



Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 3B Further Statistics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations
These are some of the traditional marking abbreviations that will appear in the mark schemes.
 - bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Qu	Scheme	Mark	AO
1 (a)	$E(X) = [-1 \times 0.2 + 0 + 1 \times 0.2] + 3 \times 0.25 + 5 \times 0.25 [= 2]$	M1	1.1b
	$E(X^2) = (-1)^2 \times 0.2 + 0 + 1 \times 0.2 + 3^2 \times 0.25 + 5^2 \times 0.25 [= 8.9]$	M1	1.1b
	$[Var(X) = 8.9 - 4 =]$ 4.9	A1	1.1b
		(3)	
	(b) $E(X^4) = (-1)^4 \times 0.2 + 0 + 1 \times 0.2 + 3^4 \times 0.25 + 5^4 \times 0.25 [= 176.9]$	M1	2.1
	$Var(X^2) = "176.9" - "8.9" ^2$	M1	1.1b
	$= 97.69$ awrt 97.7	A1	1.1b
		(3)	
		(6 marks)	
	Notes		
(a)	1 st M1 for an attempt at E(X), at least the final 2 products seen <u>or</u> an answer of 2 2 nd M1 for attempting a correct expression for E(X ²), at least 3 non-zero products seen A1 for 4.9 or exact equivalent		
(b)	1 st M1 for attempting a correct expression, at least 3 non-zero products seen (implied by 176.9) 2 nd M1 for a correct method, ft their 176.9 and their 8.9 but must be intending E(X ⁴) and E(X ²) A1 for 97.69 or awrt 97.7		

Qu	Scheme	Mark	AO	
2 (a)	(i) 0.20901...	awrt 0.209	B1	3.4
	(ii) 0.30844..	awrt 0.308	B1	1.1b
			(2)	
	(b) $H_0: \lambda = 2.4$ (or $\mu = 6$) $H_1: \lambda \neq 2.4$ (or $\mu \neq 6$)		B1	2.5
	[E = no. of errors] $E \sim \text{Po}(6)$		M1	3.3
	$P(E \leq 1) = 0.0174$ <u>or</u> $P(E \leq 2) = 0.0620$		M1	3.4
	and			
	$P(E \leq 11) = 0.980$ <u>or</u> $P(E \geq 12) = 0.0201$			
	Critical region: $E \leq 1$ or $E \geq 12$		A1	1.1b
			(4)	
(c)	[P(Type I error) = $0.0174 + 0.0201 =$] 0.0375 (Calc gives: $0.017351\dots + 0.0200919\dots = 0.037443\dots$)		B1ft	1.2
			(1)	
		(7 marks)		
Notes				
(a)	1 st B1 for awrt 0.209			

Qu	Scheme	Mark	AO
3 (a)	H_0 : There is no association between the <u>colour</u> chosen and <u>year</u> group (allow use of “independence” instead of association)	B1	2.5
	H_1 : There <u>is</u> some association between <u>colour</u> and <u>year</u> group	(1)	
	(b)(i) Need lowest <u>row total</u> and <u>column total</u> so “Black” and “10-12”	B1	2.2a
	(ii) With expected frequency $\frac{68 \times 19}{240} = 5.3833\dots$	B1	1.1b
	(c) $v = (5 - 1) \times (3 - 1) = \underline{8}$	(2)	
	cv of $\chi^2_8(1\%) = \underline{20.090}$	B1	3.4
	(significant) evidence of an association between <u>colour</u> chosen and <u>year</u> group	B1ft	1.1b
		B1ft	2.2b
		(3)	
		(6 marks)	
	Notes		
(a)	B1	for both hypotheses in context. Must mention <u>colour</u> and <u>year</u> group at least once	
	NB:	condone use of related/correlated/linked etc if recovered with independent/associated	
(b)(i)	B1	for a choosing “Black” and “10-12” with some correct reasoning mentioning <u>row and column totals</u> (allow clear equivalent, e.g. ‘lowest total frequencies’)	
(ii)	B1	for a correct expression or awrt 5.38	
(c)	1 st B1	for a correct calculation or answer of 8. May be implied by sight of cv of 20.09	
	2 nd B1ft	for 20.090 (accept 20.09) or a correct ft 1% cv using their df and correct to 2 d.p.	
	3 rd B1ft	for a correct conclusion in context (ft their cv)	
		E.g. “students in different years (tend to) have different favourite colours”	
		Do not allow contradictory statements. Condone ‘related/relationship’ in part (c)	
		NB: We ignore their hypotheses when marking (c)	

Qu	Scheme	Mark	AO
4 (a)	$X \sim \text{Geo}(\frac{1}{6})$ accept in words: <u>geometric</u> distribution with $p = \frac{1}{6}$	B1	3.3
(b)	$P(X \leq 3) = 1 - P(X > 3)$ <u>or</u> $P(X = 1) + P(X = 2) + P(X = 3)$ $= 1 - \left(\frac{5}{6}\right)^3$ or $\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{91}{216}$ (*)	(1) M1 A1cso	1.1b 1.1b
(c)	$E(X) = 6$ $\text{Var}(X) = \frac{\frac{5}{6}}{\left(\frac{1}{6}\right)^2} [=30]$ $\bar{X} \approx \sim N\left(6, \left(\sqrt{\frac{30}{64}}\right)^2\right)$ $P(5.6 < \bar{X} < 7.2) = 0.68064\dots$	(2) M1 A1	3.4 1.1b
(d)	$H_0: p = \frac{1}{6}$ $H_1: p < \frac{1}{6}$ Probability approach: $P(X \geq 16)$ $= P(X > 15) = \left(\frac{5}{6}\right)^{15} = 0.0649\dots$ (Not significant) insufficient evidence that <u>probability</u> is $< \frac{1}{6}$ or (Not significant) insufficient evidence that dice is <u>biased</u>	CR approach: $\left(\frac{5}{6}\right)^{c-1} < 0.05$ or $\left(\frac{5}{6}\right)^d < 0.05$ with use of logs CR: $X \geq 18$ or $X > 17$ A1 A1 (4)	2.5 3.4 1.1b 2.2b
		(12 marks)	
Notes			
(a)	B1 for both “geometric” or “Geo” and correct parameter of $\frac{1}{6}$. Probability must be seen in (a)		
(b)	M1 for a correct method, may be implied by a correct expression A1cso* for a correct solution leading to printed answer with no incorrect working seen		
(c)	1 st M1 for correct use of formula for $E(X)$ <u>or</u> $\text{Var}(X)$ 1 st A1 for both correct must see value of 6 for $E(X)$ and at least correct formula used for $\text{Var}(X)$ the value of 30 may be implied by 2 nd A1 2 nd M1 for use of CLT to get a normal distribution with correct mean and variance $\neq$ their 30 Condone use of X (or their letter) instead of $\bar{X}$ or seeing $N(6, \sqrt{\frac{30}{64}})$ 2 nd A1 for a correct normal distribution stated or used. May be implied by correct answer. 3 rd A1 for awrt 0.681 but allow 0.68 or 0.680 if correct distribution is seen		
(d)	B1 for both hypotheses correct in terms of p M1 for sight or use of $P(X \geq 16)$ (may be implied) or correct expression with logs (use of logs may be implied by 17.43...) 1 st A1 for 0.065 or better but accept 0.06 if a correct expression is seen, or correct CR 2 nd A1 for a correct conclusion mentioning <u>probability</u> or <u>biased</u> Condone ‘evidence suggests <u>probability</u> is equal to $\frac{1}{6}$ ’, NB: Must have correct probability or correct CR to gain final A1. NB: Send responses comparing 0.935 with 0.95 to review		

Qu	Scheme	Mark	AO
5 (a)	$H_0: p=0.03 \quad H_1: p>0.03$	B1 (1)	2.5
(b)	$D_{100} \sim B(100, 0.03) \quad [P(D_{100} \geq 5) = 1 - P(D_{100} \leq 4)]$ $= 0.18214... \quad \text{awrt } \underline{0.182}$	M1 A1 (2)	3.3 1.1b
(c)	$P(D_{80} \geq 5) + P(D_{80} = 4) \times P(D_{80} \geq 1)$ $= 0.09279... + 0.12654... \times 0.91255...$ $= 0.20826... \quad \text{awrt } \underline{0.208}$	M1 A1 A1 (3)	2.1 1.1b 1.1b
(d)(i)	Test A: $[X \sim B(100, 0.06) \quad P(X \geq 5) =] \quad 0.72322... \quad \text{awrt } \underline{0.723}$	B1	1.2
(ii)	Expected number = $80 + 80 \times P(Y = 4) \quad \text{with} \quad Y \sim B(80, 0.06)$ $= 94.87... = \underline{95}$	M1 A1 (3)	3.4 1.1b
(e)	Tests of similar size and power but Test B involves sampling fewer components so would advise to use test B.	B1 (1)	2.4
		(10 marks)	
Notes			
(a)	B1 for both hypotheses in terms of p or π		
(b)	M1 for sight or use of the correct model. Allow for $1 - 0.81785...$ or $1 - 0.91916..$ A1 for awrt 0.182		
(c)	M1 for a correct expression for required probability. May be implied by 1 st A1 1 st A1 for a correct numerical expression – values to 2 s.f. or better 2 nd A1 for awrt 0.208		
(d)(i)	B1 for power of test A = awrt 0.723		
(ii)	M1 Correct expression with $Y \sim B(80, 0.06)$ or for use of $B(80, 0.06)$ to obtain a probability of 0.814 or 0.186 Implied by sight of 95 or better A1 for 95 (accept awrt 94.9) NB: May see $0.814 \times 80 + 0.186 \times 160 = \text{awrt } 94.9 \text{ or } 95$, which is M1A1		
(e)	B1 for a choice backed up by a suitable reason: If concluding B, they need to mention <u>similar power</u> and a <u>smaller sample size</u> If concluding A, they need to mention <u>smaller size</u> and <u>greater power</u> NB: We do not ft the candidate's incorrect values for size or power		
SC	Use of Poisson approximation		
(b)	Allow M1 for Po(3) and A1 for answer of 0.1847 or better		
(c)	Allow M1 and 1 st A1 for awrt 0.21 but 2 nd A0 (Po(2.4) gives 0.2099...)		
(d)(i)	Allow B1 for 0.7149 or better from Po(6)		
(ii)	Allow M1 for expression (answer should be 94.56...) A0 for answer.		

Qu	Scheme	Mark	AO
6			
(a)	$[G_X(t) = (4 - 3t)^{-\frac{1}{2}} \Rightarrow] \quad G'_X(t) = \frac{3}{2}(4 - 3t)^{-\frac{3}{2}}; \quad [\text{So } E(X) =] \quad G'_X(t) = \frac{3}{2}$ $G''_X(t) = \frac{27}{4}(4 - 3t)^{-\frac{5}{2}}; \text{ so } G''_X(1) = \frac{27}{4}$ $\text{Var}(X) = \frac{27}{4} + \frac{3}{2} - \left(\frac{3}{2}\right)^2; = \underline{6}$	M1; A1 M1; A1ft M1; A1 (6)	2.1; 1.1b 2.1 1.1b 1.1b 1.1b
(b)	Using Maclaurin: $\frac{G''_X(0)}{2!} = \left\{\frac{1}{2}\right\} \times \frac{27}{4} \times \frac{1}{32}$ $[P(X=2) =] \quad \frac{1}{2} \times \frac{27}{4} \times \frac{1}{32} \left[= \frac{27}{256} \right]$ $[P(X=0) + P(X=1) =] \quad \frac{3}{2} \times \frac{1}{8} + \frac{1}{2}$ $P(X \leq 2) = \frac{203}{256}$ Using Binomial: $[G_X(t) =] \quad \frac{1}{2} \left(1 - \frac{3}{4}t\right)^{-\frac{1}{2}}$ $\frac{1}{2} \left(\dots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(-\frac{3}{4}t\right)^2 + \dots \right) = \left[\dots + \frac{27}{256}t^2 + \dots \right]$ $\frac{1}{2} \left(1 + \frac{3}{8}t + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(-\frac{3}{4}t\right)^2 + \dots \right)$ $P(X \leq 2) = \frac{203}{256}$	M1 A1 M1 A1 (4)	2.1 1.1b 2.1 1.1b
(c)	$[G_W(t) = G_{X_1}(t) \times G_{X_2}(t) \times t =] \quad \frac{1}{\sqrt{4-3t}} \times \frac{1}{\sqrt{4-3t}} \times t; \quad G_Y(t) = \frac{t}{4-3t}$ $G_Y(t) = \frac{\frac{1}{4}}{1 - \frac{3}{4}t} \quad \text{or} \quad G_Y(t) = \frac{\frac{1}{4}t}{1 - \frac{3}{4}t}; \quad Y \sim \text{Geo}\left(\frac{1}{4}\right)$	M1; A1 M1 A1 (4)	3.1a/2.1 1.1b 2.1 3.2a/2.2a
(d)	$P(Y > 6) = \left(1 - \frac{1}{4}\right)^6; \quad = \frac{729}{4096} = 0.177978... \quad \text{awrt } \underline{0.178}$	M1 A1 (2)	3.4,1.1b
Notes		(16 marks)	
(a)	1 st M1 for attempt to differentiate leading to $k(4 - 3t)^{-1.5}$; 1 st A1 for $E(X) = 1.5$ or exact equivalent 2 nd M1 for attempting to differentiate again leading to $m(4 - 3t)^{-2.5}$ 2 nd A1ft for $\frac{27}{4}$ or correct ft from their k provided both Ms are scored 3 rd M1 for a correct method for finding $\text{Var}(X)$; can ft their $\frac{3}{2}$ and their $\frac{27}{4}$ 3 rd A1 for 6		
(b)	1 st M1 for Maclaurin to find $P(X = 2)$ condone $\frac{27}{4} \times \frac{1}{32} \left[= \frac{27}{128} \text{ or } 0.2109375 \right]$ or putting in the form $a(1 - 0.75t)^{-0.5}$ 1 st A1 for a correct unsimplified prob for $P(X = 2)$ (may be in binomial expansion) Allow 0.105(468...) 2 nd M1 for use of pgf to find $P(X = 1)$ and $P(X = 0)$ or attempt 1 st 3 terms of bin expansion 2 nd A1 for $\frac{203}{256}$ or exact equivalent.		
(c)	1 st M1 for using product of pgf or multiplication by t 1 st A1 for correct unsimplified form of pgf 2 nd dM1 (dep. on 1 st M1) for attempting to convert pgf to form given in the formula book or for stating Geometric alongside a correct PGF for Y (may be unsimplified) 2 nd A1 for correctly deducing the distribution of Y as geometric with $p = 0.25$ [may be seen in (d)] NB: Final A1 dependent on previous marks being awarded.		
(d)	M1 for attempting to use geometric formula or their pgf (using correct coefficients); A1 for awrt 0.178		

Qu	Scheme	Marks	AO		
7(a)	$[X = \text{no. of prizes Sam wins}] \quad X \sim B(20, 0.2)$ $P(X \geq 4) = 1 - P(X \leq 3) = 0.58855\dots$ awrt 0.589	M1 A1 (2)	3.3 1.1b		
(b)	$[Y = \text{no. of game when Tessa wins her 4}^{\text{th}} \text{ prize}] \quad Y \sim \text{negB}(4, 0.2)$ $P(Y = 20) = \binom{19}{3} 0.2^3 0.8^{16} \times 0.2, = 0.043639\dots$ awrt 0.0436	M1, A1 (2)	3.3 1.1b		
(c)	$S = \text{no of prizes Sam wins in } n \text{ games} \quad S \sim B(n, 0.2)$ $\text{Profit} = (2 + 4 + 6 + \dots + 2S) - n = \frac{S}{2} [4 + 2(S - 1)] - n$ $= S^2 + S - n$ $E(S) = 0.2n$ and $\text{Var}(S) = 0.2 \times 0.8n$ $E(S^2) = 0.16n + 0.04n^2$ or $\frac{1}{25}(n^2 + 4n)$	M1 M1 A1 M1 A1	3.1b 2.1 1.1b 3.1b 1.1b		
(*)	So expected profit for Sam is $\frac{1}{25}(n^2 + 4n) + \frac{1}{5}n - n = \frac{1}{25}(n^2 - 16n)$	A1cso (6)	3.2a		
(d)	<table><tr><td><u>Using profit expression:</u> Require $P(S^2 + S - n \geq 0)$ Solving quadratic, leading to $S = \dots$ $P(S \geq 4)$ where $S \sim B(15, 0.2)$ $= 0.35183\dots$</td><td><u>Using a listing method</u> Indicates that 4 wins is first non-loss $P(S \geq 4)$ $P(S \geq 4)$ where $S \sim B(15, 0.2)$ awrt 0.352</td></tr></table>	<u>Using profit expression:</u> Require $P(S^2 + S - n \geq 0)$ Solving quadratic, leading to $S = \dots$ $P(S \geq 4)$ where $S \sim B(15, 0.2)$ $= 0.35183\dots$	<u>Using a listing method</u> Indicates that 4 wins is first non-loss $P(S \geq 4)$ $P(S \geq 4)$ where $S \sim B(15, 0.2)$ awrt 0.352	M1 M1 M1 A1 (4)	3.1b 2.1 1.1b 1.1b
<u>Using profit expression:</u> Require $P(S^2 + S - n \geq 0)$ Solving quadratic, leading to $S = \dots$ $P(S \geq 4)$ where $S \sim B(15, 0.2)$ $= 0.35183\dots$	<u>Using a listing method</u> Indicates that 4 wins is first non-loss $P(S \geq 4)$ $P(S \geq 4)$ where $S \sim B(15, 0.2)$ awrt 0.352				
(e)	$T = \text{game on which Tessa wins her } r^{\text{th}} \text{ prize} \quad T \sim \text{negB}(r, 0.2)$ or $r(r + 1)$ $\text{Profit} = (2 + 4 + 6 + \dots + 2r) - T = r(r + 1) - T$ Tessa's expected profit $= r(r + 1) - E(T)$ $= r^2 + r - \frac{r}{0.2} = \underline{\underline{r^2 - 4r}}$	M1 A1 M1 A1 (4)	3.3 1.1b 3.4 1.1b		
Notes		(18 marks)			
(a)	M1 for selecting a suitable model ($B(20, 0.2)$) A1 for awrt 0.589				
(b)	M1 for stating correct negative binomial or ${}^{19}C_3 p^3 (1 - p)^{16} \times p$ for some p A1 for awrt 0.0436				
(c)	1 st M1 Correct start to problem - sight or use of $B(n, 0.2)$. May be implied by $E(S) = 0.2n$ 2 nd M1 use of AP formula with $a = d = 2$, or $2 \times \text{AP}$ formula with $a = d = 1$, or equivalent NB: must be working in another variable, AP formulae cannot be in terms of n 1 st A1 $S^2 + S - n$ or equivalent, must be in a form from which expectation can be found 3 rd M1 for use of $E(S) = 0.2n$ and $\text{Var}(S^2) = 0.16n$ (must be labelled or used as variance) 2 nd A1 for correct value for $E(S^2)$				
(*)	3 rd A1 for a correct solution only, pulling together everything to get given answer				
(d)	1 st M1 for using a suitable prob statement or using a listing method to indicate first non-loss S 2 nd M1 for solving the inequality or using a listing method to reach $P(S \geq 4)$ NB: award first two M marks for $P(S \geq 4)$ provided it does not come from incorrect working 3 rd M1 for attempting $P(S \geq 4)$ with $B(15, 0.2)$ A1 for awrt 0.352 NB: solutions stemming from finding values of n gain no marks				
(e)	1 st M1 for sight or use of $\text{negB}(r, 0.2)$ or sight of $r(r + 1)$ 1 st A1 for $r(r + 1) - T$, where T is defined 2 nd M1 for use of $E(T) = 5r$ in an expression of “revenue” $- 5r$ 2 nd A1 for $r^2 - 4r$				

