A: Proof

Name:	_____
Class:	_____
Date:	_____

Time:	**77 minutes**
Marks:	**65 marks**
Comments:	

Q1.

Prove that

n is a prime number greater than 5 $\Rightarrow$ n^4 has final digit 1

(Total 5 marks)

Q2.

$p(x) = 30x^3 - 7x^2 - 7x + 2$

(a) Prove that $(2x + 1)$ is a factor of $p(x)$

(2)

(b) Factorise $p(x)$ completely.

(3)

(c) Prove that there are no real solutions to the equation

(5)

(Total 10 marks)

Q3.

Which of these statements is correct?

Tick **one** box.

$x = 2 \Rightarrow x^2 = 4$	
$x^2 = 4 \Rightarrow x = 2$	
$x^2 = 4 \Leftrightarrow x = 2$	
$x^2 = 4 \Rightarrow x = -2$	

(Total 1 mark)

Q4.

Prove that 23 is a prime number.

(Total 2 marks)

Q5.

Prove by contradiction that $\square$ is an irrational number.

(Total 7 marks)

Q6.

Jessica, a maths student, is asked by her teacher to solve the equation $\tan x = \sin x$, giving all solutions in the range $0° \leq x \leq 360°$

The steps of Jessica's working are shown below.

$$\tan x = \sin x$$

Step 1	$\Rightarrow$	$\frac{\sin x}{\cos x} = \sin x$	Write $\tan x$ as $\frac{\sin x}{\cos x}$
Step 2	$\Rightarrow$	$\sin x = \sin x \cos x$	Multiply by $\cos x$
Step 3	$\Rightarrow$	$1 = \cos x$	Cancel $\sin x$
	$\Rightarrow$	$x = 0°$ or $360°$	

The teacher tells Jessica that she has not found all the solutions because of a mistake.

Explain why Jessica's method is not correct.

(Total 2 marks)

Q7.

Prove that the function $f(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

(Total 6 marks)

Q8.

(a) Given that n is an even number, prove that $9n^2 + 6n$ has a factor of 12

(3)

(b) Determine if $9n^2 + 6n$ has a factor of 12 for any integer n.

(1)

(Total 4 marks)

Q9.

$p(x) = 2x^3 + 7x^2 + 2x - 3$

(a) Use the factor theorem to prove that $x + 3$ is a factor of $p(x)$

(2)

(b) Simplify the expression

(4)

(Total 6 marks)

Q10.

Prove the identity $\cot^2\theta - \cos^2\theta \equiv \cot^2\theta\cos^2\theta$

(Total 3 marks)

Q11.

A student argues that when a rational number is multiplied by an irrational number the result will always be an irrational number.

(a) Identify the rational number for which the student's argument is not true.

(1)

(b) Prove that the student is right for all rational numbers other than the one you have identified in part **(a)**.

(4)

(Total 5 marks)

Q12.

A student notices that when he adds two consecutive odd numbers together the answer always seems to be the difference between two square numbers.

He claims that this will always be true.
He attempts to prove his claim as follows:

Step 1: Check first few cases
$3 + 5 = 8$ and $8 = 3^2 - 1^2$
$5 + 7 = 12$ and $12 = 4^2 - 2^2$
$7 + 9 = 16$ and $16 = 5^2 - 3^2$

Step 2: Use pattern to predict and check a large example
$101 + 103 = 204$
subtract 1 and divide by 2 for the first number
Add 1 and divide by two for the second number
$52^2 - 50^2 = 204$ it works!

Step 3: Conclusion
The first few cases work and there is a pattern, which can be used to predict larger numbers.
Therefore, it must be true for all consecutive odd numbers.

(a) Explain what is wrong with the student's "proof".

(1)

(b) Prove that the student's claim is correct.

(3)

(Total 4 marks)

Q13.

(a) Three consecutive terms in an arithmetic sequence are $3e^{-p}$, 5, $3e^{p}$

Find the possible values of p. Give your answers in an exact form.

(6)

(b) Prove that there is no possible value of q for which $3e^{-q}$, 5, $3e^{q}$ are consecutive terms of a geometric sequence.

(4)

(Total 10 marks)