CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME: Solved

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS 9709/12

Paper 1 Pure Mathematics 1 February/March 2026

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen. Do not use correction fluid or tape.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages. Any blank pages are indicated.

DC (PQ/SG) 358735/1

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1 The equation of a curve is

$$y = x^3 - 3x^2 + 9.$$

Find the coordinates of the stationary points of the curve and determine their nature. [5]

For stationary

$\frac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$x = 0, 2$

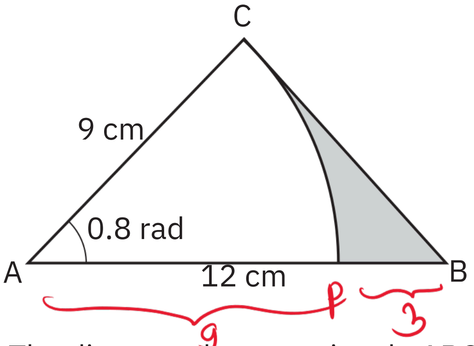
$y = 9, 5$ $\therefore$ stationary points $(0,9), (2,5)$

$\frac{d^2y}{dx^2} = 6x - 6$

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = -6 < 0 \Rightarrow (0,9)$ is maximum point

$\left.\frac{d^2y}{dx^2}\right|_{x=2} = 6 > 0 \Rightarrow (2,5)$ is minimum point

2



The diagram shows a triangle ABC with AB 12= cm, AC 9= cm and angle CAB = 0 8 . radians. Anarc with centre A and radius 9 cm is drawn inside the triangle as shown. The shaded region is bounded by this arc, the line segment BC and part of the line segment AB.

(a) Find the perimeter of the shaded region. Give your answer correct to 3 significant figures.[4]

$CB = \sqrt{9^2 + 12 - 2\cdot 9\cdot 12\cos(0\cdot 8)}$

$\overset{\frown}{PC} = 9 \times 0\cdot 8$

Perimeter $= PB + \overset{\frown}{PC} + CB$

$= 3 + 9\times 0\cdot 8 + \sqrt{9^2 + 12^2 - 2\cdot 9\cdot 12\cos(0\cdot 8)}$

Perimeter $= 18.8$

(b) Find the area of the shaded region. Give your answer correct to 3 significant figures. [3]

Area of :-

$\Delta = \frac{1}{2} \times 9 \times 12 \times \sin(0.8)$

Sector $= \frac{1}{2} \times 9^2 \times 0.8$

Shaded region $= \Delta -$ Sector $= 6.30$

3 The graph of $y=ax^2+bx+c$ has its vertex at (3, -5) and intersects the y-axis at (0, 31).

(a) Find the values of the constants a, b and c. [4]

At vertex,

$\frac{-b}{2a} = 3 \Rightarrow b = -6a$ — ①

At (0,31), $31 = a.0^2 + b.0 + c \Rightarrow c = 31$

At (3,-5), $-5 = 9a + 3b + 31 \Rightarrow -9a = -36$

$a = 4 \Rightarrow b = -24, c = 31$

$\therefore y = 4x^2 - 24x + 31$

(b) The graph is translated by the vector e-2
7o.

Find the equation of the translated graph. [2]

$y = 4(x+2)^2 - 24(x+2) + 31 + 27$

$y = 4(x^2 + 4x + 4) - 24x - 48$

$y = 4x^2 + 16x + 16 - 24x - 48$

$y = 4x^2 - 8x - 32$

4 The function f is defined by

f (x) = 2xx-+31

for x13.

(a) Find an expression for $f^{-1}(x)$. [3]

let $y = f(x) = \frac{2x+1}{x-3}$

Interchanging x and y.

$x = \frac{2y+1}{y-3}$

$xy - 3x = 2y + 1$

$y(x-2) = 1 + 3x$

$y = \frac{1+3x}{x-2}$

$\therefore f^{-1}(x) = \frac{1+3x}{x-2}$

(b) Given that

ff (x) / pr x++sxq ,

find the possible values of the integers p, q, r and s. [3]

$f(f(x))$

$= \frac{2\left(\frac{2x+1}{x-3}\right)+1}{\frac{2x+1}{x-3} - 3}$

$= \frac{4x+2+x-3}{2x+1-3x+9} = \frac{5x-1}{10-x}$

$\therefore f(f(x)) = \frac{5x-1}{10-x} = \frac{5x+(-1)}{10+(-1)x}$

5 (a) Prove the identity

$$\frac{1}{1-\sin i}+\frac{1}{1+\sin i} / \frac{2}{\cos^2 i}.$$ [3]

LHS $= \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$

$= \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$ = RHS #

(b) Hence, solve the equation

$$\frac{3}{1-3\sin i}+\frac{3}{1+3\sin i}=12$$

for 0°GiG360°. [4]

$3\left(\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}\right)=12$

$3 \cdot \frac{2}{\cos^2\theta} = 12$

$\frac{6}{12} = \cos^2\theta$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

+ve, $\theta = 45°, 315°$

-ve, $\theta = 135°, 225°$

6 The equation of a circle is

$$x2+y^2=\frac{45}{4}.$$

(a) State the centre and radius of the circle. [2]

Centre = (0,0)

Radius = $\frac{\sqrt{45}}{2} = \frac{3\sqrt{5}}{2}$

(b) The line $x+y=\frac{3}{2}$ intersects the circle at the points A and B.

Find the exact distance AB. [6]

$x+y=3/2$

$y=\frac{3}{2}-x$

$x^2+\left(\frac{3}{2}-x\right)^2=\frac{45}{4}$

$x^2+\frac{9}{4}-3x+x^2=\frac{45}{4}$

$x=-3/2,\ 3$

$y=3,\ -3/2$

$\therefore A(-3/2, 3)$ and $B(3, -3/2)$

Distance $=\sqrt{\left(3-\left(-3/2\right)\right)^2+\left(-\frac{3}{2}-3\right)^2}$

$=\frac{9\sqrt{2}}{2}$

7 The equation of a curve, C, is such that

$$\frac{dy}{dx} = \frac{(x-3)^2}{\sqrt{x}}.$$

The curve passes through the point (4, 7).

(a) Find y in terms of x. [5]

$$y = \int \frac{(x-3)^2}{\sqrt{x}}\,dx$$

$$= \int \frac{x^2-6x+9}{\sqrt{x}}\,dx$$

$$= \int x^{3/2} - 6x^{1/2} + 9x^{-1/2}\,dx$$

$$= \frac{2}{5}x^{5/2} - 4x^{3/2} + 18x^{1/2} + c$$

At (4, 7), $7 = \frac{2}{5}(4)^{5/2} - 4(4)^{3/2} + 18(4)^{1/2} + c$

$$c = -\frac{49}{5}$$

$$y = \frac{2}{5}x^{5/2} - 4x^{3/2} + 18x^{1/2} - \frac{49}{5}$$

(b) The equation of a different curve, D, is also such that $\frac{dy}{dx} = \frac{(x-3)^2}{\sqrt{x}}$. This curve instead passes through the point (4, k).

Describe, in terms of k, the transformation that maps curve C onto curve D. [2]

Translation by $\begin{pmatrix} 0 \\ k-7 \end{pmatrix}$

8 (a) Find the complete expansion of $(3-2x)^4$. [3]

$(3-2x)^4$

$= {}^4C_0\, 3^0(-2x)^4 + {}^4C_1\, 3^1(-2x)^3 + {}^4C_2\, 3^2(-2x)^2 + {}^4C_3\, 3^3(-2x)^1 + {}^4C_4\, 3^4(-2x)^0$

$= 16x^4 - 96x^3 + 216x^2 - 216x + 81$

(b) Uq saere y ionuter gaenrssw toer b teo d8e(tae)r mtoi nfiendd. `3-22j^4. Give your answer in the fo

$(3-2\sqrt{2})^4$

$= 16(\sqrt{2})^4 - 96(\sqrt{2})^3 + 216(\sqrt{2})^2 - 216(\sqrt{2}) + 81$

$= 577 + (-408)\sqrt{2}$

9 Solve the equation

$$x^{\frac{2}{3}}+(8x)^{\frac{1}{3}}=24.$$ [4]

$x^{2/3}+2x^{1/3}-24=0$

let $u=x^{1/3}$,

$u^2+2u-24=0$

$u=4,-6$

$x=64,-216$

10 Ꟊehneo 7tethd tbeyrm S o.f I at nis a griivthemn ethtiact pSrog=res8s8io0n. is 77. The sum of the first

n 11

(a) Find the 20th term of the progression. [4]

$a + 6d = 77$ —①

$\frac{11}{2}(2a + 10d) = 880$

$11a + 55d = 880$

$11(77 - 6d) + 55d = 880$

$847 - 66d + 55d = 880$

$d = -3 \Rightarrow a = 95$

$\therefore t_{20} = 95 + 19(-3) = 38$

(b) Find the smallest number of terms, N, such that $S_N < 50$. [4]

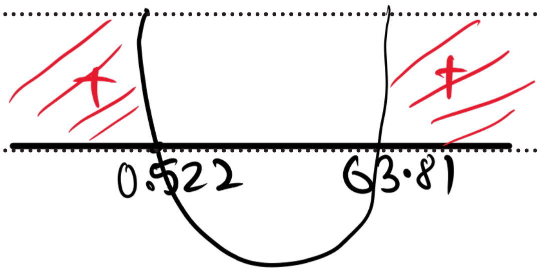
$\frac{N}{2}\left(2\times 95 + (N-3)(-3)\right) < 50$

$95N - \frac{3}{2}N^2 + \frac{3N}{2} < 50$

$3N^2 - 193N + 100 > 0$

Critical values, $N = 63.81, 0.522$

$N = 64$



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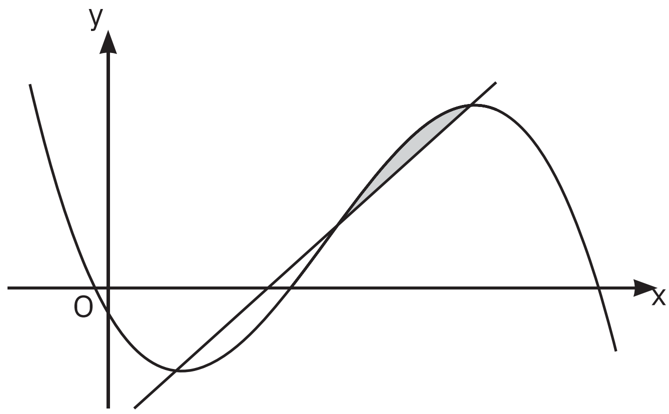

The diagram shows the curve with equation y x =- 3 2 + - 9 1 x x 5 3 - and a straight line. The straight line passes through the two stationary points of the curve.

(a) Show that the equation of the straight line is

$$y=8x-18.$$ [5]

$\frac{dy}{dx} = 0$

$-3x^2 + 18x - 15 = 0$

$x = 1, 5$

$y = -10, 22$

$\therefore (1,-10), (5,22)$ are st. points

$m = \frac{22+10}{4} = 8$

$y + 10 = 8(x-1)$

$y = 8x - 18$ — eqn

(b) Verify that the curve also meets the straight line when x=3. [1]

Curve, $y = -(3)^3 + 9(3)^2 - 15(3) - 3 = 6$

line, $y = 8(3) - 18 = 6$

∴ They intersect when $x = 3$.

(c) The shaded region is bounded by the curve and the straight line.

Find the area of the shaded region. [5]

$$\text{Area} = \int_3^5 -x^3 + 9x^2 - 15x - 3\,dx - \int_3^5 8x - 18\,dx$$

$$= \left[-\frac{x^4}{4} + 3x^3 - \frac{15x^2}{2} - 3x\right]_3^5 - \left[4x^2 - 18x\right]_3^5$$

$$= 32 - 28$$

$$= 4 \text{ sq. units}$$

Additional page

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