) = (6x - 2a)^2 + 8a(6x - 2a) + 4a^2$ $= 36x^2 + 24ax - 8a^2$ $(fg)'(x) = 72x + 24a = 0$ $x = -\frac{a}{3}, \text{ giving}$ $fg\left(-\frac{a}{3}\right) = (-4a)^2 + 8a(-4a) + 4a^2 = -12a^2$ Stationary point of fg is a minimum so range of $fg(x) \geq -12a^2$ or $[-12a^2]$	<b>B1(AO1.1)</b> <b>M1(AO1.1)</b>  <b>M1(AO2.1)</b>  <b>E1(AO2.2a)</b>  <b>[4]</b>	Accept unsimplified form  Differentiate their $fg(x) = 0$ or use square completion: $4(9x^2 + 6ax - 2a^2) = 4(3x + a)^2 - 4a^2 - 8a^2$ Solve for $x$ and substitute their value for $x$ in $fg(x)$  Must mention minimum Do not accept $x \geq -12a^2$	<b>OR</b> <b>M1</b> Complete a square on $f(x)$ <b>A1</b> Obtain $(x + 4a)^2 - 12a^2$  <b>M1</b> Substitute $g(x)$ and simplify  <b>E1</b> Obtain $(6x + 2a)^2 - 12a^2$ or equivalent form and state $fg(x) \geq -12a^2$	
	b	$144 + 48a - 8a^2 = 144$ $a = 6$	<b>M1(AO3.1a)</b> <b>M1(AO1.1)</b> <b>A1(AO1.1)</b>  <b>[3]</b>	Substitute $x = 2$ in their $fg(x)$ and equate to 144 Attempt to solve their equation Do not give this mark if $a = 0$ also given as an answer		

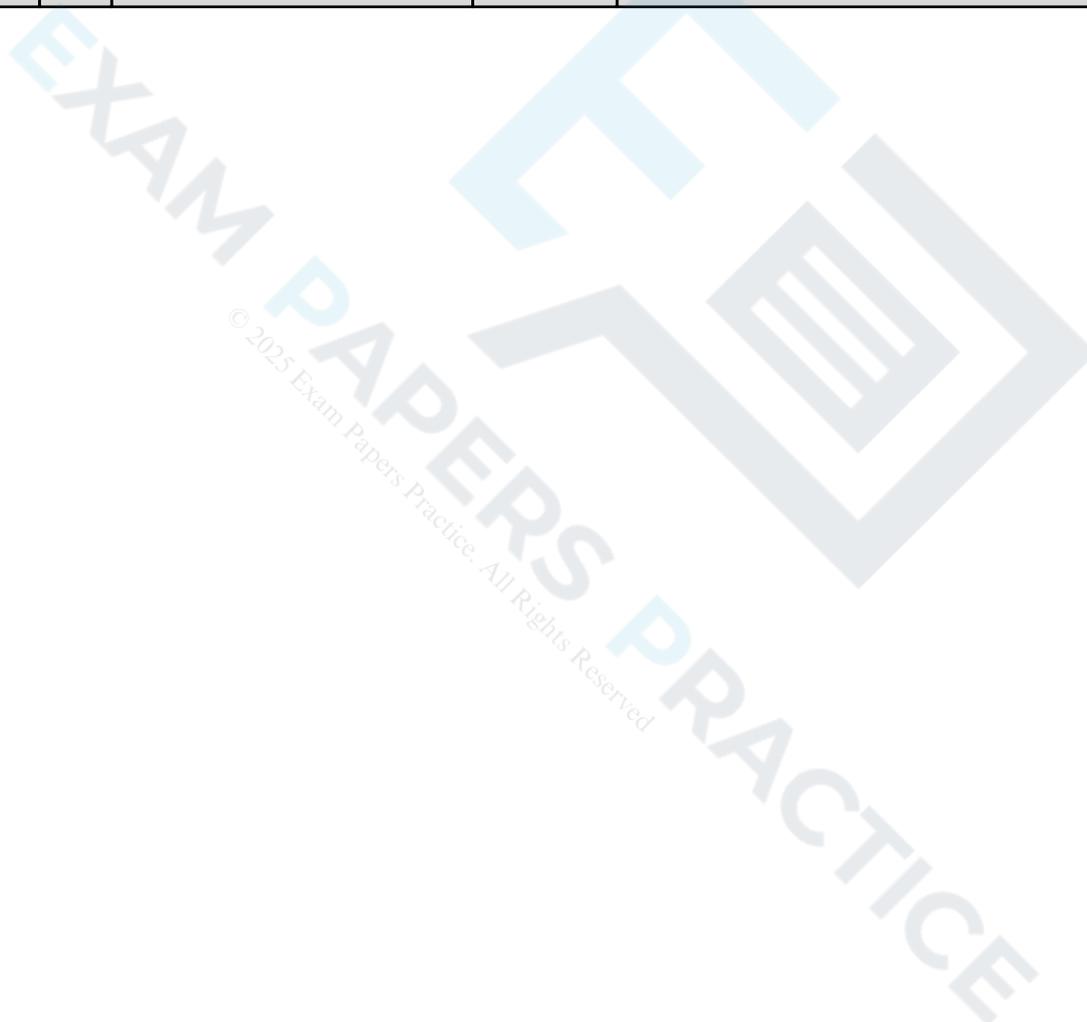
Question			Answer/Indicative content	Marks	Part marks and guidance	
		c	<p>Each <math>y</math> value in the range (<math>y &gt; -12a^2</math>) corresponds to two <math>x</math> values, e.g. <math>y = 0</math> corresponds to <math>x = 1.46</math> or <math>-5.46</math></p> <p>Therefore <math>fg</math> has no inverse</p>	<p>M1(AO2.4)</p> <p>E1(AO2.2a)</p> <p>[2]</p>	<p>An example or graph must be given, or a clear explanation that quadratic functions on the real numbers are one-to-many.</p>	
			<b>Total</b>	<b>9</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance		
50	<p>DR</p> $3x^2 + 3y^2 \frac{dy}{dx}$ $= 3y + 3x \frac{dy}{dx}$ <p>To find the stationary points</p> $\text{let } \frac{dy}{dx} = 0$ $y = x^2$ $x^3 + (x^2)^3 = 3x(x^2) + 35$ $x^6 - 2x^3 - 35 = 0$ <p>Let <math>p = x^3</math>, then <math>p^2 - 2p - 35 = 0</math></p> $p = 7 \text{ or } -5$ $\Rightarrow x = \sqrt[3]{7} \text{ or } x = -\sqrt[3]{5}$	<p>B1(AO1.1)</p> <p>M1(AO3.1a)</p> <p>A1(AO1.1)</p> <p>E1(AO2.1)</p> <p>M1(AO3.1a)</p> <p>M1(AO2.1)</p> <p>M1(AO2.1)</p> <p>M1(AO1.1)</p> <p>A1(AO3.2a)</p> <p>[9]</p>	<p>Attempt LHS derivative</p> <p>Attempt product rule on RHS</p> <p>Correct on RHS</p> <p>Explicitly set their derivative equal to zero</p> <p>Attempt to solve for their y or their x</p> <p>Substitute to get their polynomial in one variable</p> <p>Transform their disguised quadratic</p> <p>Solve their 3 term quadratic</p> <p>For both correct</p>	<p>Two non-constant terms</p> <p>Alternate <math>x = y^{\frac{1}{2}}</math></p> <p>Alternate</p> <p>A0 for decimal answer</p>	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			<b>Total</b>	<b>9</b>		
51		a	$\sqrt{16a^4}$ or $4\sqrt{a^4}$ or $a\sqrt{a} \times 4\sqrt{a}$  $= 4a^2$	<b>M1(AO1.1)</b>  <b>A1(AO1.1)</b>  <b>[2]</b>	Any correct first step	
		b	$32b^{15}$	<b>B2(AO1.1)</b>  <b>(AO1.1)</b>  <b>[2]</b>	<b>B1</b> for 32 and <b>B1</b> for $b^{15}$	
			<b>Total</b>	<b>4</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
52	a	Total profit (or $t$ ) is large when price (or $p$ ) is high	B1(AO3.5b)  [1]		
	b	Passes through (0, 0) and (12, 0) hence $t = kp(12 - p)$  $k = 200$	B1(AO3.1b)  B1(AO3.3)  [2]	Or $t = 200 p(12 - p)$ Or $t = 200(12p - p^2)$	
	c	$6400 = 200 p(12 - p)$ oe  $p^2 - 12x + 32 = 0$  $p = 4, p = 8$ $4 \leq p \leq 8$  Price must be between £4 and £8	M1(AO3.4)  A1FT(AO1.1)  A1FT(AO1.1)  A1(AO3.4)  [4]	$6400 =$ (their $k$ ) $p(12 - p)$  Any correct equation in form $ap^2 + bp + c = 0$ BC, but any method allowed Allow $4 < p < 8$	FT (b)  FT (b)

Question		Answer/Indicative content	Marks	Part marks and guidance	
	d	E.g. $p = 0$ implies giving book for free. Unrealistic. oe E.g. When $p = 0$ , $t = 0$ ; but $t$ should be negative as would make a loss. Unrealistic. oe E.g. When $p = 12.1$ , $t$ is negative. Possibly realistic as could make a loss if $p$ set too high. oe	E1(AO3.2b)  E1(AO3.2b)  [2]	Valid comment about $p = 0$	Valid comment about $p = 12.1$
		<b>Total</b>	<b>9</b>		



Question		Answer/Indicative content	Marks	Part marks and guidance	
53	a	$x^2 + y^2 = 4$  When $x = 1$ $1 + y^2 = 4 \Rightarrow y = \sqrt{3}$ $y = \frac{1}{\sqrt{3}}(4-1) \Rightarrow y = \sqrt{3}$	B1(AO1. 1) E1(AO2. 1) E1(AO2. 1)  [3]	Soi  AG Check that Q lies on the circle AG Check that Q lies on the parabola	OR B1 $x^2 + (\sqrt{3})^2 = 4 \Rightarrow x=1$ B1 $\sqrt{3} = \frac{1}{\sqrt{3}}(4-x^2) \Rightarrow x=1$

Question	Answer/Indicative content	Marks	Part marks and guidance		
	<p>b</p> $\frac{1}{\sqrt{3}} \int_{-1}^1 (4-x^2) dx$ $= \frac{22\sqrt{3}}{9}$ <p>Let <math>N</math> be the point <math>(1, 0)</math></p> <p>Area <math>OQN = \frac{\sqrt{3}}{2}</math> oe or 0.866 (3 s.f.)</p> $QON = \tan^{-1} \sqrt{3}$ $POQ = \frac{1}{3}\pi \text{ or } 60^\circ$ <p>Area sector</p> $POQ = \frac{1}{2} \times 2^2 \times \frac{1}{3}\pi \text{ oe}$ <p style="text-align: center;">(= <math>\frac{2}{3}\pi</math> oe or 2.09 (3 s.f.))</p> <p>Shaded area</p> $= \frac{22\sqrt{3}}{9} - 2 \times \frac{\sqrt{3}}{2} - \frac{2}{3}\pi \text{ oe}$ $= \frac{13\sqrt{3}}{9} - \frac{2}{3}\pi \text{ oe}$	<p>M1(AO3.1a)</p> <p>A1(AO1.1)</p> <p>B1(AO2.1)</p> <p>M1(AO3.1a)</p> <p>A1(AO1.1)</p> <p>M1(AO1.1)</p> <p>M1(AO3.2a)</p> <p>A1(AO1.1)</p> <p>[8]</p>	<p>Attempt correct integral and limits; may be implied by answer 4.23(39...)</p> <p>BC</p> <p>Or</p> $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ <p>or</p> $\cos^{-1}\left(\frac{1}{2}\right) \text{ or}$ $\frac{1}{3}\pi \text{ or } 60^\circ$ <p>M1A1 may be implied by seeing next line</p> <p>FT their angle <math>POQ</math></p> <p>Correct combination of their areas</p>	$2.1169\dots$ $A1 = \frac{11\sqrt{3}}{9}$ <p>OR</p> <p>B1 semi-circle:</p> $\int_{-1}^1 \sqrt{4-x^2} dx$ <p>by substitution, e.g. <math>x = 2\sin u</math></p> <p>M1 Use trigonometric identity e.g.</p> $\int_{-1}^1 4\cos^2 u du = \int_{-1}^1 a \cos 2u + b du$ $A1 \frac{2}{3}\pi + \sqrt{3}$ <p>M1 Shaded area</p> $= \frac{22\sqrt{3}}{9} - \frac{2}{3}\pi - \sqrt{3} \text{ oe}$ $A1 = \frac{13\sqrt{3}}{9} - \frac{2}{3}\pi \text{ oe}$	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			<b>Total</b>	<b>11</b>		
54		a	Obtain $1 + \frac{1}{3}px$ $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(px)^2$ Obtain $1 + \frac{1}{3}px - \frac{1}{9}p^2x^2$	<b>B1(AO 1.1)</b>  <b>M1(AO1. 1)</b>  <b>A1(AO1. 1)</b>  <b>[3]</b>	Must be simplified	Attempt the $x^2$ term at least in the form ${}^6C_2kx^2$
		b	$(1+qx)\left(1+\frac{1}{3}px-\frac{1}{9}p^2x^2\right)$ $=1+\left(\frac{1}{3}p+q\right)x+\left(\frac{1}{3}pq-\frac{1}{9}p^2\right)x^2$  $\frac{1}{3}p+q=1 \quad (*)$ $\frac{1}{3}pq-\frac{1}{9}p^2=-\frac{2}{9}$  $2p^2-3p-2=0$  $p=2$ or $-\frac{1}{2}$  $q=\frac{1}{3}$ or $\frac{7}{6}$	<b>M1(AO 3.1a)</b>  <b>M1(AO3. 1a)</b>  <b>M1(AO1. 1)</b> <b>A1(AO1. 1)</b> <b>A1FT(AO1.1)</b>  <b>[5]</b>	Obtain two equations in $p$ and $q$ and show evidence of substitution for $p$ or $q$ to obtain an equation in one variable Solve a 3 term quadratic equation in a single variable.  Obtain any two values  Obtain all 4 values, or FT their $p$ and (*)	Expand $(1+qx)$ and $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$ <del>their</del> compare coefficients  Or $18q^2 - 27q + 7 = 0$ Solve their quadratic  with indication of correct pairings
			<b>Total</b>	<b>8</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
55	$\frac{dy}{dx} = 2x + k + 4x^{-2}$ $2(-2) + k + 4(-2)^{-2} = 0$ $k = 3$ $\frac{d^2y}{dx^2} = 2 - 8x^{-3}$ $2 - 8x^{-3} = 0$ $x = 4^{\frac{1}{3}}$ <p>for <math>x &lt; 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} &lt; 0</math></p> <p>for <math>x &gt; 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} &gt; 0</math></p> <p>when <math>x = 4^{\frac{1}{3}}, \frac{dy}{dx} \neq 0</math> hence</p> <p>not a stationary point</p>	<p>M1(AO 1.1a)</p> <p>M1(AO3. 1a)</p> <p>A1(AO1. 1)</p> <p>M1(AO3. 1a)</p> <p>A1(AO1. 1)</p> <p>E1(AO2. 1)</p> <p>E1(AO2. 1)</p> <p>[7]</p>	<p>Attempt to differentiate</p> <p>Substitute <math>x = -2</math>, equate to 0 and attempt to solve</p> <p>Equate second derivative to 0 and attempt to solve</p> <p>Consider convex / concave either side of <math>x = 4^{\frac{1}{3}}</math> and conclude</p> <p>Consider gradient at <math>x = 4^{\frac{1}{3}}</math>, or justify that <math>x = -2</math> is the only stationary point</p>	<p>Power decreases by 1 for at least 2 terms</p>
	Total	7		

Question		Answer/Indicative content	Marks	Part marks and guidance		
56	a	DR $BE = \sqrt{3}$ from the standard triangle $BDE$  $BC = AB \cos 45$ $BC = \frac{1+\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2}$	B1(AO 2.2a)  M1(AO 2.1)  E1(AO 2.2a)  [3]	Or $AB = 1+\sqrt{3}$ seen  oe or Pythagoras' theorem AG	B0 for decimal  Must be seen $\frac{1+\sqrt{3}}{\sqrt{2}}$ must be seen.	
	b	DR Triangle $ABC$ is isosceles so $BC = AC$ but $AC = CD + \sqrt{2}$ so $CD = \frac{\sqrt{2}+\sqrt{6}}{2} - \sqrt{2}$ $= \frac{\sqrt{6}-\sqrt{2}}{2}$  $\sin 15 = \frac{CD}{BD} = \frac{\sqrt{6}-\sqrt{2}}{2} \div 2 = \frac{\sqrt{6}-\sqrt{2}}{4}$	B1(AO 2.4)  M1(AO 2.1)  A1(AO 2.2a)  [3]	State or imply that $BC = AC$  or state expression for $CD$ , may be unsimplified   Obtain expression for $\sin 15$ and simplify to answer given	M0 if decimals seen   SC1 for showing using addition formula	
		<b>Total</b>	<b>6</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
57	a	$v = 6t^2i + (10t - 4)j$ $v = 2.94i + 3j$ $9 - \tan^{-1}\left(\frac{2.94}{3}\right)$ $= 044^\circ$	<b>B1(AO1.1)</b>  <b>M1(AO3.1a)</b>  <b>A1(AO1.1)</b> <b>[3]</b>	At least one term reduces in power by 1 Substitution of $t = 0.7$ , use $\tan^{-1}\left(\frac{y}{x}\right)$ and obtain $90 - 45.578 = 44.4^\circ$ to give a 3 figure bearing	For a complete method to find a bearing
	b	$a = 12i + 10j$ $a = 8.4i + 10j$ Use $F = ma$ and use Pythagoras Obtain 1.57 N	<b>M1(AO1.1)</b> <b>A1(AO1.1)</b> <b>M1(AO3.3)</b> <b>A1FT(AO3.4)</b> <b>[4]</b>	Attempt differentiation of $v$ Substitute $t = 0.7$ FT their $a$ at $t = 0.7$	

Question		Answer/Indicative content	Marks	Part marks and guidance	
	c	$6t^2 = 10t - 4$  $6t^2 = 10t + 4 = 0$ so $t = 1$ or $\frac{2}{3}$  E.g. i component always positive so both values are valid	M1(AO2.2a)  E1(AO2.3)  [2]	Equate i and j components and solve FT their v from part (a) if it leads to a quadratic BC Must include comment on why equating components is sufficient in this case.	
		<b>Total</b>	<b>9</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance		
58	<p>Let <math>F_G</math> be the frictional force at ground level and <math>R_G</math> the reaction</p> <p>Let <math>F_W</math> be the frictional force at the wall and <math>R_W</math> the reaction</p> <p>Let <math>x</math> be the distance the man can ascend before the ladder slips</p> <p><math>F_G = \frac{1}{2} R_G</math> and <math>F_W = \frac{1}{3} R_W</math></p> <p>Resolve horizontally and vertically:  <math>F_G = R_W</math>  <math>R_G + F_W = 105g</math></p> <p><math>F_W = 15g</math>  <math>R_W = 45g = F_G</math>  <math>R_G = 90g</math></p> <p>Moments about the foot of the ladder:  <math>35g(3.5\cos 45) + (70g \cos 45)x = 45g(7\cos 45) + 15g(7\sin 45)</math></p> <p><math>x = 4.25</math></p>	<p><b>B1(AO2. 1)</b></p> <p><b>B1(AO3. 3)</b></p> <p><b>B1(AO3. 1b)</b></p> <p><b>M1(AO1. 1)</b></p> <p><b>B1(AO3. 2a)</b></p> <p><b>M1(AO3. 3)</b></p> <p><b>A1(AO3. 4)</b></p> <p><b>A1(AO1. 1)</b></p> <p><b>[8]</b></p>	<p>Either on a diagram or in words, B1 is awarded for a clear definition of the force variables used</p> <p>Both statements required</p> <p>Both resolutions required</p> <p>Accept numerical value of <math>g</math> used</p> <p>Attempt to solve the 4 equations simultaneously to obtain at least two numerical values for the variables.</p> <p>May be implied by later working</p> <p><b>B1</b> for either <math>F_W</math> and <math>R_W</math> or <math>F_G</math> and <math>R_G</math></p> <p>Allow sign errors and sin / cos confusion</p> <p>Correct statement</p> <p>cao</p>	<p>Or similarly about the top of the ladder</p>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
		<b>Total</b>	<b>8</b>			
59		$2x = x^2 + 2x - 4$ $x^2 - 4 = 0$ $x = 2$ or $-2$ $x = 2$ and $y = 4$ □□□□□ or $x = -2$ and $y = -4$	<b>M1(AO1.1a)</b> <b>A1(AO1.1)</b> <b>A1(AO1.1)</b> <b>A1(AO1.1)</b> <b>[4]</b>	or $x = 2, y = 4$ or $x = -2, y = -4$ Allow (2, 4) and (-2, -4)	Both $x$ 's or one pair $x, y$ Must be paired	
		<b>Total</b>	<b>4</b>			
60		<b>DR</b> $f(x) = ax^2 + bx + c$ $c = 3$ $a \times 4^2 + b \times 4 + 3 = -13$ $16a + 4b = -16$ $f(x) = 2ax + b$ $6a + b = -2$ eg $8a = 8$ or $16a + 4(-2 - 6a) = -16$ $a = 1$ or $b = -8$ $f(x) = x^2 - 8x + 3$	<b>B1(AO1.1)</b> <b>M1(AO3.1a)</b> <b>A1(AO1.1a)</b> <b>M1(AO1.1a)</b> <b>A1(AO1.1)</b> <b>M1(AO2.2a)</b> <b>A1(AO1.1)</b> <b>A1(AO3.2a)</b> <b>[8]</b>	Attempt sub (4, -13) in $f(x)$ oe, correct equ Attempt diff $f(x)$ Correct equ Solve & obtain a correct equ in $a$ or $b$		
		<b>Total</b>	<b>8</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
61	a	<p>(a) Moments @ A: <math>R_B a + 3aW \cos \theta = 2aR_C</math></p> <p>Resolve vertically: <math>W + R_B \cos \theta = R_C \cos \theta</math></p> <p><math>W + (2R_C - 3W \cos \theta) \cos \theta = R_C \cos \theta</math></p> <p><math>R_C = W \left( \frac{3 \cos^2 \theta - 1}{\cos \theta} \right)</math></p>	<p>M1(AO3.3)</p> <p>M1(AO3.3)</p> <p>A1(AO1.1)</p> <p>M1(AO3.4)</p> <p>A1(AO1.1)</p> <p>[5]</p>	<p>Allow sign errors and sin / cos confusion</p> <p>Allow sign errors and sin / cos confusion</p> <p>For both equations correct</p> <p>Attempt solution of simultaneous equations to find RC in terms of W and <math>\theta</math></p> <p>AG; sufficient working must be shown</p>	<p>Or moments @ B</p> <p>Or resolve    rod</p>	
	a	<p>(b) <math>R_B = W \left( \frac{3 \cos^2 \theta - 2}{\cos \theta} \right)</math></p>	<p>B1(AO1.1)</p> <p>[1]</p>	<p>oe, e.g. <math>R_B = W(3 \cos \theta - 2 \sec \theta)</math></p>		
	b	<p>For equilibrium, <math>R_B \geq 0</math> and <math>R_C \geq 0</math></p> <p>Critical case is <math>R_B = 0</math>, as this gives lower limit for <math>\theta</math></p> <p>so</p> <p><math>\cos^2 \theta = \frac{2}{3} \Rightarrow \theta_{\max} = 35.3^\circ</math> (correct to 3sf)</p>	<p>M1(AO2.1)</p> <p>E1(AO2.2a)</p> <p>[2]</p>	<p>For either considered; allow = for <math>\geq</math></p> <p>AG; sufficient reasoning required</p>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
	c	Resolve    rod: $R_A \cos \theta = W \sin \theta$  Obtain $R_A = W \tan \theta$  $W \tan \theta = W \left( \frac{3 \cos^2 \theta - 1}{\cos \theta} \right)$  $3 \sin^2 \theta + \sin \theta - 2 = 0$  $\sin \theta = \frac{2}{3}$ only, as $\sin \theta \neq -1$  $\theta = 41.8^\circ$ , but as this is greater than $35.3^\circ$ it is not possible that $R_A$ and $R_C$ are equal	<b>M1(AO3.3)</b>  <b>A1(AO2.1)</b>  <b>M1(AO2.1)</b>  <b>M1(AO2.2a)</b>  <b>A1(AO2.3)</b>  <b>E1(AO2.4)</b>  <b>[6]</b>	Allow sin / cos confusion  $R_A$ in terms of $W$ and $\theta$ correct in any form  Equate expressions for $R_A$ and $R_C$  Use of trig identities to form 3-term quadratic equation in $\sin \theta$  BC; the negative value must be seen and not given as a final answer  For correct argument justifying given result	Or moments @ C	
		<b>Total</b>	<b>14</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
62	a	$\frac{(3x)^3 \times 2x^{-1}}{9x^2} = \frac{27x^3 \times 2x^{-1}}{9x^2}$ $= \frac{54x^2}{9x^2} = 6$	B1 (AO1.1)  B1 (AO1.1)  [2]	Correctly expands $(3x)^3$ as $27x^3$	
	b	$(49x^{-4})^{-\frac{1}{2}} = \frac{1}{7}x^2$	B1 (AO1.1)  B1 (AO1.1)  [2]	For $\frac{1}{7}$ , independent  of power of $x$  For $x^2$	
		<b>Total</b>	<b>4</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
63	a	DR $f(-2) = 2(-2)^3 - (-2)^2 + k(-2) + 6 = -16 - 4 - 2k + 6$  $f(-2) = 0$ since $(x + 2)$ is a factor, so $-16 - 4 - 2k + 6 = 0 \Rightarrow k = -7$	<b>M1</b> <b>(AO2.1)</b>  <b>E1</b> <b>(AO2.1)</b>  <b>[2]</b>	Must show an intermediate line of reasoning without brackets or indices  Dependent on M1	OR divide $f(x)$ by $(x + 2)$
	b	DR $f(x) = (x + 2)(2x^2 - 5x + 3)$  $f(x) = (x + 2)(2x - 3)(x - 1)$	<b>M1</b> <b>(AO1.1a)</b>  <b>M1</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>[3]</b>	Attempt to obtain a quadratic factor by any correct method  Attempt to factorise quadratic factor	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	c	DR Equation is $f(e^y) = 0$  So factors from (b) give  $e^y = -2, \frac{3}{2}, 1$  Hence $y = \ln \frac{3}{2}$ or 0  $e^y \neq -2$ because $e^y$ is always positive	M1 (AO3.1a)  E1 (AO3.2a)  A1 (AO1.1)  E1 (AO2.4)  [4]	Use of substitution $x = e^y$ to connect the equation to $f(x)$  Interpret to give solutions to the equation  For both correct and no others  Detailed reasoning must include clear indication of why $e^y = -2$ does not give a solution	Must be explicit	
		<b>Total</b>	<b>9</b>			

Question	Answer/Indicative content	Marks	Part marks and guidance
64	$\Delta = (k - 5)^2 - 4(1)(-3k)$ $= k^2 + 2k + 25$ $= (k + 1)^2 + 24$ <p>Condition for real roots is <math>\Delta \geq 0</math></p> <p><math>(k + 1)^2 \geq 0</math> for all <math>k</math> so <math>(k + 1)^2 + 24 &gt; 0</math> and hence the equation has real roots for all values of <math>k</math></p>	<p>M1 (AO3.1a)</p> <p>A1 (AO1.1)</p> <p>M1 (AO3.1a)</p> <p>A1 (AO1.1)</p> <p>M1 (AO2.1)</p> <p>A1 (AO2.2a)</p> <p>[6]</p>	<p>Attempt at discriminant</p> <p>Obtain correct 3-term quadratic</p> <p>Complete the square on their 3-term quadratic</p> <p>OR: differentiate and solve = 0</p> <p>Obtain <math>k = -1</math></p> <p>Substitute <math>k = -1</math> and explain that the result is the minimum value of their <math>k</math>-quadratic</p> <p>Correct numerical values and complete argument using <math>24 &gt; 0</math> plus conclusion</p> <p>For '<math>b^2 - 4ac \geq 0</math>' condition OR for explanation that their <math>\Delta \geq 0</math></p> <p><math>(k + 1)^2 \geq 0</math> with complete argument and conclusion</p>
	Total	6	

Question		Answer/Indicative content	Marks	Part marks and guidance	
65	a	Centre of circle is $(-4, 2)$  $(x + 4)^2 - 16 + (y - 2)^2 - 4 + 7 = 0$  $r^2 = 13 \Rightarrow r = \sqrt{13}$	<b>B1</b> <b>(AO1.1)</b>  <b>M1</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>[3]</b>	Correct centre  $(x \pm 4)^2 - 16 + (y \pm 2)^2 - 4$ seen  $r = 3.61$ or better www	<b>OR</b> $r^2 = 4^2 + 2^2 - 7$
	b	$y = 0 \Rightarrow x^2 + 8x + 7 = 0$  $A(-7, 0)$ and $B(-1, 0)$	<b>M1</b> <b>(AO1.1a)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>[2]</b>	Substitute $y = 0$ and attempt to solve  <b>BC</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	c	$m_{OC} = -\frac{1}{2}$  Hence $m_{DE} = 2$  $y - 3 = 2\left(x + \frac{3}{2}\right) \Rightarrow y = 2x + 6$  $(x + 4)^2 + (2x + 4)^2 = 13$  $5x^2 + 24x + 19 = 0 \Rightarrow x = \dots$  $x = -\frac{19}{5}, -1$  $D$ is $\left(-\frac{19}{5}, -\frac{8}{5}\right)$ and $E$ is $(-1, 4)$	M1 (AO3.1a)  A1FT (AO1.2)  M1 (AO1.1)  M1 (AO3.1a)  M1 (AO1.1)  A1 (AO1.1)  A1 (AO3.2a)  [7]	Identify gradient of line $OC$  Use of $m_1$ $m_2 = -1$ with their $m_{OC}$  Form equation of line $DE$  Substitute to get quadratic in one variable  Expand and attempt to solve their 3-term quadratic  BC		
	d	$\text{Area} = \frac{1}{2}(6)(4) + \frac{1}{2}(6)\left(\frac{8}{5}\right)$  $= \frac{84}{5}$	M1 (AO1.1a)  A1 (AO1.1)  [2]	$\text{Area} = \frac{1}{2}(\text{their } (7-1))$ $(\text{their } (4 + \frac{8}{5}))$		

Question			Answer/Indicative content	Marks	Part marks and guidance		
			<b>Total</b>	<b>14</b>			
66		a	DR $\sqrt{36} + \sqrt{162}$ oe  $\sqrt{6^2} + \sqrt{9^2 \times 2}$ oe  $= 6 + 9\sqrt{2}$	M1(AO1.1a)  A1(AO1.1)  A1(AO1.1)  [3]	Attempt to expand bracket  Obtain 6  Obtain $9\sqrt{2}$	Must show sufficient method	
		b	DR $\frac{6(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{12-6\sqrt{2}}{2} = 6-3\sqrt{2}$	M1(AO1.1a)  A1(AO1.1)  A1(AO1.1)  [3]	Multiply numerator and denominator by $2-\sqrt{2}$  Either  numerator or denominator correct Fully correct expression	Must be simplified  Must show sufficient method	
			<b>Total</b>	<b>6</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
67	a	$2x^2 + 4x = 2(x^2 + 2x) + 5 + 5$ $= 2[(x + 1)^2 - 1] + 5$ $= 2(x + 1)^2 + 3$	<b>B1(AO2.2a)</b>  <b>B1(AO1.1)</b>  <b>M1(AO1.1a)</b>  <b>A1(AO1.1)</b>  <b>[4]</b>	$p = 2$  $q = 1$  Attempt $r$  $r = 3$	The values of $p$ , $q$ and $r$ could be stated explicitly or could be implied by an answer in completed square form	
	b	Vertex is at $(-1, 3)$	<b>B1ft(AO1.1)</b>  <b>B1ft(AO1.1)</b>  <b>[2]</b>	Correct $x$ coordinate  Correct $y$ coordinate	FT their (a)	
	c	$k < 3$	<b>B1ft(AO3.1a)</b>  <b>[1]</b>	State $k < 3$ , ft their (a)	Must be strict inequality	
		<b>Total</b>	<b>7</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
68	a	$V = x(21 - 2x)^2$ $= 4x^3 - 84x^2 + 441x$  $V' = 12x^2 - 168x + 441$  $12x^2 - 168x + 441 = 0$  $x = 3.5$ cm  when $x = 3.5$ , $V'' = 24 \times 3.5 - 60 < 0$  hence maximum	<b>B1(AO3.3)</b>  <b>M1(AO1.1a)</b>  <b>M1(AO3.1b)</b>  <b>A1ft(AO3.2a)</b>  <b>M1(AO1.1)</b> <b>A1ft(AO2.1)</b>  <b>[6]</b>	State correct expression for volume  Expand and attempt differentiation  Equate to 0 and attempt to solve  Obtain $x = 3.5$ cm only, ft on their V Use second derivative  Conclude maximum	oe  Or use product rule  <b>BC</b>  A0 if 10.5 also given  If evaluated, must be correct  Evidence required	
	b	Accept any sensible assumption	<b>B1(AO3.5b)</b>  <b>[1]</b>	E.g. Thickness of metal is assumed negligible		
		<b>Total</b>	<b>7</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
69	a	$f = (x + 1)(x^2 + 3x - 10)$ $(x)$  $= (x + 1)(x + 5)(x - 2)$	<b>M1(AO2.2a)</b>  <b>A1(AO1.1)</b>  <b>A1(AO1.1)</b>  <b>[3]</b>	Attempt complete division by $(x + 1)$  Obtain correct quotient  Obtain fully factorised $f(x)$	Allow any equiv method  Must be as product	
	b	$e^y = -1, -5, 2$  but $e^y > 0$ , so $e^y = 2$ is only valid root  hence $y = \ln 2$	<b>M1(AO2.2a)</b>  <b>E1(AO2.4)</b>  <b>A1(AO2.1)</b>  <b>[3]</b>	Link $e^y$ to attempt at roots from (a)  Explanation that $e^y > 0$  Obtain $y = \ln 2$	www	
		<b>Total</b>	<b>6</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
70	a	(0, 4]	B1(AO2.5) [1]	Do not allow $0 < f(x) \leq 4$	
	b	$f^{-1}(x) = \frac{8}{x} - 2$	M1(AO1.1a) A1(AO1.1) [2]	Obtain $\frac{8}{x} \pm 2$ Obtain correct inverse function	Allow in terms of y Must now be in terms of x
	c	$x = \frac{8}{x+2}$ $x^2 + 2x - 8 = 0$ $x = 2$	M1(AO1.1a) A1(AO2.3) [2]	Equate two of x, f(x) and $f^{-1}(x)$ and attempt to solve Obtain $x = 2$ only	AO if $x = -4$ also given
		<b>Total</b>	<b>5</b>		

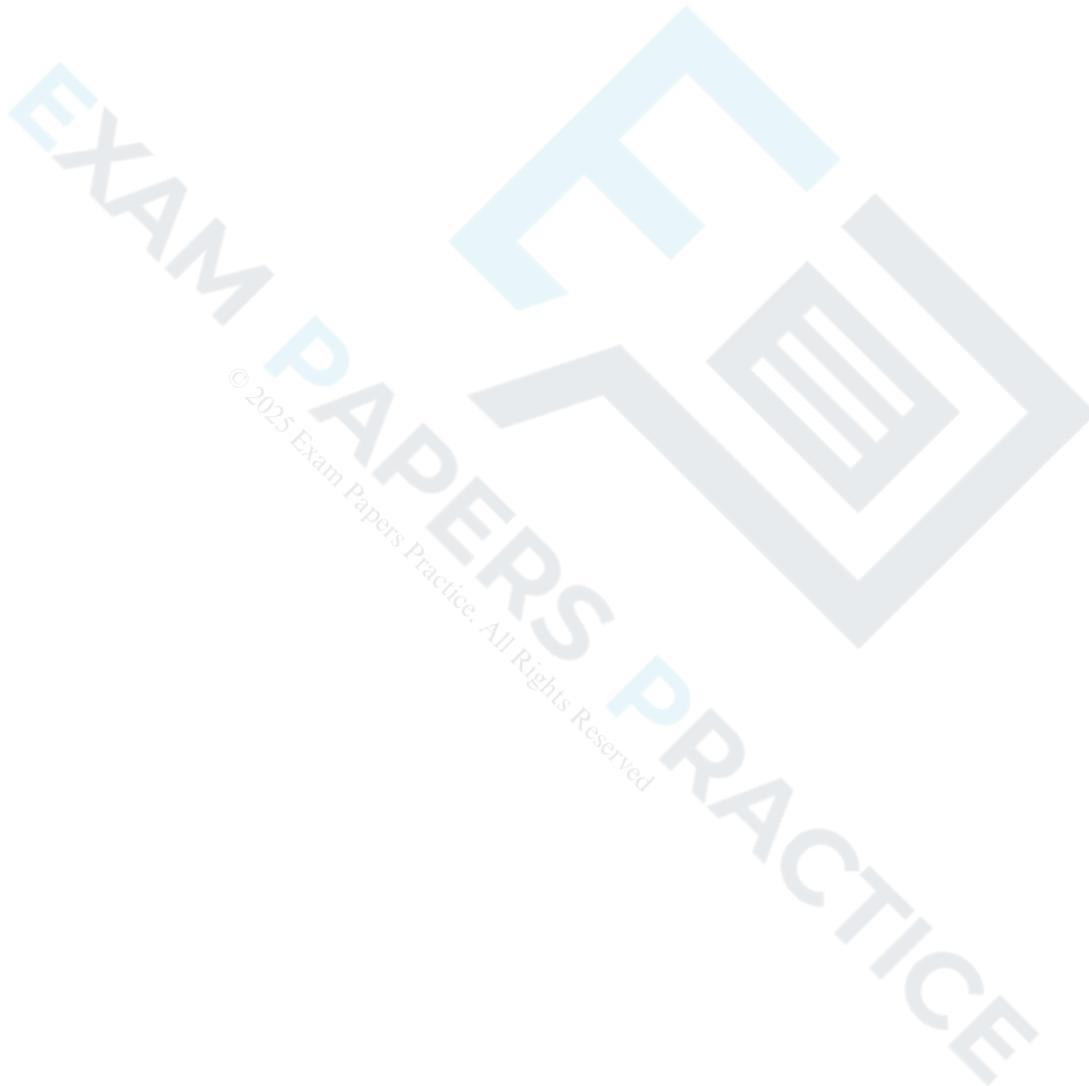
Question		Answer/Indicative content	Marks	Part marks and guidance	
71	a	$1 + x - \frac{1}{2}x^2$	B1(AO1.1) M1(AO1.1a) A1(AO1.1)  [3]	Obtain 1 + x  Attempt third term  Obtain correct third term	Terms must be simplified for B / A marks
	b	$\sqrt{1.08} \approx 1 + 0.04 - 0.5 \times 0.04^2$ $\sqrt{0.36 \times 3} \approx 1.0392$ $0.6\sqrt{3} \approx 1.0392$  $\sqrt{3} \approx 1.73$	M1(AO2.1)  M1(AO3.1a) A1(AO1.1)  [3]	Substitute 0.04 throughout  Rearrange Obtain 1.73 or better	Need $\sqrt{1.08}$ as well  Must see method
	c	Expansion is only valid for  $ x  < \frac{1}{2}$	E1(AO2.3)  [1]	Explanation must be specific	
		<b>Total</b>	<b>7</b>		



Question			Answer/Indicative content	Marks	Part marks and guidance	
					<p>ending up with a denominator of 5. A small but noticeable number obtained the correct answer but then proceeded to multiply by 3 to give integers for a and b; this error was not condoned. Likewise, a small number of those who chose to use <math>-\sqrt{7}-2</math> as their multiplier, left their final answer with a negative denominator which was not deemed appropriate at AS level.</p>	
			<b>Total</b>	<b>3</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
73	$2(x^2 - 6x) + x - 6 = 0$ $2x^2 - 11x - 6 = 0$ $(2x + 1)(x - 6) = 0$ $x = -\frac{1}{2}, x = 6$ $y = \frac{13}{4}, y = 0$	<p>M1*</p> <p>A1</p> <p>M1*dep</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>Substitute for <math>x/y</math> to eliminate one of the variables</p> <p>Correct 2/3-term quadratic in solvable form</p> <p>Attempt to solve resulting quadratic. See appendix 1.</p> <p><math>x</math> values correct</p> <p><math>y</math> values correct</p> <p>Award A1 A0 for one pair correctly found from correctly factorised quadratic</p> <p><b>Examiner's Comments</b></p> <p>Most candidates secured a large number of marks in this standard simultaneous equation question. Elimination of <math>x</math> tended to lead to more errors with the initial "<math>y =</math>" being lost and resulting in an incorrect quadratic. Whichever variable was eliminated,</p>	<p>If <math>x</math> eliminated:  <math>y = (6 - 2y)^2 - 6(6 - 2y)</math></p> <p><math>4y^2 - 13y = 0</math></p> <p><math>y(4y - 13) = 0</math></p> <p><b>Spotted solutions:</b>            If M0 DM0 SC B1 One correct pair <b>www</b>            SC B1 Second correct pair <b>www</b>            Must show on both line and curve (Can then get 5/5 if both found <b>www</b> and exactly two solutions justified)</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					difficulties with fraction arithmetic regularly led to loss of accuracy marks, particularly with the negative value of $x$ . Around two-thirds of candidates scored full marks.	
			<b>Total</b>	<b>5</b>		



Question	Answer/Indicative content	Marks	Part marks and guidance	
74	$k = t^{\frac{1}{3}}$  $4k^2 + 17k - 15 = 0$ $(4k - 3)(k + 5) = 0$  $k = \frac{3}{4}, k = -5$  $t = \frac{27}{64}, t = -125$	<p style="text-align: center;">M1*</p>  <p style="text-align: center;">M1 *dep</p>  <p style="text-align: center;">A1</p>  <p style="text-align: center;">M1</p>  <p style="text-align: center;">A1</p>  <p style="text-align: center;">[5]</p>	<p>Substitute <math>\frac{1}{t^3}</math> for <math>t^{\frac{1}{3}}</math> to obtain a quadratic expression</p>  <p>Rearrange and attempt to solve resulting quadratic equation. See appendix 1.</p>  <p>Correct values of <math>k</math></p>  <p>Attempt to cube at least one value</p>  <p>Final answers correct</p>	<p>Alternative: <b>M2</b> Rearrange and factorise into two brackets containing <math>\frac{1}{t^3}</math>. See appendix 1.</p> <p><b>SC</b> If straight to formula with no evidence of substitution at start and no cubing / cube rooting at end, then <b>B1</b> for <math>\frac{-17 \pm \sqrt{17^2 - 4 \times 4 \times -15}}{2 \times 4}</math> or better</p> <p><b>No marks</b> if whole equation cubed etc.</p> <p><b>Spotted solutions:</b></p> <p>If <b>M0 DM0</b> or <b>M1 DM0</b></p> <p><b>SC B1</b></p> <p><math>t = \frac{27}{64}</math></p> <p><b>www</b> <b>SC B1</b> <math>t = -125</math> <b>www</b> (Can then get 5/5 if both found <b>www</b>)</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					and exactly two solutions justified)	
					<p><b>Examiner's Comments</b></p> <p>Although this disguised quadratic needed rearrangement as well as substitution, this question was well approached by the vast majority of candidates, with around 70% achieving all 5 marks. As in previous sessions, some candidates did not make their choice of substitution clear which made it difficult to award marks. The question was best approached by factorisation, and those who opted to use the quadratic formula were often unable to deal with the required arithmetic. Most remembered to cube their solutions to the quadratic, although some did so inaccurately, particularly the fractional solution.</p>	
			<b>Total</b>	<b>5</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
75	<p>Turning point = (-1, 2)</p> <p>(0, 1.5)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>Negative parabola</p> <p>Turning point at (-1, 2); coordinates must be labelled on graph or clearly stated elsewhere</p> <p>Correct method to find roots*</p> <p>Correct x intercepts (1,0) and (-3, 0)</p> <p>Correct y intercept <math>(0, \frac{3}{2})</math></p> <p><b>NB</b> – Do not award 5/5 if sketch inconsistent with stated values e.g. turning point shown in wrong quadrant etc. Withhold one B1.</p>	<p>For first mark must clearly be a parabola – must not stop at or before x axis, do not allow straight line sections drawn with a ruler or tending to extra turning points etc.</p> <p>Must not be a finite plot. *If not using given form to solve, M mark only available for attempt to solve <math>k(-\frac{1}{2}x^2 - x + \frac{3}{2}) = 0</math>.</p> <p><b>See appendix 1.</b></p> <p><b>Examiner's Comments</b></p> <p>Interpreting an equation given in</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>“completed square” form proved quite taxing for many candidates with only 40% securing all five marks. Although most recognised the need to put <math>y = 0</math> to obtain the roots, many multiplied out the given squared expression and then rearranged and factorised or used the quadratic formula. Those that then obtained a different incorrect quadratic received no further credit for trying to find the <math>x</math> intercepts. Candidates who used the given form were more likely to be successful. Many also correctly spotted the turning point from this form, although others resorted to differentiation. A significant number ignored the instruction to “give the coordinates” of the turning point and credit could not be given if this was not clear. Similarly, some omitted to state the <math>y</math>-intercept. An alternative approach of trying to construct the graph through a series of transformations from <math>y = x^2</math> was seldom productive.</p>
			<b>Total</b>	<b>5</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
76		$b^2 - 4ac = (k - 6)^2 - 8k$  $k^2 - 20k + 36$  $k = 2, k = 18$  $k < 2, k > 18$  $(\infty, 0) \cup (0, 2) \cup (18, \infty)$	M1(AO1.1a)  A1(AO1.1a)  M1(AO1.2)  M1(AO1.1a)  A2(AO2.3)  (AO2.5)  [6]	Attempt discriminant  Obtain correct 3 term expression  Find critical values for $k$ , eg by attempting to solve discriminant $> 0$  Chooses 'outside region' of their inequality  A1 for stating $k \neq 0$  OR  A1 for $(\infty, 2) \cup (18, \infty)$	Allow any sign, including =  Allow 'or' but not 'and'  Any correct set notation	
		<b>Total</b>	<b>6</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
77		<p>Assume that <math>\sqrt{7}</math> is rational ie</p> $\sqrt{7} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ have no common factors}$ $b\sqrt{7} = a, \text{ so } 7b^2 = a^2$ <p>So <math>a^2</math> must be a multiple of 7, which means that <math>a</math> is a multiple of 7 as well, so <math>a = 7k</math></p> $7b^2 = (7k)^2 \wedge 7b^2 = 49k^2 \wedge b^2 = 7k^2$ <p>This implies that <math>b</math> is a multiple of 7, but it was assumed at start that <math>a</math> and <math>b</math> had no common factors, so this contradicts initial assumption. Hence <math>\sqrt{7}</math> cannot be written as <math>\frac{a}{b}</math> so it is irrational.</p>	<p>E1(AO2.1)</p> <p>M1(AO2.2a)</p> <p>E1(AO2.4)</p> <p>M1(AO2.2a)</p> <p>E1(AO2.4)</p> <p>[5]</p>	<p>Proof must start with an assumption for contradiction</p> <p>Rearrange and square both sides</p> <p>Identify that <math>a = 7k</math></p> <p>Substitute <math>a = 7k</math> and simplify</p> <p>Conclude appropriately</p>	<p>Condone not stating that <math>a</math> and <math>b</math> have no common factors</p> <p>Must have stated at start that <math>a</math> and <math>b</math> have no common factors</p>	
		<b>Total</b>	<b>5</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
78	a	$\frac{\sin \theta}{4.8} = \frac{\sin 25}{2.2}$ <p><math>\theta_1 = 67.2^\circ</math>, so bearing is <math>067^\circ</math></p> <p><math>\theta_2 = 180^\circ - 67^\circ</math></p> <p><math>\theta_2 = 112.8^\circ</math></p>	M1(AO3.1a)  A1(AO1.1)  M1(AO3.1a)  A1(AO1.1)  [4]	Attempt correct use of the sine rule to find at least one angle  Obtain $067^\circ$ , or better  Attempt correct method for $\theta_2$  Obtain $113^\circ$ , or better	Must use $25^\circ$  3 figure bearing required $180^\circ -$ their angle	
	b	$d = 4.8 \sin 25^\circ$  $= 2.03 \text{ km}$	M1(AO3.1a)  A1(AO1.1)  [2]	Attempt perpendicular distance  Obtain 2.03 km, or better	Allow all complete methods	
	c	Coastline may not be straight between <i>P</i> and <i>Q</i>	E1(AO3.5b)  [1]	Any sensible reason	Stations may not be exactly on coastline	
		<b>Total</b>	<b>7</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
79	a	$\text{Time} = \frac{500}{v}, T = \frac{500}{v} \times R$ $\text{Hence } T = \frac{500R}{v}$	<b>B1(AO 1.1)</b>  <b>[1]</b>	<b>AG Must</b> $\text{Time} = \frac{500}{v}$ <b>see</b>	
	b	$T = \frac{500}{v} \left( 270 + \frac{v^3}{200} \right) \quad \left( = \frac{135000}{v} + \frac{5v^2}{2} \right)$ $\frac{dT}{dv} = -\frac{135000}{v^2} + 5v \quad \text{oe}$ $-\frac{135000}{v^2} + 5v = 0 \quad [v^3 = 27000]$ <p>Required speed is 30 km/h</p>	<b>M1(AO 3.1a)</b>  <b>M1(AO 3.4)</b>  <b>M1(AO 1.1)</b>  <b>A1(AO 3.2a)</b>  <b>[4]</b>	Attempt diff their $T$  Their $\frac{dT}{dv} = 0$  Allow $v = 30$ km/h; not just $v = 30$ .	
	c	$T_{\min} = \frac{135000}{30} + \frac{5 \times 30^2}{2}$ <p>Min cost = £6750</p>	<b>M1(AO 1.1)</b>  <b>A1(AO 3.2a)</b>  <b>[2]</b>	Subst their '30' into their $T$  £ necessary	
		<b>Total</b>	<b>7</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
80	a	$p = 3$  $q = 1$	B1 (AO 1.1)  B1 (AO 1.1)  [2]		
	b	$x - 3 = \tan u \Rightarrow dx = \sec^2 u \, du$  $\int_3^4 \frac{1}{x^2 - 6x + 10} dx = \int_0^{\frac{1}{4}\pi} \frac{\sec^2 u}{\tan^2 u + 1} du$  $= \int_0^{\frac{1}{4}\pi} du = \frac{1}{4}\pi$	M1 (AO 1.1)  A1ft (AO 1.1)  A1 (AO 1.1)  [2]	Attempt to connect dx and du  Correct integral and limits; ft their $p$	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	C	<p>Integral is <math>\frac{1}{2} \int_3^4 \frac{2x-6}{x^2-6x+10} dx + \int_3^4 \frac{3}{x^2-6x+10} dx</math></p> <p>Use of <math>\int \frac{f'(x)}{f(x)} dx</math> and their answer to part (b)</p> $\frac{1}{2} \left[ \ln(x^2 - 6x + 10) \right]_3^4 + \frac{3}{4}\pi$ <p>Correct use of limits and correct use of logs</p> $\frac{3}{4}\pi + \ln \sqrt{2}$ <p>Alternative solution</p> $\int_0^{\frac{1}{4}\pi} \frac{3 + \tan u}{\tan^2 u + 1} \sec^2 u du$ $\int_0^{\frac{1}{4}\pi} (3 + \tan u) du$ $= [3u - \ln(\cos u)]_0^{\frac{1}{4}\pi}$ <p>Correct use of limits and correct use of logs</p> $\frac{3}{4}\pi + \ln \sqrt{2}$	<p><b>B1 (AO 2.1)</b></p> <p><b>*M1 (AO 3.1a)</b></p> <p><b>A1ft (AO 1.1)</b></p> <p><b>dep*M1 (AO 1.1)</b></p> <p><b>A1 (AO 2.2a)</b></p> <p><b>*M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>dep*M1</b></p> <p><b>A1</b></p> <p><b>[5]</b></p>	<p>Or with single numerator <math>2x - 6 + 6</math></p> <p>FT 3 × their answer to part (b)</p> <p>Attempt use of previous substitution</p> <p>Correct integral and limits</p>	<p>Limits not required</p> <p>Limits not required</p>	
		<b>Total</b>	<b>10</b>			

Question	Answer/Indicative content	Marks	Part marks and guidance			
81	DR $2x - 4y - 4x \frac{dy}{dx} + 24y^2 \frac{dy}{dx} = 0$  $\frac{dy}{dx} = 0$  $2x - 4y = 0$  $x^2 - 2x^2 - x^3 + 4 = 0$  $x^3 - x^2 - 4 = 0$  $f(2) = 0$  $(x - 2)(x^2 + x + 2) = 0$  $\Delta = -7 < 0$ so quadratic has no real roots, hence just one stationary point	M1*(AO1.1a)  A1(AO1.1)  M1d*(AO1.1a)  A1(AO1.1)  M1(AO1.1a)  A1(AO1.1)  B1(AO3.1a)  M1(AO2.1)  A1(AO2.1)  E1(AO2.4)  [10]	Attempt implicit differentiation  Obtain correct derivative  Either rearrange and use, or substitute  Obtain $2x - 4y = 0$ , or equiv  Eliminate $x$ or $y$ from eqn of curve  Obtain correct cubic  Identify $x = 2$ as root or $(x - 2)$ as factor  Attempt to factorise cubic - any valid method  Correct quadratic quotient  Justify one stationary point	Deal with at least one $y$ term correctly  OR $4y^2 - 8y^2 + 8y^3 - 4 = 0$  OR $2y^3 - y^2 - 1 = 0$  BC OR $f(1) = 0$  OR $(y - 1)(2y^2 + y + 1) = 0$ Allow for dividing by root of their cubic  Correct working only		
	Total	10				

Question		Answer/Indicative content	Marks	Part marks and guidance		
82	i	Gradient of given line = 6  Perpendicular gradient =  $-\frac{1}{6}$  $\frac{dy}{dx} = -2kx^{-3}$  $-\frac{1}{6} = -2k(-3)^{-3}$  $k = -\frac{9}{4}$	B1  M1  M1  A1 M1  A1 [6]	soi as gradient of the line  Uses product of perpendicular gradients is -1 at some point; may be implied by later working.  Attempt to differentiate ( $ax^{-3}$ seen)  Fully correct  Equates their derivative at $x = -3$ with their perpendicular gradient  Correct  value of $k$ .  Allow  $-\frac{27}{12}$ etc.	Can be implied by use of $-\frac{1}{6}$  e.g. $-\frac{27}{2k} = 6$ (implies first M1)	



Question		Answer/Indicative content	Marks	Part marks and guidance		
		$24x - 4y + 71 = 0$	A1  [4]	equation in any form – gradient 6 but ft their value of $\frac{k}{9}$ . Allow 6 $(x - -3)$ Correct equation in required form i.e. $a(24x - 4y + 71) = 0$ for integer $a$ , terms in any order. cao	of $k$ – straight line through $(-3, \text{their } \frac{k}{9})$ with correct gradient of 6 e.g. $k = 81$ leads to $y - 9 = 6(x + 3)$	
		<b>Total</b>	<b>10</b>	<b>Examiner's Comments</b> The level of arithmetical demand again proved to be the greatest obstacle to complete success in this part, even if the correct value of $k$ had been found in part (i). Another common mistake was to use the gradient of the tangent rather than the normal.		

Question		Answer/Indicative content	Marks	Part marks and guidance
83	i	$3\left(x^2 - \frac{5}{3}x\right) + 1$ $3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36}\right] + 1$ $3\left(x - \frac{5}{6}\right)^2 - \frac{13}{12}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p><math>a = 3</math></p> <p><math>b = -\frac{5}{6}</math> (not <math>-\frac{5}{3}</math>, <math>-\frac{2.5}{3}</math>)</p> <p><math>1 - 3b^2</math> or <math>3 \times \left(\frac{1}{3} - b^2\right)</math></p> <p><math>c = -\frac{13}{12}</math>. Allow <math>-\frac{39}{36}</math> etc.</p> </div> <div style="width: 45%; border-left: 1px solid black; padding-left: 5px;"> <p><math>3\left(x - \frac{5}{6}\right)^2 + \frac{13}{12}</math> B1 B1 M0 A0</p> <p><math>3\left(x - \frac{5}{6}\right) - \frac{13}{12}</math> 4/4 BOD</p> <p><math>3\left(x - \frac{5}{6}\right)^2 - \frac{13}{12}</math> B1 B0 M1 A0</p> <p><math>3\left(x^2 - \frac{5}{6}\right)^2 - \frac{13}{12}</math> B1 B0 M1 A0</p> <p><math>3x\left(x - \frac{5}{6}\right)^2 - \frac{13}{12}</math> B0 B1 M1 A0</p> <p><math>3\left(x^2 - \frac{5}{6}\right) - \frac{13}{12}</math> B1 B0 M1 A0</p> <p><math>3\left(x + \frac{5}{6}\right)^2 - \frac{13}{12}</math> B1 B0 M1 A0</p> </div> </div> <p><b>Examiner's Comments</b></p> <p>Almost all candidates recognised the need to take out a factor of 3 in order to complete the square, and most also secured the second mark. The combining of constants remains a problem for many candidates, with fraction errors, sign errors and multiplication errors all regularly seen. Nonetheless, a large proportion – over 60% – secured all four marks.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance	
	ii	$(-5)^2 - 4 \cdot 3 \cdot 1 = 13$  So 2 real roots	B1  B1ft  [2]	$\sqrt{b^2 - 4ac}$ Use of can score <b>B0 B1</b>  ft their discriminan t e.g. “- 25 - 12 = - 37 so no roots” scores <b>B0</b> <b>B1ft</b>	
		<b>Total</b>	<b>6</b>		

**Examiner's Comments**

Most candidates were successful in this part of the question, with only a few not recognising the word discriminant and a small number not stating the correct number of roots.

Question		Answer/Indicative content	Marks	Part marks and guidance		
84	a	$P(A) \times P(B) = \frac{1}{24}$ $P(A) + P(B) = \frac{1}{24} + \frac{3}{8}$ $P(A) + \frac{1}{24P(A)} = \frac{5}{12}$ $24(P(A))^2 - 10P(A) + 1 = 0$ $((6P(A) - 1)(4P(A) - 1) = 0)$ $P(A) = \frac{1}{6} \text{ and } P(B) = \frac{1}{4} \text{ or vice versa}$	M1(AO1.1a)  M1(AO1.1)  M1(AO3.1a)  A1(AO1.1)  A1(AO1.1)  [5]	Attempt equation in one P  Correct quadratic equation in one P  Allow without "vice versa"		
	b	$P(C') = 1 - P(C) \quad (= 0.4)$ $P(D \cap C') = P(C \cup D) - P(C) \quad (= 0.2)$ $P(D   C') = \frac{P(D \cap C')}{P(C')}$ $= \frac{0.2}{0.4} = 0.5$	M1(AO1.2)  M1(AO1.1)  M1(AO1.1)  M1(AO1.2)  [4]	Attempted		
		<b>Total</b>	<b>9</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance
85	a	$\frac{2}{3+x-4} \text{ or } \frac{2}{3+x+4}$ $y = \frac{2}{x-1}$	<p>M1 (AO 1.1)</p> <p>A1 (AO 1.1)</p> <p>[2]</p>	<p>Translates curve by + / - 4 parallel to the x-axis</p> <p>Fully correct, must have y =</p> <p><b>Examiner's Comments</b></p> <p>This was well done. <math>2/(3+x+4)</math> was seen, and the less satisfactory <math>2/(3+x)+4</math> also. We wanted the answer to be tidied up and 'y=' was expected.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	Stretch  Scale factor $\frac{5}{2}$ parallel to the y-axis	<b>B1</b> <b>(AO 1.2)</b>  <b>B1</b> <b>(AO 2.5)</b>  <b>[2]</b>	Must use stretch/stretching...  Allow "factor" or "SF" for "scale factor". Allow "vertically", "in the y direction". Do not accept "in/on/across/up/a long the y axis", "in the positive y direction", "SF 5/2 units"	B0B1 is possible for e.g. 'enlarge by scale factor...' etc. but not for (e.g.) 'translate by scale factor...' etc.  More than one transformation B0B0	
		<b>Total</b>	<b>4</b>	<b>Examiner's Comments</b>  Many candidates realised this was a stretch, but did not necessarily describe it well. We wanted to see scale factor $\frac{5}{2}$ and an accurate description of the direction. y- stretch was not considered acceptable. A small minority misunderstood the concept and gave a translation of 3 units as their response.		

Question		Answer/Indicative content	Marks	Part marks and guidance		
86	a	$4[x^2 - 3x] + 11$  $4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11$ $a = 4$  $(x - 3/2)^2$  $4\left(x - \frac{3}{2}\right)^2 + 2$ $c = 2$	<p style="text-align: center;"> <b>B1</b>  <b>(AO 1.1)</b>   <b>B1</b>  <b>(AO 1.1)</b>   <b>B1</b>  <b>(AO 1.1)</b>   <b>[3]</b> </p>	<p>No marks until attempt to complete the square</p> <p>Must be of the form <math>4(x \pm a)^2 \pm \dots</math></p>		
	b	No real roots	<p style="text-align: center;"> <b>B1</b>  <b>(AO 2.2a)</b>   <b>[1]</b> </p>	<p>Zero, none, 0, ... if not 'no real roots' must be consistent with their (a)</p>		
				<p><b>Examiner's Comments</b></p> <p>This was done very well. Candidates seemed to be very familiar with completing the square. The most common simple numerical error was to have <math>c = 8.75</math>. <math>(2x - 3)^2 + 2</math> was seen occasionally.</p>		
				<p><b>Examiner's Comments</b></p> <p>Many candidates did this by evaluating the discriminant rather than using the result they had just obtained. 'State' indicates neither working nor justification is required (cf Specification Document).</p>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
	c	$r = 0 \Rightarrow 1$ real root or 1 repeated root  $r < 0 \Rightarrow 2$ real roots  $r > 0 \Rightarrow$ no real roots	<b>M1</b> <b>(AO 2.4)</b>  <b>A1</b> <b>(AO 2.4)</b>  <b>[2]</b>	Attempt to relate the value of $r$ to the number of real roots (this can be implied with at least one correct statement)  All three statements correct	
				<b>Examiner's Comments</b>  This part proved less successful. Many candidates were not able to start an argument. Some attempted to evaluate $b^2 - 4ac$ but this was rarely done accurately. Those who recognised how to use the given form of the equation made the most progress, occasionally confusing the $r > 0$ and $r < 0$ cases.	
		<b>Total</b>	<b>6</b>		

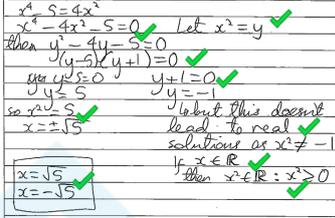
Question		Answer/Indicative content	Marks	Part marks and guidance		
87		DR $3(1 - \cos^2\theta) - 2\cos\theta - 2 = 0$  $3\cos^2\theta + 2\cos\theta - 1 = 0$  $(3\cos\theta - 1)(\cos\theta + 1) = 0$  $\cos\theta = \frac{1}{3}$ $\cos\theta = -1$  $\theta = 70.5^\circ, 289^\circ, 180^\circ$	M1(AO3.1a)  A1(AO1.1)  M1(AO1.1a)  A1(AO2.2a)  A1(AO1.2)  [5]	Attempt to use $\sin^2\theta = 1 - \cos^2\theta$  Obtain correct equation  Attempt to solve quadratic	Factorise or BC	
		<b>Total</b>	<b>5</b>			

Question	Answer/Indicative content	Marks	Part marks and guidance		
88	<p>Equations are <math>x^2 - 4y = 10</math>,  <math>x + 5y = k</math></p> <p>DR</p> <p><math>(k - 5y)^2 - 4y = 10</math></p> <p><math>25y^2 + (-4 - 10k)y + (k^2 - 10) = 0</math></p> <p>Tangent <math>\Rightarrow b^2 - 4ac = 0</math></p> <p><math>(-4 - 10k)^2 - 4(25)(k^2 - 10) = 0</math></p> <p><math>k = -\frac{127}{10}</math> (-12.7)</p> <p>OR DR</p> <p>Gradient of line = <math>-\frac{1}{5}</math></p> <p><math>\frac{dy}{dx} = \frac{1}{2}x</math></p> <p><math>\frac{1}{2}x = -\frac{1}{5}</math></p> <p><math>x = -\frac{2}{5}</math></p> <p><math>y = -\frac{123}{50}</math> (-2.46) <math>\Rightarrow k = -\frac{127}{10}</math> (-12.7)</p>	<p>M1* (AO 3.1a)</p> <p>A1 (AO 1.1)</p> <p>Dep*M1 (AO 2.1)</p> <p>A1 (AO 1.1)</p> <p>A1 (AO 2.2a)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>Substitute for <math>x/y</math> to eliminate one of the variables</p> <p>Obtain correct (un simplified) quadratic</p> <p>Uses <math>b^2 - 4ac</math> correctly for their quadratic</p> <p>Fully correct substitution must equal 0</p> <p><math>k</math> correct – with sufficient working</p> <p>Correct differentiation</p> <p>Equates their derivative with their gradient of line</p> <p><math>x</math> from</p>	<p>If <math>y</math> eliminated</p> <p><math>5x^2 + 4x - 4k - 50 = 0</math></p> <p><math>16 - 4(5)(-4k - 50) = 0</math></p> <p><math>2x - 4\frac{dy}{dx} = 0</math></p>	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					correct working only  k from correct working only	
					<p><b><u>Examiner's Comments</u></b></p> <p>The instruction requesting detailed reasoning did not prove to be an issue in this question. The two methods outlined in the mark scheme were both seen fairly often, with perhaps the 'substitution' route more popular. This required accurate algebraic work for complete success, but sign errors in the working were quite common and some petered out at the '<math>b^2 - 4ac</math>' stage. In the 'gradient' approach, candidates who worked from <math>y = x^2/4 - 10</math> or <math>y = -x/5 + k</math> to the correct value were not given full credit.</p>	
			<b>Total</b>	<b>5</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance	
89	$(x^2 - 5)(x^2 + 1) = 0$  $x^2 = 5$  $x^2 \geq 0$ , so $x^2 + 1 = 0$ has no real solutions  $x = \pm\sqrt{5}$	<p style="text-align: center;"><b>M1</b> (AO 3.1a)</p> <p style="text-align: center;"><b>A1</b> (AO 1.1)</p> <p style="text-align: center;"><b>B1</b> (AO 2.3)</p> <p style="text-align: center;"><b>A1</b> (AO 1.1)</p> <p style="text-align: center;"><b>[4]</b></p>	<p>Attempt to solve disguised quadratic, which has first been rearranged to a useable form <b>DR</b> so method must be seen</p> <p>Obtain at least <math>x^2 = 5</math></p> <p>Explicitly reject <math>x^2 + 1 = 0</math>, with reasoning</p>	<p>Substitution or direct factorisation Could use quadratic formula M0 for <math>(x - 5)(x + 1)</math>, or equiv with formula, unless clear substitution of <math>x = x^2</math></p> <p>Could be implied by their explicit substitution eg <math>u = 5</math>, where <math>u = x^2</math></p> <p>May still have <math>x^2 = -1</math> as well, but A0 if any other value for <math>x^2</math></p> <p>eg negative numbers cannot be square rooted or <math>x^2 \neq -1</math> as <math>x</math> is real <math>x^2 \neq -1</math> is insufficient without further reasoning Must be sensible reason, not just 'math</p>

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p data-bbox="821 331 901 360">Obtain</p> <p data-bbox="821 434 925 472"><math>x = \pm\sqrt{5}</math></p> <p data-bbox="986 232 1129 925">           error' or 'not possible'            Could say that there are only imaginary (or not real) roots (condone 'complex' roots)            Could say <math>x^2</math> cannot be negative, but B0 for <math>x^2</math> must be positive (or equiv as an inequality)         </p> <p data-bbox="986 965 1129 1155">           A0 if any extra roots. Both roots required, and must be exact         </p> <p data-bbox="810 1196 1082 1225"><b><u>Examiner's Comments</u></b></p> <p data-bbox="810 1263 1145 1888">           Most candidates were able to provide sufficient detail to be convincing in this 'detailed reasoning' question. It was expected that candidates would show clearly how they solved the disguised quadratic; this was usually by factorising (with or without a substitution) but some used the formula instead. There also had to be a clear reason given as to why <math>x^2 = -1</math> had no roots. Some candidates referred to the roots being imaginary (content from Further Maths) whereas others         </p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>explained that square numbers had to be greater than or equal to zero; just stating that <math>\sqrt{-1}</math> could not be done was not sufficient.</p> <p><b>Exemplar 1</b></p>  <p>This response gives a clear justification for why <math>x^2 = -1</math> does not lead to any real solutions, thus gaining full credit for this solution.</p>
			<b>Total</b>	<b>4</b>	

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Question	Answer/Indicative content	Marks	Part marks and guidance	
90	<p>DR</p> $\int_8^a 2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}} dx = 45$ $\left[ \frac{2x^{\frac{4}{3}}}{\left(\frac{4}{3}\right)} - \frac{7x^{\frac{2}{3}}}{\left(\frac{2}{3}\right)} \right]_8^a (= 45)$ $\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} - \left( \frac{3}{2}(8)^{\frac{4}{3}} - \frac{21}{2}(8)^{\frac{2}{3}} \right) (= 45)$ $\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} - (24 - 42) (= 45)$ $a^{\frac{4}{3}} - 7a^{\frac{2}{3}} - 18 = 0$ $\left( a^{\frac{2}{3}} - 9 \right) \left( a^{\frac{2}{3}} + 2 \right) = 0$ $a^{\frac{2}{3}} = 9 \quad \left( \text{and } a^{\frac{2}{3}} = -2 \right)$ <p><math>a = 27</math> only</p>	<p>M1* (AO 3.1a)</p> <p>A1 (AO 1.1)</p> <p>A1 (AO 1.1)</p> <p>Dep*M1 (AO 1.1)</p> <p>A1 (AO 1.1)</p> <p>M1 (AO 1.1)</p> <p>M1 (AO 3.1a)</p> <p>A1 (AO 1.1)</p> <p>A1 (AO 2.2a)</p> <p>[9]</p>	<p>If <math>a = 27</math> with no working then 0/9</p> <p>M1 – attempt integration (increase in power by 1 for at least 1 term) A1 – 1 term correct (accept unsimplified) A1 – both correct (accept unsimplified) <math>F(a) - F(8)</math></p> <p>oe</p> <p>Equate integrated expression to 45 – dependent on both previous M marks</p> <p>Attempt to solve quadratic in <math>a^{\frac{2}{3}}</math></p>	<p>SC if M0 for fourth M mark then award</p> <p>B1 <math>a^{\frac{2}{3}} = 9</math></p> <p>for</p> <p>B1 <math>a = 27</math> only</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					<p><u>Examiner's Comments</u></p> <p>There were some encouraging attempts at this question, with the integration done accurately. Sign errors sometimes lead to the wrong quadratic, but many realised there was a 'hidden' quadratic. This question asked for detailed reasoning so we expected to see some working to show how the quadratic solutions were obtained, either factors or some working with the formula or completing the square. A few candidates found <math>F(8) - F(a)</math> when integrating from 8 to <math>a</math> and some integrated from <math>a</math> to 8 from the start. Those who obtained <math>a = 27</math> from their calculator with no justification/detailed reasoning scored zero, although if they checked by showing the integral from 8 to 27 was equal to 45, then the first three marks were possible. There was a significant minority of candidates that attempted to differentiate rather than integrate to find the area under the curve.</p>	
			<b>Total</b>	<b>9</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
91	a	centre is $(-3, 1)$  $(x + 3)^2 - 9 + (y - 1)^2 - 1 - 10 = 0$ $(x + 3)^2 + (y - 1)^2 = 20$  $\text{radius} = 2\sqrt{5}$ or $\sqrt{20}$	<b>B1</b> <b>(AO 1.1)</b>  <b>M1</b> <b>(AO 1.1a)</b>  <b>A1</b> <b>(AO 1.1)</b>  <b>[3]</b>	Correct centre of circle  Attempt to complete the square twice  Correct radius	Allow $x = -3, y = 1$  Allow for $(x \pm 3)^2 \pm 9 + (y \pm 1)^2 \pm 1$ seen $(x \pm 3)^2 + (y \pm 1)^2 - 10 = 0$ is M0 as no evidence of subtracting the constant terms to complete the squares Or attempt to use $r^2 = g^2 + f^2 - c$  From correct working only, including correct factorisation Allow $r = 4.47$ , or better	
				<b>Examiner's Comments</b>  Solutions to this question were nearly always correct, with most candidates choosing to write the equation in factorised form. There were a few sign errors when stating the centre of the circle, and also a few errors when subtracting the constant term when completing the square each time.		



Question		Answer/Indicative content	Marks	Part marks and guidance	
	c	The line is a tangent to the circle at (1, -1)	<b>B1ft</b> <b>(AO 2.2a)</b>  <b>[1]</b>	Correct deduction Strict follow-through on their number of roots from (b)	Allow just mention of 'tangent' Allow other correct statements such as the line and the circle only touch once
				<b>Examiner's Comments</b> Part (b) shows one point of intersection so it was expected that candidates would put this information into context and conclude that the line was a tangent to the circle. If an error had happened in part (b) resulting in other than one point of intersection then candidates could still get this mark for a correct deduction from their answer.	
		<b>Total</b>	<b>8</b>		

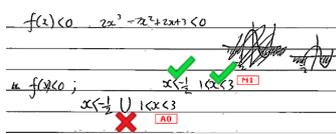
Question		Answer/Indicative content	Marks	Part marks and guidance	
92	a	$f(x) = (x - 3)(2x^2 - x - 1)$  $f(x) = (x - 3)(2x + 1)(x - 1)$	<p><b>M1</b> <b>(AO 2.2a)</b></p> <p><b>A1</b> <b>(AO 1.1)</b></p> <p><b>A1</b> <b>(AO 1.1)</b></p> <p><b>[3]</b></p>	<p>Attempt complete division by <math>(x - 3)</math></p> <p>Obtain correct quotient</p> <p>Obtain fully factorised <math>f(x)</math></p>	<p>Must be complete attempt</p> <p>Division – must be subtracting on each line (allow one error)</p> <p>Coefficient matching – valid attempt at all 3 coefficients</p> <p>Inspection – must give three correct terms on expansion</p> <p>Synthetic division – allow one error</p> <p>Could be seen in division</p> <p>Cannot be implied by <math>A = 2</math> etc</p> <p>Must be as product of all 3 linear factors</p> <p>Correct answer gets full marks, but an incorrect factorisation such as</p> <p><math>(x - 3)</math></p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					$(x + \frac{1}{2})(x - 1)$ is M0 unless method is seen  <u>Examiner's Comments</u>  Most candidates correctly found the quadratic quotient, with the most common methods being algebraic long division and the grid method, which tended to be equally successful. They could then factorise the quadratic quotient correctly. The question asks for $f(x)$ to be given in fully factorised form, so candidates were expected to give their final answer as the product of three factors. A few candidates found the roots from their calculator and then attempted to work backwards but no correct solutions were seen as the factor was given as $(x + 0.5)$ rather than the correct factorised form $(2x + 1)$ or $2(x + 0.5)$ .	
		b	Sketch of positive cubic  $(-0.5, 0), (1, 0), (3, 0), (0, 3)$	<b>B1 (AO 1.2)</b>  <b>B1 (AO 2.2a)</b>	Three roots and two stationary points  All intercepts correctly indicated	Ignore any intercepts for this mark  ft their three factors Could be given as coordinates , or just values marked on

Question	Answer/Indicative content	Marks	Part marks and guidance
		[2]	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin-bottom: 10px;">           relevant axes            BOD if coordinates transposed as long as marked on correct axis         </div> <p><b><u>Examiner's Comments</u></b></p> <p>Most candidates were able to provide a sketch of a positive cubic, including the correct behaviour at the extremities. Some candidates marked the points on the axes first resulting in a distorted graph as they then tried to fit the cubic through the points. Candidates were asked for the coordinates of any points of intersection, and most did so although a few omitted the coordinate on the y-axis.</p> <div style="text-align: center;">  </div> <p>Whilst this question asked for the coordinates, it is good practice to always provide them on a sketch graph even if not explicitly requested.</p> <p><b>Key</b></p> <div style="text-align: center;">  </div> <p>Guidance to offer for future teaching and learning practice</p>

Question		Answer/Indicative content	Marks	Part marks and guidance	
	c	$\{x : x < -0.5\} \cup \{x : 1 < x < 3\}$	<p><b>M1</b> (AO 2.2a)</p> <p><b>A1ft</b> (AO 2.5)</p> <p><b>[2]</b></p>	<p>Identify one set of values</p> <p>Fully correct solution in set notation</p>	<p>ft their cubic roots in (b), even if not 3 real, distinct, roots</p> <p>Allow notation using just inequalities</p> <p>Allow interval notation eg <math>(-\infty, -0.5)</math> and/or <math>(1, 3)</math></p> <p>If both sets of values given then ignore linking sign for this mark</p> <p>ft their cubic roots in (b), as long as 3 real, distinct, roots</p> <p>Each set should have the correct structure ie <math>\{x : \}</math> with the sets linked by <math>\cup</math></p> <p>Allow equivs</p> <p>eg <math>\{x : x &lt; -0.5\} \cup \{x : x &gt; 1\} \cap \{x : x &lt; 3\}</math></p> <p>eg <math>(-\infty,</math></p>

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p style="text-align: right;"> <math>-0.5) \cup (1, 3)</math> </p> <p>           Do not accept <math>(x &lt; -0.5) \cup (1 &lt; x &lt; 3)</math> </p> <p><b>Examiner's Comments</b></p> <p>Set notation is new to this specification and some candidates were clearly familiar with the topic whereas others were not able to use the notation correctly, achieving only partial credit.</p> <p><b>Exemplar 3</b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math display="block">x &lt; -\frac{1}{2}, 1 &lt; x &lt; 3</math> <math display="block">\{x: x &lt; -\frac{1}{2}\} \cup \{x: 1 &lt; x &lt; 3\}</math> </div> <p>This response gives a fully correct solution, using correct set notation, so gains full credit.</p> <p><b>Exemplar 4</b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math display="block">(2x + 1)(x - 1)(x - 3) &lt; 0</math> <math display="block">f(x) \in (-\infty, -0.5) \cup (1, 3)</math> </div> <p>This response demonstrates an alternative, but equally, valid use of set notation and also gains full credit.</p>

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p><b>Exemplar 5</b></p>  <p>This response gains one mark for identifying at least one correct set of values using inequalities. The use of <math>\cup</math> shows some familiarity with set notation, but the structure is not fully correct so they do not get the second mark.</p>



Question		Answer/Indicative content	Marks	Part marks and guidance		
93	a	$f(x) = c + 16 - (x - 4)^2$  $c + 16 = 19$  $c = 3$	<p><b>M1*</b> (AO 3.1a)</p> <p><b>M1d*</b> (AO 1.1a)</p> <p><b>A1</b> (AO 1.1)</p> <p>[3]</p>	<p>Attempt to identify maximum point</p> <p>Link maximum point to 19</p> <p>Solve to obtain <math>c = 3</math></p>	<p>Full attempt to complete the square Could differentiate, equate to 0 and solve to get <math>8 - 2x = 0</math>, so <math>x = 4</math></p> <p>Link the constant term of their completed square to 19 – must involve <math>c</math> Allow equation or inequality (including incorrect inequality) If using differentiation then link <math>f(\text{their } x = 4)</math> to 19</p> <p>A0 if given as inequality unless subsequently corrected Must come from fully correct working, so <math>f(x) = c + 16 - (x + 4)^2</math>, leading to <math>c + 16 = 19</math> hence <math>c = 3</math> is M1 M1 A0</p>	

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>OR</p> <p>M1* Attempt to use <math>b^2 - 4ac = 0</math> on their attempt at <math>f(x) - 19 = 0</math></p> <p>M1d* Attempt to solve their <math>64 - 4(-1)(c - 19) = 0</math></p> <p>A1 Obtain <math>c = 3</math></p> <p><b><u>Examiner's Comments</u></b></p> <p>Many fully correct solutions to this question were seen, with candidates employing a variety of different methods. The most common approaches were to write <math>f(x)</math> in completed square form and equate the maximum value to 19, or to use differentiation to identify the maximum point. Some candidates attempted rearranged to obtain <math>f(x) - 19 \leq 0</math> and attempted to use the discriminant, but only the most able identified that the condition for repeated roots should then be used.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$f(2) = c + 12$  $f(c + 12) = c + 8(c + 12) - (c + 12)^2$  $-48 - 15c - c^2 = 8$  $c^2 + 15c + 56 = 0$  $c = -7, c = -8$	<p><b>B1</b> (AO 1.1)</p> <p><b>M1*</b> (AO 1.2)</p> <p><b>M1d*</b> (AO 1.1a)</p> <p><b>A1</b> (AO 2.1)</p> <p>[4]</p>	<p>Correct <math>f(2)</math></p> <p>Attempt correct composition of <math>ff</math></p> <p>Equate to 8 and rearrange to useable form</p> <p>Both correct values for <math>c</math></p>	<p>Stated or implied by being used in later method</p> <p>Must be attempt at composition of functions so M0 for <math>\{f(2)\}^2</math></p> <p>Expand and rearrange to a three term quadratic Could be implied by the two correct roots</p> <p><b>BC</b></p> <p><b>OR</b> for the first two marks <b>M1*</b> Attempt <math>ff(x)</math> ie attempt at <math>ff(x) = c + 8(c + 8x - x^2) - (c + 8x - x^2)^2</math></p> <p><b>M1d*</b> Attempt <math>ff(2)</math> using their <math>ff(x)</math>, which may no longer be correct</p>	
				<u>Examiner's Comments</u>		

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>All candidates understood the meaning of <math>ff(2)</math> and were able to attempt the correct process for the composition of functions. The more successful method was to first find <math>f(2)</math>, simplify this to <math>c + 12</math> and then attempt <math>f(c + 12)</math>. Candidates who attempted to find <math>ff(x)</math> before substituting <math>x = 2</math> were more likely to make mistakes when simplifying their algebraic expression. It was expected that candidates would use their calculators to solve the quadratic equation, but the vast majority instead showed full detail of the method used.</p>
			<b>Total</b>	<b>7</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
94	a	(i) $fg(x) = f(x^2 + 2) = (x^2 + 2)^3$	B1(AO 1.1)E  [1]	<p><b>Examiner's Comments</b>            Nearly all candidates correctly found the composite function <math>fg(x)</math> as <math>(x^2 + 2)^3</math> although a number did expand the bracket in this part. Candidates are reminded that the number of marks available for a question or part-question are the best indicators to the amount of working and detail that is required.</p> <p>No simplification required</p> <p><b>Examiner's Comments</b>            Once again this was nearly always done correctly with the most common errors being those minority of candidates who stated that <math>(x^2)^3</math> was equal to either <math>x^5</math> or <math>x^8</math>.</p>	
		(ii) $gf(x) = g(x^3) = (x^3)^2 + 2(= x^6 + 2)$	B1(AO 1.1)E  [1]		
	b	<p><b>DR</b>  <math>(x^2 + 2)^3 = (x^2)^3 + 3(x^2)^2(2) + 3(x^2)(2)^2 + 2^3</math></p> <p><math>fg(x) = x^6 + 6x^4 + 12x^2 + 8</math></p> <p><math>fg(x) - gf(x) = 24 \Rightarrow 6x^4 + 12x^2 - 18 = 0</math></p>	<p>M1(AO 1.1)E</p> <p>A1(AO 1.1)C</p> <p>A1(AO 2.1)C</p> <p>M1(AO</p>	<p>Binomial expansion of their <math>(x^2 + 2)^3</math> – correct powers and coefficients</p>	<p>Allow one slip</p>

Question	Answer/Indicative content	Marks	Part marks and guidance	
	$x^4 + 2x^2 - 3 = 0 \Rightarrow (x^2 - 1)(x^2 + 3) = 0$ $x^2 + 3 = 0 \text{ has no real solutions}$ $x^2 - 1 = 0 \Rightarrow x = \pm 1$	1.1)C  A1(AO 2.4)A  A1(AO 2.2a)A  [6]	Correct method for solving their quadratic in $x^2$  $x^2 + 3 \neq 0$ is acceptable for this mark	If M0 next two marks become B marks
<p style="text-align: center;"><b>Examiner's Comments</b></p> <p>There were a significant number of candidates that did not employ correct bracketing and mistakenly wrote <math>(x^2 + 2)^3 - x^6 + 2 =</math> rather than <math>(x^2 + 2)^3 - (x^6 + 2) = 24 \Rightarrow (x^2 + 2)^3 - x^6 - 2 = 24</math>.</p> <p>The majority of candidates went for expanding <math>(x^2 + 2)^3</math> by writing out the bracket three times rather than using the binomial expansion of <math>(a + b)^n</math>.</p> <p>While the majority rearranged their quartic into the form <math>6(x^2)^2 + 12x^2 + 12x^2 - 18 = 0</math> there were a number of candidates did not show sufficient working in solving this quartic, even though the full question required detailed reasoning.</p> <p>Finally, it is expected at this level that as part of the detailed reasoning candidates should justify why <math>x^2 + 3 = 0</math> did not provide any solutions.</p>				

Question	Answer/Indicative content	Marks	Part marks and guidance	
		Total	8	





Question		Answer/Indicative content	Marks	Part marks and guidance	
				the given result. Candidates are reminded though that in 'show that' questions suitable working must be shown and statements such as $\frac{2}{2} \left[ \frac{1}{2} + \frac{1}{4} + 2 \left( \frac{1}{2 + \sqrt{2}} \right) \right] = \frac{11}{4} - \sqrt{2}$ are generally not acceptable.	
	b	$x = u^2 \Rightarrow dx = 2u \, du$  $\int_0^4 \frac{dx}{2 + \sqrt{x}} = \int_0^2 \frac{2u}{2 + u} \, du$ $= 2 \int_0^2 \frac{2 + u - 2}{2 + u} \, du = 2 \int_0^2 \left( 1 - \frac{2}{2 + u} \right) \, du$ $= 2 \left[ u - 2 \ln(2 + u) \right]_0^2$  $= 2 \{ (2 - 2 \ln(2 + 2)) - (0 - 2 \ln(2 + 0)) \}$  $= 2(2 - 2 \ln 2)$	<b>M1*(AO 3.1a)E</b>  <b>A1(AO 1.1)C</b>  <b>Dep*M1(AO 2.1)C</b>  <b>A1ft(AO 1.1)A</b>  <b>M1(AO 1.1)C</b>  <b>A1(AO 2.2a)A</b>  <b>[6]</b>	An attempt at integration by sub – allow any genuine attempt (as a minimum must differentiate their sub. and remove all x's) Correct integral in terms of $u$  Re-writes integral in the form $\int a + \frac{b}{1 + u} \, du$ Correctly integrates their $\int a + \frac{b}{1 + u} \, du$  Uses correct limits correctly (dependent on both previous M	Limits not required for first four marks  Or use $t = 2 + u$ to obtain integral of the form $\int a + \frac{b}{t} \, dt$ $\int 2 - \frac{4}{t} \, dt$ $= 2t - 4 \ln t$

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>marks)            oe e.g.  <math>4 - 4 \ln 4 +</math>  <math>4 \ln 2</math></p> <p><b>Examiner's Comments</b>            While nearly all candidates used the substitution correctly and re-wrote the integrand <math>\int_0^2 \frac{2u}{2+u} du</math> many could not deal with the resulting improper fraction in the integrand. The most successful candidates <math>\frac{2u}{2+u}</math> either by re-wrote using long division or realising that</p> $\frac{2u}{2+u} = \frac{4+2u-4}{2+u} = 2 - \frac{4}{2+u}$ <p>While examiners noted that some candidates employed more extreme methods (for example, further substitutions and the method of integration by parts) these were usually unsuccessful.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance
	c	$\frac{11}{4} - \sqrt{2} \approx 2(2 - 2\ln 2)$ $\ln 2 \approx \frac{5}{16} + \frac{\sqrt{2}}{4}$	M1(AO 1.1a)C  A1(AO 2.1a)A  [2]	Setting the given result approx. equal to their (b)  $k = \frac{5}{16}$  <u>Examiner's Comments</u> While some, who had struggled with part (b), left this part blank the majority of candidates equated their answers to parts (a) and (b) with nearly all who were successful in part (b) correctly determining that $k = \frac{5}{16}$ .  t
		<b>Total</b>	<b>13</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
96	a	DR $(\sqrt{3})^7$ or $\sqrt{3^7}$ or $3^3 \times \sqrt{3}$ or $3\sqrt{243}$  $27\sqrt{3}$	M1 (AO1.1a)  A1 (AO1.1)  [2]	or any correct intermediate step using $\sqrt{\quad}$  or $3^3 \times 3^{\frac{1}{2}}$  or $a = 27, b = 3$	If this step is not seen, MOAO
				<p><b>Examiner's Comments</b></p> <p>Many candidates answered this question correctly. A few made a correct first step, for example <math>\sqrt{3^7}</math>, but could not continue correctly. Some candidates gave the correct answer with no working or with incorrect working. These scored no marks.</p>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	DR $\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ $= \frac{\sqrt{2}+2}{1-2} \text{ or } \frac{\sqrt{2}+2}{-1} \text{ or } \frac{\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$ $= -2 - \sqrt{2} \quad \text{ISW}$	M1 (AO1.1a)  A1 (AO1.1)  A1 (AO1.1)  [3]	A1 for correct num OR denom  or $-2 + (-1\sqrt{2})$ or $c = -2, d = -1$ and $e = 2$	If this step is not seen, M0A0  Allow $-(2 + \sqrt{2})$	
		<b>Total</b>	<b>5</b>			

**Examiner's Comments**

Many candidates answered this question correctly. A few made a correct first step, multiplying numerator and denominator by  $1 + \sqrt{2}$ , but made a subsequent error. Some candidates gave the correct answer with no working or with incorrect working. These scored no marks.

Question		Answer/Indicative content	Marks	Part marks and guidance		
97	a	$3^2 - 4k = 0$  $k = \frac{9}{4}$ or 2.25	<p>M1 (AO1.2)</p> <p>A1 (AO1.1)</p> <p>[2]</p>	$x^2 + 3x + k$ $= (x + a)^2 =$ $x^2 + 2ax +$ $a^2$ $\Rightarrow a = 1.5$ $\Rightarrow k = 1.5^2$	or $(x + 1.5)^2 - 2.25 + k = 0$	
				<p><b>Examiner's Comments</b></p> <p>This question was answered well. Use of the discriminant was the more popular approach, but some candidates used the "completing the square" method. . A few candidates started with <math>9 - 4k &gt; 0</math>. Others used <math>b^2 + 4ac</math>. In general the "completing the square" method was less successfully applied, with mistakes in the algebraic manipulation more common.</p>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$(3 - x)(2 + x) > 0$ or $(x - 3)(x + 2) < 0$  $-2 < x < 3$ or $3 > x > -2$ ISW or $x \in (-2, 3)$	<b>M1</b> <b>(AO1.1a)</b> <b>M1</b> <b>(AO2.2a)</b>  <b>[2]</b>	oe Allow $(3 - x)(2 + x)$ or $(x - 3)(x + 2)$ Allow $x > -2, x < 3$ or $x > -2$ and $x < 3$  Correct ans: BOD M1A1	or $-2$ and $3$ seen  $x > -2$ or $x < 3$ M1A0 unless followed by ans	
		<b>Total</b>	<b>4</b>			

**Examiner's Comments**

Many candidates were unable to deal with the signs. Some wrote  $(x - 3)(x + 2) > 0$  or  $(-x - 3)(x + 2) > 0$ . Many eventually obtained either  $\{x < -2$  and  $x > 3\}$  or  $\{-3 < x < 2\}$ . A few candidates gave correct working, but gave their solution as two separate regions:  $x > -2, x < 3$ .

Question		Answer/Indicative content	Marks	Part marks and guidance	
98	a	E.g. $\log_3 x^2 = 2 \log_3 x$ ; the student has ignored the brackets and used the power rule incorrectly  E.g. $x = 3^2$ ; the student has done $2^3$	E1 (AO 2.3)  E1 (AO 2.3)  [2]	Error identified with explanation  Error identified with explanation	
	b	$(2\log_3 x + 1)(\log_3 x - 2) = 0$  $\log_3 x = -0.5, \log_3 x = 2$  $x = 3^{-0.5}$ or $x = 3^2$  $x = \frac{1}{3}\sqrt{3}$ and $x = 9$	M1 (AO 3.1a)  A1 (AO 1.1)  M1 (AO 1.1a)  A1 (AO 1.1)  [4]	Attempt to solve quadratic in $\log_3 x$  Obtain two correct roots BC  Attempt correct process to find $x$ at least once  Obtain both correct roots	soi  Any equivalent exact form
		<b>Total</b>	<b>6</b>		



Question		Answer/Indicative content	Marks	Part marks and guidance	
		$(x + 1)^2 + (y - 8)^2 = 3^2$  hence shortest distance  between line and circle is  $2\sqrt{5} - 3$	A1 (AO 3.2a)  [4]	Pythagoras to find distance between centre of circle and point of intersection  Attempt to find radius of circle  Obtain  $2\sqrt{5} - 3$	any point in solution – allow back credit to part (a) if the radius is found at the same time as the centre of circle  Allow any exact equiv
		<b>Total</b>	<b>11</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance		
100	DR $a + d = ar^2$	B1 (AO 3.1a)	Correct equation for $a_2 = g_3$	Allow $a, a_1, g_1$ or $1 + \sqrt{5}$ and may be different in each term eg $a_1 + d = g_1 r^2$	
	$a + 2d + ar^3 = 0$	B1 (AO 3.1a)	Correct equation for $a_3 + g_4 = 0$	Allow $a, a_1, g_1$ or $1 + \sqrt{5}$ and may be different in each term eg $a_1 + 2d + g_1 r^3 = 0$	
	$a + 2(ar^2 - a) + ar^3 = 0$	M1 (AO 2.1)	Eliminate $d$		
	$r^3 + 2r^2 - 1 = 0$	A1 (AO 2.1)		Could be $a, a_1, g_1$ or $1 + \sqrt{5}$ but must now be consistent throughout (soi)	
	$f(-1) = -1 + 2 - 1 = 0$ hence $(r + 1)$ is a factor	B1 (AO 2.4)	Obtain correct cubic		
	$(r + 1)(r^2 + r - 1) = 0$	M1 (AO 3.1a)	Identify $(r + 1)$ as a factor, with justification		
	$r = -1, \frac{-1 \pm \sqrt{5}}{2}$	A1 (AO 1.1a)	Attempt to find all 3 roots of cubic		
	GP is convergent so $-1 < r < 1$ , so $r = \frac{-1 + \sqrt{5}}{2}$	B1 (AO 2.4)	For all three		

Question		Answer/Indicative content	Marks	Part marks and guidance		
		$S_{\infty} = \frac{1 + \sqrt{5}}{1 - \frac{1}{2}(-1 + \sqrt{5})}$ $= \frac{2(1 + \sqrt{5})}{2 - (-1 + \sqrt{5})} = \frac{2(1 + \sqrt{5})}{3 - \sqrt{5}}$ $= \frac{2(1 + \sqrt{5})(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})} = \frac{2(3 + \sqrt{5} + 3\sqrt{5} + 5)}{9 - 5}$ $= \frac{2(8 + 4\sqrt{5})}{4} = 4 + 2\sqrt{5} \text{AG}$	<p>M1 (AO 1.1a)</p> <p>A1 (AO 2.1)</p> <p>M1 (AO 3.1a)</p> <p>A1 (AO 2.1)</p> <p>[12]</p>	<p>Identify correct value of <math>r</math>, with reason</p> <p>Attempt sum to infinity, using their <math>r</math></p> <p>Simplify to correct expression</p> <p>Rationalise denominator</p> <p>Obtain given answer www</p>	<p>Need <math>-1 &lt;</math> their <math>r &lt; 1</math></p> <p>Must also attempt expansion</p>	
		<b>Total</b>	<b>12</b>			

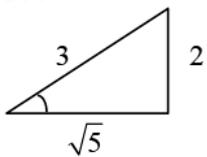
Question		Answer/Indicative content	Marks	Part marks and guidance		
101		$\frac{1 - \frac{1}{8}\theta^2}{1 + \theta} = 0.825$ $0.125\theta^2 + 0.825\theta - 0.175 = 0$ $\theta = 0.206 \text{ or } -6.81 \text{ (3 sf)}$ Discard $-6.81$ as not small. $\theta = 0.206$ (3 sf)	M1 (AO1.1a)  A1 (AO1.1) A1 (AO1.1) A1 (AO2.3) [4]	BC	Statement needed and $\theta = 0.206$ alone	
		<b>Total</b>	<b>4</b>			
102	a	$4x^2 - 12x + 3 = 4(x^2 - 3x) + 3$ $= 4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 3$ $4\left(x - \frac{3}{2}\right)^2 - 4 \times \frac{9}{4} + 3 = 4\left(x - \frac{3}{2}\right)^2 - 6$ <b>A</b>  <b>Alternative method</b> $4\left(x - \frac{3}{2}\right)^2 - 6 = 4\left[x^2 - 3x + \frac{9}{4}\right] - 6$ $= 4x^2 - 4 \times 3x + 4 \times \frac{9}{4} - 6$ $4x^2 - 12x + 3 = 4(x^2 - 3x) + 3$ <b>AG</b>	M1 (AO1.1) A1 (AO1.1)  <b>A</b> (AO2.1)  <b>M</b> (AO1.1)  A1 (AO1.1)  A1 (AO2.1) [3]	Take out a factor of 4          multiply out    square bracket   intermediate step	$4x^2 - 12x = 4(x^2 - 3x)$ $x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$ $x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$ $4x^2 - 12x = 4(x^2 - 3x)$	
	b	Minimum point is $\left(\frac{3}{2}, -6\right)$	B1 (AO1.1) B1 (AO1.1) [2]			
		<b>Total</b>	<b>5</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
103	a	$\frac{dy}{dx} = 4ax^3 + 3bx^2 - 2$ $4a(2)^3 + 3b(2)^2 = \Rightarrow 16a = 6b = 1$	<b>M1</b> <b>(AO1.1a)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO2.2a)</b> <b>[3]</b>	Attempt to differentiate – all powers reduced by 1  Correct first derivative  <b>AG</b> – sufficient working must be shown to establish given result	
	b	$\frac{d^2y}{dx^2} = 12ax^2 + 6bx$ $16a + 6b = 1 \text{ and } 4(a + b) = 0 \Rightarrow a = \dots \text{ and } b = \dots$ $a = -\frac{1}{8}, b = \frac{1}{2}$	<b>B1FT</b> <b>(AO1.1)</b>  <b>M1</b> <b>(AO2.1)</b>  <b>A1</b> <b>(AO2.2a)</b> <b>[3]</b>	Correct second derivative following through from their first derivative  Formulate two equations in $a$ and $b$ and attempt to solve for both $a$ and $b$  Both values correct	No follow through for this mark
		<b>Total</b>	<b>6</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
104	a	$2(1 - \cos^2 \theta) + \cos \theta = 4$ $\cos^2 \theta$ $2 - 2 \cos^2 \theta + \cos \theta = 4$ $\cos^2 \theta$ $6 \cos^2 \theta - \cos \theta - 2 = 0$	<b>M1</b> <b>(AO3.1a)</b>  <b>A1</b> <b>(AO2.2a)</b> <b>[2]</b>	Correctly removing square root and use of $\sin^2 \theta = 1 - \cos^2 \theta$ to obtain an equation in cos only  <b>AG</b> – sufficient working must be shown to establish given result	
	b	<b>DR</b> $(2 \cos \theta + 1)(3 \cos \theta - 2) = 0$ $\cos \theta = -\frac{1}{2} \text{ and } \cos \theta = \frac{2}{3}$ $\cos \theta = \frac{2}{3} \Rightarrow \theta = 48.2, 311.8$ $\cos \theta = -\frac{1}{2} \Rightarrow \theta = 120, 240$	<b>M1</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>A1</b> <b>(AO1.1)</b>  <b>M1</b> <b>(AO2.2a)</b> <b>[4]</b>	Correct method for solving quadratic  Any two  correct values  All four correct values	May use formula or completing the square  48.189..., 311.810...  And no others
	c	E.g. since $\cos \theta \neq -\frac{1}{2}$ in the RHS of the equation  $\sqrt{2 \sin^2 \theta + \cos \theta} = 2 \cos \theta$	<b>E1 (AO 2.3)</b> <b>[1]</b>		
		<b>Total</b>	<b>7</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
105		DR A: $C = 2m + 10$  B: $C = 2.4m + 3$  Where $C$ is the total charge in pounds and $m$ is the length of the journey in miles. $2.4m + 3 = 2m + 10$  $0.4m = 7$  $m = 17.5$  Same cost for a journey of 17.5 miles	M1(AO 3.1a) A1(AO 3.1b)   M1(AO 1.1)   A1(AO 3.2a) [4]	Either correct equation Both equations correct and variables defined.  Attempt to solve simultaneously	Attempt to solve their two equations   Units needed	Obtain 17.5 miles
		<b>Total</b>	<b>4</b>			

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Question		Answer/Indicative content	Marks	Part marks and guidance	
106	a	DR  $\cos \alpha = \pm \frac{1}{3}\sqrt{5}$	<b>M1(AO 3.1a)</b>  <b>A1 (AO 1.1)</b>  <b>[1]</b>	Attempt Pythagoras on correct right-angled triangle  Obtain $\pm \frac{1}{3}\sqrt{5}$ oe eg $\pm \sqrt{\frac{5}{9}}$	$\frac{4}{9} + \cos^2 \alpha = 1$ r $\cos^2 \alpha = \frac{5}{9}$  A0 for answer only as DR
	b	$2\sec^2 \beta - 2 - 7\sec \beta + 5 = 0$  $2\sec^2 \beta - 7\sec \beta + 3 = 0$  $(\sec \beta - 3)(2\sec \beta - 1) = 0$ $\sec \beta = 3, \sec \beta = 0.5$  $\sec \beta \geq 1$ for $0^\circ \leq \beta < 90^\circ$  hence $\sec \beta = 3$	<b>M1(AO 3.1a)</b>  <b>A1(AO 1.1)</b>  <b>M1 (AO 1.1a)</b>  <b>A1 (AO 1.1)</b>  <b>[4]</b>	Attempt to use $\tan^2 \beta = \sec^2 \beta - 1$  Obtain correct equation  Attempt to solve quadratic  Obtain $\sec \beta = 3$ only $\sec \beta = 0.5$ must be seen and discarded with a reason	Allow equiv methods involving $\cos \beta$  Allow more general reason such as $ \sec \beta  \geq 1$
<b>Total</b>			<b>6</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
107		DR $e^x = 3 + 2e^y$  $(3 + 2e^y)^2 - 4e^{2y} = 33$  $9 + 12e^y + 4e^{2y} - 4e^{2y} = 33$  $12e^y = 24$  $e^y = 2$  $y = \ln 2$  $e^x - 4 = 3$  $e^x = 7$  $x = \ln 7$	M1(AO 3.1a)  A1(AO 1.1)  M1(AO 1.1a)  A1(AO 1.1)  A1(AO 2.1)  [5]	Attempt to eliminate one variable Obtain correct equation in one variable – allow unsimplified Simplify and attempt to solve  Obtain $y = \ln 2$  Obtain $x = \ln 7$ , using either equation.	or $e^{2x} - 4(0.5e^x - 1.5)^2 = 33$  or $6e^x = 42$ etc	
		<b>Total</b>	<b>5</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
108	a	$\frac{(x-4)(x+3) + (3x+1)(x+2)}{(x+2)(x-1)(x+3)}$ $= \frac{x^2 - x - 12 + 3x^2 + 7x + 2}{(x+2)(x-1)(x+3)}$ $= \frac{4x^2 + 6x - 10}{(x+2)(x-1)(x+3)}$ $= \frac{2(2x+5)(x-1)}{(x+2)(x-1)(x+3)}$ $\frac{2(2x+5)}{(x+2)(x+3)} \text{ A.G.}$	<p><b>M1 (AO 3.1a)</b></p> <p><b>A1 (AO 2.1)</b></p> <p><b>M1 (AO 2.1)</b></p> <p><b>M1 (AO 2.1)</b></p> <p><b>A1 (AO 2.1)</b></p> <p><b>[5]</b></p>	<p>Attempt to use a common denominator</p> <p>Obtain correct unsimplified fraction</p> <p>Expand and simplify numerator</p> <p>Attempt to factorise numerator</p> <p>Obtain given answer, with sufficient detail shown</p>	<p>Could be quartic denominator if repeated factor not spotted</p>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$\int f(x)dx = 2\ln(x^2 + 5x + 6)$  $2\ln[(a + 4)^2 + 5(a + 4) + 6] - 2\ln(a^2 + 5a + 6)$  $2\ln \frac{a^2 + 13a + 42}{a^2 + 5a + 6} = 2\ln 3$  $\frac{a^2 + 13a + 42}{a^2 + 5a + 6} = 3$  $2a^2 + 2a - 24 = 0$  $2(a + 4)(a - 3) = 0$  $a = 3$	<b>M1 (AO 3.1a)</b>  <b>A1 (AO 1.1)</b> <b>M1 (AO 1.1a)</b>  <b>M1 (AO 3.1a)</b>   <b>A1 (AO 1.1)</b>  <b>M1 (AO 1.1a)</b> <b>A1 (AO 3.2a)</b>  <b>[7]</b>	Obtain integral of $k \ln(x^2 + 5x + 6)$  Obtain correct 2 $\ln(x^2 + 5x + 6)$ Attempt use of limits  Equate to $2\ln 3$ and remove logs  Obtain correct three term quadratic Attempt to solve quadratic Obtain $a = 3$ only	no need for modulus signs oe using partial fractions or $2(\ln(x+3) + \ln(x+2))$ Correct order and subtraction Using valid method          DR so method must be seen A0 if $a = -4$ also given	
		<b>Total</b>	<b>12</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
109	a	$2^{n_1-1} = 1024$  $n_1 = 11$	M1 (AO 1.1) A1 (AO 1.1) [2]		
	b	$r_2 = 4$  $4^{n_2-1} = 1024$  $n_2 = 6$	B1 (AO 1.1)  B1 (AO 2.2a) [2]		
	c	$r_3 = \sqrt{2}$  $(\sqrt{2})^{n_3-1} = 1024$  $n_3 = 21$  $S_{21} = 1 \times \frac{(\sqrt{2})^{21}-1}{\sqrt{2}-1}$ $= 2047 + 1023\sqrt{2}$ or 3490 (3 sf)	B1 (AO 1.1)  M1 (AO 3.1a) A1 (AO 2.2a)  A1FT (AO 1.1) [4]	Other correct answers score similarly, eg $r_3 = \sqrt[4]{2}$ $(\sqrt[4]{2})^{n_3-1} = 1024$ $n_3 = 41$ $S_{21} = 1 \times \frac{(\sqrt[4]{2})^{41}-1}{\sqrt[4]{2}-1}$ 6430 (3 sf)	ft their $r_3$ and $n_3$
		<b>Total</b>	<b>8</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
110	a	$1^n - 1 = 0$ , hence $(x^n - 1)$ is divisible by $(x - 1)$ by the factor theorem	<b>B1 (AO 2.2a) [1]</b>	Or by factorisation		
	b	$2^{8k} = (2^8)^k$ Let $2^8 = x$ $((2^8)^k - 1)$ is div by $(2^8 - 1)$ $(2^8 - 1) = 255$ and $255 = 15 \times 17$ Hence $2^{8k} - 1$ is div by 17	<b>M1 (AO 3.1a)</b>  <b>M1 (AO 2.4)</b> <b>M1 (AO 1.1)</b> <b>A1 (AO 2.2a) [4]</b>	May be implied  or "From (i)"  Must see last two lines, oe		
		<b>Total</b>	<b>5</b>			

Question	Answer/Indicative content	Marks	Part marks and guidance	
111	$3e^x = 1 - 2e^{\frac{1}{2}x}$ $3(e^{\frac{1}{2}x})^2 + 2e^{\frac{1}{2}x} - 1 = 0$ <p style="text-align: center;">or eg <math>3u^2 + 2u - 1 = 0</math></p> $\text{or } 3y + 2\sqrt{y} - 1 = 0$ $((3e^{\frac{1}{2}x} - 1)(e^{\frac{1}{2}x} + 1) = 0)$ $e^{\frac{1}{2}x} = -1 : \text{no solutions stated}$ $e^{\frac{1}{2}x} = \frac{1}{3} \text{ oe}$ $x = 2 \ln \frac{1}{3} \text{ or } \ln \frac{1}{9} \text{ or } -2 \ln 3$ $\text{or } -\ln 9 \text{ or } -2.20$ $y = 3 \exp(\ln \frac{1}{9}) = \frac{1}{3} \text{ or } 0.333$ Point of intersection is $(\ln \frac{1}{9}, \frac{1}{3})$	M1(AO3.1a) M1(AO3.1a)  B1(AO2.3)  A1(AO2.1) A1(AO1.1) A1(AO2.2a)  [6]	oe Attempt write quadratic equation in $e^{\frac{1}{2}x}$ or attempt a substitution and form QE Allow one sign error May not be seen eg "Cannot be negative" or "Impossible" or just "X" o $\frac{1}{2}x = \ln \frac{1}{3}$ r o $\frac{1}{2}x = -1.10$ r  or equivalent exact or (-2.20, 0.333) (3 sf) If any extra points of intersection : A0	Allow decimals throughout Allow substitute x or $y = e^{\frac{1}{2}x}$  not just eg $u = \frac{1}{3}$  Correct ans, with no working or irrelevant working: SC B2
			<u>Examiner's Comments</u>  The majority of candidates wrote a correct equation,	

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p> <math>3e^x = 1 - 2e^{\frac{1}{2}x}</math>, but then proceeded to attempt to take the logarithms of both sides. Their attempts were invariably incorrect. A few candidates wrote a correct first equation but then attempted to combine the exponential terms, obtaining equations such as <math>5e^{\frac{3}{2}x} = 1</math>.         </p> <p>           Some candidates recognised the "hidden" quadratic equation. Of these candidates, some did not substitute, but obtained the equation <math>3(e^{\frac{1}{2}x})^2 + 2e^{\frac{1}{2}x} - 1 = 0</math> and eventually succeeded. However, many who substituted, for example <math>u = e^{\frac{1}{2}x}</math>, often made errors such as obtaining <math>2u^2 + 3u - 1 = 0</math> or <math>(3u)^2 + 2u - 1 = 0</math>.         </p> <p>           The correct method leads to <math>e^{\frac{1}{2}x} = \frac{1}{3}</math>. To gain full marks candidates had to indicate in some way that the equation <math>e^{\frac{1}{2}x} = -1</math> has no solutions.         </p> <p>           Some candidates used a graphical calculator either from the start or after abandoning the incorrect logarithm method. In some cases, correct responses were seen, but preceded by incorrect working. Because the command word in this question is "Determine" rather than "Find", answers         </p>

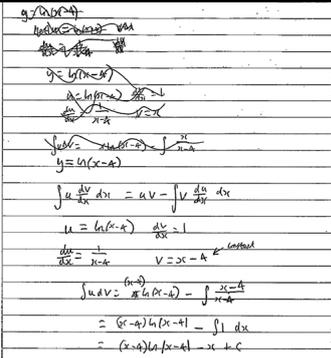
Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>obtained in this way, without working equivalent to that shown in the mark scheme, were not given full credit. They scored 2 marks (out of 6 marks), if correct to 3 significant figures</p> <p>The best solutions worked with exact values, expressed in terms of <math>e</math>. Working in terms of decimals was accepted, but the final response needed to be correct to 3 significant figures. It is worth noting that similar questions in the future might explicitly require candidates to work with exact values and in such questions, working and/or responses in terms of rounded decimals would be penalised.</p> <p> <b>Misconception</b></p> <p>A possible misconception may be that candidates expect all equations involving exponentials will be solved by using logarithms from the start.</p>
			<b>Total</b>	<b>6</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
112	a	$\int 1 \cdot \ln(x-4) dx$ so $u = \ln(x-4)$ and $v' = 1$	M1(AO1.1a)	Attempt integration by parts, with correct parts	$u$ and $v'$ correctly allocated and correct formula used M0 if $v = x - 4$ from $v' = 1$
		$x \ln x-4  - \int \frac{x}{x-4} dx$	A1(AO1.1)	Correct expression	Allow brackets not modulus
		$\int \frac{x}{x-4} dx = \int 1 + \frac{4}{x-4} dx$	M1(AO3.1a)		Allow
		$= x + 4 \ln x-4 $	A1(AO1.1)	Attempt to deal with improper fraction	$x \times \frac{1}{x-4}$ , even if subsequently spoilt Allow sign error ie $1 - \frac{4}{x-4}$ Could use substitution of $u = x - 4$ but must get as far as a proper fraction (ie $1 \pm 4u^{-1}$ ) Do not need to actually integrate for M1
		$\int \ln(x-4) dx$ $= x \ln x-4  - x - 4 \ln x-4  + c$ $= (x-4) \ln x-4  - x + c$ <b>A.G.</b>	A1(AO2.4)		
			[5]	Correct integration of fraction	Allow brackets not modulus Using a substitution

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>gives  <math>x - 4 + 4\ln x - 4 </math>;            must be in terms of <math>x</math> and not <math>u</math> for A1</p> <p>Show given answer with no errors seen</p> <p>Modulus required in final answer, as well as <math>+ c</math>            Can go from penultimate line in MS to given answer with no further detail needed            Answer from using substitution will need to justify changing <math>c</math> eg <math>c + 4</math> is a constant hence <math>c'</math> is also a constant  <b>NB</b> differentiating given answer is 0/5</p> <p><b>OR</b>            B1 use substitution of <math>v = x - 4</math>, with <math>\frac{dv}{dx} = 1</math> seen            , to obtain <math>\int</math></p>

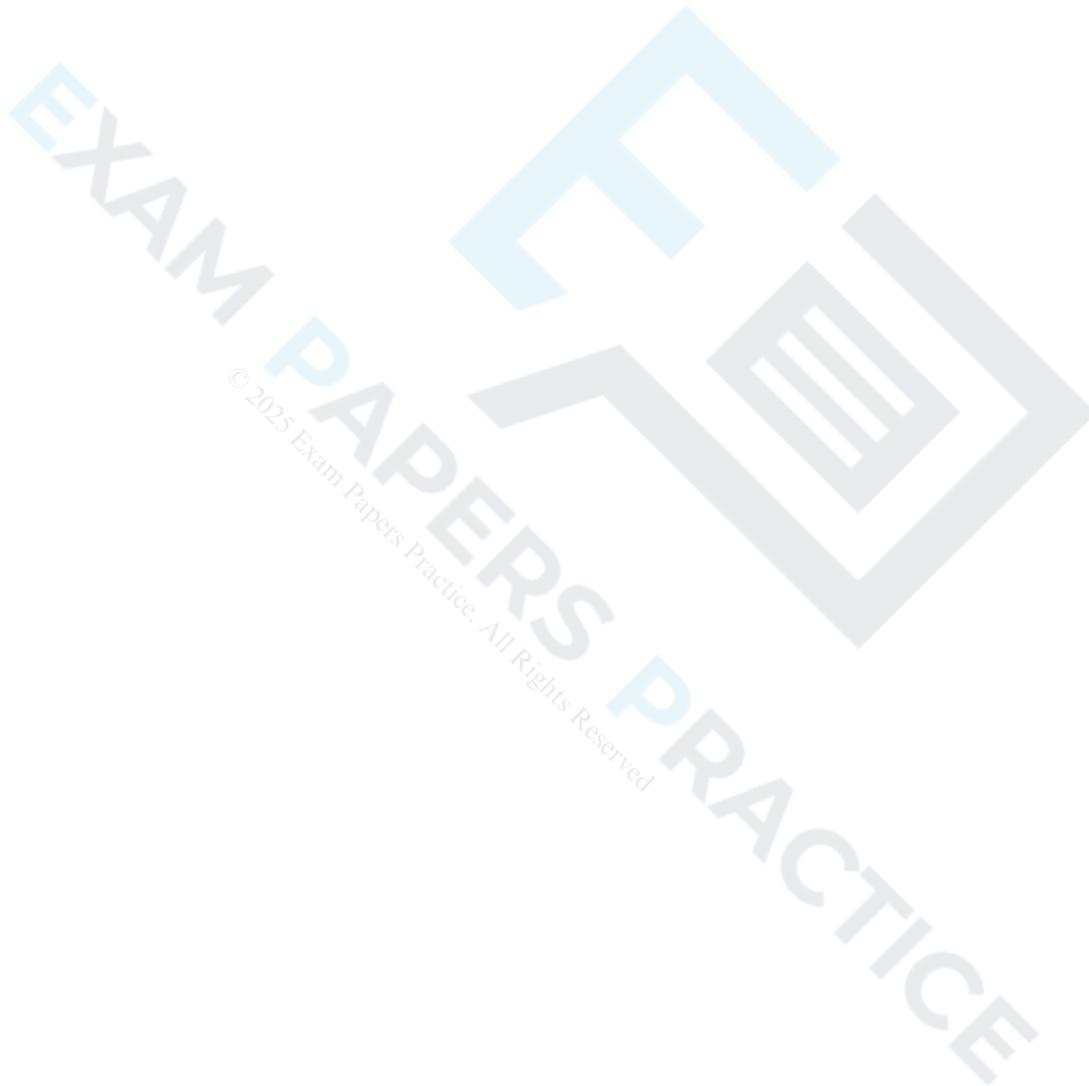
Question	Answer/Indicative content	Marks	Part marks and guidance
			<p> <math>\int \ln v \, dv</math> if            B0 as it            is not            explicit            then            next 4            marks            are still            availabl            e            M attempt i            1- ntegratio            n by            parts on  <math>\int \ln v \, dv</math>,            using            correct            parts            and            correct            formula            A1 ob            - <math>\frac{1}{v} \ln v </math>            - <math>v</math>            (allow            no            modulus            )            A1 obtain            - <math>(x - 4) \ln x - 4  - (x - 4)</math>            A1 obtain            - given            answer,            includin            g            modulus            sign,            and with            justificati            on for            their <math>c</math> b            ecoming  <math>c'</math> </p> <p><b>Examiner's Comments</b></p> <p>Many candidates could</p>

Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>attempt integration by parts and gain the first two marks available, but were then unsure as to how to deal with the improper fraction and struggled to make any further progress. The most efficient method was to carry out a division attempt, either formal algebraic division or an informal balancing method, and then integrate the resulting expression. The other common method was to use a substitution that resulted in the denominator becoming a single term, thus facilitating the subsequent division. The latter method did cause problems for some candidates as it resulted in an extra term of +4 which they had to justify being absorbed into the constant of integration; in some cases it just conveniently disappeared. Some candidates used a substitution as their first step; this approach was condoned as integration by parts was still required in this method. Candidates were expected to produce the given result and a lack of clear modulus signs in the final answer was penalised. There was also evidence that a few candidates initially used the correct method but then spoiled this by attempting to adjust their working to give the given answer straightaway.</p> <p><b>Exemplar 6</b></p>

Question		Answer/Indicative content	Marks	Part marks and guidance		
						<p>The candidate makes a good start to the question producing work that would have been worth 2 marks. Rather than trying to make further progress with this solution, they delete it and instead attempt to adjust their working to achieve the given answer straightaway. A given answer is often included to allow the candidate to make further progress in the later part of the question, and they should not allow this to influence their method.</p>
	b	State $x = 4$	B1(AO2.2a)  [1]	<div style="border-left: 1px solid black; padding-left: 5px;">           Must be an equation         </div> <p><b>Examiner's Comments</b></p> <p>This question was generally very well answered, with most candidates able to state the correct equation.</p>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
	c	$\left  \int_5^7 \ln(x-4) dx \right  + \left  \int_{4.5}^5 \ln(x-4) dx \right $ $(3\ln 3 - 2) + \left( \frac{1}{2} \ln \frac{1}{2} + \frac{1}{2} \right)$ $3\ln 3 - 2 - \frac{1}{2} \ln 2 + \frac{1}{2}$ $3\ln 3 - \frac{1}{2} \ln 2 - \frac{3}{2}$	M1(AO3.1a) A1(AO1.1) M1(AO3.1a) A1(AO1.1) [4]	Attempt sum of absolute areas Correct expression for area  Attempt to rearrange to required form  Obtain $3\ln 3 - \frac{1}{2} \ln 2 - \frac{3}{2}$	Or integral $\int_5^7 \ln(x-4) dx - \int_{4.5}^5 \ln(x-4) dx$  Any unsimplified equiv  Use $\ln \frac{1}{2} = -\ln 2$ and gather like terms Could follow M0 Allow M1 (implied) for $3\ln 3 + 0.5\ln 2 - 2.5$ , even if $-0.5\ln 0.5$ not seen first  Or $3\ln 3 - 0.5\ln 2 - 1.5$	
				<b>Examiner's Comments</b>  Only the most able candidates appreciated that the requested area existed both above and below the x-axis and were able to devise an appropriate strategy. Other candidates did calculate the two areas separately, but then simply added them together rather changing the relevant signs before combining the two integrals. However the majority of candidates simply used the two limits in the integral; many of these were still able to gain a mark for rewriting $\ln 0.5$ as $-\ln 2$ .		

Question	Answer/Indicative content	Marks	Part marks and guidance	
		Total	10	



Question			Answer/Indicative content	Marks	Part marks and guidance	
113	a	i	$(x-3)^2 - 9 + (y+2)^2 - 4 + 4 = 0 \Rightarrow (x-3)^2 + (y+2)^2 = 9$  C (3, -2)	M1(AO1.1)  A1(AO1.1)  [2]	$(x \pm 3)^2$ and $(y \pm 2)^2$ seen (or implied by correct answer) or one correct coordinate Accept $x = 3$ and $y = -2$	
					<p><b>Examiner's Comments</b></p> <p>This proved to be a good question for nearly all candidates with the vast majority correctly completing the square (twice) to find the coordinates of the centre of the circle. When errors occurred, these were nearly always down to sign errors inside the two brackets.</p>	
		ii	$r = 3$	B1(AO1.1)  [1]	Allow if stated explicitly in (a)(i) but not written down in (a)(ii) www for $r$	B0 if $r = \pm 3$ only
					<p><b>Examiner's Comments</b></p> <p>Nearly all candidates stated the radius of the circle correctly in either part (i) or part (ii) of (a).</p>	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$(x-3)^2 + (kx-3+2)^2 = 9$ or $x^2 + (kx-3)^2 - 6x + 4(kx-3) + 4 = 0$  $(1+k^2)x^2 + (-6-2k)x + 1 = 0$  $(-6-2k)^2 - 4(1+k^2)(1)$  $36 + 24k + 4k^2 - 4 - 4k^2 < 0$ $\Rightarrow 32 + 24k < 0$  $k < -\frac{4}{3}$	M1*(AO3.1a)  A1(AO1.1)  M1dep*(AO3.1a)  M1dep*(AO2.1)  A1(AO2.2a)  [5]	Substitutes the correct equation of the line into any form of their equation of the circle  oe (all terms on the same side – may not be factorised but should be simplified to 5 terms)  Correct explicit use of discriminant on their 3TQ to get an expression in $k$ only  Discriminant $< 0$ and simplify to the form $ak + b < 0$ (oe)  Fully correct (no additional values)	Each M is dependent on the previous Ms  Condone lack of equal to 0  Condone equals or incorrect inequality  $a$ and $b$ non-zero  Or exact equivalent	
				<b>Examiner's Comments</b>  Although examiners did note that some candidates left this part blank, many candidates did start this problem correctly by eliminating $y$ and obtaining		

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p> <math>(1 + k^2)x^2 + (-6 - 2k)x + 1 = 0</math>, but then many did not realise that if the line and circle did not intersect (or touch) then the discriminant of this quadratic in <math>x</math> would be negative. Of those that did realise this fact it was only arithmetical or algebraic errors that meant that the candidates could not determine that <math>k &lt; -\frac{4}{3}</math>.         </p>
			<b>Total</b>	<b>8</b>	

Question	Answer/Indicative content	Marks	Part marks and guidance	
114	DR $x = \frac{24}{3 - \sqrt{5}} = \frac{24(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})}$ $= \frac{24(3 + \sqrt{5})}{9 - 3\sqrt{5} + 3\sqrt{5} - 5} = \frac{24(3 + \sqrt{5})}{4}$ $= 18 + 6\sqrt{5}$	M1(AO 1.1)  A1(AO 1.1)  A1(AO 1.1)  [3]	Multiplying numerator and denominator by $3 + \sqrt{5}$ or $-3 - \sqrt{5}$  Correct simplified denominator  Final answer <b>cao</b> , therefore final answer of only $6(3 + \sqrt{5})$ is A0  <u>Examiner's Comments</u>  This question required detailed reasoning so candidates were expected to show full working to indicate how their final answer had been obtained. The answer needed to be in the form requested.  Those who went straight to the answer without intermediate steps had clearly just used their calculator and scored 0/3.  Candidates need to be aware that detailed reasoning questions will want substantial evidence of methods.	<b>Alternative:</b> <b>M1</b> Correct method to solve simultaneous equations formed from equating expressions to $a + b\sqrt{5}$  <b>A1</b> Either $a$ or $b$ correct  <b>A1</b> Both correct

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p><b>Exemplar 1</b></p> $x = \frac{2+3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$ $\Rightarrow \frac{2+(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$ $\Rightarrow \frac{2+3+\sqrt{5}}{9-5}$ $\Rightarrow \frac{5+\sqrt{5}}{4}$ <p><math>\therefore x = 1\frac{1}{4} + \frac{\sqrt{5}}{4}</math> // <math>a=18, b=6</math> //</p> <p>This candidate has made clear each step of their solution for full credit.</p>
			<b>Total</b>	<b>3</b>	

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Question		Answer/Indicative content	Marks	Part marks and guidance
115	a	$5[x^2 - 4x] + 3$  $= 5[(x - 2)^2 - 4] + 3$  $= 5(x - 2)^2 - 17$	$p = 5$  $(x - 2)^2$  $r = -17$  B1(AO 1.1)  B1(AO 1.1)  B1(AO 1.1)  [3]	<p>No marks until attempt to complete the square. Must be of the form <math>5(x \pm a)^2 \pm \dots</math></p> <p><b>Examiner's Comments</b></p> <p>This was frequently well done, with errors in <math>r</math> being most common, e.g. <math>5(x - 2)^2 - 4 + 3</math>. We occasionally had attempts involving <math>(x - 10)^2</math>.</p> <p> AFL</p> <p>The use of calculators to check answers should be encouraged. The quadratic solve function or finding the minimum point on a plotted graph would have confirmed the values for <math>q</math> and <math>r</math>.</p> <p>Notice that there was no specific command word used in the question to indicate any working had to be seen; candidates that knew how to obtain the completed square form from their calculator would score full marks here with correct answer only.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance	
	b	Minimum point (2, -17)	B1ft(AO 1.1) B1ft(AO 1.1)  [2]	Follow through their $-q$ Follow through their $r$	Or by differentiation  <u>Examiner's Comments</u>  'State' here really should indicate to candidates that the answer can just be written down from what they have already done, but many resorted successfully to differentiation.
	c	$x = 2$	B1ft(AO 1.1)  [1]	Follow through their $x$ coordinate in part (b)	<u>Examiner's Comments</u>  'State' again, with the same implication as in part (c).  This part was the least successful. A significant number of candidates found the tangent $y = -17$ rather than the requested normal $x = 2$ .
		<b>Total</b>	<b>6</b>		

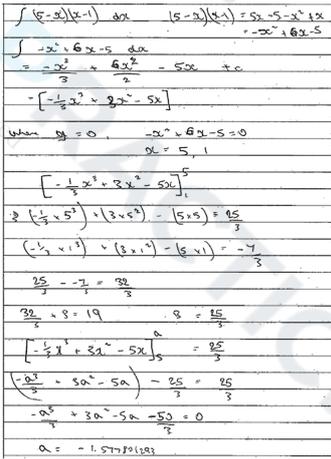
Question		Answer/Indicative content	Marks	Part marks and guidance		
116	a	DR $6(1 - \sin^2 \theta) = \frac{\sin \theta}{\cos \theta}(\cos \theta) + 4$ $6 - 6\sin^2 \theta = \sin \theta$ $+4 \Rightarrow 6\sin^2 \theta + \sin \theta - 2 = 0$	M1(AO 3.1a)  A1(AO 2.1)  [2]	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;">           Correct use of both <math>\cos^2 \theta = 1 - \sin^2 \theta</math> and <math>\tan \theta = \frac{\sin \theta}{\cos \theta}</math>            AG         </td> <td style="width: 50%; padding: 5px;">           Must show sufficient working to justify the given answer         </td> </tr> </table> <p><b><u>Examiner's Comments</u></b></p> <p>Another question requiring detailed reasoning, so examiners scrutinised the notation carefully and expected precision in presentation. For example, <math>\cos^2 \theta</math> or just <math>\cos</math> was penalised as was the omission of '=' in the final answer. To score fully candidates needed to make explicit use of trigonometric identities, for example just replacing <math>\tan \theta \cos \theta</math> with <math>\sin \theta</math> without justification of <math>\frac{\sin \theta}{\cos \theta} \cos \theta</math> was not considered sufficient reasoning.</p>	Correct use of both $\cos^2 \theta = 1 - \sin^2 \theta$ and $\tan \theta = \frac{\sin \theta}{\cos \theta}$ AG	Must show sufficient working to justify the given answer
Correct use of both $\cos^2 \theta = 1 - \sin^2 \theta$ and $\tan \theta = \frac{\sin \theta}{\cos \theta}$ AG	Must show sufficient working to justify the given answer					



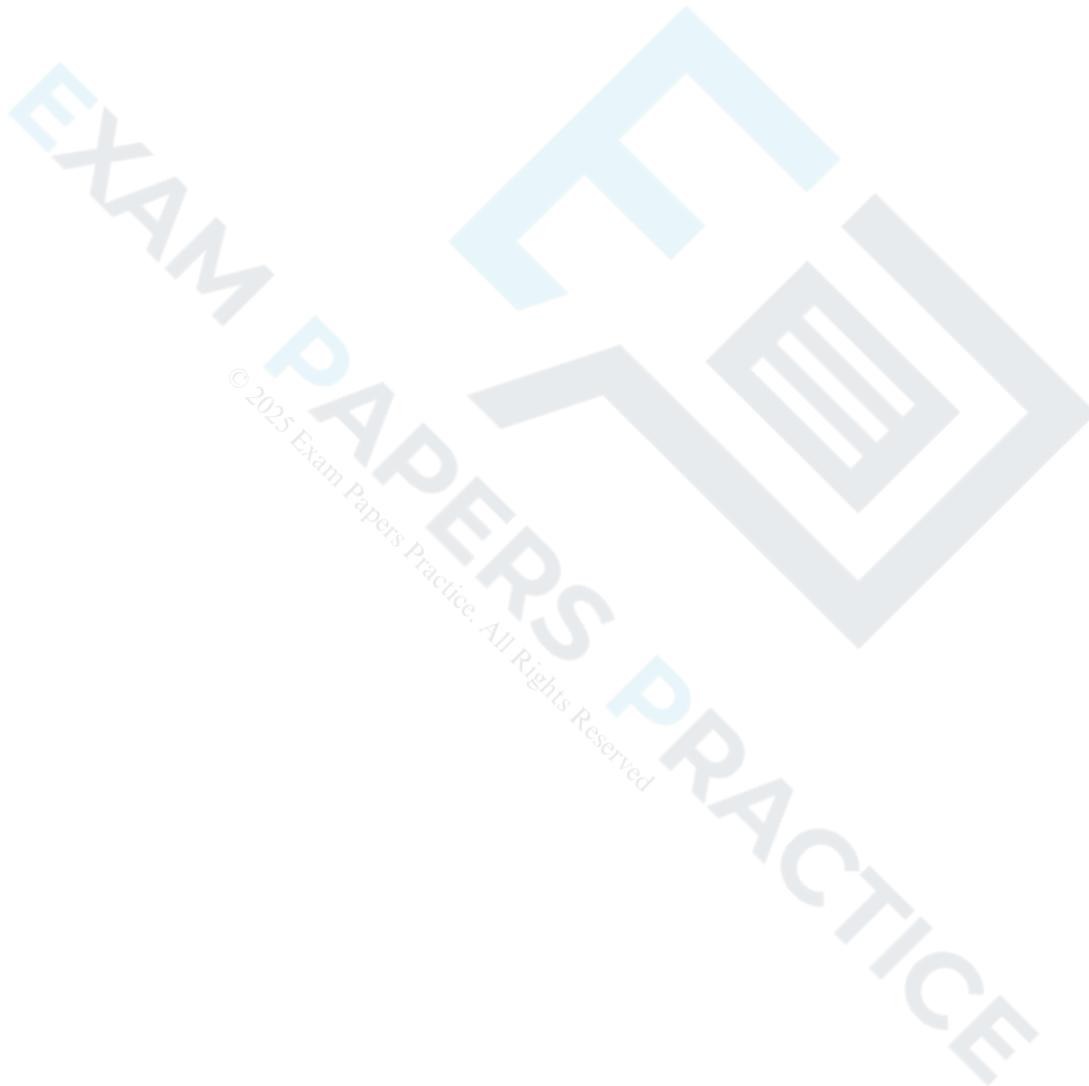
Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>intervals which are <math>\theta</math> <math>&lt; 42, 138 &lt;</math> <math>\theta &lt; 210, \theta</math> <math>&gt; 330</math> (so max. 3/5)</p> <p><b>Examiner's Comments</b></p> <p>A fully correct solution was rare. Many candidates recognised the quadratic but just found values for <math>\theta</math> and made no effort to relate them to the given graph and form inequalities. Because this is a detailed reasoning question we expected to see how the quadratic was solved and penalised those who just produced the two roots from their calculator. Often use of inequality signs was slack and recognising which quadrants angles need to be in was sometimes confused.</p> <p><b>Exemplar 3</b></p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Let <math>\sin \theta = x</math></p> <p><math>\Rightarrow 6x^2 + x - 2 = 0</math></p> <p><math>(2x - 1)(3x + 2) = 0</math></p> <p><math>x = \frac{1}{2} \quad x = -\frac{2}{3}</math></p> <p><math>\sin \theta = \frac{1}{2} \quad \sin \theta = -\frac{2}{3}</math></p> <p><math>\theta = \sin^{-1}(\frac{1}{2}) = \theta = \sin^{-1}(-\frac{2}{3})</math> ✓ ✓</p> <p><math>\theta = 30^\circ \quad \theta = 41.8^\circ</math> ✓ SA ✓</p> <p><math>\theta = 30^\circ, 150^\circ, 221.8^\circ, 318.2^\circ</math> ✓ TC ✓</p> <p>With the graph: <math>\theta &lt; 30^\circ</math> or <math>150^\circ &lt; \theta &lt; 221.8^\circ</math> or <math>\theta &gt; 318.2^\circ</math></p> </div> <p>Notice that the 'detailed reasoning' does not restrict the use of the quadratic solve function, in this exemplar the candidate has 'Let <math>\sin \theta = X</math>' with <math>6X^2 + X - 2 = 0</math> and the substitution of the roots back in terms of <math>\theta</math> to provide the required reasoning for this part of</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>their answer. This candidate avoided any potential inequality pitfalls when identifying the required three regions by utilising the initial graph provided in the question.</p> <p><b>Exemplar 4</b></p> $6 \cos^2 \theta > \tan \theta \cos \theta + 4$ $6 \sin^2 \theta + \sin \theta - 2 > 0$ <p>let <math>\sin \theta = x</math></p> $6x^2 + x - 2 > 0$ $6x^2 + 3x - 2 > 0$ $3x^2 + x - 2 = 0$ $x = \frac{1}{3} \text{ or } -\frac{2}{3}$ <p></p> $x < -\frac{2}{3} \text{ or } x > \frac{1}{3}$ $\sin \theta < -\frac{2}{3} \text{ or } \sin \theta > \frac{1}{3}$ $\theta < -41.81 \text{ or } \theta > 30$ <p><math>\ominus \text{ (A)}</math></p> <p>T.C. <math>180 - 41.81</math>  <math>= 221.81^\circ</math>  <math>360 - 41.81</math>  <math>= 318.19^\circ</math>  <math>\theta &lt; 221.81 \text{ or } \theta &lt; 318.19^\circ \text{ or } \theta &gt; 30 \text{ or } \theta &gt; 150^\circ</math></p> <p>As in many cases, the subsequent marks are not achieved due to confusions with the inequalities.</p>
			<b>Total</b>	<b>7</b>	



Question	Answer/Indicative content	Marks	Part marks and guidance
			<p>algebraic accuracy was vital. Although the integral between 1 and 5 could be done on the calculator, many did not do this and evaluated it incorrectly, often because <math>(5 - x)(x - 1)</math> was expanded incorrectly. Then the integral between 5 and a needed to be carefully done before putting everything together with the correct signs. Many cubics were wrong, so getting an exact solution proved impossible. Some obtained the correct cubic fortuitously from wrong working (which was penalised) but if all was fine by this stage we then wanted a clear explanation for the rejection of the negative root and an exact answer as requested.</p> <p><b>Exemplar 5</b></p>  <p>Exemplar 5 is a typical solution, failing because the signs have not been</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					appreciated and/or the request for an exact answer ignored.	
			<b>Total</b>	<b>8</b>		



Question		Answer/Indicative content	Marks	Part marks and guidance		
118	a	$\log_2 x^2 = \log_2(kx-1) + 3$  $\log_2 \left( \frac{x^2}{kx-1} \right) = 3$  $\frac{x^2}{kx-1} = 2^3$  $x^2 = 8(kx-1)$ $x^2 - 8kx + 8 = 0$	B1 (AO 1.2)  M1* (AO 2.1)  Dep*M1 (AO 1.1)  A1 (AO 1.1)  [4]	Using $a \log b(b^a)$  Re-arranging and correctly combining both log terms  Correctly remove logs  AG	Or re-write 3 as $\log_2 8$ and then combining e.g. $2 \log_2 x = \log_2 (8(kx-1))$  $x^2 = 8(kx-1)$  Must show sufficient working to justify the given answer (i.e. at least one more line of working from previous M mark)	
				<p><b>Examiner's Comments</b></p> <p>Many candidates here scored only one mark for <math>\log_2 x^2</math>. A confidence with using the laws of logarithms was not often seen, e.g. <math>\log_2(x^2 - kx + 1)</math> regularly stated. A few introduced <math>3^2</math> rather than <math>2^3</math>. This was 'Show that' question, with the answer given in the question, so we expected sufficient justification in the written response.</p>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$b^2 - 4ac = 0 \Rightarrow (-8k)^2 - 4(1)(8) = 0$  $k = (\pm) \frac{1}{\sqrt{2}}$  $k = \frac{1}{\sqrt{2}} \Rightarrow x = 2\sqrt{2}$  $k = -\frac{1}{\sqrt{2}} \Rightarrow x = -2\sqrt{2}$  and as $\log_2 x$ is only defined for $x > 0$ so $x \neq -2\sqrt{2}$	M1 (AO 3.1a)  A1 (AO 1.1)  A1 (AO 2.2a)  A1 (AO 3.2b)  [4]	Use of $b^2 - 4ac = 0$  oe exact  <b>BC</b> oe exact  <b>BC</b> oe statement for rejection of negative value of $x$ (allow decimal argument)	Or state equation must be of the form $(x+p)^2 = 0$ with $p^2 = 8$ so $x = (\pm)2\sqrt{2}$  reject  $x = -2\sqrt{2}$ with valid reason	
		<b>Total</b>	<b>8</b>	<b>Examiner's Comments</b>  Many recognised the need for ' $b^2 - 4ac$ ', but this was sometimes carelessly applied to give $64k - 32 = 0$ , $-8k^2 - 32 = 0$ or some such, and many more accurate tries only gave the positive value of $k$ . Often the attempt stopped at the value for $k$ with no effort to find $x$ . Presumably candidates thought $k$ was the root. We expected a valid reason for why the negative value for $x$ was rejected. Simply stating $x > 0$ was not sufficient as they needed to explain that $\log_2 x$ is only defined for $x > 0$ (not that $\log_2 x$ cannot be negative).		

Question	Answer/Indicative content	Marks	Part marks and guidance
119	$2^{2k}-1$ or $4^k-1$ (where $k$ is an integer $> 1$ )  $= (2^k)^2 - 1$ $= (2^k - 1)(2^k + 1)$  $(2^k + 1) > 1$ and $k > 1$ , hence $(2^k - 1) > 1$ Hence $(2^k - 1)(2^k + 1)$ is the product of two integers, both $> 1$ , and hence $2^{2k} - 1$ is not prime	M1(AO3. 1a)  A1(AO2. 1)  M1(AO1. 1)  A1(AO2. 2a)  [4]	<p>or <math>2^{2k+2} - 1</math> or <math>2^{2k+4} - 1</math> Allow <math>2^{2n} - 1</math>   <math>= (2^{k+1} - 1)(2^{k+1} + 1)</math> oe or <math>(2^{k+2} - 1)(2^{k+2} + 1)</math> oe</p> <p>Both statements needed</p> <p><b>Examiner's Comments</b></p> <p>Not many candidates answered this successfully, although many at least began by writing <math>2^{2k} - 1</math> and thereby gained a mark. Some gained another mark by factorising this expression correctly, but very few went on to show that neither factor was equal to 1.</p> <p>Many candidates misinterpreted the question to mean: "Prove that the following</p> <p><b>Induction:</b> Assume <math>2^k - 1</math> is <math>\div</math> by 3 (<math>k</math> even) M1 Let <math>2^k - 1 = 3p</math> (<math>p</math> integer)   <math>2^{k+2} - 1 = 4 \times 2^k - 1</math> M1  <math>= 4 \times 2^k - 4 + 3</math>  <math>= 4(2^k - 1) + 3</math>  <math>= 4 \times 3p + 3</math> A1            which is <math>\div</math> by 3            When <math>k = 2</math>: <math>2^2 - 1 = 3</math> so <math>\div</math> by 3            A1            Hence true for all even <math>n</math>. Claim true</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>statement is not true: <math>n</math> is an even number greater than 2 <math>\Rightarrow 2^n - 1</math> is prime."</p> <p>They then disproved this statement by assuming it to be true and quoting a numerical example that contradicted it and claimed to have answered the question using Proof by Contradiction.</p> <p>Some candidates calculated <math>2^2 - 1</math>, <math>2^4 - 1</math>, <math>2^6 - 1</math>, <math>2^8 - 1</math> etc., up to about <math>2^{20} - 1</math> and noted that in each case the result is not prime, and incorrectly assumed that the result has been proved by "Proof by exhaustion".</p> <p>A few candidates (who presumably were studying Further Maths) gave a proof by induction, proving that, for <math>n</math> even and greater than 2, <math>2^n - 1</math> is divisible by 3.</p>
			<b>Total</b>	<b>4</b>	

Question	Answer/Indicative content	Marks	Part marks and guidance		
120	<p>DR</p> $u^2 = 36x^4 + 12x^3 + x^2$ <p>So <math>36x^4 + 12x^3 + 7x^2 + x - 2 = u^2 + 6x^2 + x - 2</math></p> <p>Equn reduces to <math>u^2 + u - 2 = 0</math></p> <p><math>u = -2</math> or <math>1</math></p> <p><math>6x^2 + x = -2</math> has no roots because <math>\Delta = 1 - 48 &lt; 0</math></p> <p><math>6x^2 + x = 1</math> has roots</p> <p><math>x = \frac{1}{3}</math> or <math>-\frac{1}{2}</math></p>	<p>M1(AO3. 1a)</p> <p>A1(AO2. 1)</p> <p>A1(AO1. 1)</p> <p>B1(AO3. 2a)</p> <p>A1(AO1. 1)</p> <p>[5]</p>	<p><math>(36x^4 + 12x^3 + 7x^2 + x - 2) \div (6x^2 + x) = 6x^2 + x + 1</math> rem <math>-2</math></p> <p><math>((6x^2 + x)(6x^2 + x + 1) = 2)</math></p> <p><math>u(u + 1) = 2</math></p> <p><b>BC</b></p> <p>Must see correct calc'n for <math>\Delta</math> and "<math>&lt; 0</math>" for their quadratic equation</p> <p><b>BC</b> Ignore any answers from <math>u = -2</math></p> <p>SC If M1 gained but incorrect or inadequate method &amp; correct answers: M1A0A0B0 A1</p>	<p>M1 for attempt <math>(6x^2 + x)^2</math> or attempt <math>\div</math> LHS by <math>(6x^2 + x)</math>, at least 2 terms correct or obtain any correct equn in terms of <math>x</math> and <math>u</math></p> <p>or <math>x = \frac{-1 \pm \sqrt{47}i}{12}</math> given instead of "no roots" etc</p> <p>Otherwise correct ans without any correct working: no marks. (Because DR)</p>	<p><b>Examiner's Comments</b></p> <p>Candidates who started by finding <math>u^2</math> were generally successful. Many other candidates started by dividing the left hand side of the given equation by <math>6x^2 + x</math>. Those who handled the remainder correctly generally went on to be successful. Many, however,</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>after dividing and finding a remainder of 2, said that</p> $36x^4 + 12x^3 + 7x^2 + x - 2 = (6x^2 + x)(6x^2 + x + 1) - \frac{2}{6x^2 + x}$ <p>, which is incorrect.</p> <p>Some candidates arrived correctly at the two equations <math>6x^2 + x - 1 = 0</math> and <math>6x^2 + x + 2 = 0</math>. However, because this question contains the instruction "In this question you must show detailed reasoning", candidates needed to <u>show</u> (not just state) that the second equation has no real roots. Many candidates lost a mark by failing to do this.</p> <p>This question did prove to be challenging for many candidates, with a range of attempts involving a great deal of algebraic manipulation, in terms of <math>x</math> and <math>u</math>. Some used their calculator to find the roots directly, and then used these in a "fudged" attempt to provide some justification. Whilst the use of calculators to check work should be encouraged, this question required the explicit use of the substitution to be seen in order to gain credit.</p>
			<b>Total</b>	<b>5</b>	

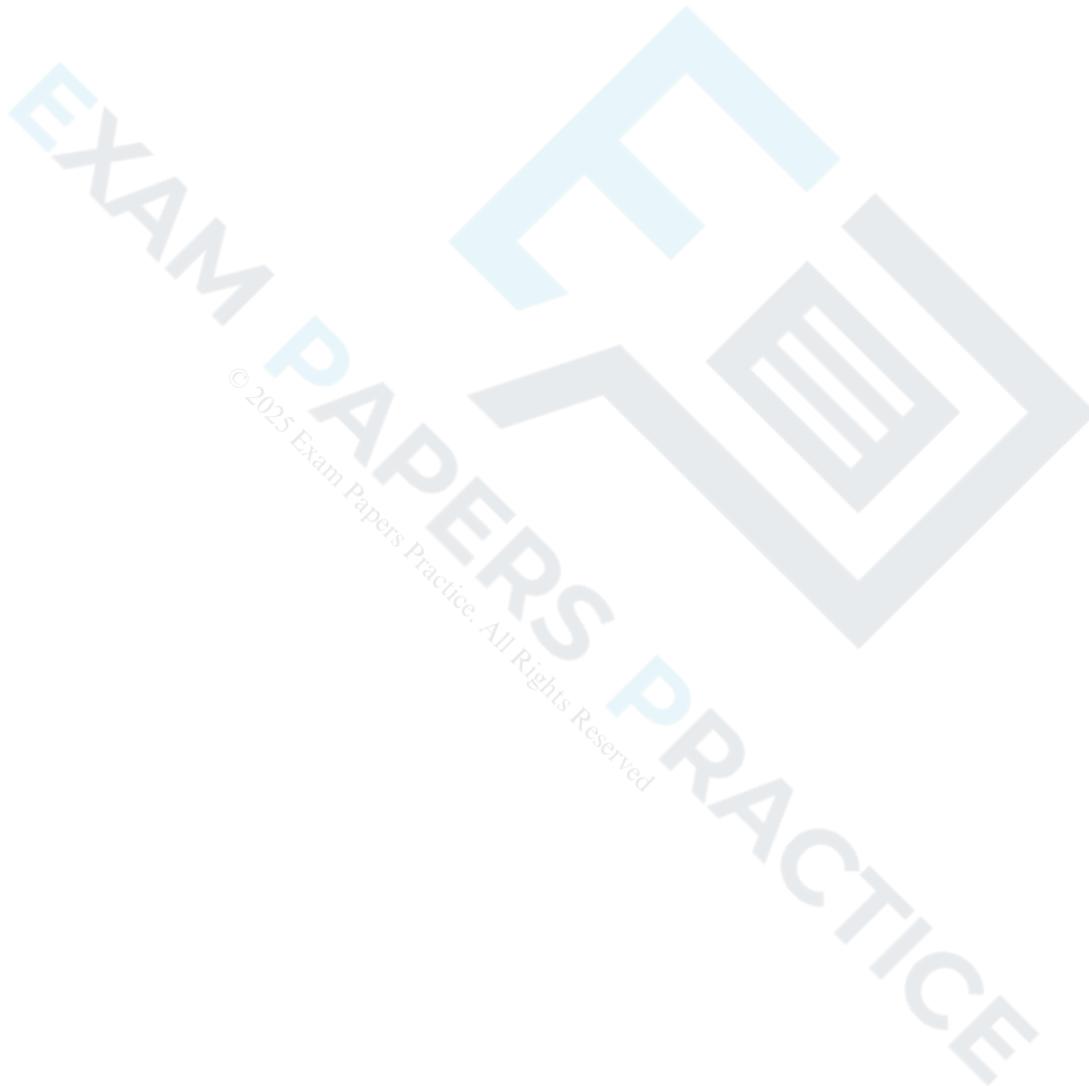
Question		Answer/Indicative content	Marks	Part marks and guidance
121		$\text{DR } (x^{\frac{3}{2}} + 1)(x^{\frac{3}{2}} - 8) = 0$  $x^{\frac{3}{2}} = -1 \text{ gives no solution}$  $x^{\frac{3}{2}} = 8 \text{ or } x^3 = 64$  $x = 4$	M1(AO1.1)  B1(AO3.2b) A1(AO1.1) A1(AO1.1) [4]	Attempt factors $(x^{\frac{3}{2}} \pm k)$ or $(y \pm k)$ of form or $(y + 1)(y - 8)$ or $y = -1$ or $y = 8$ AND $y = x^{\frac{3}{2}}$ so i Allow $(x + 1)(x - x = x^{\frac{3}{2}})$ seen 8) AND Condone inadequate reason  $y = 8$ not enough for this mark  Indep of previous A1
		<b>Total</b>	<b>4</b>	

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Question	Answer/Indicative content	Marks	Part marks and guidance
122	Example method: $3(4 - 2y)y + (4 - 2y)^2 = -14$ $12y - 6y^2 + 16 - 16y + 4y^2 = -14$ $2y^2 + 4y - 30 = 0$ $y^2 + 2y - 15 = 0 \quad (y + 5)(y - 3) = 0$ $y = -5 \text{ or } 3$ eg $x + 2(-5) = 4$ and $x + 2 \times 3 = 4$  Points of intersection are (14, -5) & (-2, 3)	M1(AO3.1a)  A1(AO1.1)  A1(AO1.1) M1(AO2.2a)  A1(AO1.1) [5]	Other methods score similarly Attempt substitution from (ii) into (i) or (i) into (ii) and obtain equation in one letter  Obtain correct eg $x^2 - 12x - 3 = 0$ quadratic equation Method may not be seen cao or $x = 14$ or $-2$  Substitute or their $x$ their $y$ values values into either equation or $x = 14, y = -5; x = -2, y = 3$ . Must be clearly paired cao <u>Examiner's Comments</u>  Many candidates answered this question well. However, others attempted to substitute from one equation into the other by simply setting the two left-hand-sides equal to each other. This did not achieve the desired result of obtaining an equation in one letter only.
	<b>Total</b>	<b>5</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
123	a	$6a^3$	B1 (AO1.1) B1 (AO1.1) [2]	Obtain 6  Obtain $a^3$	B1 only for $\pm 6a^3$
	b	$(64b^3)^{\frac{1}{3}} = 4b$ or $(4b^4)^{-\frac{1}{2}} = \frac{1}{2b^2}$  $2b^{-1} \frac{2}{b}$ or	B1 (AO1.1a)  B1 (AO1.1) [2]	Correct simplification of either term  Correct final answer	Allow $(2b^2)^{-1}$ for the second term
	c	$9^{3c} = 3^{6c}$  $27^{2c} = 3^{6c}$  $7 \times 36^c - 4 \times 3^{6c} = 3 \times 3^{6c}$  $= 3^{6c+1}$	B1 (AO1.1)  M1 (AO1.1a)  A1 (AO3.1a)  A1 (AO1.1) [4]	Either $9^c$ or $27^{2c}$ correct as a power of 3 (or 729) Attempt to write the other one of $9^c$ and $27^{2c}$ with the same base Combine to obtain correct single term  Obtain correct final answer	Ignore coefficient Index must be simplified Ignore coefficient Allow unsimplified index B2 for $27^{2c} = 9^{3c}$ Allow equiv eg $3 \times 729^c$ or $3 \times 27^{2c}$ or $3 \times 9^{3c}$ Must be single power of 3  OR B1 $9^{2c}(7 \times 9^c - 4 \times 3^{2c})$ M $9^{2c}(7 \times 3^{2c} - 4 \times 3^{2c})$ 1 $9^{2c} \times 3 \times 3^{2c}$ A13 $\times 27^{2c}$ A13 $3^{6c+1}$

Question	Answer/Indicative content	Marks	Part marks and guidance
			Total
		8	





Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$\frac{dA}{dr} = 4\pi r - 32000\pi r^{-2}$ $4\pi r - 32000\pi r^{-2} = 0$ $r^3 = 8000$ $r = 20$ Surface area = $2400\pi$ $\text{cm}^2/7540 \text{ cm}^2$ $\frac{d^2A}{dr^2} = 4\pi + 64000\pi r^{-3}$ when $r = 20$ , $\frac{d^2A}{dr^2} = 12\pi$ (or 37.7) $\frac{d^2A}{dr^2} > 0$ , hence minimum	M1 (AO1.1a) M1 (AO3.1b)  A1 (AO1.1) A1 (AO1.1)  M1 (AO2.1)  A1 (AO2.2a) [6]	Attempt diff erentiation  Equate derivative to 0 and attempt to solve for $r$ (or $h$ ) Obtain correct $r$ , units not needed Obtain correct $A$ , units not needed Attempt method to justify minimum, including substitution or consider ation of sign Correct conclusion, with justification, from correct working	Both powers decrease by 1 $-32000\pi h^{-2} + \pi\sqrt{16000}h^{-1}$  $h^{\frac{3}{2}} = \sqrt{64000}$  Allow exact or decimal (3sf or better) Could also test first derivative, or $A$ , on both sides of $r = 20$  If second derivative is evaluated, it must be correct (condone truncated decimal of 37.6)	<p><b>Examiner's Comments</b></p> <p>The first part of this Question required candidates to show the derivation of the given equation. On questions such as this, where the request is to 'show that' a given answer is correct, it is important that sufficient</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					<p>detail is given to provide a convincing solution. Examiners expected to see an equation for the volume, an expression for the total surface area and then clear substitution and manipulation to arrive at the given answer. In the second part of the question, a surprisingly common error was to find the radius but then omit to find the minimum surface area. Candidates should review a question to make sure that it is fully answered before moving on; other strategies such as highlighting or underlining key words and phrases within a question may also be helpful.</p>	
			<b>Total</b>	<b>10</b>		

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Question		Answer/Indicative content	Marks	Part marks and guidance	
125	a	$\frac{d}{dx}(e^{2x}) = 2e^{2x}$ $6xe^{2x} + (2 + 3x^2)(2e^{2x})$ $e^{2x}(6x^2 + 6x + 4)$	B1 (AO1.1) M1 (AO1.1a) A1 (AO1.1) [3]	Seen anywhere in solution Attempt product rule Obtain any fully correct expression	Could expand first
	b	$e^{2x} > 0$ for all $x$  $6x^2 + 6x + 4 = 6\left(x + \frac{1}{2}\right)^2 + \frac{5}{2}$  minimum value is $\frac{5}{2}$ so $> 0$ for all $x$  Gradient $e^{2x}(6x^2 + 6x + 4) > 0$ for all $x$ so it is increasing for all $x$	B1 (AO2.1)  M1 (AO2.1)  A1 (AO2.4)  A1 (AO2.4) [4]	Attempt to show that their 3 term quadratic factor is $> 0$ for all $x$  Full justification that quadratic factor is always positive  Justify increasing function as $f'(x) > 0$ for all $x$	B0 if clearly considering $f(x)$ or $f''(x)$ and not $f'(x)$ Complete the square or consider discriminant Could be multiple or fraction of their quadratic Show minimum point $> 0$ , or show that quadratic is always positive  <b>OR</b> B1 $e^{2x} \neq 0$ M1 Show that quadratic $\neq 0$ (detail required) M1 Show gradient

Question	Answer/Indicative content	Marks	Part marks and guidance
			<div style="display: flex; justify-content: space-between;"> <div style="width: 60%;"> <p>is positive at one point, as part of attempt to show <math>f'(x) \neq 0</math></p> <p>A1 Conclude that gradient must hence be positive for all <math>x</math>, so increasing function</p> </div> <div style="width: 35%; border-left: 1px solid black; padding-left: 5px;"> <p><b>Examiner's Comments</b></p> <p>The second part of this question proved to be challenging; some candidates did not seem familiar with the definition of an increasing function and were unsure how to proceed whereas others stated that the first derivative needed to be positive, but then continued by considering the second derivative instead. Of the minority who attempted the correct method, many gave vague statements such as 'the quadratic is always positive' but with no evidence to support this. In a 'show that' question, sufficient detail must be provided so as to be convincing. Equally, statements must be fully correct so stating that is never negative lacks the required precision. 2ex</p> </div> </div>

Question	Answer/Indicative content	Marks	Part marks and guidance
			Total
		7	



Question		Answer/Indicative content	Marks	Part marks and guidance		
126	i	$\alpha < 2$ $0 = 1.5\alpha + 2$  $\alpha = -\frac{4}{3}$  $-\frac{4}{3} < \alpha < 2$	B1 (AO3.1a) M1 (AO3.1a)  A1 (AO1.1)  A1 (AO1.1) [4]	Allow for answer of form $k < \alpha < 2$ Attempt to find value of $a$ at their $x$ intersection  Obtain $-\frac{4}{3}$ (condone any inequality sign, an equals sign or no sign) Correct final inequality	eg Use equation of line to find $\alpha$ Use gradient of line to find $\alpha$ Use a point of intersection of the two lines = their 1.5 Equate two points of intersection and solve for $\alpha$ Square both sides and link discriminant to 0 Question is 'determine' so method required for this value of $\alpha$  Formal set notation not required	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	ii	$2x - 3 = ax + 2$ $x = \frac{5}{2 - a}$ $3 - 2x = ax + 2$ $(2 + a)x = 1$ $x = \frac{1}{2 + a}$	<p>B1 (AO1.1)</p> <p>M1 (AO1.1a)</p> <p>A1 (AO1.1) [3]</p>	<p>Correct point of intersection – allow any exact equiv</p> <p>Attempt to solve linear equation with <math>2x</math> and <math>ax</math> of different signs Correct point of intersection – allow any exact equiv</p>	<p>OR M1 – square both sides and attempt to solve – as far as substituting into quadratic formula A1 A1 for each root Method may be seen in (i), only credit if answers seen in (ii) Max of 2 out of 3 if additional roots as well.</p>	
		<b>Total</b>	<b>7</b>			

**Examiner's Comments**

The most successful candidates in Question (b)(i) were those who considered a graphical approach (which the diagram was included to help with) as opposed to those who embarked on the more standard techniques for a question involving moduli.

Question			Answer/Indicative content	Marks	Part marks and guidance		
127	a	i	DR $x^2 + (mx + 2)^2 - 10x - 14(mx + 2) + 64 = 0$  $x^2 + m^2x^2 + 4mx + 4 - 10x - 14mx - 28 + 64 = 0$  $(m^2 + 1)x^2 - 10(m + 1)x + 40 = 0$ A.G.	M1 (AO1.1a)  A1 (AO1.1) [2]	Substitute eqn of tangent into eqn of circle  Expand and tidy to given answer, including '=' 0' in final answer	Could work backwards, eliminating $m$ to obtain equation of circle AG so unsimplified expansion needs to be seen	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	ii	$100(m + 1)^2 - 160(m^2 + 1) = 0$  $60m^2 - 200m + 60 = 0$  $(3m - 1)(m - 3) = 0$  $m = 3, m = \frac{1}{3}$  $y = 3x + 2$	M1* (AO3.1a) A1 (AO1.1) M1d* (AO1.1a)  A1 (AO1.1) [4]	Use $b^2 - 4ac = 0$  Obtain correct equation Attempt to solve quadratic  Obtain correct equation	M1 only awarded when '=' soi Any correct 3 term equation DR so method for solving the quadratic must be shown SC B1 for correct equation if roots not justified A0 if second equation also given OR (for first 2 marks) M1 – Attempt two equations in $m$ and $x$ (eg use lengths and gradients) and eliminate one variable A1 – correct quadratic in $m$ or $x$	

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$\text{radius} = \sqrt{10}, PC = 5\sqrt{2}$  $PA = PB = 2\sqrt{10},$  $AB = 4\sqrt{2}$ $\tan\left(\frac{1}{2}APB\right) = \frac{1}{2}$  $\tan APB = \frac{1}{1 - \frac{1}{4}}$  $\tan APB = \frac{4}{3}$	M1 (AO3.1a)  A1 (AO1.1)  M1 (AO3.1a)  A1 (AO1.1) [4]	Attempt (at least 2) useful lengths  Obtain a correct related trig ratio  Attempt $\tan APB$  Obtain $\frac{4}{3}$	NB points of intersection are (2, 8) and (6, 4)  $\cos APB = \frac{3}{5}$ , from cosine rule  DR so need to see use of identity or relevant triangle to find $\tan APB$ From explicit, exact, working <b>OR</b>  M2 – attempt $\frac{\pm m \pm n}{1 \pm mn}$ with their values for $m$ and $n$ A1FT – correct $\frac{m - n}{1 + mn}$ for their values of $m$ and $n$  A1 – obtain $\tan APB = \frac{4}{3}$	
				<b>Examiner's Comments</b>		
				This question was the only		

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>'In this question you must show detailed reasoning' question on the paper, and candidates were expected to show a detailed and complete analytical method. Part (a)(i) was generally very well answered, but the remainder of the question proved challenging. Part (ii) is a good example of where candidates have to link different topic areas – they are considering a quadratic equation and the fact that the line is tangential should indicate that the required strategy is to equate the discriminant to 0.</p> <p>Candidates struggled with the non-standard aspect of part (b), although some were able to gain some credit for working out relevant lengths and a relevant trigonometric ratio. Of the few that were so far successful, they were expected to continue to work in exact values to justify the request in the question, which not all candidates appreciated.</p>
			<b>Total</b>	<b>10</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
128	a	Stationary point at $(-3, 2) \Rightarrow b = 3, c = 2$	B2 (AO1.1) (AO1.1) [2]	B1 for one correct	
	b	Translated curve is $y = \alpha(x + (b - 4))^2 + c$  $-18 = \alpha(3 + (b - 4))^2 + c \Rightarrow \alpha = \dots$  $\alpha = -5$	M1* (AO2.1)  M1dep* (AO1.1) A1 (AO1.1) [3]	Translates y by eith $\begin{pmatrix} \pm 4 \\ 0 \end{pmatrix}$ onl er y  Substitutes $(3, -18)$ into translated curve and finds a value for $\alpha$	Possible to translate $(3, -18)$ by $\begin{pmatrix} \pm 4 \\ 0 \end{pmatrix}$ and use original curve to find $\alpha$
		<b>Total</b>	<b>5</b>		
129		DR $(x - 2)(x + 3) > 0$  $x < -3, x > 2$  $\{x: x < -3\} \cup \{x: x > 2\}$	M1(AO3.1a) A1(AO1.1) A1(AO1.1)  B1ft(AO2.5) [4]	Attempt factorise  Correct factors <u>and</u> $> 0$ or $y > 0$  Any notation. Allow "and", "or", comma etc But NOT $-3 < x > 2$ $x < -3, x > 2$ seen, with no working or muddled working; SC B1 Allow ( ) instead of { } Allow $x \in (-\infty, -3) \cup (2, \infty)$ ft their factors, dep two separate ranges.	
		<b>Total</b>	<b>4</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
130	a	$\frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta}$ $= \frac{\frac{2 \tan \theta}{1 - \tan^2 \theta} + \tan \theta}{1 - \frac{2 \tan \theta}{1 - \tan^2 \theta} \tan \theta}$ $= \frac{2 \tan \theta + \tan \theta(1 - \tan^2 \theta)}{(1 - \tan^2 \theta) - 2 \tan^2 \theta}$ $= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \text{AG}$	<p><b>B1 (AO 2.1)</b></p> <p><b>B1(AO 2.1)</b></p> <p><b>M1(AO 2.1)</b></p> <p><b>A1(AO 2.1)</b></p> <p><b>[4]</b></p>	<p>Correct expression</p> <p>Correct expression in terms of <math>\tan \theta</math></p> <p>Attempt to simplify</p> <p>Complete proof to show given identity convincingly</p>	<p>As far as clearing fractions</p>	



Question		Answer/Indicative content	Marks	Part marks and guidance		
131		$(8p - 3) - 9p = 5p - (8p - 3)$ $p = 3$ $a = 27, d = -6$ $\frac{n}{2}[2(27) + (n-1)(-6)] = -1512$ $n^2 - 10n - 504 = 0 \Rightarrow (n - 28)(n + 18) = 0$ $n = 28$ only	<b>M1 (AO 3.1a)</b> <b>A1 (AO 1.1)</b> <b>A1FT (AO 1.1)</b>  <b>M1 (AO 2.1)</b>  <b>M1 (AO 1.1)</b>  <b>A1 (AO 2.2a)</b>  <b>[6]</b>	Setting up an equation to find $p$  Using their value of $p$ to calculate $a$ and $d$  Setting up an equation using the correct formula for the sum of an AP equated to $-1512$  Expand and attempt to solve 3-term quadratic equation in $n$  This mark should be withheld if $n = -18$ appears as part of the final answer	Allow a single sign error    Solving of 3-term quadratic may be done BC	
		<b>Total</b>	<b>6</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance	
132	a	$\frac{1}{e^{2x}} = \frac{x}{x^2 + 3} \Rightarrow x^2 + 3 = xe^{2x}$ $x^2 + 3 = xe^{2x} \Rightarrow x^2 + 3 - xe^{2x} = 0$	<p>M1 (AO 1.1)</p> <p>A1 (AO 2.2a)</p> <p>[2]</p>	<p>Equate expressions and cross-multiply (to remove fractions)</p> <p>AG – sufficient working must be shown to indicate that result has been derived correctly</p>	

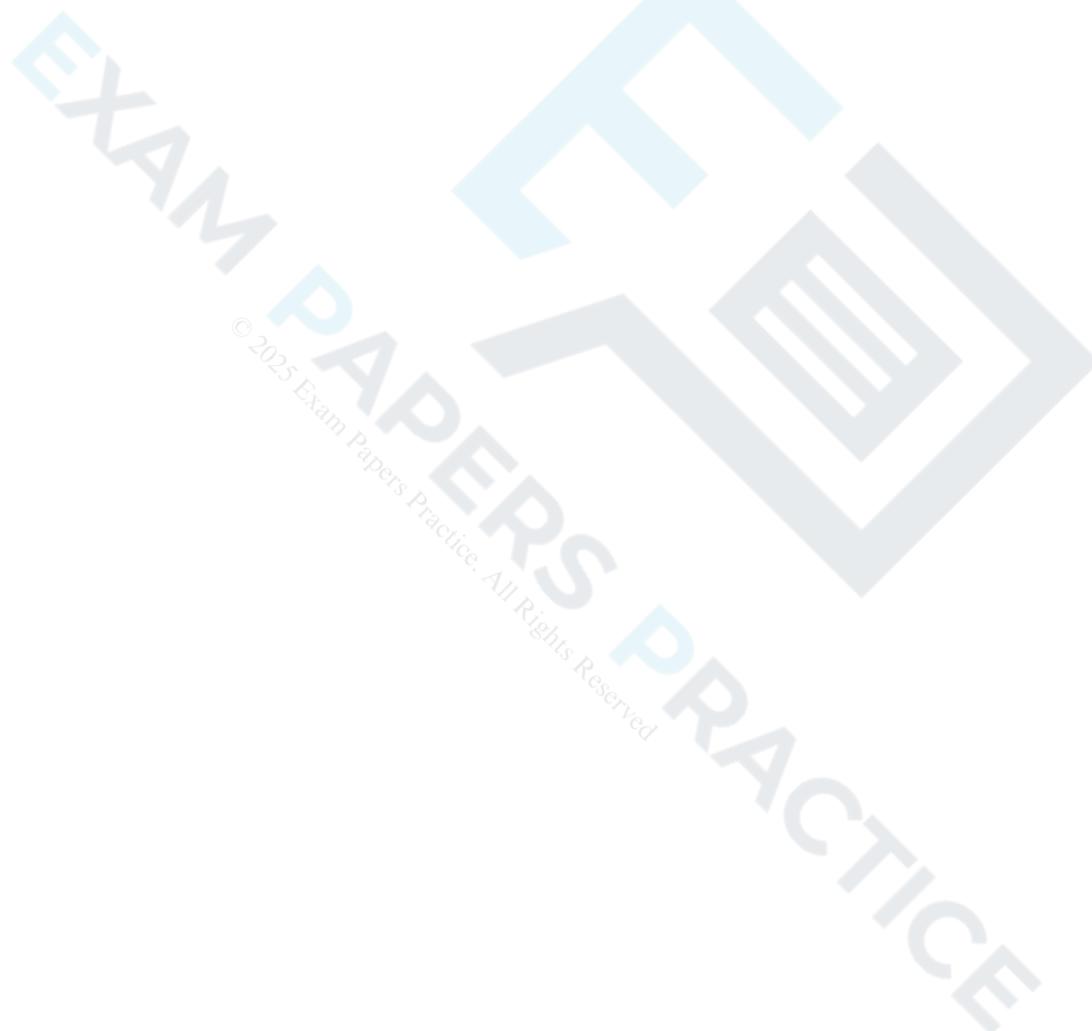
Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$\frac{d}{dx}(xe^{2x}) = e^{2x} + 2xe^{2x}$ $h'(x) = 2x - e^{2x} - 2xe^{2x}$ $x_{n+1} = x_n - \frac{x_n^2 + 3 - x_n e^{2x_n}}{2x_n - e^{2x_n} - 2x_n e^{2x_n}}$ $x_{n+1} = \frac{2x_n^2 - x_n e^{2x_n} - 2x_n^2 e^{2x_n} - x_n^2 - 3 + x_n e^{2x_n}}{2x_n - e^{2x_n}(1 + 2x_n)}$ $x_{n+1} = \frac{x_n^2 - 2x_n^2 e^{2x_n} - 3}{2x_n - e^{2x_n}(1 + 2x_n)} = \frac{x_n^2(1 - 2e^{2x_n}) - 3}{2x_n - (1 + 2x_n)e^{2x_n}}$	<p><b>M1 (AO 1.1)</b></p> <p><b>A1 (AO 1.1)</b></p> <p><b>M1* (AO 2.1)</b></p> <p><b>M1dep* (AO 1.1)</b></p> <p><b>A1 (AO 2.2a)</b></p> <p>[5]</p>	<p>Attempt at product rule for <math>xe^{2x}</math> – expression must be of the form <math>\pm e^{2x}(1 \pm kxe^{2x})</math></p> <p>Correct application of NR with their <math>h'(x)</math></p> <p>Correctly combining as a single fraction and expanding any brackets in numerator</p> <p><b>AG</b> – sufficient working must be shown as the answer is given</p>		
	c	<p>From graph eg <math>x_1 = 1</math></p> <p><math>x_2 = 0.83195181\dots, x_3 = 0.7754682\dots</math></p> <p><math>x_4 = 0.77016\dots, x_5 = 0.77011\dots</math></p> <p><math>\alpha = 0.770</math> (correct to 3 dp)</p>	<p><b>M1 (AO 3.1a)</b></p> <p><b>A1 (AO 1.1)</b></p> <p><b>A1 (AO 1.1)</b></p> <p>[3]</p>	<p>Suitable starting value chosen</p> <p>At least two correct applications of NR</p>		

Question		Answer/Indicative content	Marks	Part marks and guidance		
	d	DR $fg(x) = f(e^{-2x}) = \frac{e^{-2x}}{(e^{-2x})^2 + 3}$ $2e^{-4x} - 13e^{-2x} + 6 = 0$ $k = e^{-2x}$ $(2k - 1)(k - 6) = 0$ $e^{-2x} = \frac{1}{2} \Rightarrow x = -\frac{1}{2} \ln\left(\frac{1}{2}\right)$ $e^{-2x} \neq 6 \text{Q it is given that } x \geq 0$	M1 (AO 2.1)  A1 (AO 1.1)  M1* (AO 3.1a) M1dep* (AO 1.1) A1 (AO 2.2a) A1 (AO 2.3)  [6]	Attempt at $fg(x)$ – need not be simplified  Correct equation – fractions must be removed and powers simplified  Substitute for $e^{-2x}$ (or equivalent)  Attempt to solve resulting quadratic  www oe  Correct statement or equivalent that $e^{-2x}$ cannot be greater than 1	Or equivalent  <b>Alternative y:</b> Factorise into two brackets containing $e^{-2x}$ M2	
		<b>Total</b>	<b>16</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance		
133	a	$\frac{1}{2}xy \sin 60 = \sqrt{3}(x+y)$ Must have $xy$ , not $ab$  May be implied by next line  $\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)xy = \sqrt{3}(x+y)$ oe  $\Rightarrow 4x + 4y = xy$ (AG)	<b>M1(AO1.1a)</b>  <b>A1(AO2.1)</b>  <b>[2]</b>	Verification method: eg substitute $4x + 4y$ for $xy$ in $\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)xy = \sqrt{3}(x+y)$ M 1  obtain $\sqrt{3}(x+y) =$ $\sqrt{3}(x+y)$ A1	Condone incorrect use of brackets eg $x+y(\sqrt{3})$ , if next line shows intention is correct  Must show sufficient working to justify the given answer	
<p style="text-align: center;"><b>Examiner's Comments</b></p> <p>Most candidates answered this question correctly. A few used a verification method, rather than a linear proof, which was accepted if the mathematical argument was clear. In general it is worth noting that for questions involving "Show that . . ." using the given answer as a starting point is not guaranteed to gain full marks. Some candidates made errors <math>\frac{1}{2}xy \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}x \times \frac{\sqrt{3}}{4}y</math>, but attempted to "fudge" the remaining steps to arrive at the given equation.</p>						

Question	Answer/Indicative content	Marks	Part marks and guidance	
b	<p>Angle <math>ABC</math> is <math>90^\circ</math></p> <p><math>\cos 60 = \frac{x}{y}</math> or <math>\frac{x}{y} = \frac{1}{2}</math> or <math>y = 2x</math></p> <p><math>4x + 4(2x) = x(2x)</math> <small><math>4(\frac{y}{2}) + 4y = y \times \frac{y}{2}</math> oe</small></p> <p>oe or</p> <p><math>(\Rightarrow x(x - 6) = 0)</math> or <math>\Rightarrow y(y - 12) = 0)</math></p> <p><math>x = 6, y = 12</math></p>	<p><b>B1*(AO3.1a)</b></p> <p><b>B1(AO1.1)</b></p> <p><b>M1(AO1.1)</b></p> <p><b>A1(AO1.1)</b></p> <p><b>[4]</b></p>	<p>May be implied by correct trig statement or any correct <math>x, y</math> equn eg</p> $\frac{x}{\sin 30} = \frac{y}{\sin 90}$ $\sqrt{3}(x + 2x) = 0$ $\frac{1}{2} \times x \times 2x \times \frac{\sqrt{3}}{2}$ <p>r</p> <p>No need to consider <math>x = 0</math> or <math>y = 0</math></p>	<p>Possibly on diagram</p> <p>Dep B1*</p> <p>Obtain a correct equation in <math>x</math> or <math>y</math> only</p> <p><b>Examiner's Comments</b></p> <p>Many candidates recognised that angle <math>ABC = 90^\circ</math> and some of these went on to obtain a correct trigonometrical equation involving <math>x</math> and <math>y</math>. (Some gave it in the form <math>\frac{x}{\sin 30^\circ} = \frac{y}{\sin 90^\circ}</math> rather than the simpler <math>\cos 60^\circ = \frac{x}{y}</math>. Many then went on to use the result from the above part and obtained the correct responses. Others started again, substituting <math>y = 2x</math> into <math>\frac{1}{2}xy \frac{\sqrt{3}}{2} = (x + y)\sqrt{3}</math>. Some of these candidates made subsequent algebraic or arithmetical errors. Candidates who used the result proved in the above part were more often successful.</p> <p>The command word 'Determine' was used in this question; indicating that</p>

Question			Answer/Indicative content	Marks	Part marks and guidance	
					<p>justification should be given for any results found. A trial and improvement approach gained only one mark if the correct responses were seen.</p> <p>There were also many candidates who attempted wholly incorrect methods using, for example <math>\pi r^2</math>.</p>	
			<b>Total</b>	<b>6</b>		



Question	Answer/Indicative content	Marks	Part marks and guidance		
134	<p>DR</p> $3y + x = 7 \Rightarrow m = -\frac{1}{3}$ <p>Gradient of line <math>l</math> through centre perpendicular to given tangent is 3</p> <p>Equation of <math>l</math> is <math>y + 2 = 3(x - 3)</math></p> $\left. \begin{array}{l} 3y + x = 7 \\ y = 3x - 11 \end{array} \right\} x = 4, y = 1$ $r^2 = (4 - 3)^2 + (1 + 2)^2$ $(x - 3)^2 + (y + 2)^2 = 10$ <p><b>Alternative solution</b></p> $(x - 3)^2 + (y + 2)^2 = r^2$ $r^2 = (7 - 3y - 3)^2 + (y + 2)^2$ $10y^2 - 20y + (20 - r^2) = 0$ $(-20)^2 - 4(10)(20 - r^2)$ $(-20)^2 - 4(10)(20 - r^2) = 0 \Rightarrow r^2 = \dots$ $(x - 3)^2 + (y + 2)^2 = 10$	<p>B1 (AO2.1)</p> <p>B1FT (AO1.2)</p> <p>M1* (AO3.1a)</p> <p>M1dep* (AO1.1)</p> <p>M1 (AO1.1)</p> <p>A1 (AO2.2a)</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>M1</p> <p>A1 [6]</p>	<p>Correctly uses the result <math>m_1 m_2 = -1</math> for their <math>m</math></p> <p>Correct equation of the form <math>y + 2 = M(x - 3)</math> with any non-zero <math>M</math></p> <p>Solves simultaneous equations to find point of intersection</p> <p>Correct method to find distance (or distance squared) between centre and points of intersection</p> <p>Correct lhs of equation of circle</p> <p>Substitutes given line into equation of circle</p> <p>Correct equation in <math>r</math> and either <math>x</math> or <math>y</math> (<math>10x^2 - 80x + (250 - 9r^2)</math>)</p>	<p>M0 if no working shown but allow following M1 and A1 if earned</p> <p>Any equivalent form</p> <p>Tidied form needed</p>	

Question			Answer/Indicative content	Marks	Part marks and guidance		
					= 0) Correct use of the discriminant on their three-term quadratic in either $x$ or $y$ Set discriminant equal to zero and solve for $r$ or $r^2$ Correct rhs of equation of circle		
			<b>Total</b>	<b>6</b>			

Question		Answer/Indicative content	Marks	Part marks and guidance
135	a	$(V =) \frac{1}{2}x(2x)y = x^2y$  Slant height of the roof is  $x\sqrt{2}$  $(S =) 2xy + 2\left(\frac{1}{2}(2x)x\right) + 2(yx\sqrt{2})$  $y = \frac{600 - 2x^2}{2x(1 + \sqrt{2})} \Rightarrow V = x^2\left(\frac{300 - x^2}{x(1 + \sqrt{2})}\right)$  $V = x(300 - x^2)\left(\frac{1 - \sqrt{2}}{(1 + \sqrt{2})(1 - \sqrt{2})}\right)$  $V = x(300 - x^2)\left(\frac{1 - \sqrt{2}}{1 - 2}\right) = (\sqrt{2} - 1)x(300 - x^2)$	B1 (AO1.1)  B1 (AO3.1a) M1* (AO2.1)  M1dep* (AO3.3)  M1 (AO1.1)  A1 (AO2.2a) [6]	Correct simplified expression for the volume  Allow  $\sqrt{2x^2}$  Attempt at surface area with at least three of the five faces correct - can be unsimplified Rearranges and makes y the subject and substitutes to give an expression for V in terms of x only Rationalises the denominator correctly $a = 2, b = -1$  <u>Examiner's Comments</u>  Part (a) - $V = 2x^2y$ was seen and often the required result for the surface area was incorrect, although many candidates managed to find at least 3 correct faces and gain M1. Replacing y in V and the subsequent rationalising

Question		Answer/Indicative content	Marks	Part marks and guidance	
				was rarely seen.	
	b	$\frac{dV}{dx} = k(300 - 3x^2)$ $(k)(300 - 3x^2) = 0 \Rightarrow x = \dots$ $x = 10 \text{ cm}$	M1* (AO1.1) A1 (AO1.1)  M1dep* (AO1.1)  A1 (AO1.1) [4]	M1 for attempt at differentiation – both powers reduced by 1  Sets  $\frac{dV}{dx} = 0$ and attempts to find $x$	Allow full marks ft their values of $a$ and $b$
	c	$V = 828\text{cm}^3$	B1 (AO3.4) [1]	cao  828.42712 47...	
	d	$V$ (or $y$ ) must be positive or $300 - x^2 > 0$ so $x$ cannot exceed $\sqrt{300}$ cm	M1 (AO3.5b) A1 (AO1.1) [2]	Explanation for constraint on $x$ Correct value; accept e.g. 17.3 or better	
		<b>Total</b>	<b>13</b>		

Question	Answer/Indicative content	Marks	Part marks and guidance
136	<p>Numerator <math>\equiv (x + 1)(x - 2)(2x + 3)</math></p> <p>Denominator <math>\equiv (x + 1)(x - 2)</math></p> <p>Ans: <math>2x + 3</math></p> <p><b>Alternative method</b></p> $  \begin{array}{r}  \phantom{x^2 - x - 2} \phantom{2x^3 + x^2 - 7x - 6} \phantom{2x + 3} \\  x^2 - x - 2 \overline{) 2x^3 + x^2 - 7x - 6} \\  \underline{2x^3 - 2x^2 - 4x} \phantom{- 6} \\  3x^2 - 3x - 6 \\  \underline{3x^2 - 3x - 6} \\  - \\  2x + 3  \end{array}  $	<p>M1 (AO3.1a)</p> <p>M1 (AO1.1)</p> <p>M1 (AO1.1)</p> <p>M1 (AO1.1)</p> <p>[4]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Attempt factorise numerator into 3 linear factors</p> <p>Attempt factorise denominator into 2 linear factors</p> <p>"cancel" two common factors in num &amp; denom</p> <p>Allow no mention of <math>x \neq -1</math> or <math>x \neq 2</math> conditions.</p> <p>NB correct answer with no working or partial working: 4 marks</p> <p>SC: Answer <math>x + \frac{3}{2}</math> B3</p> <p>Attempt long division by <math>x^2 - x - 2</math> or by <math>x + 1</math> or by <math>x - 2</math></p> <p>Obtain "2x" in quotient</p> <p>Obtain "+ 3" in quotient</p> <p>Answer <math>2x + 3</math> clear (not just in the division sum)</p> <p><b>Examiner's Comments</b></p> <p>Some candidates used their calculators to solve the equation obtained by setting the numerator equal to 0. This was an acceptable method where the command word is 'Simplify', however, many candidates kept a factor of</p> <p><math>\left(x + \frac{3}{2}\right)</math> and did not replace it by the correct factor of <math>(2x + 3)</math>. Some candidates attempted long division of the cubic numerator by one</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>of the factors of the quadratic denominator, however sign errors were often seen.</p> <p>It is worth noting that the neatest way to factorise a cubic is probably to use the factor theorem (with perhaps the calculator solve function used to identify values), rather than either of the two methods mentioned above. Had the question contained the “detailed reasoning” instruction, or the command word "determine" then a written mathematical argument to confirm the factors would need to be seen.</p>
			<b>Total</b>	<b>4</b>	

Question	Answer/Indicative content	Marks	Part marks and guidance
137	DR $3\sin^4 \phi + \sin^2 \phi - 4 = 0$  $(3\sin^2 \phi + 4)(\sin^2 \phi - 1) = 0$  $\sin^2 \phi = -\frac{4}{3}$ or $\sin^2 \phi = 1$ (or $\sin \phi = \pm 1$ )  $\sin^2 \phi = -\frac{4}{3}$ is impossible  $\phi = \sin^{-1}(\pm 1)$  $\phi = \frac{1}{2}\pi, \frac{3}{2}\pi$ No extras within range  Allow "correct" extras outside range	 B1 (AO2.1)  B1 (AO1.1)  B1 (AO2.3)  M1 (AO1.1)  A1 (AO2.2a) [5]	 Attempt to solve QE in $\sin^2 \phi$ or QE in $u$ with $u = \sin^2 \phi$ so <b>Must see method</b> May be implied from $x = \sin^2 \phi$ and  $x = -\frac{4}{3}$ or 1  oe, eg $\sin \phi \neq \sqrt{-\frac{4}{3}}$ Not with  incorrect reason, eg  $\sin^2 \phi = \frac{16}{9} > 1$  solve for $\phi$ Allow $\phi = \sin^{-1}(1)$ , may be implied  Both. dep $\sin^2 \phi = -\frac{4}{3}$ and $\sin^2 \phi = 1$ (or $\sin \phi = \pm 1$ ) seen  SC $\phi = \frac{1}{2}\pi, \frac{3}{2}\pi$ with no working: B2
	<b>Total</b>	<b>5</b>	

Question	Answer/Indicative content	Marks	Part marks and guidance	
138	$f(x) = 2[x^2 + 3x] = 2\left[\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]$ $f(x) = 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} \Rightarrow -\frac{9}{2}$ <p>Range of <math>f(x) \geq -\frac{9}{2}</math></p> <p><math>f(x)</math>:</p>	<p>M1 (AO3.1a)</p> <p>A1 (AO1.1)</p> <p>A1 (AO1.2) [3]</p>	<p>Attempt to complete the square – must have <math>2\left(x + \frac{3}{2}\right)^2 \pm \dots</math> ve</p> <p>Correct completing the square and selection of n of cao – or equivalent notation e.g. set notation <math>\{f(x): f(x) \geq -\frac{9}{2} \text{ or } -\frac{9}{2}, \infty</math> [ ]</p> <p><b>Examiner's Comments</b>            There were some excellent responses to this question, although some candidates dropped marks for errors in the notation used.</p> <p>Those candidates that used completing the square in part (a) were generally more successful than those that used calculus. A significant number did not gain the final accuracy mark due to incorrectly stating <math>x \geq -\frac{9}{2}</math> rather than correctly stating the range of the function <math>f(x) \geq -\frac{9}{2}</math>.</p> <p>Some candidates appeared to misunderstand the concept of range, finding</p>	<p>Or <math>f'(x) = 4x + 6</math> and solves equal to zero Or finds x-coordinate of vertex Or by differentiation</p> <p>Allow in terms of f, y but not x</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>the roots by factorising to <math>2x(x + 3)</math> to then incorrectly conclude that <math>x \geq 3</math></p> <p> <b>Misconception</b></p> <p>A common misconception in functions is with domain and range. For a function <math>f(x)</math>, emphasis needs to be placed on the fact that domain refers to the inputs, the <math>x</math> values, and the range refers to the outputs and must be stated in terms of <math>f</math> or <math>f(x)</math> (with the use of <math>y</math> condoned but strictly should only be used if defined in the question or by the candidate as <math>y = f(x)</math>).</p>
			<b>Total</b>	<b>3</b>	

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Question		Answer/Indicative content	Marks	Part marks and guidance
139	a	DR  $4y + 4x \frac{dy}{dx} = 4x + 16y \frac{dy}{dx} - 9$ $4x \frac{dy}{dx} - 16y \frac{dy}{dx} = 4x - 4y - 9 \Rightarrow \frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$	M1 (AO1.1)  A1 (AO1.1)  A1 (AO2.1) [3]	$4xy = 2(x^2 + 4y^2) - 9x$  For correct differentiation of either LHS or RHS, even if not in an equation  AG (at least one line of working from correct differentiation to given answer)  <u>Examiner's Comments</u> This was the only 'In this question you must show detailed reasoning' on this paper. Examiners will be very focused on the individual steps of the calculation on these questions, signified by the 'DR' notation in the mark scheme.  Part (a) was generally well answered, candidates appeared to be well prepared for this type of question.  A common error was to start the mathematical argument with $\frac{dy}{dx} =$ followed by the correct set of differentiated terms with

Question		Answer/Indicative content	Marks	Part marks and guidance	
				equals sign. Some candidates dropped marks due to careless algebra manipulation.  AFL Candidates often attempt to force their algebraic manipulation to result in the given answer, ignoring fundamental rules and changing signs without justification. Method marks may not be given if the working appears random rather than simply including a clear mistake.	
	b	<b>DR</b> (At P) $4x - 16y = 0$  $x = 4y \Rightarrow 16y^2 = 2(16y^2 + 4y^2) - 36y$ $24y^2 - 36y = 0$ $y(2y - 3) = 0 \Rightarrow y = \frac{3}{2}$ $P(6, \frac{3}{2})$ (At Q) $4x - 4y - 9 = 0$ $\Rightarrow 4x(x - \frac{9}{4}) = 2x^2 + 8(x - \frac{9}{4})^2 - 9x$ $4x^2 - 24x + 27 = 0$  $Q(\frac{3}{2}, -\frac{3}{4})$ only  $PQ^2 = (6 - \frac{3}{2})^2 + (\frac{3}{2} - (-\frac{3}{4}))^2$ $PQ = \frac{9}{4}\sqrt{5}$	M1* (AO3.1a)  M1dep* (AO2.1)  A1 (AO1.1) M1* (AO3.1a) M1dep* (AO2.1)  A1 (AO3.2a)  M1 (AO1.1) A1 (AO2.2a) [3]	Forms two-term quadratic equation in y or x (if correct $x^2 - 6x = 0$ )  Forms three-term quadratic equation in y or x (if correct $16y^2 - 24y - 27 = 0$ )  Correct implies distance formula for their P and	y = 0 not required  A0 if $(\frac{9}{2}, \frac{9}{4})$ is given as possible coordinates for Q Dependent on all previous M

Question	Answer/Indicative content	Marks	Part marks and guidance		
			<p>Q www</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td style="padding: 2px;">marks</td> <td style="padding: 2px;"><math>k = \frac{9}{4}</math></td> </tr> </table> <p><b>Examiner's Comments</b>            In part (b), those candidates that recognised the correct gradients of the tangents at <math>P</math> and <math>Q</math> generally went on to use their values for the coordinates to find <math>PQ</math> and score many of the 5 method marks. However poor algebra manipulation cost accuracy marks.</p> <p style="text-align: center;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 6px;">?</span> <b>Misconception</b> </p> <p>Candidates appeared comfortable with the horizontal gradient at <math>Q</math> being found from <math>\frac{dy}{dx} = 0</math> and so using the numerator</p> <p><math>&gt;4x - 4y - 9 = 0</math>. However, a number of candidates used <math>\frac{dy}{dx} = 1</math> rather than</p> <p><math>\frac{dy}{dx} = \infty</math> for the vertical gradient at <math>P</math>.</p> <p>Candidates should recognise a vertical tangent line means that the denominator is zero and so in this question <math>4x - 16y = 0</math>.</p> <p>A number of candidates solved the quadratic equation generated to find <math>Q</math>, but then chose the coordinates <math>\left(\frac{9}{2}, \frac{9}{4}\right)</math>.</p>	marks	$k = \frac{9}{4}$
marks	$k = \frac{9}{4}$				

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>Candidates should use all information given in the question, including any sketched curves. In this case, candidates were expected to recognise that</p> <p>the two results for <math>\frac{dy}{dx} = 0</math> were valid, but that the</p> <p>diagram indicated which horizontal tangent was to be used. Note that those</p> <p>candidates that used</p> <p><math>\left(\frac{9}{2}, \frac{9}{4}\right)</math>, to find their <math>PQ</math> with no</p> <p>wrong working would have scored 6 out of 8. There was not an explicit mark given for discounting the unwanted quadratic solutions on this question, however it is good practice and may be penalised in other 'DR' questions.</p>
			<b>Total</b>	<b>11</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance
140	a	$\sin\left(2\theta + \frac{\pi}{4}\right) = 3 \cos\left(2\theta + \frac{\pi}{4}\right)$ $\sin 2\theta \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos 2\theta = 3$ $\cos 2\theta \cos \frac{\pi}{4} - 3 \sin 2\theta \sin \frac{\pi}{4}$ $4 \sin 2\theta = 2 \cos 2\theta$ $2 \frac{\sin 2\theta}{\cos 2\theta} = 1 \Rightarrow \tan 2\theta = \frac{1}{2}$ ALT: $\tan\left(2\theta + \frac{\pi}{4}\right) = 3$ $\frac{\tan 2\theta + 1}{1 - \tan 2\theta} = 3 \Rightarrow \tan 2\theta + 1 = 3(1 - \tan 2\theta)$ $\tan 2\theta = \frac{1}{2}$	M1(AO 1.1)E  A1(AO 1.1)E A1(AO 2.2a)E  [3] B1 M1  A1	<p>Correct use of compound angle formulae at least once</p> <p>Not from incorrect working  <b>AG</b> – at least one step of intermediate working seen</p> <p>Correct use of compound angle formula for tan and removal of fraction</p> <p><b>Examiner's Comments</b>            Candidates were equally split in how to tackle this part. Approximately half expanding the brackets (using the correct compound-angle formulae) while the other half <math>\tan\left(2\theta + \frac{\pi}{4}\right) = 3</math> before re-wrote as expanding. Both approaches proved equally successful in obtaining the expected result.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$\tan 2\theta = \frac{1}{2} \Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{1}{2}$ $\tan^2 \theta + 4 \tan \theta - 1 = 0$ $\tan \theta = -2 \pm \sqrt{5}$ $-2 + \sqrt{5} > 0 \text{ so}$ $\tan \theta = -2 + \sqrt{5} \text{ gives acute angle}$ $\therefore \tan \theta = -2 - \sqrt{5}$	<p>M1*(AO 3.1a)E</p> <p>Dep*M1(AO 1.1)E</p> <p>A1(AO 1.1)C</p> <p>A1(AO 2.3)A</p> <p>A1(AO 2.2a)A</p> <p>[5]</p>	<p>Double angle formula for <math>\tan 2\theta</math></p> <p>Rearranges correctly to form 3-term quadratic in <math>\tan</math></p> <p>BC – One correct exact value</p> <p>Explicit rejection and reason for rejection</p> <p>This value only</p>	<p>Allow one sign slip in formula</p>	<p><b>Examiner's Comments</b>          Many candidates did not read the question carefully and began their response by          writin <math>2\theta = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = \dots</math>eve          g n          though the question specifically asked for the exact value of <math>\tan \theta</math>. Of those candidates that used the correct double-angle formula for <math>\tan 2\theta</math> many derived the correct three-term quadratic in <math>\tan</math> with most correctly stating that  <math>\tan \theta = -2 \pm \sqrt{5}</math>. However, a significant proportion ended their response here and did not go on to determine the</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					exact value of $\tan\theta$ given that $\theta$ is an obtuse angle. A full solution needed the explicit realisation that since $-2 + \sqrt{5} > 0$ ,  $\tan\theta = -2 + \sqrt{5}$ would not give an obtuse angle and therefore the only valid solution was $\tan\theta = -2 - \sqrt{5}$
			Total	8	

Question		Answer/Indicative content	Marks	Part marks and guidance
141	a	$2(x^2 - 6x + 11.5)$ $2((x - 3)^2 + 11.5 - 9)$ $2(x - 3)^2 + 5$	B1 (AO 1.1a) B1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1)  [4]	or $a = 2$ or $b = -3$  $23 - 2(\text{their } b)^2$ or $c = 5$  <u>Examiner's Comments</u>  Most candidates answered this correctly. A few found $a$ and $b$ correctly but made an error in finding $c$ . This most frequently came from an incorrect first step such as $2(x - 3)^2 + 11.5 - 9$ or $2(x - 3)^2 + 23 - 9$ .

Question		Answer/Indicative content	Marks	Part marks and guidance		
	b	$2(x + 3)^2 + 5$ is always +ve or $2(x + 3)^2 + 5 > 0$ or $2(x + 3)^2 + 5 \geq 5$ Hence no real roots	<b>B1f</b> <b>(AO 1.1)</b>  <b>[1]</b>	or $2(x + 3)^2 = -5$ , which is impossible or "+ve quadratic" and min on $y = 5$ or "+ve" quadratic; TP at (3, 5). Both Hence no real roots Must use (a), not use D	$2(x + 3)^2 + 5 = 0$ $\Rightarrow x = \sqrt{\text{neg}}$ or $x + 3 = \sqrt{\text{neg}}$ ft their (a) (a & $c > 0$ )	
	c	$2(x - 3)^2 = 2(x^2 - 6x + 9)$ $k = 18$	<b>M1</b> <b>(AO 1.1a)</b> <b>A1</b> <b>(AO 2.2a)</b>  <b>[2]</b>	or $12^2 - 8k = 0$		
		<b>Total</b>	<b>7</b>			

**Examiner's Comments**

Most candidates answered this correctly. Some of those who discussed the turning point lost a mark because they merely stated that it is a minimum, rather than showing that this is so.

**Examiner's Comments**

This question was very well answered.