Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 01 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q), \text{ where} |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where} |pq|=|c| \text{ and } |mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q\pm c=0, \quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1 (a)	$(4,-4)$ **or e.g.** $x=4, y=-4$ o.e.	B1	1.1b
		(1)	
(b)	$(-4,6)$ **or e.g.** $x=-4, y=6$ o.e.	B1	1.1b
		(1)	
(c)	$(6, ...)$ **or** $(..., 5)$ **or** $x=6$ **or** $y=5$ o.e.	M1	1.1b
	$(6, 5)$ **or** $x=6$ **and** $y=5$	A1	1.1b
		(2)	
			(4 marks)

Notes:

General guidelines for all parts:

Remember to check answers written against the questions.
If there is any contradiction, mark the answers given in the body of the scripts.
If there is no labelling, mark the responses in the order given.
The coordinates need to be values not just a calculation e.g. **not** $6-2$ for 4
Points can be written as a coordinate pair or separately as $x=..., y=...$
Do not allow coordinates written the wrong way round but isw if necessary
e.g. $x=4, y=-4 \rightarrow (-4, 4)$ scores B1 and isw
Condone missing brackets (one or both) e.g. $x=4, y=-4$ **or** $(4, -4$ **or** $4, -4$ for $(4,-4)$
Condone a missing comma e.g. $(4\ \ -4)$ for $(4,-4)$
Condone use of a semi-colon e.g. $(4\ ;\ -4)$ for $(4,-4)$
Condone vector notation e.g. $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$ for $(4,-4)$ and condone $\left(\frac{4}{-4}\right)$

(a)
B1: $(4,-4)$ o.e. see above

(b)
B1: $(-4,6)$ o.e. see above

(c)
M1: One correct coordinate. See above.

A1: Both coordinates correct. See above.
Note that M0A1 is not a possible mark profile.
Note that in part (c), some candidates show their thinking by transforming the point piecewise e.g. $(6,-4) \rightarrow (6,4) \rightarrow (6,8) \rightarrow (6,5)$
In such cases, mark their final **pair** of coordinates

Question	Scheme	Marks	AOs
2 (a)	(i) Centre $(-3, 4)$	B1	1.1b
	(ii) States or implies that $r^2 = 24$ or $r = \sqrt{24}$	M1	1.1b
	$2\sqrt{6}$	A1	1.1b
		(3)	
(b)	Attempts a valid method e.g. Finds distance of centre from origin, Sets $y = 0$ and finds values of x	M1	3.1a
	Correct calculations, reason and conclusion (see notes)	A1	2.4
		(2)	
			(5 marks)

Notes:

Mark (i) and (ii) together

(a)(i)

B1: Centre $(-3, 4)$ Accept without brackets. May be written e.g. $x = -3,\ y = 4$

(a)(ii)

M1: States or implies that $r^2 = 24$ or $r = \sqrt{24}$. A final answer of $\sqrt{24}$ or $2\sqrt{6}$ implies the radius

May multiply out the brackets, collect terms ($x^2 + y^2 + 6x - 8y + 1 = 0$) and states the radius is $r^2 = \frac{6^2}{4} + \frac{(-8)^2}{4} - 1$ o.e. Do not condone slips for this mark.

A1: $2\sqrt{6}$ isw if they proceed to write as a decimal

(b) **Note that if their radius is incorrect in (a) then maximum score is M1A0 unless they restart in (b)**

M1: Attempts a valid method. For example

- Finds the distance (or distance ^{2}) of the centre from the origin.
  They must be attempting $(d =)\sqrt{(\pm"3")^2 + ("\pm"4")^2} = ...$ or
  $(d^2 =)(\pm"3" - 0)^2 + (\pm"4" - 0)^2 = ...$ and proceed to a value.
  May be seen as substituting the coordinates of the origin into the equation for C proceeding to a value for the left hand side e.g. 25 (to be able to compare with 24)
- Sets $y = 0$ and attempts to solve $(x+3)^2 + (-4)^2 = 24 \Rightarrow x = ...$ (at least one value)
- Sets $x = 0$ and attempts to solve $(3)^2 + (y-4)^2 = 24 \Rightarrow y = ...$ (at least one value)

In each method the starting expression or equation must be correct but do not be concerned by slips when evaluating or processing in finding the distance, the x coordinate or y coordinate.

A1: Correct calculation(s), reason and conclusion examples:

Calculation examples	Reason examples	Conclusion examples
e.g. $\left(d^2 =\right) 25$ or e.g. $(d =)\ 5$	$25 > 24$ o.e. or $5 > \sqrt{24}$ o.e. (allow 4.9 or better)	e.g. origin does NOT lie within circle / origin lies outside circle / not in circle o.e.
e.g. $(x+3)^2 + (-4)^2 = 24$ $\Rightarrow (x =)\ -3 \pm \sqrt{8}$ (allow decimals awrt -0.2 and awrt -5.8)	roots are both negative (same signs) o.e.	
e.g. $(3)^2 + (y-4)^2 = 24$ $\Rightarrow (y =)\ 4 \pm \sqrt{15}$ (allow decimals awrt 0.1 and awrt 7.9)	roots are both positive (same signs) o.e.	

Note if their reasoning is incorrect e.g. referring to the radius as 24 instead of $\sqrt{24}$ then A0 but allow referencing to "the radius of *C*" provided their radius in (a) was correct.

Note if they give a reason that the origin does not lie inside the circle *C* because e.g. $25 \neq 24$ this scores M1A0 (M1 for 25 but A0 incorrect reasoning)

Question	Scheme	Marks	AOs
3 (a)	Attempts to solve $10-6k=2k-10 \Rightarrow k=...$	M1	3.1a
	$(k=)\,\frac{5}{2}$ o.e.	A1	1.1b
		(2)	
(b)	Deduces the value of "d" $=-5$	B1 ft	2.2a
	$S_{50}=\frac{50}{2}\left(2\times"15"+49\times"-5"\right)$	M1	1.1b
	$=-\ 5375$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a) Condone using other letters for *a* and *d* (e.g. may use *r* for *d*)

M1: Attempts a valid method to solve the problem

- Uses the common difference to form a correct equation and attempts to solve to find a value for k

  e.g. Attempts to solve $10-6k=2k-10$ o.e such as $2k-6k=2(10-6k)$
- Uses 10 as the mean of $2k$ and $6k$: $\frac{2k+6k}{2}=10 \Rightarrow k=...$
- Sets up correct equations $a=6k,\ a+d=10$ and $a+2d=2k$, or may be seen as $6k+d=10$ and $10+d=2k$, **and** proceeds to find k.
- Uses the summation formula $S_3=\frac{3}{2}(6k+2k)=6k+10+2k$ **and** proceeds to find k

In each attempt the initial equation (or simultaneous equations) must be correct but do not be concerned by the mechanics of the rearrangement to find k. May be implied by $k=\frac{5}{2}$

A1: $(k=)\,\frac{5}{2}$ o.e.

(b) Work seen in (a) can only be scored if seen or used in (b)

B1ft: Common difference $=-5$ **or** ft on their value for k (even if k has been found from an incorrect method) e.g. $10-6\times"\frac{5}{2}"$ or e.g. $2\times"\frac{5}{2}"-10$ if only a numerical value is seen.

May be implied or seen in a term or summation formula.

Note that some candidates may work in terms of k throughout so only allow B1ft to be scored when they substitute in their numerical value for k, following $10-6k$ o.e. correctly embedded in a correct formula. They may make arithmetical slips before they substitute in their numerical value for k which can be condoned.

M1:	Attempts to use a **correct formula**. The expression is sufficient to score this mark but they must be using a correct value for a and $\pm d$ (or ft on their value for k for a and d) which are correctly placed in the formula. e.g. $\left(S_{50} =\right) \frac{50}{2}\left(2\times"6k"+49\times"\pm d"\right)$. Alternatively, they may find the 50th term $u_{50} = "15" + (50-1)\times"-5" = -230$ and use $\left(S_{50} =\right) \frac{50}{2}\left("15"+"-230"\right)$. If working in terms of k they must substitute in their value for k e.g. $\left(S_{50} =\right) \frac{50}{2}\left(2\times 6k+49\times(10-6k)\right) = -7050k+12250 = -7050\times"\frac{5}{2}"+12250$ Do not withhold this mark for omission of brackets around (-5) e.g. $\frac{50}{2}\left(2\times15+\left(49\right)-5\right)$ scores M1
A1:	-5375 cao

Question	Scheme	Marks	AOs
4 (a)	States or uses $a=4$	B1	1.1a
	Valid method to find $Q(x)$	M1	2.1
	$(x+4)(2x^2-5x+4)$	A1	1.1b
		(3)	
(b)	Attempts to show that their $2x^2-5x+4$ does not have any (real) roots	M1	3.1a
	Correct calculations, reason and conclusion	A1	2.1
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $a=4$ e.g. may be seen in their attempt at dividing algebraically by $x+4$

M1: Attempts to divide $\mathrm{f}(x)$ by $(x+4)$ to find a three-term quadratic $Q(x)$. There are various methods or ways to present their solution so typically methods

- by inspection look for $2x^3+3x^2-16x+16=(x+4)(2x^2+\ldots x\pm4)$
- by division look for a quadratic quotient of $2x^2-5x\pm\ldots$

A1: $(x+4)(2x^2-5x+4)$ condone the missing trailing bracket i.e. $(x+4)(2x^2-5x+4$ isw if they attempt to factorise their quadratic factor.

Allow to be scored if seen in (b).

(b)

M1: Attempts to show that their three-term quadratic "$2x^2-5x+4$" does not have any roots:

- **Attempts the discriminant**
  e.g. $b^2-4ac=25-4\times2\times4\ \left(=-7\right)$ (may be embedded in the quadratic formula)
- **Attempts to use the quadratic formula**
  e.g. $\left(x=\right)\dfrac{5\pm\sqrt{\left(-5\right)^2-4\times2\times4}}{4}$ but do not allow directly from a calculator $\dfrac{5\pm\sqrt{7}i}{4}$
- **Attempts to complete the square**
  e.g. $2x^2-5x+4=2\left(x^2-\dfrac{5}{2}x\right)+4=2\left(x-\dfrac{5}{4}\right)^2+\ldots\quad\left(=2\left(x-\dfrac{5}{4}\right)^2-\dfrac{25}{8}+4\right)$
- **Uses calculus to find the turning point**
  e.g. $\dfrac{\mathrm{d}(2x^2-5x+4)}{\mathrm{d}x}=4x-5=0\Rightarrow x=\dfrac{5}{4}\Rightarrow y=\ldots$

Note that any attempts using the discriminant or quadratic formula must have the values embedded in the correct places (may be partially evaluated) to score M1

A1: **Dependent on a correct** $Q(x) = 2x^2 - 5x + 4$

Fully correct argument that requires:

- Fully correct work
- A justification depending on strategy and no incorrect reasoning seen
- A conclusion

Examples below – Note we must see working before they proceed to a correct root or minimum value – see M1 for guidance

Strategy	**Correct work examples**	**Justification examples**	**Conclusion examples**
Via discriminant	$b^2 - 4ac = -7$	$-7 < 0$ -7 so no (real) roots but NOT $-7 \neq 0$ so no roots	so "-4 is the only (real) root" / "only one (real) root"
Via using the quadratic formula	$x = \frac{5 \pm \sqrt{7}i}{4}$ or $x = \frac{5 \pm \sqrt{-7}}{4}$	$-7 < 0$ / which is not possible / complex roots o.e. / cannot square root a negative / no (real) roots	
Via completing the square	$2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8}$ or $2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8} = 0 \Rightarrow$ $2\left(x - \frac{5}{4}\right)^2 = -\frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis $-\frac{7}{8} < 0$ / cannot square root a negative / no (real) roots	
Via calculus	$x = \frac{5}{4} \Rightarrow y = \frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis	

Note that it is possible to justify and conclude in one step by using phrases e.g. "**no more** (real) roots" or "**no other** (real) roots"

e.g. $2x^2 - 5x + 4 \Rightarrow b^2 - 4ac = 25 - 32 = -7$ so no more roots which scores M1A1

Condone $25 - 32 < 0$ as a justification that the quadratic has no real roots

Condone the conclusion -4 is the only (real) solution (instead of (real) root).

Note if $(x+4)$ is described as a root or -4 is described as a factor this scores A0

Question	Scheme	Marks	AOs
5 (a)	$\lim_{\delta x \to 0} \sum_{x=1.44}^{2.89} \frac{2}{\sqrt{x}} \delta x = \int_{1.44}^{2.89} \frac{2}{\sqrt{x}} \mathrm{d}x$	B1	1.2
		(1)	
(b)	$= \left[4\sqrt{x}\right]_{1.44}^{2.89} = 4\times1.7 - 4\times1.2$	M1	1.1b
	$= 2$	A1cso	1.1b
		(2)	
			(3 marks)

Notes:

Mark (a) and (b) together

(a)

B1: States that $\int_{1.44}^{2.89} \frac{2}{\sqrt{x}} \mathrm{d}x$ or equivalent such as $2\int_{1.44}^{2.89} x^{-\frac{1}{2}} \mathrm{d}x$ or $2\int_{1.44}^{2.89} x^{-0.5} \mathrm{d}x$ but must include the limits and the dx. Condone $\mathrm{d}x \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes.

(b)

M1: Uses $\int \frac{1}{\sqrt{x}} \mathrm{d}x = a\sqrt{x}$ or $ax^{\frac{1}{2}}$ (allow a to be 1) **and** applies the given limits to their $ax^{\frac{1}{2}}$ subtracting either way round. (Condone with the constant of integration included) You do not need to be concerned by fractions within fractions as this is still of the required form e.g. $\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}$. Only condone transcription errors of 2.89 or 1.44 when substituting the limits into the expression.

This mark can be scored for

e.g. $\left[4\sqrt{x}\right]_{1.44}^{2.89} = 4\times\sqrt{2.89} - 4\times\sqrt{1.44}$ or e.g. $\frac{2x^{\frac{1}{2}}}{\frac{1}{2}} \to \frac{2(2.89)^{\frac{1}{2}} - 2(1.44)^{\frac{1}{2}}}{\frac{1}{2}}$

May already be partially evaluated so allow e.g. $\frac{34}{5} - \frac{24}{5}$ o.e. provided it is not just 2.

A1: 2 cso

The method mark must have been awarded. Do not withhold this mark for poor notation or e.g. a missing dx in their solution.

Question	Scheme	Marks	AOs
6 (a)	Attempts to use $h = A - B\,t^{1.5}$ to form one correct equation with either $17.6 = A - B \times 4^{1.5}$ or $11.9 = A - B \times 9^{1.5}$	M1	3.1b
	Correct equations $A - 8B = 17.6$, $A - 27B = 11.9$	A1	1.1b
	Solves simultaneously to find values for A and B	dM1	1.1b
	$h = 20 - 0.3\,t^{1.5}$	A1	3.3
		(4)	
(b)	20 metres	B1ft	3.2a
		(1)	
(c)	$0 = 20 - 0.3\,T^{1.5}\ \left(\Rightarrow T^{1.5} = \frac{200}{3}\right) \Rightarrow T = \ldots$	M1	3.4
	$(T =)\ 16.4$	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Forms one correct equation either $17.6 = A - 8B$ or $11.9 = A - 27B$

May be unsimplified e.g. $17.6 = A - B \times 4^{1.5}$ or $11.9 = A - B \times 9^{1.5}$

A1: $17.6 = A - 8B$ and $11.9 = A - 27B$ which may be unsimplified

dM1: Solves simultaneously to find values for A and B. It is dependent on the previous method mark so at least one equation must be correct. Do not be concerned with the process as calculators may be used. Score if values for A and B are reached from a pair of simultaneous equations.

A1: $h = 20 - 0.3\,t^{1.5}$ o.e. e.g. $t = \left(\frac{10}{3}(20 - h)\right)^{\frac{2}{3}}$ isw once a correct equation is found.

Requires the complete equation including $h = \ldots$

Just stating the values for A and B is A0 but allow A1 to be scored if the correct equation is seen in (b) or (c)

(b)

B1ft: 20 metres but ft on their A metres (provided $A > 0$). Condone if they have a value for A which is given to a greater degree of accuracy than 3sf but they round this to 3sf or better in (b).

Requires units as well (allow m for metres)

(c) Do not be concerned with the use of *t* or *T*. If awrt 16.4 is seen in (a) it must be seen or used in (c) to score.

M1: Sets $"20"-"0.3"T^{1.5}=0$ and proceeds to a value or expression for T e.g. $\left(\frac{"20"}{"0.3"}\right)^{\frac{2}{3}}$

Do not be concerned by the use of any inequalities instead of "="
You do not need to be concerned by the mechanics of the rearrangement, they just need to achieve a value or expression.

If no equation is seen then may be implied by a correct value for T (to the nearest integer e.g. awrt "16") or a correct expression for T for their A and B. You may need to check $\left(\frac{"20"}{"0.3"}\right)^{\frac{2}{3}}$ on your calculator. They may also achieve this by trial and improvement.

A1: $(T=)$ awrt 16.4 including $\left(\frac{200}{3}\right)^{\frac{2}{3}}$ or exact equivalent e.g. $\sqrt[\frac{3}{2}]{\frac{200}{3}}$ ignore any units for time. If an exact value is given condone the radical symbol not fully covering the fraction provided it is not clearly e.g. $\frac{\sqrt[\frac{3}{2}]{200}}{3}$

Condone $(0\le)\ T\le$ awrt 16.4 o.e. but not $(0\le)\ T<$ awrt 16.4

Do not accept any greater than (or equal) inequalities e.g. $T\ge$ awrt 16.4

isw if there is a written response to (c) once a correct value or valid expression for T is seen.

Question	Scheme	Marks	AOs
7 (a)	Either $x \le -1$ or $2 \le x \le 5$	M1	2.2a
	Both $\{x : x \in \mathbb{R}, x \le -1\} \cup \{x : x \in \mathbb{R}, 2 \le x \le 5\}$ o.e.	A1	2.5
		(2)	
(b)	States $(y =)\ \alpha(x+1)^2(x-5)^2$ or $(\mathrm{f}(x) =)\ \alpha(x+1)^2(x-5)^2$	M1	1.1b
	Substitutes $(0, -75)$ into $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α	dM1	3.1a
	$y = -3(x+1)^2(x-5)^2$ o.e.	A1	2.1
		(3)	
(c)	Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2 \Rightarrow y = (-243)$	M1	2.1
	$0 < k < 243$	A1ft	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Either

- $x \le -1$ o.e. e.g. $-1 \ge x$
- $2 \le x \le 5$ o.e.

but condone use of strict inequalities anywhere for this mark.
e.g. $2 < x < 5$ or $2 < x \le 5$ or $2 \le x < 5$ May also write e.g. $x < 5$ and $x > 2$ which scores M1 but not "$x < 5$ or $x > 2$"

Allow interval notation such as e.g. $[2,5]$ or $(-\infty, -1]$ or condone e.g. $(2,5)$

Ignore incorrect inequality statements not related to the one which is valid.
e.g. "$2 \leqslant x < 5$ and $x > -1$" which scores M1 for the first inequality.

A1: Requires $\{\ \}$ and $\cup$

$\{x : x \le -1\} \cup \{x : 2 \le x \le 5\}$ or $\{x \mid x \le -1\} \cup \{x \mid 2 \le x \le 5\}$ either way round

but condone $\{x \le -1\} \cup \{2 \le x \le 5\}$, $\{x \le -1 \cup 2 \le x \le 5\}$.

Allow e.g. $\{x : x \le -1\} \cup \{x : 2 \le x \cap x \le 5\}$

Use of $\cap$ to join the two separate regions is A0
It is acceptable (but not required) to mention $\mathbb{R}$
e.g. $\{x : x \in \mathbb{R}, x \le -1\} \cup \{x : x \in \mathbb{R}, 2 \le x \le 5\}$

Condone use of a lower limit written as e.g. $\{x : -\infty \le x \le -1\} \cup \{x : 2 \le x \le 5\}$

(b) Note a correct equation written down scores all 3 marks.
A correct expression but missing e.g. $y=...$ or $f(x)=...$ scores M1dM1A0

M1: Forms the equation of the form $(y=)\ \alpha(x+1)^2(x-5)^2$. Condone $\alpha=1$

Award for sight of $\alpha(x+1)^2(x-5)^2$ even with $\alpha=1$ i.e. $(x+1)^2(x-5)^2$

dM1: Substitutes $(0,-75)$ into the form $y=\alpha(x+1)^2(x-5)^2$ and attempts to find the value for α. It is dependent on the previous method mark.

A1: $y=-3(x+1)^2(x-5)^2$ o.e. (e.g. $y=-3x^4+24x^3-18x^2-120x-75$)

isw after a correct answer. Condone $f(x)=-3(x+1)^2(x-5)^2$ but not $C=-3(x+1)^2(x-5)^2$

A correct equation scores all 3 marks. Allow if seen in (c)
isw if they attempt to multiply out.

Alternative I part (b):
Using the form $y=ax^4+bx^3+cx^2+dx+e$, then setting up and solving simultaneous equations.
There are various versions of this but can be marked similarly.

M1: Sets e equal to -75 (may just be seen in their equation) **and** forms **three correct** different equations in a, b, c and d which may be unsimplified.

Note that the form $y=ax^4+bx^3+cx^2+dx+e$ is M0 until e is set equal to -75

There are 5 equations that can be formed, only 3 are necessary for this mark.
Do not condone slips.

Using $(-1,0)$ $\Rightarrow 0=a-b+c-d-75$ o.e.

Using $(5,0)$ $\Rightarrow 0=625a+125b+25c+5d-75$ o.e.

Using $\frac{dy}{dx}=0$ at $x=2$ $\Rightarrow 0=32a+12b+4c+d$ o.e.

Using $\frac{dy}{dx}=0$ at $x=-1$ $\Rightarrow 0=-4a+3b-2c+d$ o.e.

Using $\frac{dy}{dx}=0$ at $x=5$ $\Rightarrow 0=500a+75b+10c+d$ o.e.

dM1: Forms **four correct** different equations and solves to find values for a, b, c and d. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y=-3x^4+24x^3-18x^2-120x-75$ o.e. isw if they attempt to factorise but withhold this mark if they e.g. divide all terms by 3.

Condone $f(x)=...$ but not $C=...$

A correct equation scores all 3 marks. Allow if seen in (c)

Alternative II part (b): Uses the form $y=(x+1)(x-5)(ax^2+bx+c)$

M1: Substitutes $x=0,\ y=-75\quad -75=-5c\Rightarrow c=15$, multiplies out, differentiates

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=(2x-4)(ax^2+bx+15)+(x^2-4x-5)(2ax+b)$$

and forms **a correct equation** in a and b which may be unsimplified.

Using $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ at $x=2$ $\Rightarrow 0=4a+b$ o.e.

Using $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ at $x=-1$ $\Rightarrow 0=a-b+15$ o.e.

Using $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ at $x=5$ $\Rightarrow 0=5a+b+3=0$ o.e.

dM1: Forms **two correct** different equations and solves to find values for a and b. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y=(x+1)(x-5)(-3x^2+12x+15)$ o.e. isw if they attempt to multiply out or factorise Condone $\mathrm{f}(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

Alternative III part (b): Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=\beta(x+1)(x-2)(x-5)$ (β may be 1) and integrates.

M1: Integrates $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\beta(x+1)(x-2)(x-5)$ to $(y=)\ \beta\left(\frac{1}{4}x^4-2x^3+\frac{3}{2}x^2+10x+k\right)$ and forms **one correct** equation using either $(0,-75)$: $-75=\beta k$ (allow $-75=k$)

$(-1,0)$: $0=\beta\left(\frac{1}{4}+2+\frac{3}{2}-10+k\right)$ $\quad(5,0)$: $0=\beta\left(\frac{625}{4}-250+\frac{75}{2}+50+k\right)$

dM1: Forms a different equation using one of $(0,-75),(-1,0),(5,0)$ and solves to find values for β and k. You do not need to be concerned by the process of solving . A calculator can be used to solve the equations.

A1: $y=-12\left(\frac{1}{4}x^4-2x^3+\frac{3}{2}x^2+10x+\frac{25}{4}\right)$ o.e. isw if they attempt to multiply out or factorise

Condone $\mathrm{f}(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

(c)

M1: Substitutes $x=2$ into their $y=-3(x+1)^2(x-5)^2$ (must be a quartic in any form) and proceeds to find a value for y. Sight of their $\pm y$ (or ± 243) scores M1.
You may need to check this on your calculator if only a value is seen.

A1ft: $0<k<243$ o.e. ft on their negative y value at $x=2$.
Allow use of set notation, interval notation and allow e.g. $k<243,\ k>0$ but do not allow OR or $\cup$. Do not accept $0\le k\le 243$ o.e.
If there are multiple attempts at describing the region, mark what appears to be their final answer.
This mark can only be scored if they have a negative quartic graph function
i.e. $\alpha<0$ for their $y=\alpha(x+1)^2(x-5)^2$ or $a<0$ for their $y=ax^4+bx^3+cx^2+dx+e$

Question	Scheme	Marks	AOs
8 (a)	e.g. The last line should start $25k^2 + 20k + 4$	B1	2.3
		(1)	
(b)	Considers one of the missing calculations $m = 5k + 3$ and attempts $m^2 = (5k+3)^2 = \ldots$ **or** $m = 5k + 4$ and attempts $m^2 = (5k+4)^2 = \ldots$	M1	2.1
	Achieves one correct statement $m^2 = (5k+3)^2 = 25k^2 + 30k + 9 = 5\left(5k^2 + 6k + 2\right) - 1$ **or** $m^2 = (5k+4)^2 = 25k^2 + 40k + 16 = 5\left(5k^2 + 8k + 3\right) + 1$	A1	1.1b
	Considers **both** of the missing calculations	dM1	1.1b
	Achieves both correct statements with final concluding remark (see notes)	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Corrects the error for the case when $m = 5k + 2$. The correction may be on the proof in the box or may be described in the main body of the text. May just see the $10k$ crossed out and replaced with $20k$ in the box or described in the main body of the work.
Sight of the quadratic $\left(25k^2\right) + 20k\ (+4)$ scores B1 and isw e.g. if they make other errors in the expansion.
If B1 is not scored in (a) then allow to score if seen correct in (b) e.g. they may attempt the case $m = 5k + 2$ as part of their proof in (b).

(b) **Main scheme method uses $m = 5k + 3$ and $m = 5k + 4$**

You will need to look at both cases and mark the one which is fully correct first.
Allow a different variable to k and may be different letters for the two cases.
If a candidate attempts repeated cases e.g. $m = 5k + 4$ and $m = 5k - 1$ then mark both and award the higher mark of the two.

Condone use of *m* as a variable for the first three marks.

There should be no errors in the algebra for the A marks including invisible brackets but do not be concerned with any re-attempt at doing the case $m = 5k + 2$

Note that there are other allowable valid pairs of combinations covering the final two distinct cases e.g. $m = 5k - 2$ **and** $m = 5k + 4$**, or** $m = 5k - 1$ **and** $m = 5k + 3$
but NOT e.g. $m = 5k + 3$ **and** $m = 5k - 2$

Note that we are not expecting candidates to state what set of numbers *k* belongs to but we will condone pairs such as $m = -5k - 1$ **and** $m = -5k - 2$

Typically candidates will show the algebraic steps as in the main scheme but for this particular pair they may justify equivalence using the results in the box for $m = 5k + 1$ **and** $m = 5k + 2$ without requiring calculations which is acceptable.

M1: Considers one valid case e.g. $m = 5k + 3$ and attempts $m^2 = (5k+3)^2$ **or** $m = 5k + 4$ and attempts $m^2 = (5k+4)^2$

Look for expanding out the brackets and simplifying to a 3TQ. Condone slips.

A1: Achieves one correct statement which includes the case, the quadratic multiplied out and written in the required form

e.g. $m^2 = (5k+3)^2 = 25k^2 + 30k + 9 = 5(5k^2 + 6k + 2) - 1$ **or**

e.g. $m^2 = (5k+4)^2 = 25k^2 + 40k + 16 = 5(5k^2 + 8k + 3) + 1$

dM1: Considers **both cases** for a valid pair (see first M1 for guidance). It is dependent on the previous method mark. Condone slips.

A1: Full proof with correct statements for both cases for a valid pair. Each must include the case, the quadratic multiplied out and written in the form which is not in terms of m (we do not need the "where $n = ...$" at the end of the statements - you can ignore these)

Requires a minimal overall conclusion eg. Proven, QED, tick

Condone recovery of interchanging of variables.

m	m^2	$5n \pm 1$
$5k+3$	$25k^2 + 30k + 9$	$5(5k^2 + 6k + 2) - 1$
$5k+4$	$25k^2 + 40k + 16$	$5(5k^2 + 8k + 3) + 1$
$5k-1$	$25k^2 - 10k + 1$	$5(5k^2 - 2k) + 1$
$5k-2$	$25k^2 - 20k + 4$	$5(5k^2 - 4k + 1) - 1$

Ignore any additional cases that are not required to complete the proof

(and ignore replications of the ones given in the box in the question)

Question	Scheme	Marks	AOs
9 (a)	$0=15T-T\,\mathrm{e}^{0.2T}\Rightarrow \mathrm{e}^{0.2T}=15$	M1	3.4
	$(T=)$ awrt 13.5	A1	1.1b
		(2)	
(b)	Attempts to differentiate using the product rule $\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(t\,\mathrm{e}^{0.2t}\right)=\right)\ 0.2t\,\mathrm{e}^{0.2t}+\mathrm{e}^{0.2t}$	M1	1.1b
	$v=15t-t\,\mathrm{e}^{0.2t}\Rightarrow\left(\frac{\mathrm{d}v}{\mathrm{d}t}=\right)15-\left(0.2t\,\mathrm{e}^{0.2t}+\mathrm{e}^{0.2t}\right)$	A1	1.1b
	Sets their $\frac{\mathrm{d}v}{\mathrm{d}t}=0\Rightarrow \mathrm{e}^{0.2t}(0.2t+1)=15\Rightarrow \mathrm{e}^{0.2t}=\frac{15}{0.2t+1}$	dM1	3.1b
	$\Rightarrow t=5\ln\left(\frac{75}{t+5}\right)$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2=5\ln\left(\frac{75}{8+5}\right)$	M1	1.1b
	awrt 8.478	A1	1.1b
	(ii) awrt 8.55 seconds **(including units)**	A1	3.2a
		(3)	
			(9 marks)

Notes:

(a) May use t or another variable which is acceptable.

M1: Uses the model with $v=0$ and proceeds to $\mathrm{e}^{0.2T}=15$ Do not be concerned by the use of an inequality sign instead of an equals.
May be implied by awrt 13.5

A1: $(T=)$ awrt 13.5 units are not required but if given they must be seconds (or e.g. secs or s)

(b)	**If no attempt is seen for (b) then allow differentiation seen in (a) to score in (b)**
M1:	Attempts to use the product rule to differentiate $t\,\mathrm{e}^{0.2t}$ achieving the form $A\mathrm{e}^{0.2t}+Bt\mathrm{e}^{0.2t}$ (*A* and *B* both non zero but may be 1) which may be unsimplified. It is likely to be part of an expression.
A1:	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}=\right)15-\left(0.2t\,\mathrm{e}^{0.2t}+\mathrm{e}^{0.2t}\right)$ o.e. which may be unsimplified. (Condone a missing trailing bracket) Do not allow recovery of signs to score this mark if they initially write e.g. $15-0.2t\,\mathrm{e}^{0.2t}+\mathrm{e}^{0.2t}=0$ and on a later line correct this. e.g. $15-0.2t\,\mathrm{e}^{0.2t}-\mathrm{e}^{0.2t}=0$
dM1:	Sets $15\pm A\mathrm{e}^{0.2t}\pm Bt\mathrm{e}^{0.2t}=0$ (the =0 may be implied), attempts to make $\mathrm{e}^{\pm0.2t}$ (or $C\mathrm{e}^{\pm0.2t}$) the subject **and** proceeds to the form $C\mathrm{e}^{0.2t}=\frac{D}{E+Ft}$ or $C\mathrm{e}^{-0.2t}=\frac{E+Ft}{D}$ (where *C* can be 1 and $A, B, C, D, E, F\neq0$) It is dependent on the previous method mark. May see $15-"0.2t\,\mathrm{e}^{0.2t}-\mathrm{e}^{0.2t}"=0\Rightarrow\mathrm{e}^{0.2t}=\frac{15}{0.2t+1}$ which scores dM1 They must take out a factor of $\mathrm{e}^{\pm0.2t}$ (or $C\mathrm{e}^{\pm0.2t}$) and divide by their bracket. Condone sign slips in their rearrangement, however, if they take logs of both sides first, the rearrangement must be correct (with no sign slips). Allow invisible brackets to be implied by further work which is not the given answer.
A1*:	Achieves the given answer with **no errors seen including use of invisible brackets** (but condone a missing trailing bracket) All previous marks in (b) must have been scored. The **= 0** must have been seen somewhere in their solution. Do not allow this mark to be scored for proceeding directly from $\mathrm{e}^{0.2t}(0.2t+1)=15\Rightarrow t=5\ln\left(\frac{75}{t+5}\right)$ which is A0* We must see either $\mathrm{e}^{0.2t}=\frac{15}{0.2t+1}$ o.e. or an unsimplified expression for *t* e.g. $t=5\ln\left(\frac{15}{0.2t+1}\right)$ before achieving the given answer. Condone $t=5\ln\frac{75}{t+5}$
(c) (i)	**Check by the question. If there is a contradiction between answers, the answer in the main body of the script takes precedence.**
M1:	Attempts to use the iteration formula at least once. $t_2=5\ln\left(\frac{75}{8+5}\right)$. May be implied by awrt 8.76 or awrt 8.48 or awrt 8.58. It is not implied by awrt 8.55
A1:	awrt 8.478 (on its own can score M1A1)
(c)(ii)	**This mark can only be scored provided in (c)(i) M1 has been scored so M0A0A1 is not a possible mark profile.**
A1:	awrt 8.55 **seconds** (e.g. **s** or **secs**) **Requires units.** If the candidate lists their iterations but does not select an answer then take the final value, which still requires units to be stated (which in many cases is likely to be omitted) Note that awrt 8.55 does not imply M1**.**

Question	Scheme	Marks	AOs
10 (a)	Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$	M1	1.1b
	$\left(\overrightarrow{PR} =\right) 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	Attempts to show the triangle is isosceles (**or** right-angled) e.g. Attempts $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2 + 8^2 + (-2)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{6^2 + 6^2}$	M1	3.1a
	Shows e.g. $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{72}$ $(= 6\sqrt{2})$	A1	1.1b
	Attempts to show the triangle is isosceles **and** right-angled e.g. attempts to find the lengths of all three sides AND e.g. attempts to compare lengths via use of "$a^2 + b^2 = c^2$"	M1	1.1b
	e.g. Shows that $\left\|\overrightarrow{PQ}\right\|^2 + \left\|\overrightarrow{QR}\right\|^2 = \left\|\overrightarrow{PR}\right\|^2$ as $72 + 72 = 144$ so PQR is a right-angled triangle	A1	2.1
		(4)	
		(6 marks)	

Notes:

(a) If part (a) is not attempted and the correct $\overrightarrow{PR}$ is seen in part (b) then M1A1 can be awarded

M1: Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$ with at least one correct component of $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$. A typical misread of $\overrightarrow{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ can score for at least one correct component of $8\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$

A1: Correct <u>vector</u>. Allow $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ but **not** $\begin{pmatrix} 8\mathbf{i} \\ 8\mathbf{j} \\ 4\mathbf{k} \end{pmatrix}$ and **not** (8, 8, 4)

Condone $\begin{matrix} 8 \\ 8 \\ 4 \end{matrix}$ for $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ Do not apply isw here but award for e.g. $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = \begin{pmatrix} 8\mathrm{i} \\ 8\mathrm{j} \\ 4\mathrm{k} \end{pmatrix}$

E.g. if they obtain $\overrightarrow{PR} = 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ and then say $\overrightarrow{PR} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ then award A0

(b) **Note that M1A0M1A1 is not possible. If they have an incorrect vector in part (a) then the maximum score is M1A1M1A0. A misread of $\overrightarrow{QR}$ as $6\mathrm{i} + 6\mathbf{j}$ in (b) can only score a maximum M1A1M1A0**

They may attempt to show the triangle is either isosceles or right-angled in either order. You will need to look through their solution and award the order which scores most marks. Usually it will be isosceles first. Condone slips to be recovered.

To show the triangle is isosceles they only need to show **two sides** (or **two angles**) are the same. They do not need to consider the other side to show it is isosceles.

M1: Attempts to show that the triangle is either isosceles (**or** right-angled). See table below.

A1: Fully shows that the triangle is isosceles (or right-angled). See table below. Allow slips in their method if recovered as long as they proceed to correct lengths or values. **A conclusion that the triangle is isosceles or right-angled is not required for this mark**.

Isosceles	**Requirement for M1 examples**	**Requirement for A1 examples**
Using lengths	Attempts to find the length or length^2 of PQ and QR: $\left(\lvert\overrightarrow{PQ}\rvert=\right)\sqrt{2^2+8^2+(-2)^2}\ \left(=\sqrt{72}=6\sqrt{2}\right)$ or seen as e.g. $2\sqrt{1^2+4^2+(-1)^2}\ \left(=2\sqrt{18}\right)$ $\left(\lvert\overrightarrow{QR}\rvert=\right)\sqrt{6^2+6^2}\ \left(=\sqrt{72}=6\sqrt{2}\right)$ or may be seen as e.g. $3\sqrt{2^2+2^2}\ \left(=3\sqrt{8}\right)$ May be implied by e.g. $6\sqrt{2}$ Condone missing brackets around $(-2)^2$ provided the intention is clear to square and add implied by e.g. $6\sqrt{2}$	States or shows that $\lvert\overrightarrow{PQ}\rvert=\lvert\overrightarrow{QR}\rvert\ \left(=\sqrt{72}\ (=6\sqrt{2})\right)$, or equivalent. Accept e.g. $PQ^2=QR^2$ or "both are 72" Condone poor notation and/or labelling of lengths provided they are not clearly referring to the longest length. e.g. achieves $6\sqrt{2}$ for both PQ and QR and states they are the same scores M1A1 **Only stating isosceles without a comparison of *PQ* and *QR* is A0**
		Uses the sine rule with the lengths and angles embedded in the correct places e.g. $\dfrac{\sin\angle QPR}{QR}=\dfrac{\sin\angle PRQ}{PQ}$ Deduces $\sin\angle QPR=\sin\angle PRQ$ so the angles are the same o.e.
Right-angled	**Requirement for M1 examples**	**Requirement for A1 examples**
Using lengths	Attempts to find all three lengths or lengths^2 $\left(\lvert\overrightarrow{PR}\rvert=\right)\sqrt{"8"^2+"8"^2+"4"^2}\ \ (=12)$ or may be seen as e.g. $4\sqrt{"2"^2+"2"^2+"1"^2}$ or implied by their 12 **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2\Rightarrow\right)"72"+"72"="144"$	States or shows that $\lvert PQ\rvert^2+\lvert QR\rvert^2=\lvert PR\rvert^2$, or equivalent Condone poor notation and/or labelling of lengths.
	Attempts to find all three lengths or lengths^2 (see above for guidance) **AND ATTEMPTS** the cosine rule correctly to find $\cos PQR=\dfrac{"72"+"72"-"144"}{2\times"\sqrt{72}"\times"\sqrt{72}"}$ o.e.	States or shows $\cos PQR=0$
Scalar dot product (Further Maths)	Attempts e.g. $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$ oe **Must see the calculation for M1**	States or shows that $(2\times6)+(8\times0)+(-2\times6)=0$ oe (they do not need to write anything more for this mark)
	$\left(\cos PQR=\right)\dfrac{(2\times6)+(8\times0)+(-2\times6)}{"\sqrt{72}"\times"\sqrt{72}"}$ oe	$\cos PQR=\dfrac{(2\times6)+(8\times0)+(-2\times6)}{\sqrt{72}\times\sqrt{72}}=0$

M1: Attempts to show that the triangle is both isosceles **and** right-angled. Usually they will have shown the triangle is isosceles and attempt to show that it is right-angled. **Here are some examples but there will be others**:

Right-angled	Method examples (required for second method mark)
Using lengths	Attempts to find all three lengths or lengths^2 **AND ATTEMPTS** $\left(PQ^2 + QR^2 = PR^2 \Rightarrow\right)$ "72"+"72"="144"
	Attempts to find all three lengths or lengths^2 (see above for guidance) **AND ATTEMPTS** the cosine rule in an attempt to find $\cos PQR = \dfrac{\text{"72"+"72"−"144"}}{2\times\text{"}\sqrt{72}\text{"}\times\text{"}\sqrt{72}\text{"}}$
Scalar dot product	Attempts $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix} = (2\times6)+(8\times0)+(-2\times6)$
	$\left(\cos PQR =\right)\dfrac{(2\times6)+(8\times0)+(-2\times6)}{\text{"}\sqrt{72}\text{"}\times\text{"}\sqrt{72}\text{"}}$ oe

Alternatively, having already shown the triangle is right-angled they may:

- use trigonometry to show that the two base angles are both $45°$
- attempt to find the required lengths or lengths^2 of *PQ* and *PR* if not already found (Note they may also find the length or length^2 of *QR* and may use the sine rule or cosine rule)

Note if they have shown either property then we will condone making an assumption of the other property to justify the size of angle *PRQ* and/or angle *QPR* and may use trigonometry to show either

$\left(\sin\theta = \dfrac{6\sqrt{2}}{12} \Rightarrow\right)\theta = \arcsin\left(\dfrac{6\sqrt{2}}{12}\right) = 45°$ (we must see arcsin or $\sin^{-1}$)

$\sin\theta = \dfrac{6\sqrt{2}}{12} \Rightarrow \sin\theta = \dfrac{\sqrt{2}}{2}\left(\text{or } \dfrac{1}{\sqrt{2}}\right) \Rightarrow \theta = 45°$ (since this is a known angle)

$\sin 45° = \dfrac{\sqrt{2}}{2}\left(\text{or } \dfrac{1}{\sqrt{2}}\right) = \dfrac{6\sqrt{2}}{12}$ (This requires all 3 equivalences (can be in any order))

Using trigonometry (SOHCAHTOA) to show the second property they will e.g.

- Use triangle *PQR* and assume right angled
- Split triangle *PQR* into two right angled congruent triangles using the isosceles property

If they use triangle *PQR* then it requires

- For M1: Calculations showing the required property to score M1
- For A1: To either draw a labelled right angled triangle or state the assumption that it is right angled and conclude

If they split the triangle *PQR* in half then they have formed two right angled congruent triangles which (provided they had two lengths the same already) will not be an assumption of a right angle. **Then it requires**

- For M1: Calculations to find either at least one of the base angles of $45°$ OR to find the right angle. It should be clear whether they
  - found a base angle of PQR and doubled it or
  - found half of the right angle and doubled it.
- For A1: Fully correct calculations and conclusion
  Note if they find a 45 degree angle and double it then it needs to be clear whether this is the right angle or if it is the sum of two base angles because there are four angles of size 45 degrees via this method.
  They would have to e.g. mention about the angle sum of a triangle oe or show a clearly labelled diagram and calculations with labels that match the diagram.

  Note that use of tan is unlikely to score because if they just use two equal lengths for the two shorter sides of their right angled triangle, then $\tan(A)=1$, so the angles will always be $45°$, $45°$ and $90°$ – it is the use of the length of 12 (or 6) which is going to lead to showing the triangle is right angled via these routes.

A1: Fully shows that the triangle is isosceles and right-angled **and concludes** that the triangle is **both isosceles and right-angled**. These conclusions may appear at separate stages of their solution. Condone poor notation and/or labelling of lengths provided the intention is clear. Condone slips if recovered. If they have a preamble then they must have a minimal conclusion e.g. proven, tick, QED

Note if they have an incorrect vector in part (a) then the maximum score is M1A1M1A0

Question	Scheme	Marks	AOs
11 (a)	Uses $t=0, V=20\,000 \Rightarrow 20\,000 = 1500 + A\mathrm{e}^{0}$	M1	3.4
	$A = 18500$	A1	1.1b
	$t=2.5, V=12\,000 \Rightarrow 12\,000 = 1500 + 18\,500\mathrm{e}^{-k\times 2.5}$ $\Rightarrow 10500 = 18500\mathrm{e}^{-k\times 2.5} \Rightarrow k = ...\left(= -\frac{2}{5}\ln\frac{21}{37} = \text{awrt}\,0.227\right)$	dM1	3.1b
	$V = 1500 + 18500\,\mathrm{e}^{-0.227t}$	A1	3.3
		(4)	
(b)	Achieves $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right) -kA\mathrm{e}^{-kt}$ or $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	B1	1.1b
	Substitutes $A\mathrm{e}^{-kt} = V - 1500$ into $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right) -kA\mathrm{e}^{-kt}$ or $"18500"\mathrm{e}^{"-0.227"t} = V - 1500$ into $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	M1	3.4
	Rate of change in value of car is $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right) -k(V-1500)$ or $"-0.227"(V-1500)$ *	A1*	2.1
		(3)	
(c)	Suggests a suitable limitation of the model (see notes)	B1	3.5b
		(1)	
			(8 marks)

Notes:

Mark (a) and (b) together

(a)

M1: Uses the equation of the model with $t=0, V=20\,000 \Rightarrow 20000 = 1500 + A\mathrm{e}^{0}$ o.e.
May be implied by 18500

A1: $A = 18500$ (18500 with no working seen scores M1A1) Ignore £ if present.

dM1: Attempts to use the equation of the model $t=2.5, V=12\,000$
$\Rightarrow 12000 = 1500 + "18500"\mathrm{e}^{-2.5k}$ and proceeds to $C\mathrm{e}^{\pm k\times 2.5} = D$ (where $C\times D > 0$ and allow $C=1$) **before proceeding** to find a value for *k*. Allow them to have a non-numerical *C* for this mark.
Note it cannot be implied by their awrt ± 0.227 so
$12000 = 1500 + "18500"\mathrm{e}^{-2.5k} \Rightarrow k = 0.227$ scores dM0A0 as we need to see the intermediate stage $C\mathrm{e}^{\pm k\times 2.5} = D$. (typically look for $18\,500\mathrm{e}^{-k\times 2.5} = 10\,500 \Rightarrow k = ...$ or condone to be implied by a correct expression involving logarithms for their *A*)
It is dependent on the previous method mark.

A1: $V = 1500 + 18500\,\mathrm{e}^{\text{awrt}-0.227t}$ o.e. e.g. $t = \dfrac{\ln\left(\dfrac{V-1500}{18500}\right)}{\text{awrt}-0.227}$ Allow *k* to be exact.

(b)

B1: Differentiates to a form $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right)-kA\mathrm{e}^{-kt}$ where k and A may be their values from (a)

e.g. $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$ May just see e.g. (using an exact k) $"-4191.3..."\mathrm{e}^{"-0.227"t}$ or e.g. (using k to 3sf) $"-4199.5"\mathrm{e}^{"-0.227"t}$ but do not be too concerned by over rounding provided the intention is clear that it is their $-kA$
Do not be too concerned by the left hand side / poor labelling of the derivative.

M1: Substitutes $A\mathrm{e}^{-kt}=V-1500$ into their $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right)\pm kA\mathrm{e}^{-kt}$ to form an expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of V. May see e.g. $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}\Rightarrow"-0.227"(V-1500)$.

Condone recovery of a sign slip on the index on $\mathrm{e}^{-kt}\to\mathrm{e}^{kt}$ provided it is before they substitute $A\mathrm{e}^{-kt}=V-1500$ for this mark.

Using the given answer: substitutes $V-1500=A\mathrm{e}^{-kt}$ and shows that $-k(V-1500)=-kA\mathrm{e}^{-kt}$ (may be in terms of their A and k)

A1*: Full and complete proof with sight of $\frac{\mathrm{d}V}{\mathrm{d}t}$ oe seen somewhere in their solution and no errors seen. Must see $-kA\mathrm{e}^{-kt}$ (or using their values) before proceeding to the given answer which may be written using their numerical value for k

Using the given answer they must conclude that $\frac{\mathrm{d}V}{\mathrm{d}t}=-k(V-1500)$ which may be written using their numerical value for k

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Alt (b) Separating the variables – may be in terms of their numerical value for *k*

B1ft: Separates the variables correctly and integrates to $\ln(V-1500)=-kt$ with or without $+c$

$$\int\frac{1}{(V-1500)}\mathrm{d}V=\int -k\,\mathrm{d}t\Rightarrow\ln(V-1500)=-kt\ \ (+c)\ \ \text{oe}$$

M1: Proceeds from $...\ln(V-1500)=...kt+c$ o.e. and rearranges to make V the subject. Condone slips.

e.g. $\ln(V-1500)=-kt+c\Rightarrow V-1500=A\mathrm{e}^{-kt}\Rightarrow V=1500+A\mathrm{e}^{-kt}$ or $V=1500+\mathrm{e}^{-kt+c}$
(just look for proceeding to V=… for this mark though.) May be in terms of their numerical values for k and A

A1*: Achieves $V=1500+A\mathrm{e}^{-kt}$ with no errors seen and concludes that $\frac{\mathrm{d}V}{\mathrm{d}t}=-k(V-1500)$

May be in terms of their numerical values for k and A

Alt (b) Rearranging to make *t* the subject

B1ft: For a correct rearrangement to $t=-\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)$ ft on their k

M1: Differentiates $...\ln(V-1500)$ to $\frac{...}{V-1500}$ and then finding $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{\left(\frac{\mathrm{d}t}{\mathrm{d}V}\right)}$

Typically look for $t=-\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)\Rightarrow\frac{\mathrm{d}t}{\mathrm{d}V}=-\frac{1}{k}\left(\frac{1}{V-1500}\right)$ o.e. so they may have an unsimplified version of this e.g.

$$t=-\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)\Rightarrow\frac{\mathrm{d}t}{\mathrm{d}V}=-\frac{1}{k}\left(\frac{\frac{1}{18500}}{\frac{V-1500}{18500}}\right)\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}t}=-k\left(\frac{\frac{V-1500}{18500}}{\frac{1}{18500}}\right)$$

May be in terms of their numerical *k*. Condone slips.

A1*: Achieves the given answer with no errors and $\frac{\mathrm{d}V}{\mathrm{d}t}$ seen somewhere in their solution

(c) **Note the question asks for a limitation of the model. If there is ambiguity over whether the response is referring to the model then try putting "the model suggests" or "the model" in front of their comment to see if this is a valid limitation.**
Ignore comments which do not contradict a valid limitation.
If values are given then they must be correct and if it is the value of the car then it must have units (£ or pounds)

B1: **Suitable limitations in context referring to the limitation of the model which score B1**
e.g. (the model suggests) the value/price of the car will never go below £1500
e.g. (the model suggests) the rate of decrease of the value of the car is proportionally the same each year
e.g. (the model suggests) after a certain period of time the car will no longer lose value (condone this property of the model that it will tend to a limit)
e.g. (the model) only takes account of age
e.g. (the model) does not take into account damage / alteration to the car/ mileage
e.g. (the model) predicts that the car's value will always go down
e.g. after many years the car may become worthless whereas the model does not allow for this (valid limitation comparing the car to the model)
e.g. the value of the car may go up where as it only decreases according to the model

Do not accept vague/incorrect/irrelevant or non-contextual comments which score B0
e.g. after many years the car may become worthless
e.g. the value of the car may increase
e.g. damage or alterations to the car may impact the value
e.g. the car will still have value when it is very old (should refer to £1500)
e.g. (the model suggests) the minimum value is 1500 (no units for money)
e.g. (the model suggests) the (value of the) car cannot be negative
e.g. car value will fluctuate which the model will not show (a model is not for this purpose)
e.g. the rate of decrease is proportionally the same each year (no context)

Question	Scheme	Marks	AOs
12 (a)	e.g. Substitutes $x=6$ into **both** $(y=)\dfrac{15x}{(2x+3)(x-3)}$ **and** $(y=)\ 2x-10$ and finds the y values for both	M1	1.1b
	e.g. Achieves 2 for both and makes a valid conclusion *	A1*	2.4
		(2)	
(b)	Sets $\dfrac{15x}{(2x+3)(x-3)}=2x-10$ and attempts to cross multiply	M1	1.1b
	$4x^3-26x^2-3x+90=0$ *	A1*	2.1
		(2)	
(c)	Deduces that $(x-6)$ is a factor and attempts to divide	M1	2.1
	$4x^3-26x^2-3x+90=(x-6)\left(4x^2-2x-15\right)$	A1	1.1b
	Solves their $4x^2-2x-15$ using suitable method	M1	1.1b
	Deduces $x=\dfrac{1-\sqrt{61}}{4}$ (see note)	A1	2.2a
		(4)	
			(8 marks)

Notes:

(a) Must be seen in (a) to score

M1: Examples to verify include:

- substitutes $x=6$ into **both** $(y=)\dfrac{15x}{(2x+3)(x-3)}$ **and** $(y=)\ 2x-10$ and finds the y values for both
- substitutes $x=6$ into one of the two equations to find $y=2$ and then substitutes this into the other equation and solves to find x
- sets $\dfrac{15x}{(2x+3)(x-3)}=2x-10 \Rightarrow$ cubic equation (=0 implied) and either substitutes $x=6$ into the expression, attempts $f(6)$ or else attempts to divide the cubic = 0 by $(x-6)$.

Condone without calculations for this mark only. i.e. $f(6)=0$

Condone slips for this mark in any of the outlined approaches. There may be variations of these as well.

A1*: Correct calculations must be seen with a minimal conclusion that they intersect

e.g. $\dfrac{15\times 6}{(2\times 6+3)(6-3)}=2$ and $12-10=2$ so they intersect (allow eg meet/cross/intercept)

e.g. $\dfrac{15\times 6}{(2\times 6+3)(6-3)}=2 \Rightarrow 2=2x-10 \Rightarrow x=6$ so they intersect

Acceptable alternatives are:

$f(x)=4x^3-26x^2-3x+90,\ f(6)=4\times 6^3-26\times 6^2-3\times 6+90=0 \Rightarrow$ so they intersect

$f(x)=4x^3-26x^2-3x+90 \Rightarrow (x-6)(4x^2-2x-15)$ so $x=6$ is a root so they intersect

OR $(x-6)$ is a factor hence they intersect

Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.

Special case: ($f(x)=4x^3-26x^2-3x+90=0$ o.e.) $\Rightarrow$ awrt -1.7, 6, awrt 2.2 Scores M1A0 with or without a conclusion. Requires all 3 roots.

(b) Work seen in (a) must be used in (b) to score

M1: Sets $\dfrac{15x}{(2x+3)(x-3)}=2x-10$ and attempts to cross multiply to form a cubic equation.

Score for $15x=(2x-10)(2x+3)(x-3)$ or may be implied by an attempt to multiply two of the three brackets out e.g. $15x=(2x-10)\left(2x^2-3x-9\right)$ or

$15x=(x-3)\left(4x^2-14x-30\right)$ or $15x=(2x+3)(2x^2-16x+30)$ (may be unsimplified)

Condone slips provided the intention is clear.
Condone invisible brackets to be implied by further work which is not the given answer.
Do not allow this mark to be scored for proceeding straight from $\dfrac{15x}{(2x+3)(x-3)}=2x-10$ to a simplified expression of $15x=4x^3-26x^2+12x+90$

A1*: Achieves the given answer with sufficient intermediate steps seen and **no errors including invisible brackets** in the main body of their solution (ignore "side workings")
Expect to see on the right hand side the product of a linear and a quadratic expression and an expression with all of the brackets multiplied out before simplifying to the given answer.
e.g.

$$\frac{15x}{(2x+3)(x-3)}=2x-10\Rightarrow 15x=\left(4x^2-14x-30\right)(x-3)\Rightarrow 15x=4x^3-26x^2+12x+90$$

$\Rightarrow 4x^3-26x^2-3x+90=0$ * which is A1*

(c) Work seen in (a) or (b) must be used in (c) to score

M1: For the key step in dividing the cubic by $(x-6)$. There are various methods or ways to present their solution so typically methods

- by inspection look for first and last terms e.g. $(x-6)(4x^2\pm\ldots x\pm15)$
- by division look for a three term quadratic quotient of $4x^2-2x\pm\ldots$

A1: $4x^3-26x^2-3x+90=(x-6)\left(4x^2-2x-15\right)$ This may be implied by sight of $\left(4x^2-2x-15\right)$ in their working.

M1: **Whilst not dependent on the previous method mark, it does require an attempt at dividing by $(x-6)$ however poor to achieve a three term quadratic factor**

Solves their $4x^2-2x-15=0$ using either the quadratic formula or completing the square. Usual rules apply for solving a quadratic. **It cannot be scored for stating the roots directly via use of a calculator which is M0A0**

A1: $(x=)\ \dfrac{1-\sqrt{61}}{4}$ or exact equivalent e.g. $\dfrac{1}{4}-\sqrt{\dfrac{61}{16}}$

The root $(x=)\ \dfrac{1+\sqrt{61}}{4}$ must be rejected if found. Isw if they put their exact value in decimal form.

Question	Scheme	Marks	AOs
13	$\int \frac{x}{(2x+1)^3}\,dx = \frac{x(2x+1)^{-2}}{-4} + \int \frac{(2x+1)^{-2}}{4}(dx)$	M1	3.1a
	$= \ldots + \frac{(2x+1)^{-1}}{-8}$	dM1	1.1b
	$-\frac{x}{4(2x+1)^2} - \frac{1}{8(2x+1)}$	A1	1.1b
	e.g. $= \left(-\frac{2}{4(2\times2+1)^2} - \frac{1}{8(2\times2+1)}\right) - \left(-\frac{0}{4(2\times0+1)^2} - \frac{1}{8(2\times0+1)}\right)$	ddM1	1.1b
	e.g. $-\frac{2}{100} - \frac{1}{40} + \frac{1}{8} = \frac{2}{25}$ *	A1*	2.1
		(5)	
13 Alt I	$\int \frac{x}{(2x+1)^3}\,dx = \int \frac{u-1}{4u^3}(du)$ where $u = 2x+1$	M1	3.1a
	$= \int \frac{1}{4}u^{-2} - \frac{1}{4}u^{-3}\,du = \ldots$	dM1	1.1b
	$= -\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}$	A1	1.1b
	$\int_0^2 \frac{x}{(2x+1)^3}\,dx = \left[-\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}\right]_1^5 = \left(-\frac{1}{20} + \frac{1}{200}\right) - \left(-\frac{1}{4} + \frac{1}{8}\right)$	ddM1	1.1b
	e.g. $-\frac{1}{20} + \frac{1}{200} + \frac{1}{8} = \frac{2}{25}$ *	A1*	2.1
		(5)	
			(5 marks)

Notes:

M1: Obtains $\alpha x(2x+1)^{-2} \pm \beta\int(2x+1)^{-2}\,(dx)$ o.e. where $\alpha, \beta \neq 0$ but may be equal to each other (you do not need to be concerned about how they arrive at this)

dM1: Uses a correct method to integrate an expression of the form $\pm\beta\int(2x+1)^{-2}\,(dx) \to \pm\gamma(2x+1)^{-1}$, $\beta, \gamma \neq 0$

It is dependent on the previous method mark.

A1: $-\frac{x}{4(2x+1)^2} - \frac{1}{8(2x+1)}$ o.e. Allow this to be unsimplified

Watch out for the DI method

	D	I
+	x	$(2x+1)^{-3}$
−	1	$-\frac{(2x+1)^{-2}}{4}$
+	0	$\frac{(2x+1)^{-1}}{8}$

Giving correct integration e.g. $\displaystyle\int \frac{x}{(2x+1)^3}\,\mathrm{d}x = -\frac{x(2x+1)^{-2}}{4} - \frac{(2x+1)^{-1}}{8}$

Score M1dM1 for obtaining $\pm px(2x+1)^{-2} \pm q(2x+1)^{-1},\ \ p,q \neq 0$ and A1 for both correct.

ddM1: Substitutes 0 and 2 into an expression of the form $\pm px(2x+1)^{-2} \pm q(2x+1)^{-1},\ \ p,q \neq 0$ or equivalent and subtracts either way round. Evidence of limits used cannot be the given answer. Condone slips but not directly evaluating the expression as 0 when x is 0. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the dx throughout

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Alternative method I – substitution using $u = 2x+1$

M1: Uses a suitable substitution e.g. $u = 2x+1$ and proceeds to $A\displaystyle\int \frac{u-1}{u^3}\,(\mathrm{d}u)$ o.e.

dM1: Splits into separate fractions and attempts to integrate $A\displaystyle\int u^{-2} - u^{-3}\,(\mathrm{d}u)$ Look for at least one correct index for one of the two terms i.e. $u^{-2} \to u^{-1}$ or $u^{-3} \to u^{-2}$. It is dependent on the previous method mark.

A1: $-\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}$ o.e.

ddM1: Substitutes correct limits (1 and 5 if in terms of u) into an expression of the correct form $...u^{-1} + ...u^{-2}$ (or may have substituted back in terms of x and substitutes in 0 and 2 into an expression of the correct form $\pm p(2x+1)^{-2} \pm q(2x+1)^{-1},\ \ p,q \neq 0$ o.e and subtracts either way round – see above for ddM1). Evidence of limits used cannot be the given answer. Condone slips. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the du throughout

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Alternative method II – partial fractions

M1: Writes $\dfrac{x}{(2x+1)^3}$ as $\dfrac{0.5}{(2x+1)^2} - \dfrac{0.5}{(2x+1)^3}$ o.e. Allow $\pm\dfrac{M}{(2x+1)^2} \pm \dfrac{N}{(2x+1)^3}$ (where M and N are constants)

dM1: $\displaystyle\int \frac{x}{(2x+1)^3}\,(\mathrm{d}x) = \int \frac{"0.5"}{(2x+1)^2}\,(\mathrm{d}x) - \int \frac{"0.5"}{(2x+1)^3}\,(\mathrm{d}x) = \pm...(2x+1)^{-1} \pm ...(2x+1)^{-2}$

A1: $-\dfrac{1}{4(2x+1)^1} + \dfrac{1}{8(2x+1)^2}$ o.e.

ddM1: Substitutes 0 and 2 into an expression of the correct form $\pm...(2x+1)^{-1} \pm ...(2x+1)^{-2}$ and subtracts either way round. Evidence of limits used cannot be the given answer. Condone slips but not directly evaluating the expression as 0 when x is 0. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the dx throughout.

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\sin(x+30°)=\sin x\cos 30°\pm\cos x\sin 30°$, $\cos(x+30°)=\cos x\cos 30°\mp\sin x\sin 30°$ $\Rightarrow \pm\sin x\cos 30°\pm\cos x\sin 30°\pm\sqrt{3}(\pm\cos x\cos 30°\pm\sin x\sin 30°)$	M1	2.1
	Correct expression $\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)$	A1	1.1b
	States or implies that $\sin 30°=\frac{1}{2}$ and $\cos 30°=\frac{\sqrt{3}}{2}$ $=\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)=2\cos x$ *	A1*	2.1
		(3)	
(b)	$2\cos\theta=3\sin 2\theta\Rightarrow 2\cos\theta=6\sin\theta\cos\theta$	M1	2.1
	$\sin\theta=\frac{1}{3}$	A1	1.1b
	$\theta=\arcsin\frac{1}{3}\Rightarrow\theta=...$	dM1	1.1b
	$(\theta=)$ awrt $19.5°$, awrt $160.5°$, $90°$	A1	2.2a
		(4)	
			(7 marks)

Notes:

(a) **Condone a complete proof entirely in θ (or another variable) instead of x.**
Do not be concerned with the omission of degrees.

M1: Attempts to use both compound angle expansions to set up an expression in $\sin x$ and $\cos x$
i.e. $\pm\sin x\cos 30°\pm\cos x\sin 30°\pm\sqrt{3}(\pm\cos x\cos 30°\pm\sin x\sin 30°)$
The terms must be correct but condone sign errors and a slip on the multiplication of the $\sqrt{3}$ if they attempt to multiply out the brackets. (The $\sqrt{3}$ may be omitted entirely)
This mark may be implied by further work
e.g. $\pm\frac{\sqrt{3}}{2}\sin x\pm\frac{1}{2}\cos x\pm\sqrt{3}\left(\pm\frac{\sqrt{3}}{2}\cos x\pm\frac{1}{2}\sin x\right)$

A1: Correct expression $\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)$ o.e.
May be implied by
$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)$ (or implied if multiplied out)

A1*: Proceeds to the given answer with **no errors seen including invisible brackets** (condone a missing trailing bracket). We must see the exact numerical values used for $\sin 30°$ and $\cos 30°$ before proceeding to the given answer.
Minimum required
$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)=2\cos x$ which scores M1A1A1*

$\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)=2\cos x$ scores M1A1A0*
There should not be any notational or bracketing errors and no mixed or missing variables.

Alternative for part (a)

M1: Writes the left hand side of the equation as

$$\frac{1}{2}\sin\left(x+30°\right)+\frac{\sqrt{3}}{2}\cos\left(x+30°\right)=\sin 30°\sin\left(x+30°\right)+\cos 30°\cos\left(x+30°\right)$$

A1: Correct expression for $\sin 30°\sin\left(x+30°\right)+\cos 30°\cos\left(x+30°\right)=\cos x$

A1*: Proceeds to the given answer $\sin\left(x+30°\right)+\sqrt{3}\cos\left(x+30°\right)=2\cos\left(x+30°-30°\right)=2\cos x$ with **no errors seen including invisible brackets** (condone a missing trailing bracket). There should not be any notational or bracketing errors and no mixed or missing variables.

Alternative part (a) – using the *R*-alpha method

M1: States e.g. $R\cos(x+30\pm\alpha)=\sqrt{3}\cos(x+30)\mp\sin(x+30)$

and attempts to find either R or α correctly. (may be implied)

A1: Achieves $2\cos(x+30-30)$

A1*: Achieves $2\cos x$ with no errors seen and both stages of working shown e.g.

- States $\left(R\cos(x+30\pm\alpha)=\right)\sqrt{3}\cos(x+30)\cos\alpha\mp\sin(x+30)\sin\alpha$ oe
- Shows $R=\sqrt{1+3}=2$ and $\tan\alpha=\left(\frac{1}{\sqrt{3}}\right)$ o.e. e.g. using cosine or sine

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(b) Condone the use of *x* for θ and mixed variables

M1: Sets $2\cos\theta=3\sin 2\theta$ and proceeds to $A\cos\theta=B\sin\theta\cos\theta$. (allow $A=B$)
May be implied by $\sin\theta=k$ ($k\neq 0,1$)

A1: $\sin\theta=\frac{1}{3}$

dM1: Finds at least one of their values of θ for their $\sin\theta=k$ ($k\neq 0,1$) It is dependent on the previous method mark. You may need to check their value(s) (in degrees or radians) but may be implied by e.g. awrt$19°$ (or awrt$20°$) or awrt$161°$ (or awrt$160°$)
(awrt 0.34 or awrt 2.8 in radians)

A1: Deduces that $(\theta=)$ $90°$ as well as giving $(\theta=)$ awrt$19.5°$, awrt$160.5°$ with no other values in the given range (ignore any found outside of the range)
The degree symbol is not required. (Note the angles are 19.4712206…and 160.528779…)
Answers in radians score A0.

Question	Scheme	Marks	AOs
15 (a)	$\frac{1}{2}r^2\theta+\frac{1}{10}r^2=240 \Rightarrow r\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ or $\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r^2}$	M1 A1	3.4 1.1b
	Substitutes into the expression for P $r\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ into $(P=)\ r\theta+2r+\frac{1}{5}r$	dM1	3.4
	$P=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}+2r+\frac{1}{5}r=\frac{480}{r}-\frac{1}{5}r+2r+\frac{1}{5}r=2r+\frac{480}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dP}{dr}=\right)2-\frac{480}{r^2}$	M1	1.1b
	Sets $\frac{dP}{dr}=0 \Rightarrow r^2=240$ $r=$ awrt 15.5	dM1 A1	2.1 1.1b
		(3)	
(c)	$\left(\frac{d^2P}{dr^2}=\right)\frac{960}{r^3}$	M1	1.1b
	$\left(\frac{d^2P}{dr^2}=\right)$ awrt $0.26>0$ proving a minimum value of P	A1	1.1b
		(2)	
			(9 marks)

Notes:

(a) **Note that just finding a correct equation for the area and/or a correct equation for the perimeter (before any substitution) is insufficient to score any marks.**

M1: Uses area formulae to form an equation of the form $\alpha r^2\theta+\beta r^2=240$ o.e. ($\alpha,\beta\neq 0$) and rearranges to make $r\theta$, θ or $r\theta+\frac{1}{5}r$ the subject. Look for:

$r\theta=\dfrac{M\pm Nr^2}{r}\left(=\dfrac{M}{r}\pm Nr\right)$ o.e. or $\theta=\dfrac{M\pm Nr^2}{r^2}\left(=\dfrac{M}{r^2}\pm N\right)$ o.e. where $M,N\neq 0$

or $r\theta+\frac{1}{5}r=\frac{L}{r}$ $L\neq 0$ o.e. May work in degrees.

A1: A correct rearrangement for θ or $r\theta$ or $r\theta+\frac{1}{5}r$ which may be unsimplified (may be in degrees)

$r\theta=\dfrac{240-\frac{1}{10}r^2}{\frac{1}{2}r}$ o.e. e.g. $r\theta=\dfrac{2400-r^2}{5r}$ or $r\theta=\dfrac{480-0.2r^2}{r}$

or $r\theta+\frac{1}{5}r=\frac{480}{r}$ o.e.

or $\theta = \dfrac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2}$ o.e. e.g. $\theta = \dfrac{2400 - r^2}{5r^2}$ or $\theta = \dfrac{480}{r^2} - \dfrac{1}{5}$ or $\theta = 480r^{-2} - 0.2$

dM1: Substitutes their $r\theta = \dfrac{M \pm Nr^2}{r}$ o.e. or $\theta = \dfrac{M \pm Nr^2}{r^2}$ o.e. or $r\theta + \dfrac{1}{5}r = \dfrac{L}{r}$ into an expression of the form $(P =)\ r\theta + Qr$, $Q \neq 0$ (typically $P = r\theta + \dfrac{11}{5}r$) which may be unsimplified or in degrees. It is dependent on the previous method mark. It is acceptable for their valid expression for θ, $r\theta$ or $r\theta + \dfrac{1}{5}r$ to be substituted into the perimeter expression directly (without first seeing them in the perimeter expression).

A1*: $P = 2r + \dfrac{480}{r}$ following a correct method (condone slips to be recovered) and all previous marks scored. Condone invisible brackets to be recovered.
$P =$, Perimeter $=$ must be seen at least once in their solution in the correct place.

(b) **Mark (b) and (c) together. There is no requirement to see the notation $\dfrac{\mathrm{d}P}{\mathrm{d}r}$ in part (b). It may even be called $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Allow use of e.g. P' or e.g. y'**

M1: $\left(\dfrac{\mathrm{d}P}{\mathrm{d}r} =\right) p \pm \dfrac{q}{r^2}$ where p and q are non-zero constants

dM1: Sets or implies that their $\dfrac{\mathrm{d}P}{\mathrm{d}r} = 0$ and proceeds to $mr^{\pm 2} = n,\quad m \times n > 0$. It is dependent on the previous method mark. Do not be concerned by the mechanics of the rearrangement. This mark may be implied by a correct answer to their $p - \dfrac{q}{r^2} = 0$. You may need to check this on your calculator.

A1: $r =$ awrt 15.5 or $\sqrt{240}\ \left(= 4\sqrt{15}\right)$ Do not accept $\pm$ (ignore any units if given)

(c) **Condone other letters used instead of *P* and *r* for $\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2}$ e.g. $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for M1 only.**

Just using $\dfrac{\mathrm{d}P}{\mathrm{d}r}$ and considering a sign change is M0A0

M1: Differentiates and finds $\left(\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2} =\right) \pm \dfrac{f}{r^3}$ (do not be concerned about the sign)

A1: **Note if they score A0 in (b) then this mark cannot be scored.**
Requires

- a correct a correct expression for $\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2}$
- a correct value for $\left(\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2} =\right) \dfrac{960}{r^3} =$ awrt 0.26 using awrt 15.5 (but allow 0.23(43..) if using 16)
- a correct comparison with 0 **and** a conclusion e.g. minimum

The expression for the second derivative does not need to be labelled but if it is then it must be $\dfrac{\mathrm{d}^2P}{\mathrm{d}r^2}$ o.e. or accept e.g. P'' BUT $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ used in their conclusion is A0

Question	Scheme	Marks	AOs
16 (a)	$R=\int_1^{\sqrt{3}} \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x$		
	$\int \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x = \int \frac{1}{4\sin^2 u\sqrt{4-4\sin^2 u}}\,2\cos u\,(\mathrm{d}u)$	M1	3.1a
	Uses $1-\sin^2 u=\cos^2 u \Rightarrow \sqrt{4-4\sin^2 u}=2\cos u$	dM1	1.1b
	$=\int \frac{1}{4\sin^2 u\times 2\cos u}\,2\cos u\,(\mathrm{d}u)=\int \frac{1}{4}\operatorname{cosec}^2 u\,\mathrm{d}u$	A1	2.1
	Deduces $a=\frac{\pi}{6}$ and $b=\frac{\pi}{3}$	B1	2.2a
		(4)	
(b)	$\int \frac{1}{4}\operatorname{cosec}^2 u\,(\mathrm{d}u)=-\frac{1}{4}\cot u\ \ (+c)$	B1ft	1.2
	$=\left[-\frac{1}{4}\cot u\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=-\frac{1}{4}\cot\frac{\pi}{3}+\frac{1}{4}\cot\frac{\pi}{6}$	M1	2.1
	$=\frac{1}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{6}$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a)

M1: Proceeds to $\frac{1}{A\sin^2 u\sqrt{\pm C\pm C\sin^2 u}}\times B\cos u$ o.e. or may be implied by $\frac{\cos u}{D\sin^2 u\cos u}$ o.e in terms of u only $A, B, C, D\neq 0$

Requires $\mathrm{d}x\to \ldots\cos u\ (\mathrm{d}u)$ o.e. Condone the omission of the integral sign and du for this mark. Condone the $\pm B\cos u$ appearing after the du if present

It cannot be implied by $\frac{1}{D\sin^2 u}$ or $\frac{\operatorname{cosec}^2 u}{D}$ where $D\neq 0$

dM1: Attempts to use $\pm 1\pm\sin^2 u=\pm\cos^2 u$ to change $\sqrt{\pm C\pm C\sin^2 u}$ to $p\cos u$ o.e. (Must be seen somewhere in their solution which may be part of side-workings)

May be seen as e.g. $\frac{1}{A\sin^2 u\sqrt{C\cos^2 u}}\times B\cos u$ or $\frac{1}{A\sin^2 u\times F\cos u}\times B\cos u$

It is dependent on the previous method mark.

A1: $\int \frac{1}{4}\operatorname{cosec}^2 u\,\mathrm{d}u$ Ignore any limits or the absence of them for this mark.

Requires the integral sign and du. Do not isw. Must be seen in (a)

B1: Deduces $a=\frac{\pi}{6}$ and $b=\frac{\pi}{3}$ (in radians) May be with their final integral in the correct positions or stated separately which value is a and which value is b. Must be seen in (a).

Alternative part (a) (Further Maths)

M1: $u = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{M}{\sqrt{1-Nx^2}}$ It cannot be implied by $\frac{1}{D\sin^2 u}$ or $\frac{\mathrm{cosec}^2 u}{D}$, $D \neq 0$

dM1: Uses $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{\sqrt{4-x^2}}$ o.e , substitutes $x = 2\sin u$ and proceeds to $\frac{1}{4\sin^2 u}$ It is dependent on the previous method mark.

A1: $\int \frac{1}{4}\mathrm{cosec}^2 u \,\mathrm{d}u$ Ignore any limits or the absence of them for this mark.

Requires the integral sign and du. Do not isw. Must be seen in (a)

B1: Deduces $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$ (must be in radians)

May be with their final integral in the correct positions or stated separately which value is a and which value is b. Must be seen in (a).

(b) Condone x instead of u provided the appropriate limits are substituted into the function

B1ft: $k\mathrm{cosec}^2 u \to -k\cot u$ (may be left in terms of $k \neq 0$)

e.g. $-2\mathrm{cosec}^2 u \to 2\cot u$ is B1 (look for the sign to change as well)

Note some candidates may prefer to change

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{4}\mathrm{cosec}^2 u\,(\mathrm{d}u) = -\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{1}{4}\mathrm{cosec}^2 u\,(\mathrm{d}u) = \frac{1}{4}\cot u\ (+c)$ which scores B1ft

M1: Uses the limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$ either way round (or condone use of $30°$ and $60°$) in an expression of the form $\pm q\cot u$ and subtracts (either way round).

Allow $q = 1$. May be implied by their final answer provided B1ft has been scored.

May write as e.g. $-\frac{1}{4}\frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} + \frac{1}{4}\frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$

Do not condone a sign slip here e.g. $-\frac{1}{4}\cot\frac{\pi}{3} - \frac{1}{4}\cot\frac{\pi}{6}$ is M0.

The expression is sufficient but if just a value is stated following integration to the required form then you may need to check this on your calculator.

If no algebraic integration is seen then M0

If only decimal values are seen then M0

A1: $\frac{1}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{6}$ provided the previous two marks have been scored.

Note that incorrect integration e.g. $\int k\,\mathrm{cosec}^2 u\,\mathrm{d}u = k\cot u \to \frac{1}{2\sqrt{3}}$ scores B0ftM1A0

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