IB Maths: AA HL

Advanced Integration

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.9 Advanced Integration
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Advanced Integration

Question 1

Find the following indefinite integrals:

(a)

$\int 4\sec^2 2x\,dx$

[2 marks]

(b)

$\int \sec\frac{x}{3}\tan\frac{x}{3}\,dx$

[2 marks]

(c)

$\int \frac{1}{\sin^2\left(x+\frac{\pi}{4}\right)}\,dx$

[3 marks]

Question 2

Find the following indefinite integrals:

(a)

$\int (\ln 3)3^x dx$

[2 marks]

(b)

$\int \frac{12}{9+x^2}\,dx$

[2 marks]

(c)

$$\int \frac{3}{5\sqrt{16-x^2}}\,dx$$

[2 marks]

(d)

Using a sketch, briefly describe the family of graphs corresponding to all the possible specific solutions to the integral in part (a).

[3 marks]

Question 3

(a)

Show that

$$\frac{5}{x^2+9x+14} = \frac{A}{x+2} + \frac{B}{x+7}$$

where A and B are constants to be found.

[2 marks]

(b)

Hence, find the indefinite integral

$$\int \frac{5}{x^2+9x+14}\,dx$$

using laws of logarithms to simplify your answer as far as possible.

[4 marks]

(c)
Show that

$$\int_{-1}^{5} \frac{5}{x^2+9x+14}\,\mathrm{d}x = \ln 7 - \ln 2.$$

[4 marks]

Question 4

(a)
Use the substitution $u = x-3$ to find the following indefinite integral:

$$\int x\sqrt{x-3}\,\mathrm{d}x.$$

[4 marks]

(b)
Hence find the value of the definite integral

$$\int_{4}^{7} x\sqrt{x-3}\,dx$$

(i)
by evaluating the definite integral entirely in terms of x

(ii)
by converting the integral limits to appropriate values of u and evaluating the definite integral entirely in terms of u.

Verify that the two methods give the same result for the value of the integral.

[4 marks]

Question 5

(a)
Show that $x^2-10x+29$ may be written in the form $p+(x-q)^2$, where p and q are constants to be determined.

[2 marks]

(b)
Using your results from part (a) along with the substitution $u = x - q$, show that

$$\int \frac{1}{x^2 - 10x + 29}\,\mathrm{d}x = \frac{1}{2}\arctan\left(\frac{x-5}{2}\right) + c\,.$$

[5 marks]

(c) Find the exact value of the definite integral

$$\int_5^7 \frac{1}{x^2 - 10x + 29}\,dx$$

[3 marks]

Question 6

Use the substitution $u = 1 + \sin^3 x$ to show that

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \sqrt{1 + \sin^3 x}\,\mathrm{d}x = \frac{2}{9}\left(2\sqrt{2} - 1\right).$$

[6 marks]

Question 7

(a)
Use integration by parts to find the indefinite integral

$$\int x\mathrm{e}^{2x}\,\mathrm{d}x.$$

[4 marks]

(b)
Hence find the exact value of the definite integral

$$\int_0^3 xe^{2x}\,dx.$$

[3 marks]

(c)
Use technology to evaluate the integral in part (b), and compare this to the exact value you found.

[1 mark]

Question 8

(a)
Use integration by parts twice to show that

$$\int 32x^2e^{4x}\,dx = e^{4x}(px^2+qx+r)+c$$

where p, q and r are constants to be found, and where c is a constant of integration.

[6 marks]

Let f be a function defined for all $x \in \mathbb{R}$. Consider the graph of $y = f(x)$.

(b)
Given that

$$\frac{dy}{dx} = 32x^2e^{4x}$$

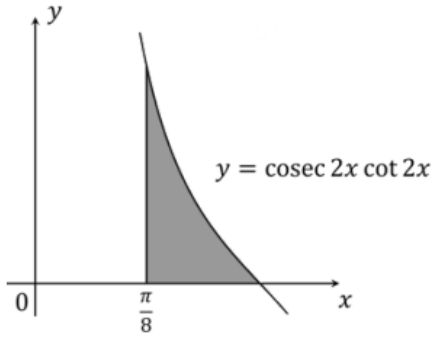
and that the graph passes through the point $\left(\frac{1}{4}, \frac{e+7}{2}\right)$, find an expression for $f(x)$.

[3 marks]

Question 9

Let f be the function defined by $f(x) = \operatorname{cosec} 2x \cot 2x,\ 0 < x < \frac{\pi}{2}$.

The diagram below shows a part of the graph of the curve $y = f(x)$. The shaded region is the region bounded by the curve, the positive x-axis and the line $x = \frac{\pi}{8}$.

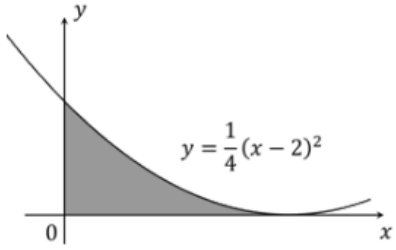


Find the exact area of the shaded region.

[6 marks]

Question 10

The following diagram shows a part of the graph of the curve $y = \frac{1}{4}(x-2)^2$. The shaded region is the region enclosed by the graph and the positive x- and y-axes.



(a)

(i)

Find the coordinates of the points where the graph intersects the coordinate axes.

(ii)

For the part of the curve that forms the boundary of the shaded region, show that $x = 2 - 2\sqrt{y}$.

[3 marks]

(b)
Find the area of the shaded region

(i)
by calculating it as an area between the curve and the x-axis.

(ii)
by calculating it as an area between the curve and the y-axis.

[6 marks]

(c)
Find the volume of the solid formed when the shaded region is rotated 2π radians about the x-axis.

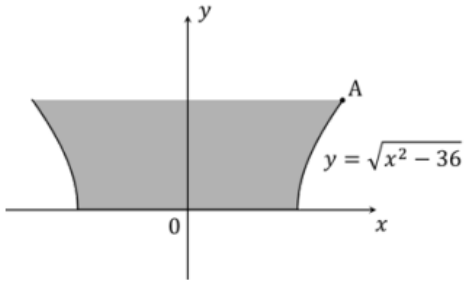
[5 marks]

(d)
Find the volume of the solid formed when the shaded region is rotated 2π radians about the y-axis.

[5 marks]

Question 11

The diagram below shows the cross-section of a bowl that a company is planning to begin producing.



As indicated on the diagram, one of the sides of the bowl in the cross-section may be described by the curve $y=\sqrt{x^2-36}$, where units for x and y are centimetres. The cross-section is entirely symmetrical about the y-axis. The flat circular bottom of the bowl has a diameter of 12 cm, and the vertical depth of the bowl is 6 cm. For purposes of answering this question, the thickness of the bottom and sides of the bowl may be regarded as negligible.

(a)
Find the exact coordinates of the point marked A on the diagram.

[3 marks]

(b)
Show that the capacity of the bowl in cm^3 is given by

$$\pi \int_0^b (y^2 + 36) dy$$

where b is a constant to be determined.

[4 marks]

(c)
Hence find the capacity of the bowl.

[2 marks]