AQA

A-level
FURTHER MATHEMATICS
7367/1

Paper 1

Mark scheme

June 2025

Version: 1.0 Final

2 5 6 A 7 3 6 7 / 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles 4th answer.	1.1b	B1	$c > 4$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles 3rd answer.	2.2a	B1	$-z^*$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles 1st answer.	2.2a	B1	$\lim_{x \to 0}\left(\frac{\sqrt{x}}{\cos x}\right)$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Ticks 2nd answer.	1.1b	B1	$\text{sech}^2 x + \tanh^2 x = 1$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Obtains a correct expression or value for $\arg z_2$ Condone use of degrees. or Obtains $\frac{z_1}{z_2} = k(-0.7 + 0.9\text{i})$ PI AWRT 2.23 rad or AWRT 128 degrees	1.1b	B1	$\arg z_1 = \tan^{-1}\left(\frac{1}{5}\right) = 0.19739\ldots$ $\arg z_2 = \tan^{-1}\left(\frac{-4}{-2}\right) - \pi =$ $-2.03444\ldots$ $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = 2.23$
	Uses $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$ or Obtains argument of their $\frac{z_1}{z_2}$ PI AWRT 2.23 rad or AWRT 128 degrees	1.1a	M1	
	Obtains **AWRT** 2.23	1.1b	A1	
	Question total		**3**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="3">6</td><td>Obtains exactly five values of y</td><td>1.1a</td><td>M1</td><td rowspan="3">

<table>
<tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr>
<tr><td>y</td><td>1</td><td>0.9701</td><td>0.7071</td><td>0.4061</td><td>0.2425</td></tr>
<tr><td>y</td><td>1</td><td>$\frac{4\sqrt{17}}{17}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{4\sqrt{97}}{97}$</td><td>$\frac{\sqrt{17}}{17}$</td></tr>
</table>

$\frac{1}{3} \times \frac{1}{2}(1 + 0.2425 + 4(0.9701 + 0.4061) + 2 \times 0.7071)$
$= 1.36025$
$= 1.360$ (3dp)</td></tr>
<tr><td>Uses the Simpson's rule formula correctly for their five y values and their h</td><td>1.1a</td><td>M1</td></tr>
<tr><td>Obtains 1.360
Condone 1.36
Condone AWRT 1.360</td><td>1.1b</td><td>A1</td></tr>
<tr><td></td><td>Question total</td><td></td><td>3</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
7	Obtains $\frac{w+1}{2}$ Accept any letter for w or Obtains $2(\alpha+\beta+\gamma)-3$ PI by $-\frac{23}{5}$	1.1b	B1	$w = 2z - 1 \Rightarrow z = \frac{w+1}{2}$ $5\left(\frac{w+1}{2}\right)^3 + 4\left(\frac{w+1}{2}\right)^2 - \left(\frac{w+1}{2}\right)$ $+3=0$ $\frac{5}{8}(w^3 + 3w^2 + 3w + 1) + (w^2 + 2w$ $+ 1) - \frac{1}{2}(w + 1) + 3 = 0$ $\frac{1}{8}(5w^3 + 15w^2 + 15w + 5 + 8w^2 +$ $16w + 8 - 4w - 4 + 24) = 0$ $5w^3 + 23w^2 + 27w + 33 = 0$
	Substitutes $\frac{w\pm1}{2}$ or $\frac{w}{2}\pm1$ into the cubic expression. Accept any letter for w or Writes the new pairwise sum of roots as $4(\alpha\beta+\beta\gamma+\gamma\alpha)-4(\alpha+\beta+\gamma)+3$ PI by $\frac{27}{5}$	3.1a	M1	
	Obtains a four-term cubic expression from the substitution method or Writes the new product of roots as $8\alpha\beta\gamma-4(\alpha\beta+\beta\gamma+\gamma\alpha)$ $+2(\alpha+\beta+\gamma)-1$ PI by $-\frac{33}{5}$	1.1a	M1	
	Obtains $k(5w^3+23w^2+27w+33)=0$ Where k is an integer	1.1b	A1	
	Question total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8	Obtains $\frac{(x-1)^2}{4}+\frac{(y-3)^2}{25}=1$	1.2	B1	$C_2:\frac{(x-1)^2}{4}+\frac{(y-3)^2}{25}=1$ $y=0\Rightarrow\frac{(x-1)^2}{4}+\frac{9}{25}=1$ $25(x-1)^2+36=100$ $25(x-1)^2=64$ $(x-1)^2=\frac{64}{25}$ $x=1\pm\frac{8}{5}$ $\left(\frac{13}{5},0\right),\left(-\frac{3}{5},0\right)$
	Substitutes $y=0$ into their equation for C_2 **and** solves to obtain at least one value of x	1.1a	M1	
	Obtains $\left(\frac{13}{5},0\right)$ and $\left(-\frac{3}{5},0\right)$ Must be coordinates.	1.1b	A1	
	Question total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
9	Obtains $-\sqrt{3}=7\lvert\mathbf{b}\rvert\cos\theta$ OE or $-\sqrt{3}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ OE	1.2	B1	$-\sqrt{3}=7\lvert\mathbf{b}\rvert\cos\theta$ $\cos\theta=\frac{-\sqrt{3}}{7\lvert\mathbf{b}\rvert}$ $12=7\lvert\mathbf{b}\rvert\sin\theta$ $\sin\theta=\frac{12}{7\lvert\mathbf{b}\rvert}$ $1=\cos^2\theta+\sin^2\theta$ $=\frac{3}{49\lvert\mathbf{b}\rvert^2}+\frac{144}{49\lvert\mathbf{b}\rvert^2}$ $=\frac{3}{\lvert\mathbf{b}\rvert^2}$ $\lvert\mathbf{b}\rvert^2=3$ $\lvert\mathbf{b}\rvert=\sqrt{3}$
	Obtains $12=7\lvert\mathbf{b}\rvert\sin\theta$ OE or $12=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$ OE	1.1b	B1	
	Obtains an equation in only $\lvert\mathbf{b}\rvert^2$ or $\lvert\mathbf{b}\rvert$ from a valid method	3.1a	M1	
	Deduces that $\lvert\mathbf{b}\rvert=\sqrt{3}$ Accept AWRT 1.73	2.2a	A1	
	Question total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)(i)	States that Astrid did not show that the result is true for $n = 1$	2.3	E1	Astrid did not show that the result is true for $n = 1$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)(ii)	**Shows** that the result is true for $n = 1$ for **both** definition and formula	2.4	E1	$S_1 = 2$ and $S_1 = 1 \times (1+1) = 2$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(b)(i)	Ticks 2nd answer.	2.3	E1	It is true for $n = k$, and also true for $n = k + 1$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(b)(ii)	Writes a correct statement to replace the incorrect statement.	2.1	R1	If true for $n = k$, it is also true for $n = k + 1$
	Subtotal		**1**	

	Question total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)	Obtains $\dfrac{-e^x}{(1+e^x)^2}$ OE	1.1b	B1	$f(x)=\dfrac{1}{1+e^x}$ $f'(x)=\dfrac{-e^x}{(1+e^x)^2}$ $f''(x)=\dfrac{(1+e^x)^2(-e^x)-(-e^x)(2e^x)(1+e^x)}{(1+e^x)^4}$ $=\dfrac{-e^x(1+e^x)+2e^{2x}}{(1+e^x)^3}$ $=\dfrac{-e^x+e^{2x}}{(1+e^x)^3}$
	Uses a correct method to obtain the second derivative.	1.1a	M1	
	Completes a reasoned argument to obtain $f''(x)=\dfrac{-e^x+e^{2x}}{(1+e^x)^3}$ AG	2.1	R1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(b)	Obtains $f(0)=\dfrac{1}{2}$ OE $f'(0)=-\dfrac{1}{4}$ OE $f''(0)=0$	1.1b	B1	$f^{(3)}(x)=$ $\dfrac{(1+e^x)^3(-e^x+2e^{2x})-(-e^x+e^{2x})(3e^x)(1+e^x)^2}{(1+e^x)^6}$ $f(0)=\dfrac{1}{2}$ $f'(0)=-\dfrac{1}{4}$ $f''(0)=0$ $f^{(3)}(0)=\dfrac{2^3(-1+2)-0}{2^6}=\dfrac{8}{64}=\dfrac{1}{8}$ $f(x)=\dfrac{1}{2}-\dfrac{1}{4}x+\dfrac{1}{48}x^3+...$
	Obtains $f^{(3)}(0)=0.125$ OE	1.1b	B1	
	Substitutes their four values into Maclaurin's series.	1.1a	M1	
	Obtains $\dfrac{1}{2}-\dfrac{1}{4}x+\dfrac{1}{48}x^3$	1.1b	A1	
	Subtotal		**4**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Forms correct characteristic equation and solves. PI by $\lambda = 1$ & $\lambda = 0.5$ Condone $\lambda = 20$ & $\lambda = 10$ from $0 = (19 - \lambda)(11 - \lambda) - 9$	1.1a	M1	$0 = \left(\frac{19}{20} - \lambda\right)\left(\frac{11}{20} - \lambda\right) - \frac{9}{400}$ $0 = 2\lambda^2 - 3\lambda + 1$ $\lambda = 1$ & $\lambda = 0.5$ $\lambda = 1$: $\mathbf{0} = \begin{bmatrix} \frac{-1}{20} & \frac{3}{20} \\ \frac{3}{20} & \frac{-9}{20} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ $\lambda = 1$: $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\lambda = 0.5$: $\mathbf{0} = \begin{bmatrix} \frac{9}{20} & \frac{3}{20} \\ \frac{3}{20} & \frac{1}{20} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ $\lambda = 0.5$: $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$
	Obtains $\lambda = 1$ & $\lambda = 0.5$	1.1b	A1	
	Uses correct equation to find eigenvector for one of their two eigenvalues. PI by a correct eigenvector.	1.1a	M1	
	Obtains a correct eigenvector for one of their two eigenvalues. Allow any scalar multiple.	1.1b	A1	
	Obtains both correct eigenvectors paired with the corresponding correct eigenvalue. Allow any scalar multiple.	1.1b	A1	
	Subtotal		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(b)	Explains that for a line of invariant points $\lambda = 1$ or $\mathbf{Mv} = \mathbf{v}$	2.4	M1	As $\lambda = 1$ for a line of invariant points. $y = \frac{1}{3}x$
	Deduces $y = \frac{1}{3}x$ OE from $\lambda = 1$	2.2a	R1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(c)	Deduces their correct **U** with no zero column.	2.2a	B1F	$\mathbf{U}=\begin{bmatrix}3 & -1\\ 1 & 3\end{bmatrix}$
	Deduces their correct **D** Must be compatible with their **U**	2.2a	B1F	$\mathbf{D}=\begin{bmatrix}1 & 0\\ 0 & 0.5\end{bmatrix}$
	Obtains $\mathbf{U}^{-1}$, ft their **U**	1.1b	B1F	$\mathbf{U}^{-1}=\frac{1}{10}\begin{bmatrix}3 & 1\\ -1 & 3\end{bmatrix}$
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(d)	Multiplies their matrices in correct order with powers inside $\mathbf{D}^n$	1.1a	M1	$\mathbf{M}^n=\frac{1}{10}\begin{bmatrix}3 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}1^n & 0\\ 0 & 0.5^n\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 3\end{bmatrix}$
	Deduces that as $n\to\infty$ limit of $\mathbf{D}^n$ is $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ or $\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$	2.2a	B1	$\mathbf{L}=\frac{1}{10}\begin{bmatrix}3 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 3\end{bmatrix}$ $\mathbf{L}=\frac{1}{10}\begin{bmatrix}9 & 3\\ 3 & 1\end{bmatrix}$
	Completes a reasoned argument to obtain correct **L**	2.1	R1	
	Subtotal		**3**	

	Question total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Obtains $-4-3\text{i}$	1.2	B1	One root is $-4-3\text{i}$ Let the other root $=\gamma$ Then $(-4+3\text{i})(-4-3\text{i})+\gamma(-4+3\text{i})$ $+\gamma(-4-3\text{i})=\frac{92}{4}=23$ $25-8\gamma=23$ $\gamma=\frac{1}{4}$ So the other roots are $-4-3\text{i}$ and $\frac{1}{4}$
	Selects a correct method; for example, uses the pairwise sum of roots, or substitutes the given root into the equation, or expands $\left(z-(-4+3\text{i})\right)\left(z-(-4-3\text{i})\right)$	3.1a	M1	
	Sets the pairwise sum = $\pm\frac{92}{4}$ or Finds $\text{f}\left(z\right)$ as a product of a quadratic and a linear factor or in expanded form PI by $\left(4z-1\right)$ PI by correct values for r and s	1.1a	M1	
	Obtains $\frac{1}{4}$	2.2a	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(b)	Forms an equation in r, eg uses the sum of roots. or Expands their $\left(z^2+8z+25\right)\left(4z-1\right)$	3.1a	M1	$\frac{-r}{4}=-4+3\text{i}-4-3\text{i}+\frac{1}{4}=\frac{-31}{4}$ $r=31$ $(-4+3\text{i})(-4-3\text{i})\left(\frac{1}{4}\right)=\frac{-s}{4}$ $\frac{25}{4}=\frac{-s}{4}$ $s=-25$
	Forms an equation in s, eg uses the product of roots. or Compares coefficients	3.1a	M1	
	Obtains $r=31$ and $s=-25$	1.1b	A1	
	Subtotal		**3**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Uses definitions of sinh and cosh. PI $x=\frac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$ or $x=\frac{e^{2y}+1}{e^{2y}-1}$	1.1a	M1	Let $y=\coth^{-1}(x)$ Then $x=\coth y$ $x=\frac{\frac{1}{2}(e^{y}+e^{-y})}{\frac{1}{2}(e^{y}-e^{-y})}=\frac{e^{2y}+1}{e^{2y}-1}$ $x(e^{2y}-1)=e^{2y}+1$ $xe^{2y}-x=e^{2y}+1$ $e^{2y}(x-1)=x+1$ $e^{2y}=\frac{x+1}{x-1}$ $2y=\ln\left(\frac{x+1}{x-1}\right)$ $y=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ $\coth^{-1}(x)=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$
	Rearranges to make e^{2y} or e^{2x} the subject. or Uses the quadratic equation formula	3.1a	M1	
	Completes a reasoned argument, starting from the full definitions of sinh and cosh, to obtain $\coth^{-1}(x)=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ AG	2.1	R1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(b)	Uses the result from part **(a)** to form an equation and applies at least one law of logs correctly	3.1a	M1	$\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)=-\ln 5=\ln\frac{1}{5}$ $\ln\left(\frac{x+1}{x-1}\right)=2\ln\frac{1}{5}=\ln\left(\frac{1}{5}\right)^{2}$ $\frac{x+1}{x-1}=\frac{1}{25}$ $25(x+1)=x-1$ $25x+25=x-1$ $x=-\frac{13}{12}$
	Obtains $\frac{x+1}{x-1}=\frac{1}{25}$ OE	1.1b	A1	
	Obtains $-\frac{13}{12}$ OE	1.1b	A1	
	Subtotal		**3**	

	Question total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Selects an appropriate method Eg eliminates one variable to form at least one equation	3.1a	M1	$x + 2y - z = 9$ (1) $x - 3y + 3z = t$ (2) $3x + y + z = 4t$ (3) $-5y + 4z = t - 9$ $(2)-(1)$ $-5y + 4z = 4t - 27$ $(3)-3(1)$ $t - 9 = 4t - 27$ $t = 6$
	Obtains two correct equations in the same two variables and t	1.1b	A1	
	Forms and solves an equation in t	1.1a	M1	
	Obtains 6	1.1b	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(b)	Selects an appropriate method eg equates one variable to zero and forms simultaneous equations in two variables. or Sets one variable equal to a multiple of a parametric variable	3.1a	M1	$x + 2y - z = 9$ $x - 3y + 3z = 6$ Let $x = 0$: $2y - z = 9$ $-3y + 3z = 6$ $-y + z = 2$ $y = 11$, $z = 13$ Let $y = 0$: $x - z = 9$ $x + 3z = 6$ $z = -\frac{3}{4}$, $x = \frac{33}{4}$ Two points on the line: $A(0,11,13)$, $B\left(\frac{33}{4},0,-\frac{3}{4}\right)$ $\overrightarrow{AB} = \begin{bmatrix} \frac{33}{4} \\ -11 \\ -\frac{55}{4} \end{bmatrix}$ Direction vector $= \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}$ $\mathbf{r} = \begin{bmatrix} 0 \\ 11 \\ 13 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}$
	Obtains one correct point on the line. or Correctly obtains a second variable in terms of the parametric variable	1.1b	A1	
	Uses a correct method to obtain a direction vector eg Uses two points on their line to form a direction vector or Obtains the cross product of two normal vectors. or substitutes to find the third variable in terms of the parametric variable	1.1a	M1	
	Obtains a correct direction vector. or Correctly obtains the third variable in terms of the parametric variable	1.1b	A1	
	Completes a reasoned argument to obtain a correct vector equation.	2.2a	R1	
	Subtotal		**5**	

	Question total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
16	Differentiates in order to obtain $\frac{dy}{dx}$	3.1a	M1	$y^2 = 0.8x$ $2y\frac{dy}{dx} = 0.8 \Rightarrow \frac{dy}{dx} = \frac{0.4}{y}$ $\left(\frac{dy}{dx}\right)^2 = \frac{0.16}{y^2} = \frac{0.16}{0.8x} = \frac{0.2}{x}$ $S = 2\pi \int_0^{0.25} y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ $= 2\pi \int_0^{0.25} \sqrt{0.8x}\sqrt{1+\frac{0.2}{x}}\,dx$ $= 2\pi\sqrt{0.8}\int_0^{0.25} (x+0.2)^{\frac{1}{2}}\,dx$ $= 2\pi\sqrt{0.8}\left[\frac{2}{3}(x+0.2)^{\frac{3}{2}}\right]_0^{0.25}$ $= \frac{4}{3}\pi\sqrt{0.8}\left(0.45^{\frac{3}{2}} - 0.2^{\frac{3}{2}}\right)$ = 0.79587… = 0.796 m^2 to 3 decimal places
	Obtains a correct expression for $\left(\frac{dy}{dx}\right)^2$ or $\frac{dy}{dx}$ in terms of x or y	1.1b	A1	
	Substitutes their y and their $\frac{dy}{dx}$ into the formula for surface area in terms of one variable. Condone missing or incorrect limits	1.1a	M1	
	Writes integrand in the form $k(x+0.2)^{1/2}$ or $k(5x+1)^{1/2}$ or $k(0.8x+0.16)^{1/2}$ or $k\cosh^2 u \sinh u$ or $k\,y(y^2+0.16)^{1/2}$ OE	2.2a	M1	
	Obtains $k(x+0.2)^{3/2}$ or $k(5x+1)^{3/2}$ or $k(0.8x+0.16)^{3/2}$ or $k\cosh^3 u$ or $k(y^2+0.16)^{3/2}$ OE	1.1b	A1	
	Substitutes correct upper and lower limits into an integrated expression of the form $k(ax+b)^{3/2}$ or $k\cosh^3 u$ or $k(ay^2+b)^{3/2}$ and subtracts.	1.1a	M1	
	Completes a reasoned argument to obtain 0.796 Must see 0.7958… or $\frac{19\pi}{75}$ Condone omission of units. AG	2.1	R1	
	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Obtains $5e_A = 3e_B + 0.6 \times 10$ OE	3.1b	B1	In equilibrium position $5e_A = 3e_B + 0.6 \times 10$ $2 = e_A + e_B$ $e_A = 1.5$, $e_B = 0.5$ After release $3(e_B - x) + 0.6\times 10 - \frac{4}{\sqrt{5}}\dot{x} - 5(e_A + x) = 0.6\ddot{x}$ $3(0.5 - x) + 0.6\times 10 - \frac{4}{\sqrt{5}}\dot{x} - 5(1.5 + x) = 0.6\ddot{x}$ $1.5 - 3x + 6 - \frac{4}{\sqrt{5}}\dot{x} - 7.5 - 5x = 0.6\ddot{x}$ $0.6\ddot{x} + \frac{4}{\sqrt{5}}\dot{x} + 8x = 0$ $0.6\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{4}{\sqrt{5}}\frac{\mathrm{d}x}{\mathrm{d}t} + 8x = 0$
	Obtains $3(e_B - x)$ or $5(e_A + x)$ Condone their incorrect e_A or e_B	2.2a	B1F	
	Forms **five**-term equation of motion in terms of x (with at least two terms correct). Condone "a" for $\ddot{x}$ and "v" for $\dot{x}$ Condone sign errors on the terms. Condone their incorrect e_A or e_B	3.1b	M1	
	Forms correct equation of motion in terms of x. Can be in terms of e_A and e_B Condone "a" for $\ddot{x}$ and "v" for $\dot{x}$ Condone their incorrect e_A or e_B	1.1b	A1F	
	Completes a reasoned argument to obtain $0.6\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{4}{\sqrt{5}}\frac{\mathrm{d}x}{\mathrm{d}t} + 8x = 0$ Accept $0.6\ddot{x} + \frac{4}{\sqrt{5}}\dot{x} + 8x = 0$ AG	2.1	R1	
	Subtotal		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(b)	Obtains complex solutions of the Auxiliary Equation.	3.1a	M1	$0.6\lambda^2+\frac{4}{\sqrt{5}}\lambda+8=0$
	Obtains $e^{\frac{-2\sqrt{5}}{3}t}\left(A\cos\left(\frac{10}{3}t\right)+B\sin\left(\frac{10}{3}t\right)\right)$	1.1b	A1	$\lambda=\frac{-2\sqrt{5}}{3}\pm\frac{10}{3}\mathrm{i}$ $x=e^{\frac{-2\sqrt{5}}{3}t}\left(A\cos\left(\frac{10}{3}t\right)+B\sin\left(\frac{10}{3}t\right)\right)$
	Obtains $A=0.5$ Condone $A=-0.5$	3.3	B1	$x=0.5, t=0 \Rightarrow A=0.5$
	Sets their $\dot{x}=0$ when $t=0$ Must use the product rule	3.3	M1	$\dot{x}=\frac{-2\sqrt{5}}{3}e^{\frac{-2\sqrt{5}}{3}t}\left(A\cos\left(\frac{10}{3}t\right)+B\sin\left(\frac{10}{3}t\right)\right)$ $+e^{\frac{-2\sqrt{5}}{3}t}\left(\frac{-10}{3}A\sin\left(\frac{10}{3}t\right)+\frac{10}{3}B\cos\left(\frac{10}{3}t\right)\right)$
	Obtains $B=\frac{\sqrt{5}}{10}$	1.1b	A1	$\dot{x}=0, t=0$ $0=\frac{-2\sqrt{5}}{3}A+\frac{10}{3}B$ $\Rightarrow B=\frac{\sqrt{5}}{10}$
	Completes a reasoned argument to obtain $x=e^{\frac{-2\sqrt{5}}{3}t}\left(\frac{1}{2}\cos\left(\frac{10}{3}t\right)+\frac{\sqrt{5}}{10}\sin\left(\frac{10}{3}t\right)\right)$	2.1	R1	$x=e^{\frac{-2\sqrt{5}}{3}t}\left(\frac{1}{2}\cos\left(\frac{10}{3}t\right)+\frac{\sqrt{5}}{10}\sin\left(\frac{10}{3}t\right)\right)$
	Subtotal		**6**	

	Question total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(a)	Forms and solves a quadratic equation or inequality in $\sinh x$ or Forms a quartic expression in e^x	3.1a	M1	Let $s = \sinh x$ $\frac{15}{s} = 4s + 0.5$ $8s^2 + s - 30 = 0$ $\sinh x = -2, \frac{15}{8}$ $x = \ln 4, \ln(\sqrt{5} - 2)$ Solution of inequality: $\ln(\sqrt{5} - 2) < x < 0, x > \ln 4$
	Obtains -2 and $\frac{15}{8}$ or Obtains $4e^{4x} + e^{3x} - 68e^{2x} - e^x + 4$	1.1b	A1	
	Obtains at least one of $\ln 4$ or $\ln(\sqrt{5} - 2)$ OE	1.1a	M1	
	Obtains $\ln(\sqrt{5} - 2) < x < 0, x > \ln 4$ OE	2.2a	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(b)	Uses the chain rule.	3.1a	M1	$f(x) = \ln\left(\tanh\left(\frac{1}{2}x\right)\right)$ $f'(x) = \dfrac{\frac{1}{2}\operatorname{sech}^2\left(\frac{1}{2}x\right)}{\tanh\left(\frac{1}{2}x\right)}$ $= \dfrac{\cosh\left(\frac{1}{2}x\right)}{2\sinh\left(\frac{1}{2}x\right)\cosh^2\left(\frac{1}{2}x\right)}$ $= \dfrac{1}{2\sinh\left(\frac{1}{2}x\right)\cosh\left(\frac{1}{2}x\right)}$ $= \dfrac{1}{\sinh x}$ $= \operatorname{cosech} x$
	Obtains a correct expression for the derivative.	1.1b	A1	
	Uses hyperbolic identities to express their $f'(x)$ in terms of only $\sinh\left(\frac{1}{2}x\right)$ **and** $\cosh\left(\frac{1}{2}x\right)$	2.2a	M1	
	Completes a reasoned argument to show that $f'(x) = \operatorname{cosech} x$ AG	2.1	R1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(c)	Deduces that the area of R is the sum of two integrals (at least one correct).	2.2a	B1	Area of $R = A_1 + A_2$, where $A_1 = \int_0^{\ln 4} \left(4\sinh x + \frac{1}{2}\right)dx = \left[4\cosh x + \frac{1}{2}x\right]_0^{\ln 4}$ $= \left(4\cosh(\ln 4) + \frac{1}{2}\ln 4\right) - 4$ $= \frac{9}{2} + \ln 2$ $A_2 = \int_{\ln 4}^{\ln 9} 15\text{cosech}\, x\, dx = 15\left[\ln\left(\tanh\left(\frac{1}{2}x\right)\right)\right]_{\ln 4}^{\ln 9}$ $= 15\left\{\ln\left(\tanh(\ln 3)\right) - \ln\left(\tanh(\ln 2)\right)\right\}$ $= 15\left\{\ln\frac{4}{5} - \ln\frac{3}{5}\right\} = 15\ln\frac{4}{3}$ Area of $R = \frac{9}{2} + \ln 2 + 15\ln\frac{4}{3}$
	Deduces the correct limits for A_1 and A_2	2.2a	B1	
	Obtains $\pm\left(4\cosh x + \frac{1}{2}x\right)$	1.1b	B1	
	Obtains $\pm 15\ln\left(\tanh\left(\frac{1}{2}x\right)\right)$	1.1b	B1	
	Obtains $\pm\left(\frac{9}{2} + \ln 2\right)$ or AWRT ± 5.19	1.1a	M1	
	Obtains $\pm 15\ln\frac{4}{3}$ or AWRT ± 4.32	1.1a	M1	
	Obtains $\frac{9}{2} + \ln 2 + 15\ln\frac{4}{3}$	2.1	R1	
	Subtotal		**7**	

	Question total		**15**	

	Paper Total		**100**	