

Mark schemes

Q1.

	Marking Instructions	AO	Marks	Typical Solution
	Investigates last digit of n . Allow M1 for investigation of $2k + 1$	AO3.1a	M1	Last digit of n determines last digit of n^4 All even numbers divide by 2, so are not prime Any number ending in 5 is a multiple of 5 so is not prime Primes > 5 end in 1, 3, 7 or 9
	Deduces that only need to investigate numbers ending in 1, 3, 7, 9 Condone inclusion of 5 at this stage	AO2.2a	M1	
	Considers each in turn to show that n^4 will end in a 1	AO1.1a	M1	If n ends in 1, 1^4 is 1 so n^4 ends in a 1 If n ends in 3, 3^4 is 81 so n^4 ends in a 1 If n ends in 7, 7^4 is 2401 so n^4 ends in a 1 If n ends in 9, 9^4 is 6561 so n^4 ends in a 1
	Provides evidence that 1^4 , 3^4 , 7^4 , 9^4 all end in a 1	AO1.1b	A1	
	Constructs rigorous mathematical argument to show the required result Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips. Must include clear statement that final digit of n determines final digit of n^4	AO2.1	R1	Statement proved by exhaustion
	Total 5 marks			

Q2.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Begins a proof using a valid method Eg. Factor theorem, algebraic division, multiplication of correct factors	AO1.1a	M1	$p\left(-\frac{1}{2}\right) = 30 \times \left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) + 2$ $= 0$ $\therefore 2x + 1 \text{ is a factor of } p(x)$
	Constructs rigorous mathematical proof. To achieve this mark: Factor theorem the student must clearly substitute and state that $p(-1/2) = 0$ and clearly state that this implies that $2x + 1$ is a factor Algebraic division OR Multiplication of correct factors The method must be completely correct with a concluding statement	AO2.1	R1	
(b)	Obtains quadratic factor PI	AO1.1a	M1	$p(x) = (2x + 1)(15x^2 - 11x + 2)$ $= (2x + 1)(5x - 2)(3x - 1)$
	Obtains second linear factor	AO1.1b	A1	
	Writes $p(x)$ as the product of the correct three linear factors. NMS correct answer 3/3	AO1.1b	A1	
(c)	Rearranges to achieve a cubic equation in $\sec x$ (or $\cos x$)	AO3.1a	M1	$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$ $\Rightarrow 30\sec^2 x + 2\cos x = 7\sec x + 7$ $\Rightarrow 30\sec^3 x + 2 = 7\sec^2 x + 7\sec x$ $30\sec^3 x - 7\sec^2 x - 7\sec x + 2 = 0$ $\Rightarrow (2\sec x + 1)(5\sec x - 2)(3\sec x - 1) = 0$ $\Rightarrow \sec x = -\frac{1}{2}, \frac{2}{3}, \frac{1}{3}$

				These values do not fall within the range of $\sec x$ as they are between -1 and 1 $\therefore \frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$ has no real solutions.
	Equates to zero and uses result from (b) or factorises	AO1.1a	M1	
	Deduces that if solutions exist they must be of the form $\sec x = -\frac{1}{2}$, $\sec x = \frac{1}{3}$ or $\sec x = \frac{2}{5}$ OE	AO2.2a	A1	
	Explains that the range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$ OE OE for $\cos x$	AO2.4	E1	
	Completes argument explaining that there cannot be any real solutions as values are outside of the function's range	AO2.1	R1	
	Total 10 marks			

Q3.

Marking Instructions	AO	Marks	Typical Solution
Ticks correct option	AO2.5	B1	$x = 2 \Rightarrow x^2 = 4$
Total 1 mark			

Q4.

Marking Instructions	AO	Marks	Typical Solution
Begins checking for factors to start proof by exhaustion or makes a statement about numbers which don't need to be checked	AO3.1a	M1	so only need to check 2 and 3
Completes rigorous argument , for example: Only need to check primes less than 23 is not divisible by 2 or 3	AO2.1	R1	23 is odd so no need to check 2. 23 is not a multiple of 3 $\therefore$ 23 is prime.

	therefore 23 is prime or checks all possible factors or checks more factors than necessary, but argument must be complete.			
	Total 2 marks			

Q5.

	Marking Instructions	AO	Marks	Typical Solution
	Begins proof by contradiction, assumes that $\sqrt[3]{2}$ is rational OE	AO3.1a	M1	Assume $\sqrt[3]{2}$ is rational $\sqrt[3]{2} = \frac{a}{b},$ a and b have no common factors $\Rightarrow \sqrt[3]{2b^3} = a$ $\Rightarrow 2b^3 = a^3$ $\therefore a$ is even let $a = 2d$ then $2b^3 = 8d^3$ $\Rightarrow b^3 = 4d^3$ $\therefore b$ is even Hence, a and b have a common factor of 2. This is a contradiction. $\therefore$ the assumption that $\sqrt[3]{2}$ is rational must be incorrect and it is proved that $\sqrt[3]{2}$ is an irrational number
	Uses language and notation correctly to state initial assumptions	AO2.5	B1	
	Manipulates fraction including cubing.	AO1.1a	M1	
	Deduces a is even	AO2.2a	R1	
	Deduces b is even	AO2.2a	R1	
	Explains why there is a	AO2.4	E1	

	contradiction			
	Completes rigorous argument to show that ☐ is irrational	AO2.1	R1	
	Total 7 marks			

Q6.

	Marking Instructions	AO	Marks	Typical Solution
	Demonstrates a clear understanding that $\sin x = 0$ is a solution, and that this has not been properly taken into account.	AO2.3	R1	$\sin x = 0$ leads to a solution, but when she cancelled $\sin x$ she effectively assumed it was not equal to 0 and hence lost a number of solutions.
	Explains that cancelling $\sin x$ is not allowed if it is zero / only allowed if it is non-zero	AO2.4	E1	
	Total 2 marks			

Q7.

	Marking Instructions	AO	Marks	Typical Solution
	Explains clearly that $f(x)$ is increasing $\Leftrightarrow f'(x) > 0$ (for all values of x) or Explains $\Rightarrow f(x)$ is increasing $f'(x) > 0$ for all values of x This may appear at any appropriate point in their argument	AO2.4	E1	For all x, $f'(x) > 0 \Rightarrow f(x)$ is an increasing function $f(x) = x^3 - 3x^2 + 15x - 1$ $\Rightarrow f'(x) = 3x^2 - 6x + 15$ $\Rightarrow f'(x) = 3(x-1)^2 + 12$ $\therefore f'(x) \text{ has a minimum value of } 12 \quad \left. \vphantom{\begin{matrix} f(x) \\ f'(x) \end{matrix}} \right\} (*)$ therefore $f'(x) > 0$ for all values of x OR for $f'(x)$, $b^2 - 4ac = -144$ $\therefore f'(x) \neq 0$ for any real x, so $f'(x)$ is either always positive or always negative.

				$f'(0) = 15$ therefore $f'(x) > 0$ for all values of x OR $f''(x) = 6x - 6$, which $= 0$ when $x = 1$ so $\min f'(x)$ is $f'(1) = 12$ therefore $f'(x) > 0$ for all values of x Thus, since, $f'(x) > 0$ for all values of x it is proven that $f(x)$ is an increasing function.
	Differentiates – at least two correct terms	AO1.1a	M1	
	All terms correct	AO1.1b	A1	
	Attempts a correct method which could lead to $f'(x) > 0$	AO3.1a	M1	
	Correctly deduces $f'(x) > 0$ (for all values of x)	AO2.2a	A1	
	Writes a clear statement that links the steps in the argument together, the deduction about a positive gradient for all values of x proves that the given function is increasing for all values of x	AO2.1	R1	
	Total 6 marks			

Q8.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Begins to construct a rigorous mathematical proof by generalising the form of an even number and substituting it into the given expression	AO2.1	M1	Let $n = 2m$ $9n^2 + 6n = 9(2m)^2 + 6(2m)$ $= 36m^2 + 12m$ $= 12(3m^2 + m)$
	Simplifies expression and	AO1.1b	A1	Hence 12 is a factor of the

	extracts 12 as a common factor			expression $9n^2 + 6n$ when n is any even number
	Completes rigorous proof – well explained. A statement is required that links the factor of 12 to the expression $9n^2 + 6n$ when n is an even number	AO2.1	R1	
(b)	Uses a counter example by substituting any odd number into the expression and shows that the resulting value is not a multiple of 12	AO2.2a	R1	Let $n = 1$ $9(1)^2 + 6(1) = 15$ 12 is not a factor of 15 and hence statement is not true for all integers n
Total 4 marks				

Q9.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Demonstrates $p(-3) = 0$	AO1.1b	B1	$p(-3) = 2(-3)^3 + 7(-3)^2 + 2(-3) - 3$ $= -54 + 63 - 6 - 3 = 0$ $p(-3) = 0 \Rightarrow x + 3$ is a factor
	Constructs rigorous mathematical proof (to achieve this mark, the student must clearly calculate and state that $p(-3) = 0$ and clearly state that this implies that $x + 3$ is a factor)	AO2.1	R1	
(b)	Factorises the numerator and denominator (this mark is achieved for any reasonable attempt at factorisation through the selection of an appropriate method, eg long division)	AO1.1a	M1	$\frac{(x+3)(2x^2+x-1)}{(2x+1)(2x-1)}$ $= \frac{(x+3)(2x-1)(x+1)}{(2x+1)(2x-1)}$ $= \frac{(x+3)(x+1)}{(2x+1)}, x \neq \pm \frac{1}{2}$
	Finds second factor in numerator or fully factorises denominator (PI by complete factorisation)	AO1.1b	A1	

	Finds fully correct factorised expression (PI by complete factorisation)	AO1.1b	A1	
	Obtains a completely correct solution with restriction on domain stated	AO1.1b	A1	
	Total 6 marks			

Q10.

	Marking Instructions	AO	Marks	Typical Solution
	Recalls a correct trig identity, which could lead to a correct answer	AO1.2	B1	$ \begin{aligned} &(\text{LHS} \equiv) \\ &\cot^2 \theta - \cos^2 \theta \\ &= \frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta \\ &= \cos^2 \theta \left(\frac{1}{\sin^2 \theta} - 1 \right) \\ &\equiv \cos^2 \theta (\operatorname{cosec}^2 \theta - 1) \\ &\equiv \cos^2 \theta \cot^2 \theta \\ &(\equiv \text{RHS}) \\ &\mathbf{AG} \end{aligned} $
	Performs some correct algebraic manipulation and uses second identity to commence proof (at least two lines of argument)	AO2.1	R1	
	Concludes a rigorous mathematical argument to prove given identity AG Must start with one side and through clear logical steps arrive at the other side. In order to be sufficiently clear, each line should be a single step, unless clear further explanation is given.	AO2.1	R1	
	Total 3 marks			

Q11.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Identifies zero as number for which student's argument is not true	AO1.2	B1	0
(b)	Uses 'proof by contradiction' Must see commencement of argument including stated assumption and at least two lines of argument	AO2.1	M1	Let a be irrational, and b be a non-zero rational, so $b = \frac{c}{d}$, where $c, d \in \mathbb{Z}$; $c, d \neq 0$ Assume ab is rational, so $ab = \frac{p}{q}$ where $p, q \in \mathbb{Z}$; $q \neq 0$ $\therefore \frac{ac}{d} = \frac{p}{q}$ $\therefore a = \frac{pd}{qc}$ so a is rational, which is a contradiction Hence ab must be irrational
	Represents product of rational and irrational numbers in symbolic form	AO2.5	M1	
	Correctly deduces that the product must be irrational	AO2.2a	A1	
	Completes a rigorous mathematical argument, proving that a non-zero rational multiplied by an irrational is irrational Must start with initial assumptions and prove the result convincingly Must define p, q, c, d as integers	AO2.1	R1	
				Total 5 marks

Q12.

	Marking Instructions	AO	Marks	Typical Solution
(a)	States any correct reason	AO2.3	B1	Just checking a few cases only proves it for those cases
(b)	Commences an argument, writing the sum of two consecutive odd numbers algebraically (at least two lines of argument)	AO2.1	R1	Two consecutive odd numbers can be written as $2n + 1$ and $2n + 3$ Their sum is $2n + 1 + 2n + 3 \equiv 4n + 4$
	At some point in the argument correctly writes the difference of two appropriate square numbers algebraically	AO2.5	R1	n^2 and $(n + 2)^2$ are two square numbers Their difference is $(n + 2)^2 - n^2$ $\equiv n^2 + 4n + 4 - n^2$ $\equiv 4n + 4$ Therefore the sum of two consecutive odd numbers can always be written as the difference of two square numbers
	Correctly deduces the result from correct working	AO2.2a	R1	
Total 4 marks				

Q13.

	Marking Instructions	AO	Marks	Typical Solution
(a)	Finds a difference between 2 terms	AO3.1a	M1	$3e^p - 5 = 5 - 3e^{-p}$ (*) $3e^p - 10 + 3e^{-p} = 0$ $3e^{2p} - 10e^p + 3 = 0$ $e^p = \frac{1}{3}, 3$ $p = \ln \frac{1}{3}, \ln 3$ ALT to (*)

				$2(5 - 3e^{-p}) = 3e^p - 3e^{-p}$ Or $2(3e^{-p} - 5) = 3e^p - 3e^{-p}$
	Forms an equation using two differences	AO3.1a	M1	
	Forms a quadratic equation in e^p	AO1.1a	M1	
	Obtains a correct quadratic equation	AO1.1b	A1	
	Obtains 2 correct solutions for e^p from 'their' quadratic FT only applies if previous mark has been awarded	AO1.1b	A1F	
	Obtains final answers in an exact form FT applies if previous mark has been awarded	AO2.2a	A1F	
(b)	Finds a ratio between two consecutive terms (no requirement to use a and r)	AO3.1a	M1	Assume it is possible that $3e^{-q}$, 5 and $3e^q$ are three consecutive terms of a geometric sequence $a = 3e^{-q}, ar = 5, ar^2 = 3e^q$ $\frac{ar}{a} = \frac{5}{3e^{-q}} \Rightarrow r = \frac{5e^q}{3}$ $\frac{ar^2}{ar} = \frac{3e^q}{5} \Rightarrow r = \frac{3e^q}{5}$ $\frac{5}{3e^{-q}} = \frac{3e^q}{5} \Rightarrow 25 = 9$ This is a contradiction therefore $3e^{-q}$, 5 and $3e^q$ cannot form three consecutive terms of a geometric sequence.
	Compares two ratios (could be ratios of successive terms, no requirement to use a and r)	AO3.1a	M1	
	Identifies a contradiction	AO2.1	R1	
	Draws a conclusion about k	AO2.4	R1	
	Total 10 marks			



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