AQA

A-level
MATHEMATICS
7357/1

Paper 1

Mark scheme

June 2026

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
FT	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
ISW	ignore subsequent working
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks second box	1.2	B1	$\cos^2\theta + \sin^2\theta \equiv 1$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles first answer	2.2a	B1	$\begin{pmatrix} 0 \\ -7 \end{pmatrix}$
	Question 2 Total		**1**	

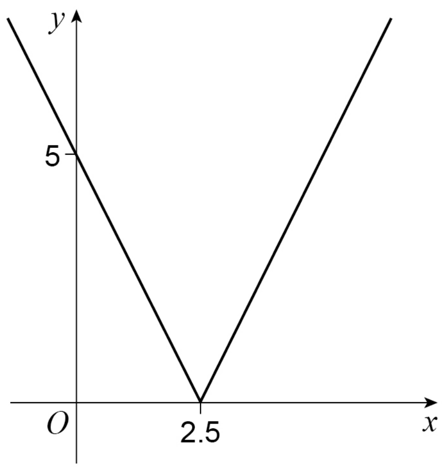
Q	Marking instructions	AO	Marks	Typical solution
3	Circles first answer	1.1b	B1	$\dfrac{8-4\sqrt{n}}{4-n}$
	Question 3 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Circles first answer	1.1b	B1	0
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Circles fourth answer	2.2a	B1	periodic
	Subtotal		**1**	

	Question 4 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5	States or uses $h = 0.3$ OE Accept 0.15 as the multiplier. PI by the correct answer	1.1b	B1	$h = 0.3$ $\frac{0.3}{2}\begin{pmatrix}0.48238+0.18213+\\2(0.38203+0.30018+0.23441)\end{pmatrix}$ $= 0.3747$
	Substitutes the given y values to achieve 0.48238 + 0.18213 + 2(0.38203 + 0.30018 + 0.23441) Ignore h Accept correct exact values or values to more than 5 decimal places. PI by the correct answer or 2.49775 or 2.49773(2505…) from exact values	1.1a	M1	
	Obtains AWRT 0.3747	1.1b	A1	
	Question 5 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6	Sketches “V” shape, resting on the x-axis Can be free hand for this mark. Condone poor symmetry. Condone extension of one line to the negative y-axis	1.1a	M1	
	Sketches “V” shape, resting on the positive x-axis AND Labels either 2.5 on the x-axis at the vertex of their “V” shape, or 5 on the y-axis at the point of intersection	1.1b	A1	
	Obtains correct straight-line sketch, extending into the 2nd quadrant, with both axis labels correct. Condone dotted line extension of $y = 2x - 5$ for x < 2.5 Condone poor symmetry.	1.1b	A1	
	Question 6 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Obtains $2e^{2x}$	1.1b	B1	$\frac{dy}{dx} = 2e^{2x}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Substitutes $x = 0$ into their $\frac{dy}{dx}$ to obtain a value PI by $\frac{dy}{dx} = 2$, m = 2, which could be embedded in a straight-line equation	1.1a	M1	$x = 0$, $\frac{dy}{dx} = 2$ $y = 2x + 4$
	Uses 2 or their stated value of $\frac{dy}{dx}$ and the point (0, 4) to form an equation of a straight line.	1.1a	M1	
	Obtains $y = 2x + 4$ ACF	1.1b	A1	
	Subtotal		**3**	

	Question 7 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8	Obtains 17	1.1b	B1	$R = 17$ $R\sin\alpha = 15,\ R\cos\alpha = 8$ $\tan\alpha = \frac{15}{8}$ 17 cos(x – 61.9°)
	Obtains a correct equation in α For example $\tan\alpha = \frac{15}{8}$ or *their* $R\sin\alpha = 15$ or *their* $R\cos\alpha = 8$	1.1a	M1	
	Obtains 17 cos(x – 61.9°) AWRT 61.9 Condone missing units.	1.1b	A1	
	Question 8 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Selects an appropriate method to determine A and B eg forms $x + 3 = A(1 - x) + B(1 - 3x)$ **and** substitutes a value for x or compares coefficients. Or Uses cover-up method/inspection PI by correct A or B	3.1a	M1	$x + 3 = A(1 - x) + B(1 - 3x)$ $x = 1 \Rightarrow 4 = -2B$ $B = -2$ $A = 5$
	Obtains correct A=5 and B=–2	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Obtains 2 correct terms ACF	1.1a	M1	$(1 - x)^{-1} \approx$ $1 + (-1)(-x) + \dfrac{(-1)(-2)(-x)^2}{2}$ $= 1 + x + x^2$
	Obtains $1 + x + x^2$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Obtains $1 + 3x + 9x^2$	2.2a	R1	$1 + 3x + 9x^2$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(i)	Forms their $A(1 + 3x + 9x^2) + B(1 + x + x^2)$ Or Expands their $(x + 3)(1 + x + x^2)(1 + 3x + 9x^2)$ with two brackets correctly expanded up to terms in x^2	3.1a	M1	$5(1 + 3x + 9x^2) - 2(1 + x + x^2)$ $= 3 + 13x + 43x^2$
	Obtains $3 + 13x + 43x^2$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(ii)	Deduces range of values for x $\lvert x \rvert < \dfrac{1}{3}$ OE	2.2a	R1	$-\dfrac{1}{3} < x < \dfrac{1}{3}$
	Subtotal		**1**	

	Question 9 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Takes natural logs of both sides to obtain $\ln N = \ln (A \times B^t)$ or $\ln \frac{N}{A} = \ln B^t$	3.4	B1	$\ln N = \ln (A \times B^t)$ $\ln N = \ln A + \ln B^t$ $\ln N = \ln A + t \ln B$
	Uses a log rule correctly for example $\ln \frac{N}{A} = \ln N - \ln A$ $\ln (A \times B^t) = \ln A + \ln B^t$ $\ln B^t = t \ln B$	1.1a	M1	
	Completes reasoned argument to show the linear relationship For example, obtains $\ln N = \ln A + t \ln B$ with no errors Typical solution with first line omitted scores B0M1R1 Typical solution with second line omitted scores B1M1R0	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	Obtains 0.3	1.1b	B1	$\frac{4.2-3}{5-1} = 0.3$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Uses $\ln B =$ their 0.3	1.1a	M1	$\ln B = 0.3$ $B = e^{0.3}$ $= 1.35$
	Obtains AWRT 1.35 Accept exact value.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(iii)	Writes one of • ln $A = k$ • ln $N = k$ • e^k where k is AWFW (2.6,2.8) PI by AWFW (13.46,16.45)	3.4	M1	3 = ln A + 1 × 0.3 ln A = 2.7 A = 14.9 Initial number of newts released was 15
	Obtains AWFW (13.46,16.45)	1.1b	A1	
	States 14 or 15 Either of these must come from use of k = 2.7 or 2.68 unless NMS when 14 or 15 scores M1 A1 A1	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(iv)	Forms an equation of the form $10Q = Q \times 1.35^t$ where Q is their A or their A rounded to the nearest integer and using their 1.35 Allow for example $150 = 14.9 \times 1.35^t$ Or ln 150 = ln 14.9 + t ln 1.35 OE for example ln 150 = 2.7 + 0.3t using their 14.9 or 15 and 1.35 Or $10 = 1.35^t$ using their 1.35 Accept use of inequality	3.4	M1	150 = 15 × 1.35^t 8 years
	Obtains 8 Must come from a correct value of B=1.35 or $e^{0.3}$ OE and their value of A	3.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	States an appropriate factor related to resource limitation or death and no incorrect factors For example Limited food Disease Limited habitat Death Predation	3.5c	E1	The model could be refined to account for disease.
	Subtotal		**1**	

	Question 10 Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
11	Selects a correct method to find p(x) For example Integrates to obtain at least one correct term in x Or Differentiates a general quartic and obtains at least one correct coefficient	3.1a	M1	$\text{p}(x) = 2x^4 - x^3 - 10x + c$ $\text{p}\left(\frac{1}{2}\right) = 0$ $\Rightarrow 2\left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^3 - 10\left(\frac{1}{2}\right) + c = 0$ $\Rightarrow c = 5$ $\Rightarrow \text{p}(x) = \frac{8x^4}{4} - \frac{3x^3}{3} - 10x + 5$
	Obtains at least two correct terms in x Or Compares and obtains at least two correct coefficients	1.1a	M1	
	Obtains $\frac{8x^4}{4} - \frac{3x^3}{3} - 10x + c$ OE $2x^4 - x^3 - 10x + c$ Missing +c can be recovered if they go on to obtain $(2x - 1)(x^3 - 5)$ or $2x^4 - x^3 - 10x + 5$	1.1b	A1	
	Selects a correct method to find the constant term of p(x) For example Substitutes $\frac{1}{2}$ into their quartic $2x^4 - x^3 - 10x$ (+ c = 0) must not be equated to anything other than 0. Or Uses algebraic division or inspection to obtain a cubic factor with at least two terms with the correct coefficient of x^3 for their $2x^4 - x^3 - 10x$ (+ c) PI by correct answer	3.1a	M1	
	Deduces $(\text{p}(x) =)\ \frac{8x^4}{4} - \frac{3x^3}{3} - 10x + 5$ OE $(\text{p}(x) =)\ 2x^4 - x^3 - 10x + 5$ may be in factorised form $(2x - 1)(x^3 - 5)$	2.2a	A1	
	Question 11 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Explains that there are more intersections/solutions Must be definite AND that would be seen on the graph if it was extended or that lie outside of the range of values shown	2.3	E1	The graphs will continue to intersect for values of $x < 0$ and $x > 4$. The equation will have lots of solutions.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Uses $\sin 2a \approx 2a$ Or Uses $\sin 2a = 2\sin a \cos a$ AND $\sin a \approx a$ AND $\cos a \approx 1 - \frac{a^2}{2}$	3.1a	M1	$\frac{1}{\sin 2a} = 4 - a$ $\frac{1}{2a} \approx 4 - a$ $1 \approx 8a - 2a^2$ $2a^2 - 8a + 1 \approx 0$
	Completes reasoned argument to show the given result. If $\sin 2a = 2\sin a \cos a$ approach is used they must explain that higher powers can be ignored for small values. Condone equality throughout Must have conclusion in terms of a Condone α	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Obtains at least one solution to $2a^2 - 8a + 1 = 0$ $\frac{4 \pm \sqrt{14}}{2}$, 3.87(0…), 0.129(1…)	1.1a	M1	$2a^2 - 8a + 1 = 0$ $a = \frac{4 \pm \sqrt{14}}{2}$ $a = 0.129$
	Infers the correct decimal value AWRT 0.129	2.2b	R1	
	Subtotal		**2**	

	Question 12 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)(i)	Forms an equation for the area of the form $Axw + B\pi x^2 = 100$,	3.3	M1	$4wx + \frac{1}{2}\pi x^2 = 100$ $4wx = 100 - \frac{1}{2}\pi x^2$ $4wx = \frac{200 - \pi x^2}{2}$ $w = \frac{200 - \pi x^2}{8x}$
	Completes reasoned argument with at least one more line of working to show $width = \frac{200 - \pi x^2}{8x}$	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)(ii)	Obtains $6x + 2w + \pi x$ Can be unsimplified ISW Accept a consistently used alternative letter for width.	3.3	B1	$P = 6x + 2w + \pi x$ $P = 6x + 2\left(\frac{200 - \pi x^2}{8x}\right) + \pi x$ $= 6x + \frac{50}{x} - \frac{\pi x}{4} + \pi x$ $= \left(6 + \frac{3\pi}{4}\right)x + \frac{50}{x}$
	Uses the given result from part (a)(i) to eliminate w from their expression for perimeter in the form $Ax + Bw + C\pi x$ to form an expression for P in terms of x	3.4	M1	
	Completes reasoned argument with at least one more line of working, with no errors seen, to show $P = \left(6 + \frac{3\pi}{4}\right)x + \frac{50}{x}$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)(i)	Explains a minimum or stationary point will occur when $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$ or $\frac{\mathrm{d}C}{\mathrm{d}x} = 0$ where C is the cost.	2.4	E1	The minimum perimeter will occur when $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$ $\frac{\mathrm{d}P}{\mathrm{d}x} = 6 + \frac{3\pi}{4} - \frac{50}{x^2}$ $6 + \frac{3\pi}{4} - \frac{50}{x^2} = 0$ $x^2 = \frac{200}{24 + 3\pi}$ $x = 2.45$ Cost $=$ $\left(\left(6 + \frac{3\pi}{4}\right) \times 2.45 + \frac{50}{2.45}\right) \times 12.75$ $= 521.23$ £521.23 $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{100}{x^3} = \frac{100}{2.45^3} = 6.8 > 0$ $\therefore$ P is a min So the minimum cost is £521.23
	Uses the model to minimise perimeter or cost. For example Differentiates P w.r.t. x with either $6 + \frac{3\pi}{4}$ or $-\frac{50}{x^2}$ correct. Could be multiplied by 12.75	3.4	M1	
	Obtains $6 + \frac{3\pi}{4} - \frac{50}{x^2}$ Could be multiplied by 12.75	1.1b	A1	
	Uses their $6 + \frac{3\pi}{4} - \frac{50}{x^2} = 0$ to obtain a positive value for x	1.1a	M1	
	Obtains cost of £521.23 or £521.24	3.2a	A1	
	Justifies that the cost/perimeter is a minimum. For example Shows that $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{100}{x^3} > 0$, since $x > 0$ Or Compares sign of gradient either side of $x =$ AWRT 2.45	2.4	E1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)(ii)	Gives a reason connected with the length why the cost is likely to be more: • Unlikely to be able to purchase exactly the right length of material. • There might be wastage if the material has to be cut • Corners/curves may cause increased length, possibly due to thickness of material.	3.5a	E1	It is unlikely the material can be bought in exactly the required length.
	Subtotal		**1**	
	Question 13 Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
14	If log rule applied to start with, give first two marks for integration by parts applied to $x^3 \ln x$ and ignore $x^3 \ln 2$ term. Begins integration by parts by writing $u = \ln 2x \qquad v' = x^3$ $u' = \frac{P}{x} \qquad v = Qx^4$ PI by $Qx^4 \ln 2x - \int \frac{P}{x} \times Qx^4 \, dx$ Or $u = x^3 \qquad v' = \ln 2x$ $u' = Rx^2 \qquad v = f(x) \neq \ln 2x$	3.1a	M1	$u = \ln 2x \qquad v' = x^3$ $u' = \frac{1}{x} \qquad v = \frac{x^4}{4}$ $\left[\frac{x^4}{4}\ln 2x\right]_{\frac{1}{2}}^{2} - \int_{\frac{1}{2}}^{2} \frac{1}{x}\frac{x^4}{4} dx$ $= \left[\frac{x^4}{4}\ln 2x - \frac{x^4}{16}\right]_{\frac{1}{2}}^{2}$ $= \left(\frac{2^4}{4}\ln 4 - 1\right) - \left(\frac{\left(\frac{1}{2}\right)^4}{4}\ln 1 - \frac{\left(\frac{1}{2}\right)^4}{16}\right)$ $= \left(4\ln 2^2 - 1\right) - \left(-\frac{1}{256}\right)$ $= 8\ln 2 - \frac{255}{256}$
	Substitutes their u, u', v, v' of either of the above forms into the integration by parts formula eg $Qx^4 \ln 2x - \int \frac{P}{x} \times Qx^4 \, dx$ $x^3 f(x) - \int Rx^2 f(x) \, dx$ PI by $\frac{x^4}{4} \ln 2x - \frac{x^4}{16}$ OE	1.1a	M1	
	Obtains $\frac{x^4}{4} \ln 2x - \int \frac{1}{x} \frac{x^4}{4} \, dx$ Condone missing dx PI by $\frac{x^4}{4} \ln 2x - \frac{x^4}{16}$	1.1b	A1	
	Completes integration by parts to obtain $\frac{x^4}{4} \ln 2x - \frac{PQx^4}{4}$	1.1a	M1	
	Substitutes correct limits into $\frac{x^4}{4} \ln 2x - Kx^4$ and subtracts condone poor bracketing.	1.1a	M1	
	Completes reasoned argument to obtain $8\ln 2 - \frac{255}{256}$ With at least one line of substituted values before final answer.	2.1	R1	
	Question 14 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Obtains an expression for either sector *OCD* or *OBC* For example $\frac{1}{2}r^2\theta$, $\frac{1}{2}OD^2\left(\frac{\pi}{2}-\theta\right)$ Condone 90 in place of $\frac{\pi}{2}$	1.1b	B1	Area of sector OBC= $\frac{1}{2}r^2\theta$ Area of triangle OAC = $\frac{1}{2}OA\times OC\sin\theta$ $\frac{1}{2}r^2\theta-\frac{1}{2}r^2\cos\theta\sin\theta=\frac{1}{2}\frac{\pi r^2}{4}$ $\frac{1}{2}r^2\theta-\frac{1}{4}r^2\times 2\cos\theta\sin\theta=\frac{\pi r^2}{8}$ $\frac{1}{2}r^2\theta-\frac{1}{4}r^2\sin 2\theta=\frac{\pi r^2}{8}$ $4r^2\theta-2r^2\sin 2\theta=\pi r^2$ $4\theta-2\sin 2\theta=\pi$
	Obtains an expression for the area of the triangle *OAC* in terms of θ For example $\frac{1}{2}r\times r\times\cos\theta\sin\theta$ $\frac{1}{2}r\times OA\sin\theta$ Do not award for $\frac{1}{2}ab\sin\theta$ unless a and b are clearly linked to the diagram	1.1b	B1	
	Selects a method to obtain an expression for the area of gold or the area of silver in terms of θ and **one** other variable eg r For example for gold Subtracts their triangle *OAC* from their sector *OBC* to obtain an expression for the area of *ABC* Or for silver adds their triangle *OAC* to their sector *OCD* to obtain an expression for the area of *OACD*	3.1a	M1	
	Forms a correct equation in terms of θ and one other variable For example $\frac{1}{2}r^2\left(\frac{\pi}{2}-\theta\right)+\frac{1}{4}r^2\sin 2\theta=\frac{\pi r^2}{8}$ $\frac{1}{2}r^2\theta-\frac{1}{2}r^2\cos\theta\sin\theta=\frac{1}{2}\frac{\pi r^2}{4}$ $\frac{1}{2}r^2\left(\frac{\pi}{2}-\theta\right)+\frac{1}{4}r^2\sin 2\theta=\frac{1}{2}r^2\theta-\frac{1}{4}r^2\sin 2\theta$	3.4	A1	
	Completes a reasoned argument to show $4\theta-2\sin 2\theta=\pi$	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Evaluates $4\theta - 2\sin 2\theta - \pi$ Or Evaluates $\pi + 2\sin 2\theta - 4\theta$ at 1.1 ($\pm$ 0.358…) and at 1.2 ($\mp$ 0.307…) Or Evaluates using any two other appropriate values inside the interval but either side of root.	1.1a	M1	$4\theta - 2\sin 2\theta = \pi \Rightarrow 4\theta - 2\sin 2\theta - \pi = 0$ Let $f(\theta) = 4\theta - 2\sin 2\theta - \pi$ $f(1.1) = -0.358... < 0$ $f(1.2) = 0.307... > 0$ Hence the solution lies between 1.1 and 1.2
	Completes reasoned argument with reference to change of sign and the values 1.1 and 1.2 **and** correct rearrangement eg $4\theta - 2\sin 2\theta - \pi = 0$ **and** evaluation, accepting values rounded or truncated to 1 sf	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)	Differentiates $2\sin 2\theta$ to obtain $4\cos 2\theta$ OE PI $4\cos 2\theta$ or $4\cos 2\times 1.1$ seen in denominator PI by correct θ_1	1.1b	B1	$\theta_{n+1} = \theta_n - \dfrac{4\theta_n - 2\sin 2\theta_n - \pi}{4 - 4\cos 2\theta_n}$ $\theta_1 = 1.1564$
	Obtains an expression of the form $\theta_n - \dfrac{\pm(4\theta_n - 2\sin 2\theta_n - \pi)}{\pm(4 - 4\cos 2\theta_n)}$ OE where θ_n can be θ_0 θ x or 1.1 condone missing subscripts PI by correct θ_1	1.1a	M1	
	Obtains AWRT 1.1564	1.1b	A1	
	Subtotal		**3**	

	Question 15 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	Obtains $\frac{1}{2}$	1.1b	B1	0.5
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(i)	Obtains $\dfrac{\left(\dfrac{e^{1-(n+1)} \text{ OE}}{2^{n+1}}\right)}{\left(\dfrac{e^{1-n}}{2^n}\right)}$ Or $\dfrac{e^{1-(n+1)} \text{ OE}}{2^{n+1}} \times \dfrac{2^n}{e^{1-n}}$ Or substitutes an integer value for $n \geq 1$ correctly obtains $\dfrac{1}{2e}$ with no errors	2.1	B1	$\dfrac{u_{n+1}}{u_n} = \dfrac{\left(\dfrac{e^{1-(n+1)}}{2^{n+1}}\right)}{\left(\dfrac{e^{1-n}}{2^n}\right)}$ $= \dfrac{2^n e^{1-(n+1)}}{2^{n+1} e^{1-n}}$ $= 2^{-1}e^{-1}$ $= \dfrac{1}{2e}$
	Simplifies to obtain correct power of e or correct power of 2 starting from $\dfrac{\left(\dfrac{e^{1-(n+1)}}{2^{n+1}}\right)}{\left(\dfrac{e^{1-n}}{2^n}\right)}$ or $\dfrac{e^{1-(n+1)}}{2^{n+1}} \times \dfrac{2^n}{e^{1-n}}$ OE with at least one line of correct working before reaching $2^{-1}e^{-1}$ or $\dfrac{1}{2e}$ condone starting from $\dfrac{\left(\dfrac{e^{1-n+1}}{2^{n+1}}\right)}{\left(\dfrac{e^{1-n}}{2^n}\right)}$	1.1a	M1	
	Completes reasoned argument to show $\dfrac{u_{n+1}}{u_n} = \dfrac{1}{2e}$ CSO	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(ii)	States $-1<\frac{1}{2\mathrm{e}}<1$ Or states r = $\frac{1}{2\mathrm{e}}$ and $-1<r<1$ Accept $0<\frac{1}{2\mathrm{e}}<1$ OE	2.4	E1	$-1<\frac{1}{2\mathrm{e}}<1$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16(c)	Substitutes their u_1 or $\frac{1}{2\mathrm{e}}$ into sum to infinity formula	1.1a	M1	$\dfrac{\frac{1}{2}}{1-\frac{1}{2\mathrm{e}}}$ $=\dfrac{1}{2\left(1-\frac{1}{2\mathrm{e}}\right)}=\dfrac{\mathrm{e}}{2\mathrm{e}-1}$
	Substitutes $\frac{1}{2}$ and $\frac{1}{2\mathrm{e}}$ into sum to infinity formula	1.1b	A1	
	Completes reasoned argument to obtain $\frac{\mathrm{e}}{2\mathrm{e}-1}$	2.1	R1	
	Subtotal		**3**	

	Question 16 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)	Separates variables to obtain an equation of the form $\int \frac{P}{h^2}\,\mathrm{d}h = \int \frac{1}{Q}\left(2+\sin\frac{t}{4}\right)\mathrm{d}t$ where $PQ = -400$ Must have integral signs and consistent dh and dt	3.1a	B1	$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h^2}{400}\left(2+\sin\frac{t}{4}\right)$ $\int \frac{400}{h^2}\,\mathrm{d}h = \int -\left(2+\sin\frac{t}{4}\right)\mathrm{d}t$ $-\frac{400}{h} = -2t + 4\cos\frac{t}{4} + c$ $t=0, h=10 \Rightarrow -40 = 4 + c$ $c = -44$ $h = \frac{400}{2t+44-4\cos\frac{t}{4}} = \frac{200}{t+22-2\cos\frac{t}{4}}$
	Integrates the $\frac{P}{h^2}$ term to obtain $-\frac{P}{h}$	1.1a	M1	
	Integrates $\sin\frac{t}{4}$ to obtain $R\cos\frac{t}{4}$	1.1a	M1	
	Completes integration to obtain $-\frac{400}{h} = -2t + 4\cos\frac{t}{4} + c$ OE Condone missing constant of integration	1.1b	A1	
	Uses t = 0 and h = 10 to determine constant of integration for their equation of the form $\frac{D}{h} = Et + F\cos\frac{t}{4} + c$ OE	3.4	M1	
	Obtains $-\frac{400}{h} = -2t + 4\cos\frac{t}{4} - 44$ OE	1.1b	A1	
	Completes reasoned argument to obtain $h = \frac{200}{t+22-2\cos\frac{t}{4}}$ This mark can be scored without scoring B1 for missing dh etc at the start	2.1	R1	
	Subtotal		**7**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)	Substitutes $t = 4$ into their model from part (a) of the form $h = \dfrac{A}{Bt + C - 2\cos\frac{t}{4}}$ to obtain a value for h PI by AWRT 8.03	3.4	M1	$\dfrac{200}{4 + 22 - 2\cos\frac{4}{4}} = 8.03$ 8.03 is approximately 8 so the model is valid
	Compares AWRT 8.03 to 8 or compares AWRT 1.97 to 2 AND comments that the model is valid.	3.5a	A1	
	Subtotal		**2**	

	Question 17 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
18(a)	Obtains $x^4 - y^4$	1.1b	B1	$x^4 - y^4$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)	Begins argument from LHS or RHS applying one useful manipulation from the following: Uses $(x^2 + y^2)(x^2 - y^2) = x^4 - y^4$ Or Converts between cos/sin and cosec/cot Or Uses $\cot^2\theta - \text{cosec}^2\theta = -1$ Or Uses $1 - \cos^2\theta = \sin^2\theta$	3.1a	M1	$\cot^4\theta - \text{cosec}^4\theta$ $\equiv (\cot^2\theta + \text{cosec}^2\theta)(\cot^2\theta - \text{cosec}^2\theta)$ $\equiv -(\cot^2\theta + \text{cosec}^2\theta)$ $\equiv -\dfrac{\cos^2\theta + 1}{\sin^2\theta}$ $\equiv -\dfrac{\cos^2\theta + 1}{1 - \cos^2\theta}$ $\equiv \dfrac{\cos^2\theta + 1}{\cos^2\theta - 1}$
	Makes progress with their argument applying a second technique listed above.	3.1a	M1	
	Makes further progress with their rigorous argument applying $(x^2 + y^2)(x^2 - y^2) = x^4 - y^4$ **AND** Converts between cos/sin and cosec/cot **AND** at least one of Uses $\cot^2\theta - \text{cosec}^2\theta = -1$ Or Uses $1 - \cos^2\theta = \sin^2\theta$ If working from LHS to RHS correctly obtaining a single fraction in terms of $\cos^2\theta$ scores M1M1M1	3.1a	M1	
	Completes rigorous argument to prove the given result condone starting the argument with “LHS” or “RHS”	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
18(c)	Begins an argument starting from $\dfrac{\cos^2\theta+1}{\cos^2\theta-1}$ For example Explains that $\cos^2\theta + 1 > 0$ Condone inclusive inequality or Explains that $\cos^2\theta - 1 < 0$ Condone inclusive inequality	2.4	M1	$\cos^2\theta + 1 > 0$ $\cos^2\theta - 1 < 0$ $\cot^4\theta - \text{cosec}^4\theta = \dfrac{\text{positive}}{\text{negative}} < 0$
	Completes reasoned argument For example Explains that $\cos^2\theta + 1 > 0$ AND Explains that $\cos^2\theta - 1 < 0$ Completes reasoned argument stating that $\cot^4\theta - \text{cosec}^4\theta = \dfrac{\text{positive}}{\text{negative}} < 0$	2.1	R1	
	Subtotal		**2**	

	Question 18 Total		**7**	

	Question Paper Total		**100**	