1 Find the coordinates of the points on the curve $y = \frac{1}{3}x^3 + \frac{9}{x}$ at which the tangent is parallel to the line $y = 8x + 3$. **[10]**

2 The quadratic equation $kx^2 + (3k - 1)x - 4 = 0$ has no real roots. Find the set of possible values of k. **[7]**

3

(i) The line joining the points $(-2, 7)$ and $(-4, p)$ has gradient 4. Find the value of p. **[3]**

(ii) The line segment joining the points $(-2, 7)$ and $(6, q)$ has mid-point $(m, 5)$. Find m and q. **[3]**

(iii) The line segment joining the points $(-2, 7)$ and $(d, 3)$ has length $2\sqrt{13}$. Find the two possible values of d. **[4]**

4

(i) Solve the simultaneous equations

$$y = 2x^2 - 3x - 5, \quad 10x + 2y + 11 = 0.$$

[5]

(ii) What can you deduce from the answer to part (i) about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$? **[1]**

5 Solve the equations

(i) $3^n = 1$, **[1]**

(ii) $t^{-3} = 64$, **[2]**

(iii) $(8p^6)^{\frac{1}{3}} = 8$. **[3]**

6

(i) Solve the equation $x^2 - 6x - 2 = 0$, giving your answers in simplified surd form.

[3]

(ii) Find the gradient of the curve $y = x^2 - 6x - 2$ at the point where $x = -5$.

[2]

7 The curve $y = (1 - x)(x^2 + 4x + k)$ has a stationary point when $x = -3$.

(i) Find the value of the constant k.

[7]

(ii) Determine whether the stationary point is a maximum or minimum point.

[2]

(iii) Given that $y = 9x - 9$ is the equation of the tangent to the curve at the point A, find the coordinates of A.

[5]

8

(i) Sketch the curve $y = 2x^2 - x - 6$, giving the coordinates of all points of intersection with the axes.

[5]

(ii) Find the set of values of x for which $2x^2 - x - 6$ is a decreasing function.

[3]

(iii) The line $y = 4$ meets the curve $y = 2x^2 - x - 6$ at the points P and Q. Calculate the distance PQ.

[4]

9

(i) Express $3x^2 + 9x + 10$ in the form $3(x + p)^2 + q$.

[3]

(ii) State the coordinates of the minimum point of the curve $y = 3x^2 + 9x + 10$.

[2]

(iii) Calculate the discriminant of $3x^2 + 9x + 10$.

[2]

10 It is given that $f(x) = \frac{6}{x^2} + 2x$.

(i) Find f′(*x*).

[3]

(ii) Find f″(*x*).

[2]

11 Solve the equation $8x^6 + 7x^3 - 1 = 0$.

[5]

12 Express each of the following in the form $a\sqrt{5}$, where *a* is an integer.

(i) $4\sqrt{15} \times \sqrt{3}$

[2]

(ii) $\frac{20}{\sqrt{5}}$

[1]

(iii) $5^{\frac{3}{2}}$

[1]

13 Express each of the following in the form $k\sqrt{3}$, where *k* is an integer.

(i) $\frac{6}{\sqrt{3}}$

[1]

(ii) $10\sqrt{3} - 6\sqrt{27}$

[2]

(iii) $3^{\frac{5}{2}}$

[2]

14 A circle with centre C has equation $(x - 2)^2 + (y + 5)^2 = 25$.

(i) Show that no part of the circle lies above the x-axis. [3]

(ii) The point P has coordinates $(6, k)$ and lies inside the circle. Find the set of possible values of k. [5]

(iii) Prove that the line $2y = x$ does not meet the circle. [4]

15 Given that $y = 6x^3 + \frac{4}{\sqrt{x}} + 5x$, find

(i) $\frac{dy}{dx}$, [4]

(ii) $\frac{d^2y}{dx^2}$. [2]

16 Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. [5]

17 Express $5x^2 + 10x + 2$ in the form $p(x + q)^2 + r$, where p, q and r are integers. [4]

18 The curve $y = 2x^3 - ax^2 + 8x + 2$ passes through the point B where $x = 4$.

(i) Given that B is a stationary point of the curve, find the value of the constant a. [5]

(ii) Determine whether the stationary point B is a maximum point or a minimum point. [2]

(iii) Find the x-coordinate of the other stationary point of the curve. [3]

19

(i) Sketch the curve $y = 2x^2 - x - 3$, giving the coordinates of all points of intersection with the axes. **[4]**

(ii) Hence, or otherwise, solve the inequality $2x^2 - x - 3 > 0$. **[2]**

(iii) Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of the constant k. **[3]**

20 Solve the simultaneous equations

$$2x + y - 5 = 0, \qquad x^2 - y^2 = 3.$$

[5]

21 Solve the equation $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$. **[5]**

22 Express each of the following in the form 5^k.

(i) 25^4 **[1]**

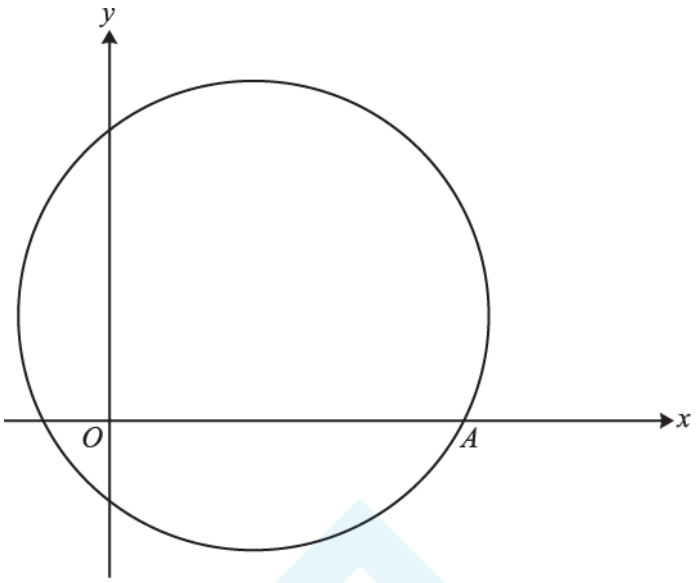
(ii) $\frac{1}{\sqrt[4]{5}}$ **[2]**

(iii) $(5\sqrt{5})^3$ **[2]**

23 Express $\frac{8}{\sqrt{3}-1}$ in the form $a\sqrt{3}+b$, where a and b are integers. **[3]**

24 The curve $y = 4x^2 + \frac{a}{x} + 5$ has a stationary point. Find the value of the positive constant a given that the y-coordinate of the stationary point is 32. **[8]**

25



The diagram shows the circle with equation $x^2 + y^2 - 8x - 6y - 20 = 0$.

(i) Find the centre and radius of the circle. [3]

The circle crosses the positive x-axis at the point A.

(ii) Find the equation of the tangent to the circle at A. [6]

(iii) A second tangent to the circle is parallel to the tangent at A. Find the equation of this second tangent. [3]

(iv) Another circle has centre at the origin O and radius r. This circle lies wholly inside the first circle. Find the set of possible values of r. [2]

26 Find the set of values of k for which the equation $x^2 + 2x + 11 = k(2x - 1)$ has two distinct real roots. [7]

27

(i) Express $4 + 12x - 2x^2$ in the form $a(x + b)^2 + c$. [4]

(ii) State the coordinates of the maximum point of the curve $y = 4 + 12x - 2x^2$. [2]

28 Express the following in the form 2^p.

(i) $(2^5 \div 2^7)^3$

[2]

(ii) $5\times4^{\frac{2}{3}}+3\times16^{\frac{1}{3}}$

[3]

29 Solve the equation $2y^{\frac{1}{2}}-7y^{\frac{1}{4}}+3=0$.

[5]

30 Solve the simultaneous equations

$$x^2 + y^2 = 34,\ 3x - y + 4 = 0.$$

[5]

31 Express $\frac{3+\sqrt{20}}{3+\sqrt{5}}$ in the form $a+b\sqrt{5}$.

[4]

32 The positive constant a is such that $\int_a^{2a}\frac{2x^3-5x^2+4}{x^2}\,dx=0$.

(i) Show that $3a^3 - 5a^2 + 2 = 0$.

[6]

(ii) Show that $a = 1$ is a root of $3a^3 - 5a^2 + 2 = 0$, and hence find the other possible value of a, giving your answer in simplified surd form.

[6]

33

(i) The first three terms of an arithmetic progression are $2x$, $x + 4$ and $2x - 7$ respectively. Find the value of x.

[3]

(ii) The first three terms of another sequence are also $2x$, $x + 4$ and $2x - 7$ respectively.

(a) Verify that when $x = 8$ the terms form a geometric progression and find the sum to infinity in this case.

[4]

(b) Find the other possible value of x that also gives a geometric progression.

[4]

34

(i) Show that the equation $2\sin x = \frac{4\cos x - 1}{\tan x}$ can be expressed in the form

$$6\cos^2 x - \cos x\, 2 = 0.$$

[3]

(ii) Hence solve the equation $2\sin x = \frac{4\cos x - 1}{\tan x}$, giving all values of x between 0° and 360°.

[4]

35

(a) The first term of a geometric progression is 50 and the common ratio is 0.8. Use logarithms to find the smallest value of k such that the value of the kth term is less than 0.15.

[4]

(b) In a different geometric progression, the second term is -3 and the sum to infinity is 4. Show that there is only one possible value of the common ratio and hence find the first term.

[8]

36 The cubic polynomial f(x) is defined by f(x) = 12 – 22x + 9x^2 – x^3.

(i) Find the remainder when f(x) is divided by (x + 2).

[2]

(ii) Show that (3 – x) is a factor of f(x).

[1]

(iii) Express f(x) as the product of a linear factor and a quadratic factor.

[3]

(iv) Hence solve the equation f(x) = 0, giving each root in simplified surd form where appropriate.

[3]

37

(i) Show that the equation

$$\sin x - \cos x = \frac{6\cos x}{\tan x}$$

can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0.$$

[2]

(ii) Hence solve the equation $\sin x - \cos x = \frac{6\cos x}{\tan x}$ for $0° \leqslant x \leqslant 360°$.

[4]

38 In an arithmetic progression the first term is 5 and the common difference is 3. The nth term of the progression is denoted by u_n.

(i) Find the value of u_{20}. [2]

(ii) Show that $\sum_{n=10}^{20} u_n = 517$. [3]

(iii) Find the value of N such that $\sum_{n=N}^{2N} u_n = 2750$. [6]

39

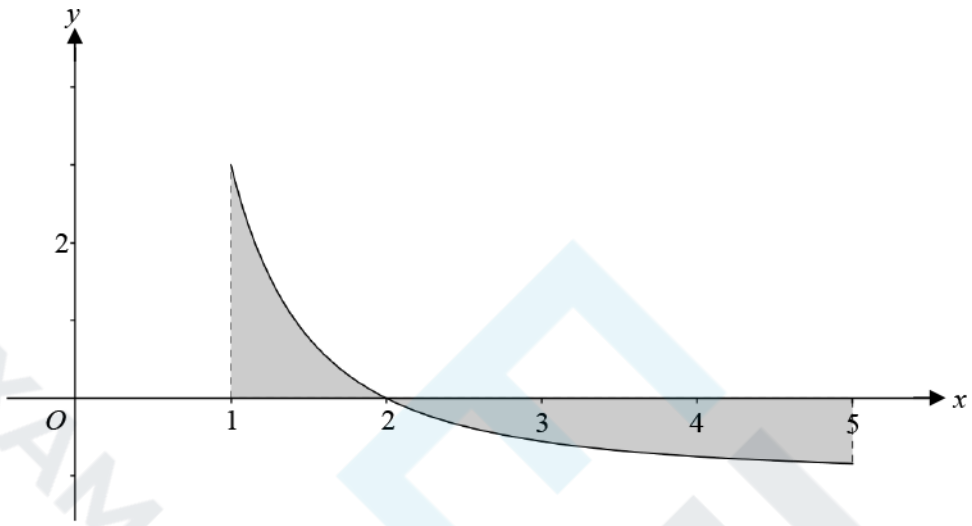
(i) Express $2\log_3 x - \log_3(x + 4)$ as a single logarithm. [2]

(ii) Hence solve the equation $2\log_3 x - \log_3(x + 4) = 2$. [4]

40 **(a)** Find $\int (x^3 - 6x) \, dx$. [3]

(b) (i) Find $\int \left(\frac{4}{x^2} - 1 \right) dx$. [3]

(ii) The diagram below shows part of the curve $y = \frac{4}{x^2} - 1$. The curve crosses the x-axis at (2, 0). The shaded region is bounded by the curve, the x-axis, and the lines $x = 1$ and $x = 5$.



Calculate the area of the shaded region. **[3]**

41 **In this question you must show detailed reasoning.**

The cubic polynomial f(x) is defined by f(x) = $4x^3 + 4x^2 + 7x - 5$.

(a) Show that $(2x - 1)$ is a factor of f(x). **[2]**

(b) Hence solve the equation $4\sin^3\theta + 4\sin^2\theta + 7\sin\theta - 5 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. **[7]**

42 **(a)** Sketch the curve $y = 2x^2 - x - 3$. **[3]**

(b) Hence, or otherwise, solve $2x^2 - x - 3 < 0$. **[2]**

(c) Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of k. **[3]**

43 (a) Write down and simplify the first four terms in the expansion of $(x + y)^7$.
Give your answer in ascending powers of x. [2]

(b) Given that the terms in $x^2 y^5$ and $x^3 y^4$ in this expansion are equal, find the value of $\frac{x}{y}$. [2]

(c) A hospital consultant has seven appointments every day. The number of these appointments which start late on a randomly chosen day is denoted by L. The variable L is modelled by the distribution $\mathrm{B}\left(7, \frac{3}{8}\right)$.
Show that, in this model, the hospital consultant is equally likely to have two appointments start late or three appointments start late. [3]

44 **In this question you must show detailed reasoning.**

Solve the equation $2\cos^2 x = 2 - \sin x$ for $0° \leq x \leq 180°$. **[5]**

45 A student is attempting to model the flight of a boomerang. She throws the boomerang from a fixed point O and catches it when it returns to O. She suggests the model for the displacement, s metres, after t seconds is given by $s = 9t^2 - \frac{3}{2}t^3$, $0 \leq t \leq 6$. For this model

(a) Determine what happens at $t = 6$, **[2]**

(b) Find the greatest displacement of the boomerang from O, **[4]**

(c) Find the velocity of the boomerang 1 second before the student catches it, [2]

(d) Find the acceleration of the boomerang 1 second before the student catches it. [2]

46 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

Distance is measured in metres and time in seconds.

A ship of mass 100 000 kg is being towed by two tug boats. The cables attaching each tug to the ship are horizontal. One tug produces a force of (350**i** + 400**j**) N and the other tug produces a force of (250**i** – 400**j**) N. The total resistance to motion is 200 N. At the instant when the tugs begin to tow the ship, it is moving east at a speed of 1.5 m s^{-1}.

(a) Explain why the ship continues to move directly east. **[2]**

(b) Find the acceleration of the ship. **[2]**

(c) Find the time which the ship takes to move 400 m while it is being towed. Find its speed after moving that distance. **[6]**

47 Solve the simultaneous equations.

$$x^2 + 8x + y^2 = 84$$
$$x - y = 10$$

[4]

48 Business A made a £5000 profit during its first year.
In each subsequent year, the profit increased by £1500 so that the profit was £6500 during the second year, £8000 during the third year and so on.

Business B made a £5000 profit during its first year.
In each subsequent year, the profit was 90% of the previous year's profit.

(a) Find an expression for the total profit made by business A during the first n years. **[2]**
Give your answer in its simplest form.

(b) Find an expression for the total profit made by business B during the first n years. **[3]**
Give your answer in its simplest form.

(c) Find how many years it will take for the total profit of business A to reach £385 000. [3]

(d) Comment on the profits made by each business in the long term. [2]

49 For all real values of x, the functions f and g are defined by f $(x) = x^2 + 8ax + 4a^2$ and g$(x) = 6x - 2a$, where a is a positive constant.

(a) Find fg(x) . Determine the range of fg(x) in terms of a. **[4]**

(b) If fg(2) =144, find the value of a. **[3]**

(c) Determine whether the function fg has an inverse. [2]

50 **In this question you must show detailed reasoning.**

Find the exact values of the x-coordinates of the stationary points of the curve $x^3 + y^3 = 3xy + 35$. [9]

51 Simplify fully.

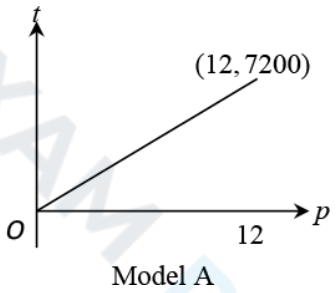
(a) $\sqrt{a^3} \times \sqrt{16a}$. [2]

(b) $(4b^6)^{\frac{5}{2}}$ [2]

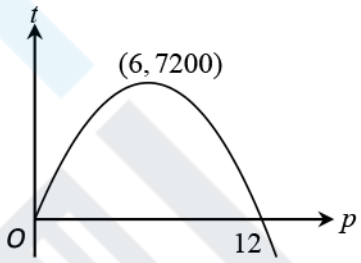
52 A publisher has to choose the price at which to sell a certain new book. The total profit, £t, that the publisher will make depends on the price, £p. He decides to use a model that includes the following assumptions.

- If the price is low, many copies will be sold, but the profit on each copy sold will be small, and the total profit will be small.
- If the price is high, the profit on each copy sold will be high, but few copies will be sold, and the total profit will be small.

The graphs below show two possible models.



Model A



Model B

(a) Explain how model A is inconsistent with one of the assumptions given above. **[1]**

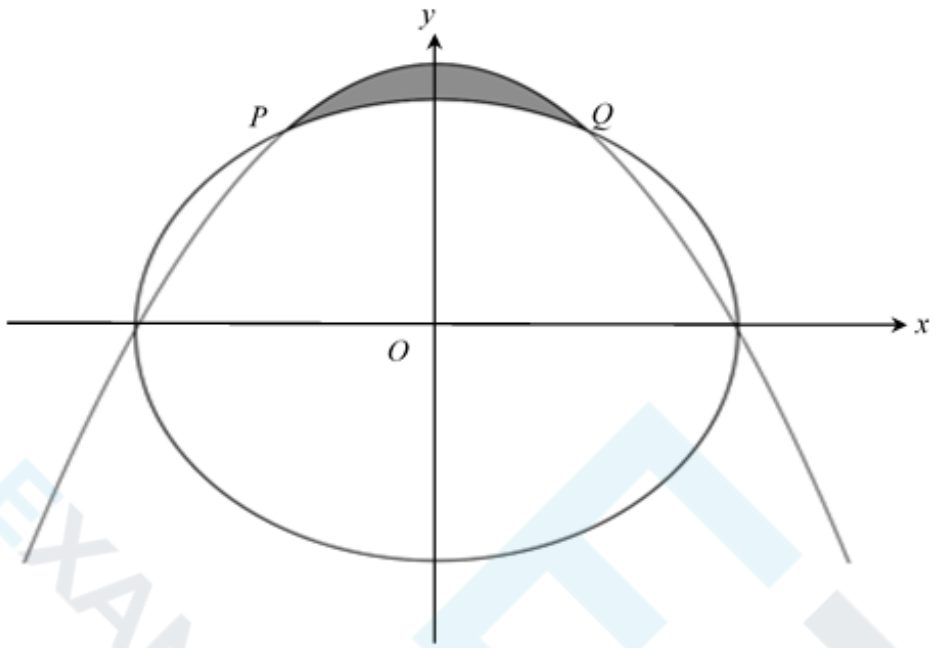
(b) Given that the equation of the curve in model B is quadratic, show that this equation is of the form $t = k(12\,p - p^2)$, and find the value of the constant k. **[2]**

(c) The publisher needs to make a total profit of at least £6400. Use the equation found in part **(b)** to find the range of values within which model B suggests that the price of the book must lie. **[4]**

(d) Comment briefly on how realistic model B may be in the following cases. **[2]**

- $p = 0$
- $p = 12.1$

53 The diagram shows the circle with centre O and radius 2, and the parabola $y = \frac{1}{\sqrt{3}}\left(4 - x^2\right)$.



The circle meets the parabola at points P and Q, as shown in the diagram.

(a) Verify that the coordinates of Q are $\left(1, \sqrt{3}\right)$. **[3]**

(b) Find the **exact** area of the shaded region enclosed by the arc PQ of the circle and the parabola. **[8]**

54

(a) Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x. [3]

(b) Given that the expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is

$$1+x-\tfrac{2}{9}x^2+\ldots$$

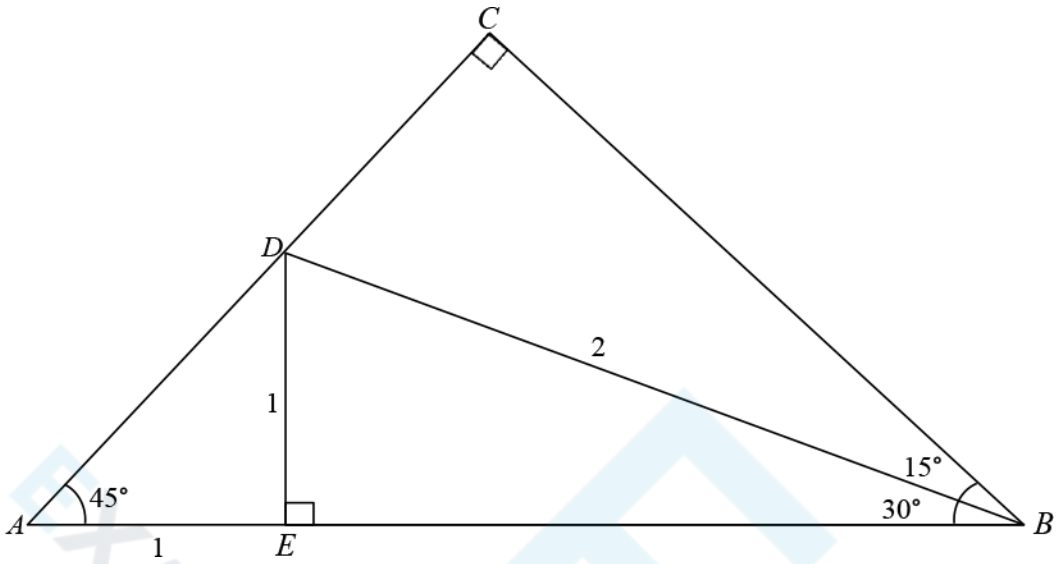
find the possible values of p and q. [5]

55 A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant. Given that the curve has a minimum point when $x = -2$

- find the value of k,
- show that the curve has a point of inflection which is not a stationary point.

[7]

56 **In this question you must show detailed reasoning.**



The diagram shows triangle *ABC*. The angles *CAB* and *ABC* are each 45°, and angle *ACB* = 90°. The points *D* and *E* lie on *AC* and *AB* respectively, such that *AE* = *DE* =1, *DB* = 2 and angle *BED* = 90°. Angle *EBD* = 30° and angle *DBC* = 15°.

(a) Show that $BC = \frac{\sqrt{2}+\sqrt{6}}{2}$. **[3]**

(b) By considering triangle *BCD*, show that $\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$. **[3]**

57 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector **r** metres at time t seconds is given by $\mathbf{r} = 2t^3\mathbf{i} + (5t^2 - 4t)\mathbf{j}$.

(a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately 044°. **[3]**

(b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**

(c) Determine the times at which the particle is moving on a bearing of 045°. **[2]**

58 A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall. The ladder is inclined at an angle of 45° to the horizontal. A man of mass 70 kg is standing on the ladder at a point C, which is x metres from A. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$.

The system is in limiting equilibrium.

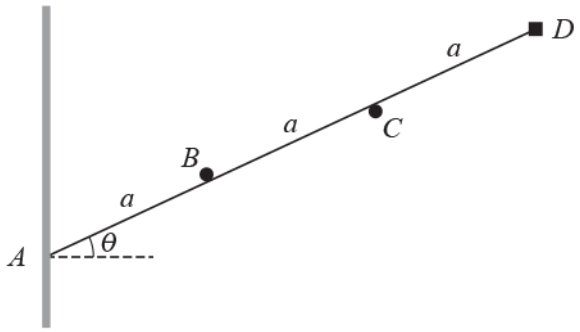
Find x. **[8]**

59 Solve the simultaneous equations $y = 2x$ and $y = x^2 + 2x - 4$. **[4]**

60 **In this question you must show detailed reasoning.**

A curve has equation $y = f(x)$, where $f(x)$ is a quadratic polynomial in x. The curve passes through $(0, 3)$ and $(4, -13)$. At the point where $x = 3$ the gradient of the curve is -2. Find $f(x)$. **[8]**

61



A thin light rod AD has length $3a$. The end A is in contact with a smooth vertical wall which is perpendicular to the vertical plane containing the rod. The rod carries a load of weight W at the end D. The rod is held in equilibrium by two fixed smooth pegs B and C, where $AB = BC = CD = a$. The rod passes under peg B and over peg C, and makes an angle i with the horizontal (see diagram).

(a) **(i)** Show that the normal contact force at C may be expressed as $W\left(\frac{3\cos^2\theta - 1}{\cos\theta}\right)$. **[5]**

(ii) Find the normal contact force at B in terms of W and θ. **[1]**

(b) Hence show that the value of θ is at most 35.3°, correct to 3 significant figures. **[2]**

(c) Show that it is not possible for the magnitude of the reaction at A to equal the magnitude of the reaction at C. **[6]**

62 Simplify

(a) $\dfrac{(3x)^3 \times 2x^{-1}}{9x^2}$, [2]

(b) $(49x^{-4})^{-\frac{1}{2}}$. [2]

63 **In this question you must show detailed reasoning.**

The cubic polynomial $2x^3 - x^2 + kx + 6$ is denoted by f(x). It is given that $(x + 2)$ is a factor of f(x).

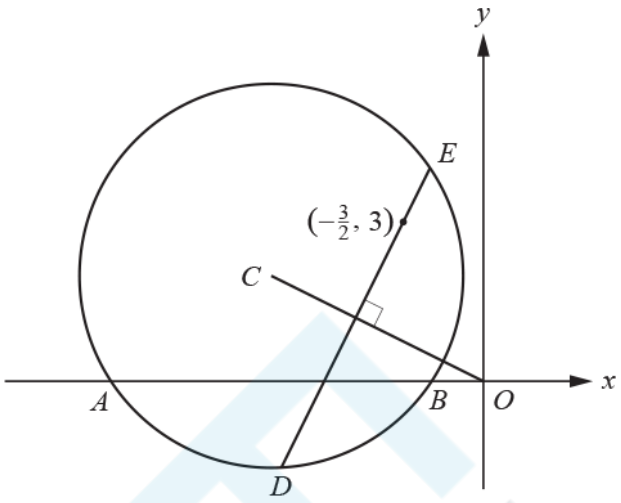
(a) Show that $k = -7$. **[2]**

(b) Factorise f(x) completely. **[3]**

(c) Hence solve the equation $2e^{3y} - e^{2y} - 7e^{y} + 6 = 0$, giving each root in an exact form. **[4]**

64 Show that, for all values of k, the equation $x^2 + (k - 5)x - 3k = 0$ has real roots. **[6]**

65



A circle with centre C has equation $x^2 + y^2 + 8x - 4y + 7 = 0$, as shown in the diagram. The circle meets the x-axis at A and B.

(a) Find

- the coordinates of C,
- the radius of the circle. **[3]**

(b) Find the coordinates of the points A and B. **[2]**

The chord DE passes through the point $\left(-\frac{3}{2}, 3\right)$ and is perpendicular to OC, where O is the origin.

(c) Find the coordinates of the points D and E. **[7]**

(d) Hence find the area of the quadrilateral $BEAD$. **[2]**

66 **In this question you must show detailed reasoning.**

Express each of the following in the form $a+b\sqrt{2}$, where *a* and *b* are integers.

(a) $\sqrt{3}(\sqrt{12}+\sqrt{54})$ **[3]**

(b) $\dfrac{6}{2+\sqrt{2}}$ **[3]**

67 **(a)** Express $2x^2 + 4x + 5$ in the form $p(x + q)^2 + r$, where p, q and r are integers. **[4]**

(b) State the coordinates of the turning point on the curve $y = 2x^2 + 4x + 5$. **[2]**

(c) Given that the equation $2x^2 + 4x + 5 = k$ has no real roots, state the set of possible values of the constant k. **[1]**

68 A sheet of metal is a square of side 21 cm. Equal squares of side x cm are cut from each corner, and the sheet

is then folded to make an open box with vertical sides.

(a) Use calculus to find the value of x that maximises the volume of the box. Justify that the volume is a maximum. **[6]**

(b) State an assumption that is needed when answering part **(a)**. **[1]**

69 The cubic polynomial f(x) is defined by f(x) = $x^3 + 4x^2 - 7x - 10$.

(a) Given that f(-1) = 0, express f(x) in a fully factorised form. **[3]**

(b) Show the equation $e^{3y} + 4e^{2y} - 7e^{y} - 10 = 0$ has exactly one real root. State the exact value of this root. **[3]**

70 The function f is defined as $f(x) = \frac{8}{x+2}$ for $x \geqslant 0$.

(a) State the range of f. **[1]**

(b) Find an expression for $f^{-1}(x)$. **[2]**

(c) Solve the equation $f(x) = f^{-1}(x)$. **[2]**

71 **(a)** Find the first three terms in the expansion of $(1+2x)^{\frac{1}{2}}$ in ascending powers of x. **[3]**

(b) Obtain an estimate of $\sqrt{3}$ by substituting $x = 0.04$ into your answer to part **(a)**. **[3]**

(c) Explain why using $x = 1$ in the expansion would not give a valid estimate of $\sqrt{3}$. **[1]**

72 Express $\frac{2+\sqrt{7}}{\sqrt{7}-2}$ in the form $a+b\sqrt{7}$, where *a* and *b* are rational numbers. **[3]**

73 Solve the simultaneous equations

$$y = x^2 - 6x, \qquad 2y + x - 6 = 0.$$ **[5]**

74 Find the roots of the equation $4t^{\frac{2}{3}} = 15 - 17t^{\frac{1}{3}}$. [5]

75 Sketch the curve $y = -\frac{1}{2}(x+1)^2 + 2$, giving the coordinates of the turning point and indicating all points of intersection with the axes. [5]

76 The equation $kx^2 + (k - 6)x + 2 = 0$ has two distinct real roots. Find the set of possible values of the constant k, giving your answer in set notation. **[6]**

77

Prove by contradiction that $\sqrt{7}$ is irrational. **[5]**

78 Two lifeboat stations, P and Q, are situated on the coastline with Q being due south of P. A stationary ship is at sea, at a distance of 4.8 km from P and a distance of 2.2 km from Q. The ship is on a bearing of 155° from P.

(a) Find any possible bearings of the ship from Q. **[4]**

(b) Find the shortest distance from the ship to the line PQ. **[2]**

(c) Give a reason why the actual distance from the ship to the coastline may be different to your answer to part (b). **[1]**

79 On a particular voyage, a ship sails 500 km at a constant speed of v km/h. The cost for the voyage is £R per hour. The total cost of the voyage is £T.

(a) Show that $T = \frac{500R}{v}$. **[1]**

The running cost is modelled by the following formula.

$$R = 270 + \frac{v^3}{200}$$

The ship's owner wishes to sail at a speed that will minimise the total cost for the voyage. It is given that the graph of T against v has exactly one stationary point, which is a minimum.

(b) Find the speed that gives the minimum value of T. **[4]**

(c) Find the minimum value of the total cost. **[2]**

80 **(a)** Determine the values of p and q for which

$$x^2 - 6x + 10 \equiv (x - p)^2 + q.$$ **[2]**

(b) Use the substitution $x - p = \tan u$, where p takes the value found in part **(a)**, to evaluate

$$\int_3^4 \frac{1}{x^2 - 6x + 10}\,\mathrm{d}x.$$ **[3]**

(c) Determine the value of

$$\int_3^4 \frac{x}{x^2 - 6x + 10}\,\mathrm{d}x,$$

giving your answer in the form $a + \ln b$, where a and b are constants to be determined. **[5]**

81 **In this question you must show detailed reasoning.**

Show that the curve with equation $x^2 - 4xy + 8y^3 - 4 = 0$ has exactly one stationary point. **[10]**

82 The normal to the curve $y = \frac{k}{x^2}$ at the point where $x = -3$ is parallel to the line $\frac{1}{2}y = 2 + 3x$.

(i) Determine the value of the constant k. [6]

(ii) Find the equation of the normal where $x = -3$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

83 **(i)** Express $3x^2 - 5x + 1$ in the form $a(x + b)^2 + c$. **[4]**

(ii) Work out the value of the discriminant of $3x^2 - 5x + 1$ and hence state the number of real roots of the equation $3x^2 - 5x + 1 = 0$. **[2]**

84

(a) Events A and B are independent, and $\mathrm{P}(A \cap B) = \frac{1}{24}$ and $\mathrm{P}(A \cup B) = \frac{3}{8}$.

Find P(A) and P(B). **[5]**

(b) Events C and D are such that P(C) = 0.6, P(D) = 0.3 and P ($C \cup$ D) = 0.8. Find P ($D \,|\, C'$). **[4]**

85

(a) The curve $y = \frac{2}{3+x}$ is translated by four units in the positive x-direction. State the equation of the curve after it has been translated. [2]

(b) Describe fully the single transformation that transforms the curve $y = \frac{2}{3+x}$ to $y = \frac{5}{3+x}$. [2]

86 **(a)** Express $4x^2 - 12x + 11$ in the form $a(x + b)^2 + c$. **[3]**

(b) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$. **[1]**

(c) Explain fully how the value of r is related to the number of real roots of the equation $p(x + q)^2 + r = 0$ where p, q and r are real constants and $p > 0$. **[2]**

87 **In this question you must show detailed reasoning.**

Solve the equation $3\sin^2\theta - 2\cos\theta - 2 = 0$ for $0° \leq \theta \leq 360°$. **[5]**

88 **In this question you must show detailed reasoning.**

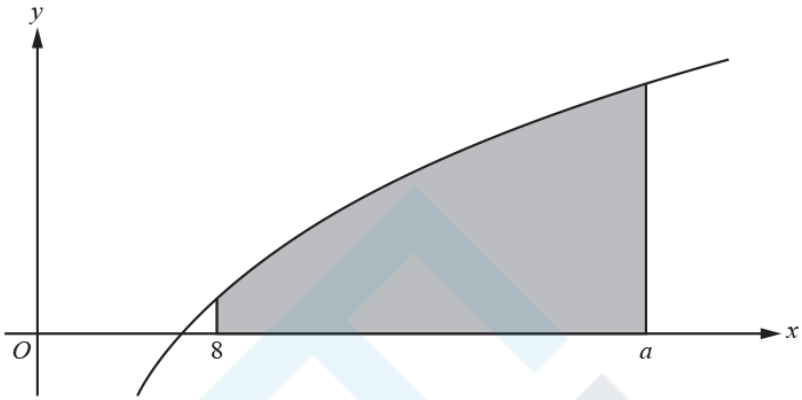
The line $x + 5y = k$ is a tangent to the curve $x^2 - 4y = 10$. Find the value of the constant k. **[5]**

89 **In this question you must show detailed reasoning.**

Find the two real roots of the equation $x^4 - 5 = 4x^2$. Give the roots in an exact form. **[4]**

90 **In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x-axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a. **[9]**

91 The equation of a circle is $x^2 + y^2 + 6x - 2y - 10 = 0$.

(a) Find the centre and radius of the circle. **[3]**

(b) Find the coordinates of any points where the line $y = 2x - 3$ meets the circle $x^2 + y^2 + 6x - 2y - 10 = 0$. **[4]**

(c) State what can be deduced from the answer to part **(b)** about the line $y = 2x - 3$ and the circle $x^2 + y^2 + 6x - 2y - 10 = 0$. **[1]**

92 The cubic polynomial f(x) is defined by f(x) = $2x^3 - 7x^2 + 2x + 3$.

(a) Given that $(x - 3)$ is a factor of f(x), express f(x) in a fully factorised form. **[3]**

(b) Sketch the graph of y = f(x), indicating the coordinates of any points of intersection with the axes. **[2]**

(c) Solve the inequality f(x) < 0, giving your answer in set notation. [2]

(d) The graph of y = f(x) is transformed by a stretch parallel to the x-axis, scale factor $\frac{1}{2}$.
Find the equation of the transformed graph. [2]

93 The function f is defined for all real values of x as $f(x) = c + 8x - x^2$, where c is a constant.

(a) Given that the range of f is $f(x) \leq 19$, find the value of c. **[3]**

(b) Given instead that $ff(2) = 8$, find the possible values of c. **[4]**

94 **In this question you must show detailed reasoning.**

The functions f and g are defined for all real values of x by

$$f(x) = x^3 \text{ and } g(x) = x^2 + 2.$$

(a) Write down expressions for

(i) $fg(x)$, **[1]**

(ii) $gf(x)$. **[1]**

(b) Hence find the values of x for which $fg(x) - gf(x) = 24$. **[6]**

95 **(a)** Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,\mathrm{d}x \approx \frac{11}{4} - \sqrt{2}\,.$$ **[5]**

(b) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,\mathrm{d}x\,.$$ **[6]**

(c) Using your answers to parts **(a)** and **(b)**, show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. **[2]**

96 **In this question you must show detailed reasoning.**

(a) Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where *a* is an integer and *b* is a prime number. **[2]**

(b) Express $\frac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c+d\sqrt{e}$, where *c* and *d* are integers and *e* is a prime number. **[3]**

97 (a) The equation $x^2 + 3x + k = 0$ has repeated roots. Find the value of the constant k. **[2]**

(b) Solve the inequality $6 + x - x^2 > 0$. **[2]**

98 A student was asked to solve the equation $2(\log_3 x)^2 - 3\log_3 x - 2 = 0$. The student's attempt is written out below.

$$2(\log_3 x)^2 - 3\log_3 x - 2 = 0$$
$$4\log_3 x - 3\log_3 x - 2 = 0$$
$$\log_3 x - 2 = 0$$
$$\log_3 x = 2$$
$$x = 8$$

(a) Identify the two mistakes that the student has made. **[2]**

(b) Solve the equation $2(\log_3 x)^2 - 3\log_3 x - 2 = 0$, giving your answers in an exact form. **[4]**

99 A line has equation $y = 2x$ and a circle has equation $x^2 + y^2 + 2x - 16y + 56 = 0$.

(a) Show that the line does not meet the circle. **[3]**

(b) **(i)** Find the equation of the line through the centre of the circle that is perpendicular to the line $y = 2x$. **[4]**

(ii) Hence find the shortest distance between the line $y = 2x$ and the circle, giving your answer in an exact form. **[4]**

100 **In this question you must show detailed reasoning.**

The *n*th term of a geometric progression is denoted by g_n and the *n*th term of an arithmetic progression is denoted by a_n. It is given that $g_1 = a_1 = 1 + \sqrt{5}$, $g_3 = a_2$ and $g_4 + a_3 = 0$.

Given also that the geometric progression is convergent, show that its sum to infinity is $4 + 2\sqrt{5}$. **[12]**

101

Use small angle approximations to estimate the solution of the equation $\frac{\cos\frac{1}{2}\theta}{1+\sin\theta} = 0.825$, if θ is small enough to neglect terms in θ^3 or above. **[4]**

102

(a) Show that $4x^2 - 12x + 3 = 4\left(x - \frac{3}{2}\right)^2 - 6$. [3]

(b) State the coordinates of the minimum point of the curve $y = 4x^2 - 12x + 3$. [2]

103 A curve has equation $y = ax^4 + bx^3 - 2x + 3$.

(a) Given that the curve has a stationary point where $x = 2$, show that $16a + 6b = 1$. **[3]**

(b) Given also that this stationary point is a point of inflection, determine the values of a and b. **[3]**

104 **(a)** Given that $\sqrt{2\sin^2\theta+\cos\theta} = 2\cos\theta$, show that $6\cos^2\theta - \cos\theta - 2 = 0$. **[2]**

(b) In this question you must show detailed reasoning. **[3]**

Solve the equation

$$6\cos^2\theta - \cos\theta - 2 = 0,$$

giving all values of i between 0° and 360° correct to 1 decimal place. **[4]**

(c) Explain why not all the solutions from part **(b)** are solutions of the equation

$$\sqrt{2\sin^2\theta + \cos\theta} = 2\cos\theta.$$

[1]

105 **In this question you must show detailed reasoning.**

Andrea is comparing the prices charged by two different taxi firms.

Firm **A** charges £20 for a 5 mile journey and £30 for a 10 mile journey, and there is a linear relationship between the price and the length of the journey.

Firm **B** charges a pick-up fee of £3 and then £2.40 for each mile travelled.

Find the length of journey for which both firms would charge the same amount. **[4]**

106 **In this question you must show detailed reasoning.**

(a) Given that $\sin\alpha = \frac{2}{3}$, find the exact values of cos α. **[2]**

(b) Given that $2\tan^2\beta - 7\sec\beta + 5 = 0$, find the exact value of sec β. **[4]**

107 **In this question you must show detailed reasoning.**

Solve the simultaneous equations

$e^x - 2e^y = 3$
$e^{2x} - 4e^{2y} = 33.$

Give your answer in an exact form. **[5]**

108 **In this question you must show detailed reasoning.**

A function f is given by $f(x) = \frac{x-4}{(x+2)(x-1)} + \frac{3x+1}{(x+3)(x-1)}$.

(a) Show that f(x) can be written as $\frac{2(2x+5)}{(x+2)(x+3)}$. **[5]**

(b) Given that $\int_a^{a+4} \mathrm{f}(x)\,\mathrm{d}x = 2\ln 3$, find the value of the positive constant a. **[7]**

109 The table shows information about three geometric series. The three geometric series have different common ratios.

	First term	Common ratio	Number of terms	Last term
Series 1	1	2	n_1	1024
Series 2	1	r_2	n_2	1024
Series 3	1	r_3	n_3	1024

(a) Find n_1. **[2]**

(b) Given that r_2 is an integer less than 10, find the value of r_2 and the value of n_2. **[2]**

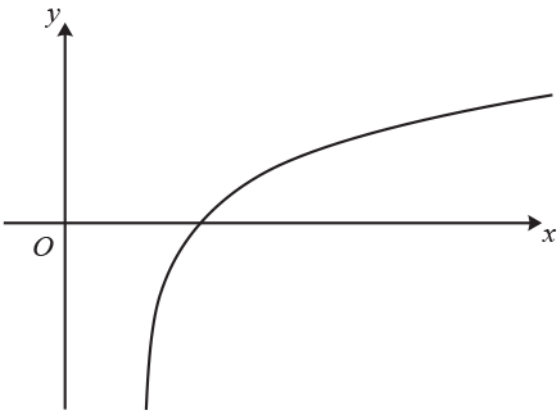
(c) Given that r_3 is **not** an integer, find a possible value for the sum of all the terms in Series 3. **[4]**

110 **(a)** Show that, if n is a positive integer, then $(x^n - 1)$ is divisible by $(x - 1)$. **[1]**

(b) Hence show that, if k is a positive integer, then $2^{8k} - 1$ is divisible by 17. **[4]**

111 Find the coordinates of the point of intersection of the curves $y = 3e^x$ and $y = 1 - 2e^{\frac{1}{2}x}$. [6]

112(
a)



The diagram shows part of the curve $y = \ln(x - 4)$.

Use integration by parts to show that $\int \ln(x-4)\mathrm{d}x = (x-4)\ln|x-4| - x + c$. **[5]**

(b) State the equation of the vertical asymptote to the curve $y = \ln(x - 4)$. **[1]**

(c) Find the total area enclosed by the curve $y = \ln(x - 4)$, the x-axis and the lines $x = 4.5$ and $x = 7$. Give your answer in the form $a\ln 3 + b\ln 2 + c$ where a, b and c are constants to be found. **[4]**

113(A circle with centre C has equation $x^2 + y^2 - 6x + 4y + 4 = 0$.

a)

Find

(i) the coordinates of C, **[2]**

(ii) the radius of the circle. **[1]**

(b) Determine the set of values of k for which the line $y = kx - 3$ does not intersect or touch the circle. **[5]**

114 **In this question you must show detailed reasoning.**

Solve the equation $x(3-\sqrt{5}) = 24$, giving your answer in the form $a+b\sqrt{5}$, where a and b are positive integers. **[3]**

115(a) Express $5x^2 - 20x + 3$ in the form $p(x + q)^2 + r$ where p, q and r are integers. **[3]**

(b) State the coordinates of the minimum point of the curve $y = 5x^2 - 20x + 3$. **[2]**

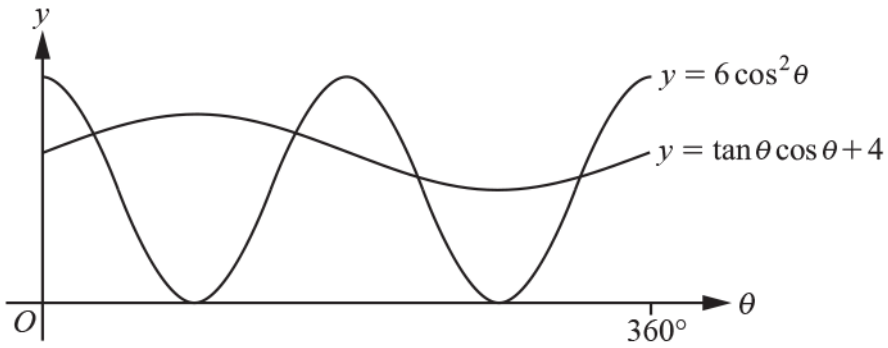
(c) State the equation of the normal to the curve $y = 5x^2 - 20x + 3$ at its minimum point. **[1]**

116(a) **In this question you must show detailed reasoning.**

Show that the equation $6\cos^2\theta = \tan\theta\cos\theta + 4$

can be expressed in the form $6\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

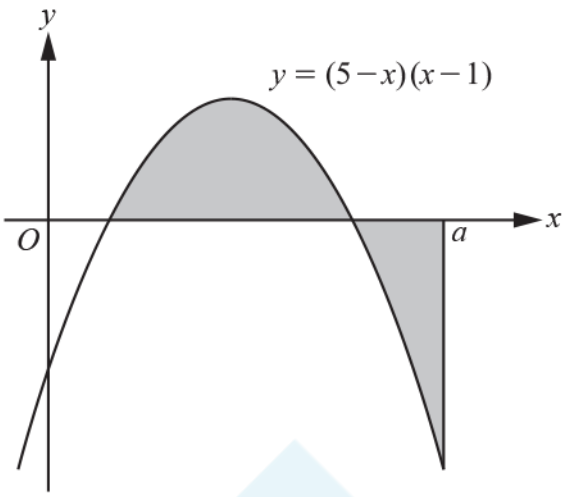
(b)



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = \tan\theta\cos\theta + 4$, where θ is in degrees.

Solve the inequality $6\cos^2\theta > \tan\theta\cos\theta + 4$ for $0° < \theta < 360°$. **[5]**

117



The diagram shows part of the curve $y = (5–x)(x–1)$ and the line $x = \alpha$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of α. **[8]**

118(a) Show that the equation $2 \log_2 x = \log_2(kx - 1) + 3$, where k is a constant, can be expressed in the form $x^2 - 8kx + 8 = 0$. **[4]**

(b) Given that the equation $2 \log_2 x = \log_2(kx - 1) + 3$ has only one real root, find the value of this root. **[4]**

119 Shona makes the following claim.

"n is an **even** positive integer greater than $2 \Rightarrow 2^n - 1$ is **not** prime"

Prove that Shona's claim is true. **[4]**

120 **In this question you must show detailed reasoning.**

Use the substitution $u = 6x^2 + x$ to solve the equation $36x^4 + 12x^3 + 7x^2 + x - 2 = 0$. **[5]**

121 **In this question you must show detailed reasoning.**

Solve the equation $x^3 - 7x^{\frac{3}{2}} - 8 = 0$. **[4]**

122 Determine the points of intersection of the curve $3xy + x^2 + 14 = 0$ and the line $x + 2y = 4$. **[5]**

123(Simplify fully.

a)

$\sqrt{12a} \times \sqrt{3a^5}$ [2]

(b)

$(64b^3)^{\frac{1}{3}} \times (4b^4)^{-\frac{1}{2}}$ [2]

(c) $7 \times 9^{3c} - 4 \times 27^{2c}$ [4]

124(A cylindrical metal tin of radius r cm is closed at both ends. It has a volume of $16000\pi \text{cm}^3$.

a)

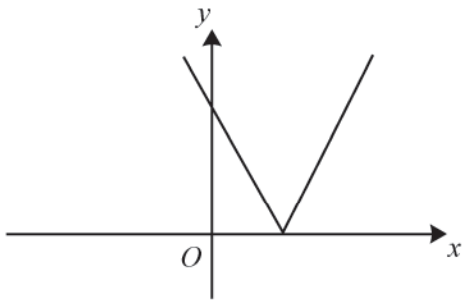
Show that its total surface area, A cm^2, is given by $A = 2\pi r^2 + 320007\pi r^{-1}$. **[4]**

(b) Use calculus to determine the minimum total surface area of the tin. You should justify that it is a minimum. **[6]**

125(a) Differentiate $(2 + 3x^2)\, e^{2x}$ with respect to x. **[3]**

(b) Hence show that $(2 + 3x^2)\, e^{2x}$ is increasing for all values of x. **[4]**

126



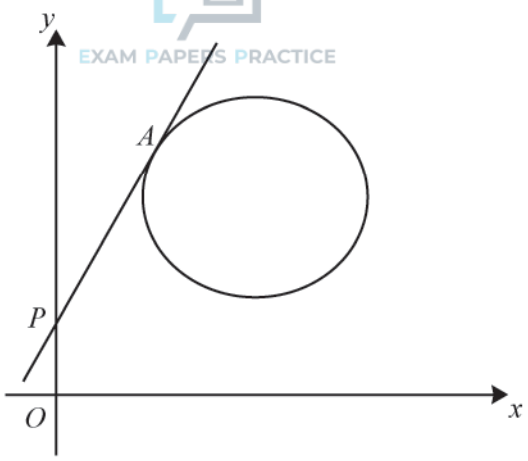
The diagram shows the graph of $y = |2x - 3|$.

Given that the graphs of $y = |2x - 3|$ and $y = \alpha x + 2$ have two distinct points of intersection, determine

(i) the set of possible values of α, **[4]**

(ii) the x-coordinates of the points of intersection of these graphs, giving your answers in terms of α. **[3]**

127(
a)



The diagram shows a circle with equation $x^2 + y^2 - 10x - 14y + 64 = 0$. A tangent is drawn from the point $P(0,2)$ to meet the circle at the point A. The equation of this tangent is of the form $y = mx + 2$, where m is a constant **greater than** 1.

(i) Show that the x-coordinate of A satisfies the equation $(m^2 + 1)x^2 - 10(m + 1)x + 40 = 0$. **[2]**

(ii) Hence determine the equation of the tangent to the circle at A which passes through P. **[4]**

(b) A second tangent is drawn from P to meet the circle at a second point B. The equation of this tangent is of the form $y = nx + 2$, where n is a constant **less than** 1.

Determine the exact value of tan APB. **[4]**

128(a) A curve has equation $y = a(x + b)^2 + c$, where a, b and c are constants. The curve has a stationary point at (–3, 2).

State the values of b and c. **[2]**

(b) When the curve is translated by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ the transformed curve passes through the point (3, –18).

Determine the value of a. **[3]**

129 **In this question you must show detailed reasoning.**

Solve the inequality $x^2 + x - 6 > 0$, giving your answer in set notation. **[4]**

130

(a) By first writing tan 3θ as tan $(2\theta + \theta)$, show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. **[4]**

(b) Hence show that there are always exactly two different values of θ between 0° and 180° which satisfy the equation

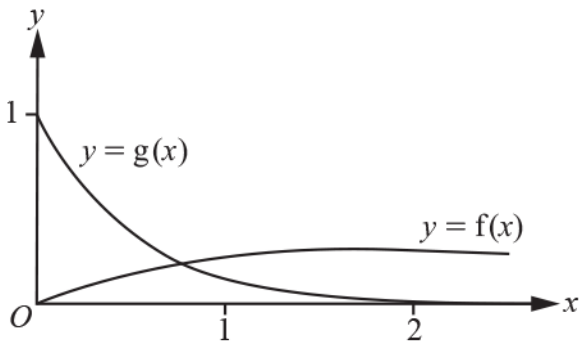
$3 \tan 3\theta = \tan \theta + k$,

where k is a non-zero constant. **[5]**

131 The first three terms of an arithmetic series are $9p$, $8p - 3$, $5p$ respectively, where p is a constant.

Given that the sum of the first n terms of this series is -1512, find the value of n. **[6]**

132



The functions f(x) and g(x) are defined for $x \geq 0$ by $f(x) = \frac{x}{x^2+3}$ and g(x) = e^{-2x}. The diagram shows the curves y = f(x) and y = g(x). The equation f(x) = g(x) has exactly one real root α.

(a) Show that α satisfies the equation h(x) = 0, where h(x) = $x^2 + 3 - xe^{2x}$. **[2]**

(b) Hence show that a Newton-Raphson iterative formula for finding α can be written in the form

$$x_{n+1} = \frac{x_n^2(1-2e^{2x_n})-3}{2x_n-(1+2x_n)e^{2x_n}}.$$

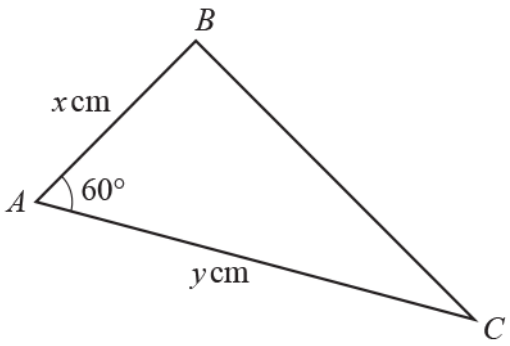
[5]

(c) Use this iterative formula, with a suitable initial value, to find α correct to 3 decimal places. Show the result of each iteration. [3]

(d) **In this question you must show detailed reasoning.**

Find the exact value of x for which $\mathrm{fg}(x) = \frac{2}{13}$. [6]

133(
a)



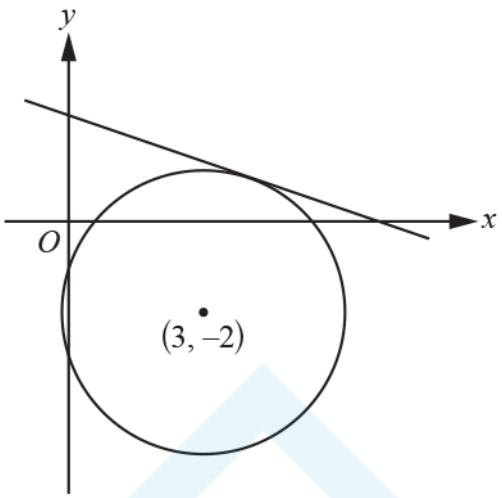
The diagram shows triangle *ABC*, with *AB* = *x* cm, *AC* = *y* cm and angle *BAC* = 60°. It is given that the area of the triangle is $(x+y)\sqrt{3}\,\text{cm}^2$.

Show that $4x + 4y = xy$. **[2]**

(b) When the vertices of the triangle are placed on the circumference of a circle, *AC* is a diameter of the circle.

Determine the value of *x* and the value of *y*. **[4]**

134 **In this question you must show detailed reasoning.**

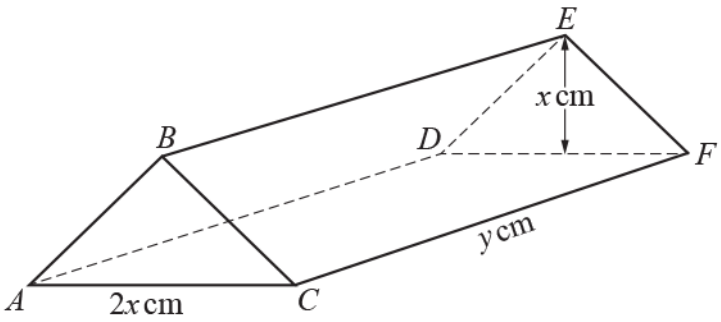


The diagram shows the line $3y + x = 7$ which is a tangent to a circle with centre (3, –2).

Find an equation for the circle. **[6]**

135(

a)



The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism *ABCDEF*. The base *ACFD* of the roof is a horizontal rectangle, and the cross-section *ABC* of the roof is an isosceles triangle with *AB* = *BC*.

The lengths of *AC* and *CF* are 2*x* cm and *y* cm respectively, and the height of *BE* above the base of the roof is *x* cm.

The total surface area of the **five** faces of the roof is 600 cm^2 and the volume of the roof is *V* cm^3.

Show that $V = kx(300 - x^2)$, where $k = \sqrt{a} + b$ and *a* and *b* are integers to be determined. **[6]**

(b) Use differentiation to determine the value of x for which the volume of the roof is a maximum. **[4]**

(c) Find the maximum volume of the roof. Give your answer in cm^3, correct to the nearest integer. **[1]**

(d) Explain why, for this roof, x must be less than a certain value, which you should state. **[2]**

136

Simplify fully $\frac{2x^3+x^2-7x-6}{x^2-x-2}$. **[4]**

137 **In this question you must show detailed reasoning .**

Solve the equation $3\sin^4 \varphi + \sin^2 \varphi = 4$, for $0 \leq \varphi < 2\pi$, where φ is measured in radians. **[5]**

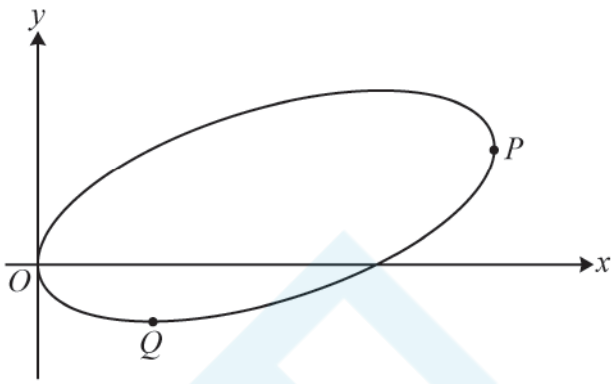
138 The functions f and g are defined for all real values of x by

$f(x) = 2^{x^2} + 6x$ and $g(x) = 3x + 2$.

Find the range of f. **[3]**

139(**In this question you must show detailed reasoning.**

a)



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

(b) At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

140

It is given that the angle θ satisfies the equation $\sin\left(2\theta+\frac{1}{4}\pi\right)=3\cos\left(2\theta+\frac{1}{4}\pi\right)$.

(a) Show that $\tan 2\theta=\frac{1}{2}$. **[3]**

(b) Hence find, in surd form, the exact value of tan θ, given that θ is an obtuse angle. **[5]**

141 **(a)** Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. **[4]**

(b) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. **[1]**

(c) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. **[2]**

END OF QUESTION PAPER