AQA

AS
MATHEMATICS
Paper 1

Thursday 15 May 2025   Afternoon   Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

Section A

Answer **all** questions in the spaces provided.

1   Identify the expression that is equivalent to $\tan x$

Circle your answer.

[1 mark]

$\sin^2 x + \cos^2 x$ $\sin^2 x - \cos^2 x$ $\dfrac{\cos x}{\sin x}$ $\boxed{\dfrac{\sin x}{\cos x}}$

2   Find the value of $\log_b \dfrac{1}{b^2}$

Circle your answer.

[1 mark]

$\boxed{-2}$ $-\dfrac{1}{2}$ $\dfrac{1}{2}$ 2

$$\log_b \frac{1}{b^2} = \log_b b^{-2} = -2\log_b b = -2$$

3   The polynomial $p(x)$ is given by

$$p(x) = 2x^3 - ax^2 + 6x + 2a$$

It is given that $(x-2)$ is a factor of $p(x)$

Find the value of a by using the factor theorem.

[3 marks]

$x - 2 = 0$ $p(2) = 0$

$x = 2$ $2(2)^3 - a(2)^2 + 6(2) + 2a = 0$

$16 - 4a + 12 + 2a = 0$

$16 + 12 = 4a - 2a$

$28 = 2a$

$2a = 28$

$a = 14$

Turn over for the next question

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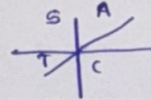**4** Solve the equation

$$2\tan 3\theta - 3 = 0$$

for $0° \le \theta \le 180°$

Give your answers to the nearest degree.

[3 marks]

$\tan 3\theta = \frac{3}{2}$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$

$\therefore 3\theta = 56.3°, 180° + 56.3°, 360° + 56.3°$

$3\theta = 56.3°, 236.3°, 416.3°$

$\theta = 18.76°, 78.77°, 138.77°$

$\therefore \theta \simeq 19°, 79°, 139°$

5 Jayven claims that for two real numbers a and b

if $a > b$, then it must be true that $\frac{a}{b} > 1$

By using a counter example, show that Jayven is **not** correct.

[2 marks]

Consider $a = 2, b = -1$

$a > b \quad [2 > -1]$

$\frac{a}{b} = \frac{2}{-1} = -2$ is not greater than 1

$\therefore$ Claim is false.

6 It is given that $p = \log_2 x$ and $q = \log_2 y$

6 (a) Express

$$\log_2\left(\frac{x^2}{y}\right)$$

in terms of p and q

[2 marks]

$$\log_2\left(\frac{x^2}{y}\right) = \log_2 x^2 - \log_2 y$$
$$= 2\log_2 x - \log_2 y$$
$$= 2p - q$$

6 (b) Express

$$\log_2\left(16x^3\sqrt{y}\right)$$

in terms of p and q

Give your answer in a form not involving logarithms.

[3 marks]

$$\log_2\left(16x^3\sqrt{y}\right) = \log_2 16 + \log_2 x^3 + \log_2 \sqrt{y}$$
$$= 4\log_2 2 + 3\log_2 x + \frac{1}{2}\log_2 y$$
$$= 4 + 3p + \frac{1}{2}q.$$

7 A circle has equation

$$(x - a)^2 + (y - b)^2 = 49$$

7 (a) State the radius of the circle.

[1 mark]

$$r^2 = 49$$
$$r = 7$$

7 (b) The circle crosses the x-axis at two distinct points.

Find the range of possible values for b

[2 marks]

At x-axis, $y = 0$

$$(x-a)^2 + (y-b)^2 = 49$$
$$x^2 - 2ax + a^2 + b^2 = 49$$

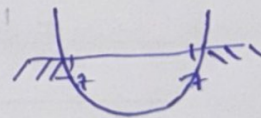$$b^2 - 4ac < 0$$
$$(2a)^2 - 4(a^2 + b^2 - 49) < 0$$
$$4a^2 - 4a^2 - 4b^2 + 196 < 0$$
$$4b^2 - 196 > 0$$
$$b^2 - 49 > 0$$
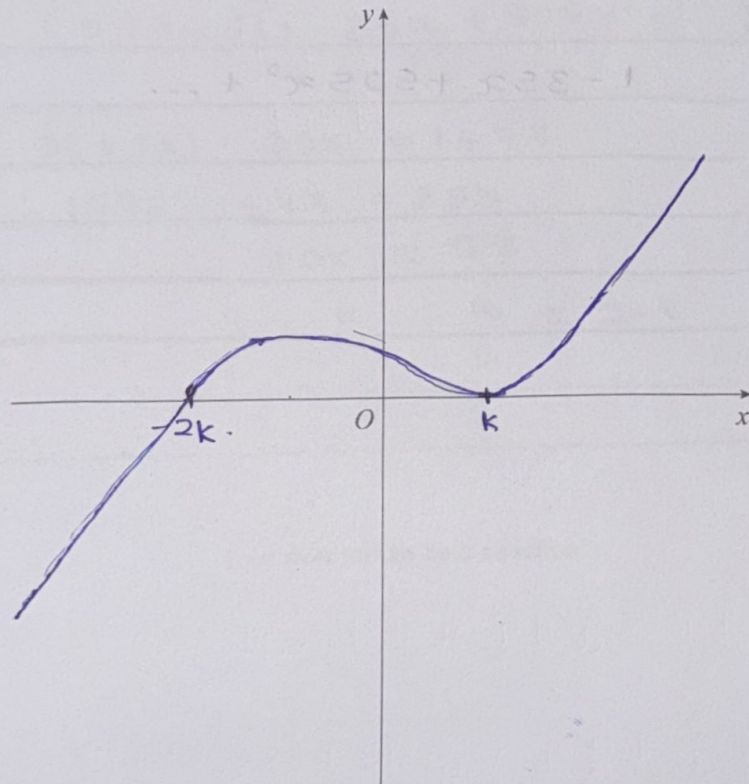$$(b-7)(b+7) > 0$$

$$b < -7 \quad \& \quad b > 7$$

8 Sketch the graph of
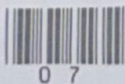
$$y = (x - k)^2(x + 2k)$$

where k is a positive constant.

Label the coordinates of the points where the graph meets the axes.

[4 marks]

Turn over for the next question

9 (a) Find, in ascending powers of x, the first three terms in the expansion of

$$(1-5x)^7$$

[3 marks]

$$^7C_0(1)^7(-5x)^0 + {}^7C_1(1)^6(-5x)^1 + {}^7C_2(1)^5(-5x)^2+\ldots$$

$$1 + (-35)x + 525x^2 + \ldots$$

$$1 - 35x + 525x^2 + \ldots$$

9 (b) The coefficient of x^2 in the expansion of

$$(3 + kx)(1 - 5x)^7$$

is 1477

Find the value of k

[3 marks]

$$\frac{(3+kx)(1-5x)^7}{(3+kx)(1-35x+525x^2+...)}$$

$$3(525) - 35k = 1477$$
$$1575 - 1477 = 35k$$
$$35k = 98$$
$$k = \frac{14}{5} = 2.8$$

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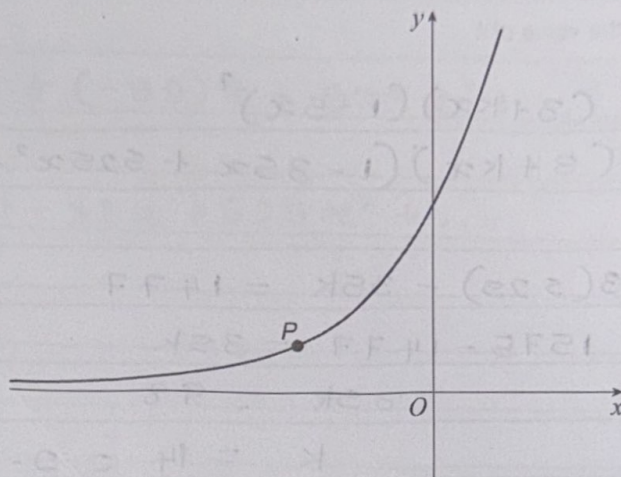
10 A curve has the equation

$$y = e^{3x}$$

and the point P lies on the curve, as shown in the diagram.

The tangent to the curve at P is parallel to the line with equation $x - 9y = 23$

Find the x-coordinate of P

[5 marks]

$x - 9y = 23$

$9y = x - 23$

$y = \frac{1}{9}x - \frac{23}{9}$ $\therefore$ $m = \frac{1}{9}$

$y = e^{3x}$ $\Rightarrow$ $\frac{dy}{dx} = 3e^{3x}$

$3e^{3x} = \frac{1}{9}$

$e^{3x} = \frac{1}{27}$

$\ln e^{3x} = -\ln 27$

$3x = -\ln 27$

$x = -\frac{1}{3}\ln 27 = -\ln 3$ \\

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11 (a) Expand $x(x-2)(x+4)$

[1 mark]

$$x[x^2+4x-2x-8]$$
$$x[x^2+2x-8]$$
$$x^3+2x^2-8x$$

11 (b) Show that

$$\int_{-4}^{2} x(x-2)(x+4)\,dx = 36$$

Fully justify your answer.

[4 marks]

$$\int_{-4}^{2} x^3+2x^2-8x\,dx$$

$$\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{2}$$

$$\left[\frac{(2)^4}{4}+\frac{2(2)^3}{3}-4(2)^2\right]-\left[\frac{(-4)^4}{4}+\frac{2(-4)^3}{3}-4(-4)^2\right]$$

$$\left(4+\frac{16}{3}-16\right)-\left(64-\frac{128}{3}-64\right)$$

$$-\frac{20}{3}+\frac{128}{3}$$

$$= 36 \quad (\text{shown})$$

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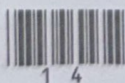**11 (c)**   The curve C has equation

$$y = x(x - 2)(x + 4)$$

A sketch of C is shown in the diagram.

11 (c) (i)   Explain why your answer to part **(b)** will not give the total area of the shaded region bounded by C and the x-axis.

[1 mark]

Because the part from 0 to 2 will give a negative value since its below the x-axis.

11 (c) (ii) Find the total area of the shaded region bounded by C and the x-axis.

[2 marks]

$$A = \int_{-4}^{0} x^3 + 2x^2 - 8x \, dx + \left| \int_{0}^{2} x^3 + 2x^2 - 8x \, dx \right|$$

$$= \left[0 + \frac{20}{3} \right] + \left| \left[-\frac{128}{3} - 0 \right] \right|$$

$$= \frac{20}{3} + \left| -\frac{128}{3} \right|$$

$$= \frac{20}{3} + \frac{128}{3} = \frac{148}{3}$$

Turn over for the next question

Turn over ▶

12 (a) It is given that

$$^{40}C_{18} = \frac{40!}{18!\,p!}$$

Write down the value of p

[1 mark]

$$p = 40 - 18 = 22.$$

12 (b) Use the result from part **(a)** to show that

$$\frac{^{40}C_{18}}{^{40}C_{17}} = \frac{23}{q}$$

where q is an integer to be found.

[2 marks]

$$\frac{40!}{18!\,22!} \div \frac{40!}{17!\,23!} = \frac{23}{q}$$

$$\frac{40!}{18!\,22!} \times \frac{17!\,23 \times 22!}{40!} = \frac{23}{q}$$

$$\frac{17! \times 23}{18 \times 17!} = \frac{23}{q}$$

$$\frac{23}{18} = \frac{23}{q}$$

$$q = 18$$

13 A computer game awards points to a player based on the time taken to complete a level.

The points awarded decrease as the time taken to complete a level increases.

Rebekah believes that the points awarded, P, can be modelled by the equation

$$P = Ae^{kt}$$

where t is the time, in seconds, taken to complete the level and A and k are constants.

13 (a) Explain, in context, the meaning of the value of A

[1 mark]

A is the number of points awarded
if a player completes the level instantly
(in 0 seconds)

13 (b) Show that

$$\ln P = \ln A + kt$$

[2 marks]

$$P = Ae^{kt}$$
$$\ln P = \ln(Ae^{kt})$$
$$\ln P = \ln A + \ln e^{kt}$$
$$\ln P = \ln A + kt \ln e$$
$$\ln P = \ln A + kt \qquad (\text{shown})$$

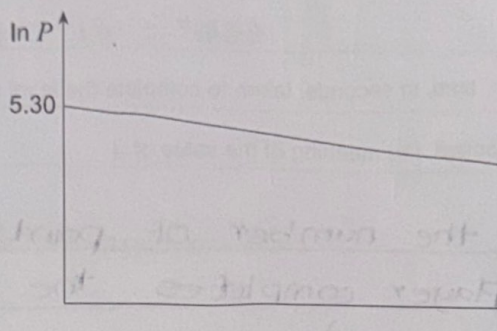**Question 13 continues on the next page**

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13 (c)   Rebekah records the points and the time taken for her to complete each level.

She plots the values of ln P against t

Rebekah obtains a straight-line graph with a gradient of -0.08 and a vertical intercept of 5.30, as shown in the diagram.

ln P

5.30

t

Find the value of A and the value of k

[3 marks]

Gradient is -0.08 ∴ $K = -0.08$

ln $A = 5.3$
$A = e^{5.3} = 200.34$

13 (d)   Rebekah scores 20 points for completing a particular level.

Find, to the nearest second, the time taken to complete this level.

[3 marks]

$$P = A e^{kt}$$
$$20 = e^{5.3} \cdot e^{-0.08t}$$
$$20 = e^{5.3 - 0.08t}$$
$$\ln 20 = 5.3 - 0.08t$$
$$0.08t = 5.3 - \ln 20$$
$$t = \frac{5.3 - \ln 20}{0.08} \approx 28.8$$

$$\therefore \quad 29 \text{ seconds}$$

END OF SECTION A

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

14   A particle of mass 2 kg is moving on a smooth horizontal surface under the action of a single horizontal force of 5 N

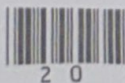Find the acceleration of the particle.

Circle your answer.

[1 mark]

0.4 m s^{-2} 2.5 m s^{-2} 10 m s^{-2} 24.5 m s^{-2}

15   Three forces F_1, F_2 and F_3 are given by:

$$F_1 = (-2i + 4j) \text{ N}$$

$$F_2 = (-3i + 6j) \text{ N}$$

$$F_3 = (-6i + 9j) \text{ N}$$

One of the following statements is true.

Identify the correct statement.

Tick (✓) **one** box.

[1 mark]

F_1 and F_2 are parallel forces.   ✓

F_1 and F_3 are parallel forces.   ☐

F_2 and F_3 are parallel forces.   ☐

None of the forces F_1, F_2 and F_3 are parallel.   ☐

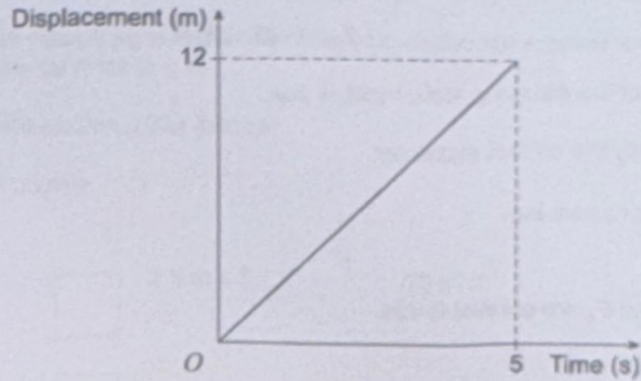**Turn over for the next question**

Turn over ▶

16   The displacement–time graph, **Figure 1**, shows the first 5 seconds of the motion of a
      car which starts from rest and travels 12 metres.

Figure 1

Displacement (m)

12

O   5   Time (s)

16 (a)   Find the speed of the car.

[1 mark]

speed = displacement / time

speed = 12/5 = 2.4 m/s

16 (b)   On **Figure 2**, draw a more realistic displacement–time graph to show the first
          5 seconds of motion of the car.

[1 mark]

Figure 2

Displacement (m)

12

O   5   Time (s)

17   **In this question use** $g = 9.8$ **m s**$^{-2}$

Lamic has mass 60 kg.

He is standing on the floor of a lift.

The lift is accelerating upwards at 1.2 m s^{-2}

The reaction of the floor on Lamic is R newtons.

Find the value of R

[2 marks]

$$\text{Mass} \quad m = 60 \text{ kg}. \quad \text{Acceleration upwards}$$
$$g = 9.8 \text{ m/s}^2 \qquad\qquad = 1.2 \text{ m/s}^2$$

$$R - mg = ma \Rightarrow R = m(g + a)$$

$$R = 60(9.8 + 1.2) = 660N$$

Turn over for the next question

Turn over ▶

2 3

18   **In this question use** $g = 10 \text{ m s}^{-2}$

A ball is thrown vertically upwards from a height of 1.8 metres above the ground.

The initial velocity of the ball is 12 m s^{-1}

The greatest height reached by the ball above the ground is h metres.

Find the value of h

[3 marks]

$$V^2 = u^2 + 2as.$$

$$0 = (12)^2 + 2(-10)s$$
$$144 = 20s.$$

$$S = 7.2 \text{ m}$$
$$h = 1.8 + 7.2 = 9.0 \text{ m}$$

19 A force **F** is $\begin{bmatrix} p \\ -0.5 \end{bmatrix}$ newtons, where p is a constant.

Given that the magnitude of **F** is 1.3 newtons, find the possible values of p

[3 marks]

$$F = \begin{bmatrix} p \\ -0.5 \end{bmatrix} \qquad |\bar{F}| = 1.3$$

$$|\bar{F}| = \sqrt{p^2 + (-0.5)^2} = 1.3$$

$$\sqrt{p^2 + 0.25} = 1.3$$

$$p^2 = 1.44$$

$$p = \pm\sqrt{1.44} = \pm 1.2$$

$$p = \pm 1.2$$

Turn over for the next question

Turn over ▶

20   A sports scientist is modelling the speed of an athlete who ran a 100-metre race.

The speed, v m s^{-1}, of the athlete at time t seconds after the start of the 100-metre race is given by

$$v = 1.8 + 3.8t - 0.25t^2$$

20 (a)   State the initial speed of the athlete according to the model.

[1 mark]

At time t=0 V=1·8 all to eg ^4
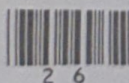
Therefore, initial speed = 1·8 m/s

20 (b) (i)   Find an expression, in terms of t, for the acceleration of the athlete.

[2 marks]

$a = \dfrac{dv}{dt} = 3.8 - 0.25(2)(t)$

$a = 3.8 - 0.5t$

20 (b) (ii) Hence find the maximum speed of the athlete.

Fully justify your answer.

[4 marks]

At max speed $a = 0.80$:

$3.8 - 0.5t = 0 \Rightarrow t = 7.6 s$

$V = 1.8 + 3.8(7.6) - 0.25(7.6)^2$

$V = 16.24$ m/s

$\Rightarrow$ Substitute $t = 7.6$ into the original speed equation

20 (c) The official maximum speed recorded, by the scientist, for the athlete was 12.4 m s^{-1}

Evaluate the accuracy of the model used by the scientist.

[1 mark]

The model predicts a max speed of 16.24 m/s while the measured value is 12.4. Therefore, the model overestimates the max speed by a significant margin. External factors like friction, air resistance etc cut accounted for.

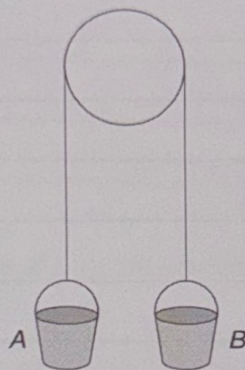**Turn over for the next question**

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21 Two buckets, A and B, each have mass 0.5 kg

Each bucket is attached to one end of a light inextensible rope.

The rope is hung over a smooth fixed pulley.

The system is in equilibrium with both buckets hanging freely at rest, as shown in the diagram.

A builder then places a brick of mass 3 kg inside bucket A

Bucket A, with the brick inside, then moves vertically downwards.

During the subsequent motion, the magnitude of the acceleration of each bucket is a m s^{-2} and the magnitude of the tension in the rope is T N

Assume the buckets and brick can be modelled as particles.

21 (a) Find a and T, leaving your answers in terms of g

Fully justify your answer.

[6 marks]

Using Newton's second law:

Net force on A $= m_A g - T = m_A a$

① $\Rightarrow 3.5g - T = 3.5a$

② Net force on B $= T - m_B g = m_B a$

② $\Rightarrow T - 0.5g = 0.5a$

Add equations ① and ②

$$3.5g - T + T - 0.5g = 3.5a + 0.5a$$

$$3g = 4a \qquad \boxed{a = \frac{3g}{4}}$$

Substitute $a = \frac{3g}{4}$ into eq^n ②

$$T = 0.5g + 0.5a = \boxed{\frac{7g}{8} = T}$$

21 (b) Explain a limitation of modelling the buckets as particles.

[1 mark]

Modelling the buckets as particles ignores their speed and shape, so effects such as air resistance or rotational motion are not accounted for.
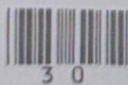
END OF QUESTIONS

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