Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 03 Further Pure Mathematics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	3.35 cao	B1	1.1b
		(1)	
(b)	Step length = 0.25	B1	1.1b
	$\frac{1}{3}\times"0.25"\left[2.73+16.31+2\left("3.35"\right)+4\left(2.81+5.22\right)\right]$	M1	1.1b
	$=4.8$	A1	1.1b
		(3)	
(c)	$\int_{1.5}^{2.5}\mathrm{e}^{\cot^2 x}\,\mathrm{d}x=\int_{1.5}^{2.5}\mathrm{e}^{\operatorname{cosec}^2 x-1}\,\mathrm{d}x=\frac{1}{\mathrm{e}}\int_{1.5}^{2.5}\mathrm{e}^{\operatorname{cosec}^2 x}\,\mathrm{d}x=\frac{1}{\mathrm{e}}\times"4.8"$	M1	3.1a
	= awrt 1.8	A1ft	1.1b
		(2)	
			(6 marks)

Notes

(a)
B1: Correct value 3.35 only, this may be seen in the table or in the script
(b)
B1: Correct step length, may be seen in the application of Simpson's rule formula. Could be seen as 1.75 – 1.5 or $\frac{2.5-1.5}{4}$
M1: Correct application of Simpson's rule using their step length. If no step length stated and uses $\frac{1}{3}\left[\text{ends}+2\left(\text{evens}\right)+4\left(\text{odd}\right)\right]$ this is M0
A1: Awrt 4.8 must be using 3.35 from (a)
Note calculator answer is 4.67 is no marks
Note Answer only with no working is no marks,
Note: If they write $\frac{1}{3}\times h\left[\text{ends}+2\left(\text{evens}\right)+4\left(\text{odd}\right)\right]$ which leads to awrt 4.8 this gets B1M1A1
If their answer is incorrect this would be B0 (if no h stated) M0A0
(c)
M1: Uses a correct identity to express $\int \mathrm{e}^{\cot^2 x}\,\mathrm{d}x$ in terms of $\int \mathrm{e}^{\operatorname{cosec}^2 x}\mathrm{d}x$ and then uses correct index work to allow $\int_{1.5}^{2.5}\mathrm{e}^{\cot^2 x}\,\mathrm{d}x$ to be evaluated using part (b) $\frac{1}{\mathrm{e}}\times"4.8"$
A1ft: Awrt 1.8 or follow through $\frac{1}{\mathrm{e}}\times\text{their}\,(\text{b})$ which must be evaluated and correct for their (b) rounded to 1 d.p.
If uses Simpson's rule again this is M0 A0

Question	Scheme	Marks	AOs
2(a)	$\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+Ax\frac{dy}{dx}+By=0 \quad A,B\neq 0$ $\frac{d^3y}{dx^3}=-3\frac{d^2y}{dx^2}+Ax\frac{dy}{dx}+By \quad A,B\neq 0$	M1	2.1
	$\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-2y=0$ or $\frac{d^3y}{dx^3}=-3\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+2y$	A1	1.1b
	$\frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}-2x\frac{d^2y}{dx^2}-2\frac{dy}{dx}-2\frac{dy}{dx}=0\Rightarrow\frac{d^4y}{dx^4}=...$ Or $\frac{d^4y}{dx^4}=-3\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2\frac{dy}{dx}$ or $\frac{d^4y}{dx^4}=-3\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}+4\frac{dy}{dx}$	M1	1.1b
	$\frac{d^4y}{dx^4}=4\frac{dy}{dx}+2x\frac{d^2y}{dx^2}-3\frac{d^3y}{dx^3}$ cso	A1	2.2a
		(4)	
(b)	$x=2, y=1,\Rightarrow\frac{dy}{dx}=1\Rightarrow\left(\frac{d^2y}{dx^2}\right)_2=4+4-3=5$	B1	2.2a
	$\left(\frac{d^3y}{dx^3}\right)_2=2(2)(1)+2(1)-3(5)=-9$ $\left(\frac{d^4y}{dx^4}\right)_2=4(1)+2(2)(5)-3(-9)=51$	M1	1.1b
	$y=y(2)+(x-2)\left(\frac{dy}{dx}\right)_2+\frac{(x-2)^2}{2!}\left(\frac{d^2y}{dx^2}\right)_2+$ $\frac{(x-2)^3}{3!}\left(\frac{d^3y}{dx^3}\right)_2+\frac{(x-2)^4}{4!}\left(\frac{d^4y}{dx^4}\right)_2+...$	M1	2.5
	$y=1+(x-2)+\frac{5}{2}(x-2)^2-\frac{3}{2}(x-2)^3+\frac{17}{8}(x-2)^4+...$ Or $y=x-1+\frac{5}{2}(x-2)^2-\frac{3}{2}(x-2)^3+\frac{17}{8}(x-2)^4+...$	A1	1.1b
		(4)	
			(8 marks)

Notes
(a) M1: Attempts to differentiate including an attempt at the product rule to achieve the correct form in any order A1: Correct differentiation M1: Continues to differentiate using the product rule again to reach the 4th derivative to achieve the correct form $\frac{d^4y}{dx^4} = A\frac{d^3y}{dx^3} + Bx\frac{d^2y}{dx^2} + C\frac{dy}{dx} + D\frac{dy}{dx}$ in any order A1: Completes the process to obtain the correct expression (a = 4, b = 2, c = −3) with no errors seen and correct notation throughout and = 0 where applicable cso (b) B1: Deduces the correct value for $y''(2)$ M1: Finds the values of the derivatives at x = 2. There must be evidence of the correct values of $x = 2, y = 1, \frac{dy}{dx} = 1$ being used at least once M1: Uses the correct Taylor series expansion with their derivatives. Must be using $y = 1, \frac{dy}{dx} = 1$ in their series expansion. If no method is shown they must have all terms correct for their derivatives A1: Correct expansion must include y = or f(x) = somewhere

Question	Scheme	Marks	AOs
3	$x=5$ or $x\neq 5$	B1	2.2a
	$\{x>5\}$ $x^2-4=(8+4x)(x-5)\Rightarrow 3x^2-12x-36=0\Rightarrow x=...$ **or** $\{x<5\}$ $x^2-4=(8+4x)(5-x)\Rightarrow 5x^2-12x-44=0\Rightarrow x=...$	M1	1.1b
	$\{x>5\}$ $x^2-4=(8+4x)(x-5)\Rightarrow 3x^2-12x-36=0\Rightarrow x=...$ **And** $\{x<5\}$ $x^2-4=(8+4x)(5-x)\Rightarrow 5x^2-12x-44=0\Rightarrow x=...$	M1	3.1a
	Any 2 of: $x=-2,\ \frac{22}{5},\ 6$	A1	1.1b
	$x<-2,\ \frac{22}{5}<x<5,\ \ 5<x<6$ o.e.	A1 A1	1.1b 3.2a
		(6)	
	Alternative 1 by squaring:		
	$x=5$ or $x\neq 5$	B1	2.2a
	$\frac{x^2-4}{\lvert x-5\rvert}=4x+8\Rightarrow\frac{x^4-8x^2+16}{x^2-10x+25}=16x^2+64x+64$ $x^4-8x^2+16=(16x^2+64x+64)(x^2-10x+25)$	M1	3.1a
	$15x^4-96x^3-168x^2+960x+1584=0\Rightarrow x=...$	M1	1.1b
	Any 2 of: $x=-2,\ \frac{22}{5},\ 6$	A1	1.1b
	$x<-2,\ \frac{22}{5}<x<5,\ \ 5<x<6$ o.e.	A1 A1	1.1b 3.2a
	Alternative 2		
	$x=5$	B1	2.2a
	$(x^2-4)(x-5)=(8+4x)(x-5)^2\Rightarrow(x-5)(3x^2-12x-36)=0\Rightarrow x=..$ **or** $\frac{(x^2-4)-(4x+8)(x-5)}{x-5}=\frac{3x^2-12x-36}{x-5}=0\Rightarrow x=...$ **OR** $(x^2-4)(5-x)=(8+4x)(5-x)^2\Rightarrow(5-x)(5x^2-12x-44)\Rightarrow x=..$ **or** $\frac{(x^2-4)-(4x+8)(5-x)}{5-x}=\frac{5x^2-12x-44}{5-x}=0\Rightarrow x=...$	M1	3.1a
	$(x^2-4)(x-5)=(8+4x)(x-5)^2\Rightarrow(x-5)(3x^2-12x-36)=0\Rightarrow x=..$ **or**	M1	1.1b

	$\dfrac{\left(x^2-4\right)-\left(4x+8\right)\left(x-5\right)}{x-5}=\dfrac{3x^2-12x-36}{x-5}=0\Rightarrow x=...$ **AND** $\left(x^2-4\right)\left(5-x\right)=\left(8+4x\right)\left(5-x\right)^2\Rightarrow\left(5-x\right)\left(5x^2-12x-44\right)=0\Rightarrow$ **or** $\dfrac{\left(x^2-4\right)-\left(4x+8\right)\left(5-x\right)}{5-x}=\dfrac{5x^2-12x-44}{5-x}=0\Rightarrow x=...$		
	Any 2 of: $x=-2,\ \dfrac{22}{5},\ 6$	A1	1.1b
	$x<-2,\ \dfrac{22}{5}<x<5,\ \ 5<x<6$ o.e.	A1 A1	1.1b 3.2a
			(6 marks)

Notes
B1: Deduces that $x = 5$ is a critical value. May be seen stated or as part of their final answer $x \neq 5$ M1: Attempts to find the critical values by solving a quadratic equation for $x > 5$ or for $x < 5$ M1: A complete method to find all the critical values by solving two quadratic equations. E.g. considers both $x > 5$ and $x < 5$ A1: Obtains any 2 correct critical values as long as one of the previous method marks has been scored A1: Obtains any 2 correct regions. Note that $\frac{22}{5} < x < 6, \quad x \neq 5$ counts as 2 correct regions. A1: Obtains the fully correct regions with no extras. Note that $x < -2, \ \frac{22}{5} < x < 6, \quad x \neq 5$ is also fully correct. If uses set notation it must be correct, or, $\cup$ not $\cap$ isw **Alternative 1** B1: Deduces that $x = 5$ is a critical value. May be seen stated or as part of their final answer $x \neq 5$ M1: Attempts to find square both sides and multiples up or vice versa in order to obtain a quartic equation that will give the critical values M1: A complete method to find all four of the critical values by solving a quartic equation may use a calculator A1: Obtains any 2 correct critical values, as long as they have scored at least one of the previous method marks. May be seen as an inequality. A1: Obtains any 2 correct regions. Note that $\frac{22}{5} < x < 6, \quad x \neq 5$ counts as 2 correct regions. A1: Obtains the fully correct regions with no extras. Note that $x < -2, \ \frac{22}{5} < x < 6, \quad x \neq 5$ is also fully correct. If uses set notation it must be correct, or, $\cup$ not $\cap$ isw **Alternative 2** B1: Deduces that $x = 5$ is a critical value. May be seen stated or as part of their final answer $x \neq 5$ M1: Either attempts to multiply both sides by $(x-5)^2$ or $(5-x)^2$ and solves to find the critical values. Or collects onto one side with a denominator of $x-5$ or $5-x$, uses a common denominator to combine and then finds the critical values may use a calculator M1: Attempts to multiply both sides by $(x-5)^2$ and $(5-x)^2$ and solves to find the critical values. may use a calculator Or collects onto one side with a denominator of $x-5$ and $5-x$, uses a common denominator to combine and then finds the critical values A1: Obtains any 2 correct critical values, as long as they have scored one of the previous method marks and using a correct equation A1: Obtains any 2 correct regions. Note that $\frac{22}{5} < x < 6, \quad x \neq 5$ counts as 2 correct regions. A1: Obtains the fully correct regions with no extras. Note that $x < -2, \ \frac{22}{5} < x < 6, \quad x \neq 5$ is also fully correct

Question	Scheme	Marks	AOs
4(a)	$y=\left(15+\mathrm{e}^{2x}\right)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(15+\mathrm{e}^{2x}\right)^{-\frac{1}{2}}\times 2\mathrm{e}^{2x}$ or $y^2=15+\mathrm{e}^{2x} \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}$	B1	1.1b
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\left(15+\mathrm{e}^{2x}\right)^{\frac{1}{2}}\times A\mathrm{e}^{2x}+B\left(15+\mathrm{e}^{2x}\right)^{-\frac{1}{2}}\times\mathrm{e}^{2x}\times\mathrm{e}^{2x}}{15+\mathrm{e}^{2x}}$ $\left\{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\left(15+\mathrm{e}^{2x}\right)^{\frac{1}{2}}\times 2\mathrm{e}^{2x}-\frac{1}{2}\left(15+\mathrm{e}^{2x}\right)^{-\frac{1}{2}}\times 2\mathrm{e}^{2x}\times\mathrm{e}^{2x}}{15+\mathrm{e}^{2x}}\right\}$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=A\mathrm{e}^{2x}\left(15+\mathrm{e}^{2x}\right)^{-\frac{1}{2}}+B\mathrm{e}^{2x}\times\mathrm{e}^{2x}\left(15+\mathrm{e}^{2x}\right)^{-\frac{3}{2}}$ $\left\{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\mathrm{e}^{2x}\left(15+\mathrm{e}^{2x}\right)^{-\frac{1}{2}}-\frac{1}{2}\times 2\mathrm{e}^{2x}\times\mathrm{e}^{2x}\left(15+\mathrm{e}^{2x}\right)^{-\frac{3}{2}}\right\}$ Or $A\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+By\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=C\mathrm{e}^{2x}$ $\left\{2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\mathrm{e}^{2x}\right\}$	M1	1.1b
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\left(15+\mathrm{e}^{2x}\right)\times 2\mathrm{e}^{2x}-\mathrm{e}^{4x}}{\left(15+\mathrm{e}^{2x}\right)^{\frac{3}{2}}}=\frac{\mathrm{e}^{2x}\left(\mathrm{e}^{2x}+30\right)}{\left(15+\mathrm{e}^{2x}\right)^{\frac{3}{2}}}$ *	A1*	2.1
		(3)	
(b)	$x=0\Rightarrow y=4,\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4},\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{31}{64}$ leading to $\Rightarrow y=4+\frac{x}{4}+\frac{31}{64}\frac{x^2}{2!}+\ldots=4+\frac{x}{4}+\frac{31}{128}x^2$	M1 A1	1.1b 1.1b
		(2)	
(c)	$\sin 3x=3x-\frac{9}{2}x^3+\ldots$	B1	2.2a
		(1)	
(d)	$\left\{\lim_{x\to 0}\right\}\frac{\sqrt{\left(15+\mathrm{e}^{2x}\right)}-4}{\sin 3x}=\left\{\lim_{x\to 0}\right\}\frac{4+\frac{x}{4}+\frac{31}{128}x^2-4}{3x-\frac{9}{2}x^3}=\left\{\lim_{x\to 0}\right\}\frac{\frac{1}{4}+\frac{31}{128}x}{3-\frac{9}{2}x^2}$	M1	2.1

	$$\lim_{x\to 0}\frac{\frac{1}{4}+\frac{31}{128}x}{3-\frac{9}{2}x^2}\left\{=\frac{1}{4}\div 3\right\}=\frac{1}{12}$$ * cso	A1*	1.1b
		(2)	
(8 marks)			

Notes

(a)
B1: Correct first derivative in any form, may square first and use implicit differentiation
M1: Differentiates again applying the quotient or product rule correctly to achieve the correct form or uses implicit differentiation to achieve a correct form
A1*: Correct proof with no errors. Must show sufficient working.
If uses the product rule they must show a common denominator

If uses the quotient rule there must be evidence to show the multiplication of $\left(15+e^{2x}\right)^{\frac{1}{2}}$ top and bottom

If uses implicit differentiation we need to see the substitution for $\frac{dy}{dx}$ and y and rearranging.

(b)
M1: A full method to obtain the required expansion. I.e. finds the values up to the second derivative and uses a correct formula
A1: Correct simplified expansion with $y =$ or $f(x) =$ seen somewhere in their solution

(c)
B1: Deduces the correct simplified expansion, ignore any extra terms, whether correct or not

(d)
M1: A rigorous argument using their answers to (b) and (c), showing the cancelling constant term and the division by x in the numerator and denominator to establish the limiting behaviour.
A1*: Correct proof with no errors including correct limiting notation seen in the final stage cso

Note: Using L'Hospital's Rule with the functions $\sqrt{\left(15+e^{2x}\right)}-4$ and $\sin 3x$ is M0A0

Using L'Hospital's Rule with their expansions $\dfrac{4+\frac{x}{4}+\frac{31}{128}x^2-4}{3x-\frac{9}{2}x^3}$ leading to $\dfrac{\frac{1}{4}+\frac{31}{64}x}{3-\frac{27}{2}x^2}$ could score M1A1

Question	Scheme	Marks	AOs
5(a)	$1+\text{cosec}\,x=1+\frac{1}{\sin x}=1+\frac{1}{\frac{2t}{1+t^2}}$ or $1+\frac{1+t^2}{2t}\left(=\frac{t^2+2t+1}{2t}\right)$ or $1+\text{cosec}\,x=\frac{\sin x+1}{\sin x}=\frac{\frac{2t}{1+t^2}+1}{\frac{2t}{1+t^2}}$	B1	1.1a
	$\int\frac{1}{1+\text{cosec}\,x}\,\text{d}x=\int\frac{2t}{t^2+2t+1}\times\frac{2}{1+t^2}\,\text{d}t$	M1	2.1
	$\int\frac{1}{1+\text{cosec}\,x}\,\text{d}x=\int\frac{4t}{(1+t^2)(1+t)^2}\,\text{d}t$ *	A1*	1.1b
		(3)	
(b)	$\frac{4t}{(1+t^2)(1+t)^2}\equiv\frac{At+B}{1+t^2}+\frac{C}{1+t}+\frac{D}{(1+t)^2}$ or $\frac{4t}{(1+t^2)(1+t)^2}\equiv\frac{At+B}{1+t^2}+\frac{Ct+D}{(1+t)^2}$ or $\frac{4t}{(1+t^2)(1+t)^2}\equiv\frac{At+B}{1+t^2}+\frac{C}{1+t}+\frac{Dt+E}{(1+t)^2}$ Leading to finding the value for at least one constant	M1	3.1a
	$\frac{4t}{(1+t^2)(1+t)^2}\equiv\frac{2}{1+t^2}-\frac{2}{(1+t)^2}$	A1	1.1b
	$\int\frac{2}{1+t^2}-\frac{2}{(1+t)^2}\,\text{d}t=2\arctan t+\frac{2}{1+t}$	A1	2.2a
	$=2\arctan\left(\tan\frac{x}{2}\right)+\frac{2}{1+\tan\frac{x}{2}}+k=x+\frac{2}{1+\tan\frac{x}{2}}+k$ cso	M1 A1	1.1b 2.1
		(5)	
			(8 marks)

Notes
(a) B1: Selects the correct formulae to express $1+\operatorname{cosec} x$ in terms of *t*. May be seen within the integral M1: Makes a complete substitution to obtain an integral in terms of *t* only, including $\mathrm{d}x = \frac{2}{1+t^2}\mathrm{d}t$ A1*: Correct proof with no errors or omissions, must see the factorisation if the denominator is expanded first. Long division or equivalent with t^2+1 or $(t+1)^2$ or $t+1$ (b) M1: Realises the need to express the integrand in terms of partial fractions and attempts the correct form, condone the use of *x* instead of *t* on their numerator with *t*'s on the denominator. Must attempt to find at least one constant term A1: Correct partial fractions. A1: Correct integration, must have scored the previous marks, no need for the constant of integration Note an incorrect form such as $\frac{A}{1+t^2}+\frac{B}{1+t}+\frac{C}{(1+t)^2}$ can lead to a correct answer but is M0A0A0 M1: Reverses the substitution to express in terms of *x* A1: Correct answer with no errors including "+ *k*" must have scored all the previous marks in part (b) **Note Verification method using $\frac{A}{1+t^2}+\frac{B}{(1+t)^2}$ to find the values for *A* and *B*, please send to review**

Question	Scheme	Marks	AOs
6(i)	$\left\{\lim_{x\to 0}\right\}\dfrac{1-\cos 7x}{x\sin 9x}=\left\{\lim_{x\to 0}\right\}\dfrac{A\sin 7x}{\sin 9x+Bx\cos 9x}$	M1	1.1b
	$\left\{\lim_{x\to 0}\right\}\dfrac{1-\cos 7x}{x\sin 9x}=\left\{\lim_{x\to 0}\right\}\dfrac{7\sin 7x}{\sin 9x+9x\cos 9x}$	A1	1.1b
	$\left\{\lim_{x\to 0}\dfrac{7\sin 7x}{\sin 9x+9x\cos 9x}=\right\}\left\{\lim_{x\to 0}\right\}\dfrac{49\cos 7x}{9\cos 9x+9\cos 9x-81x\sin 9x}$	M1	3.1a
	$\lim_{x\to 0}\dfrac{49\cos 7x}{9\cos 9x+9\cos 9x-81x\sin 9x}=\dfrac{49}{9+9}=\dfrac{49}{18}$ *	A1*	1.1b
		(4)	
(ii)(a)	$u=x^2\Rightarrow\dfrac{\mathrm{d}u}{\mathrm{d}x}=2x,\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}=2,\left\{\dfrac{\mathrm{d}^3u}{\mathrm{d}x^3}=0\right\}$	M1	1.1b
	$v=\mathrm{e}^{3x}\Rightarrow\dfrac{\mathrm{d}v}{\mathrm{d}x}=3\mathrm{e}^{3x},\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2}=9\mathrm{e}^{3x},\dfrac{\mathrm{d}^3v}{\mathrm{d}x^3}=27\mathrm{e}^{3x},\ldots,\left\{\dfrac{\mathrm{d}^kv}{\mathrm{d}x^k}=3^k\mathrm{e}^{3x}\right\}$	M1	2.1
	$\dfrac{\mathrm{d}^ky}{\mathrm{d}x^k}=3^k\mathrm{e}^{3x}x^2+k\times 2x\times 3^{k-1}\mathrm{e}^{3x}+\dfrac{k(k-1)}{2}\times 2\times 3^{k-2}\mathrm{e}^{3x}$	M1 A1	3.1a 1.1b
	$=3^{k-2}\mathrm{e}^{3x}\left(9x^2+6kx+k(k-1)\right)$	A1	2.2a
		(5)	
(b)	$\dfrac{\mathrm{d}^9y}{\mathrm{d}x^9}=3^{9-2}\mathrm{e}^{3x}\left(9x^2+6\times 9x+9\times 8\right)=0$ $\Rightarrow 9x^2+54x+72=0\Rightarrow x=\ldots$	M1	1.1b
	$x=-4,\,-2$ cso	A1	2.2a
		(2)	
			(11 marks)

Notes
(i) M1: Differentiates numerator and denominator to the correct form, **may be done separately** A1: Correct derivatives M1: Recognises the requirement to differentiate numerator and denominator again and achieves the correct form of the derivatives may be seen **separately** or as a fraction $\frac{A\cos 7x}{B\cos 9x + C\cos 9x + Dx\sin 9x}$ A1*: Completes the proof, including correct limit notation seen when $x = 0$ is substituted with as a minimum $\frac{49}{9+9} = \frac{49}{18}$ or $\lim_{x\to 0}\frac{49\cos 7x}{18\cos 9x - 81x\sin 9x} = \frac{49}{18}$ (ii)(a) M1: Differentiates $u = x^2$ twice, may be seen as part of the expression for the k^{th} derivative M1: Uses $v = \mathrm{e}^{3x}$ to establish the form of the derivatives $\frac{\mathrm{d}^k v}{\mathrm{d}x^k} = 3^k \mathrm{e}^{3x}$. Look for multiples of e^{3x} increasing by a factor of 3 each time, need at least three. This may be seen as part of the expression for the k^{th} derivative M1: A correct strategy for the k^{th} derivative. This requires the correct derivative of x^2 combined with the correct derivative of e^{3x} in terms of k together with the correct binomial coefficient. Condone using $k(k-1)$ for the third term A1: A correct unsimplified expression A1: Correct expression in the required form with correct values of A, B and C . Condone missing trailing bracket (NB $A = 9$, $B = 6$, $C = 1$) Repeated differentiation to find at least the third derivative $y = x^2\mathrm{e}^{3x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2x\mathrm{e}^{3x} + 3x^2\mathrm{e}^{3x} \Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\mathrm{e}^{3x} + 6x\mathrm{e}^{3x} + 6x\mathrm{e}^{3x} + 9x^2\mathrm{e}^{3x}$ $\Rightarrow \frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = 6\mathrm{e}^{3x} + 12\mathrm{e}^{3x} + 36x\mathrm{e}^{3x} + 18x\mathrm{e}^{3x} + 27x^2\mathrm{e}^{3x}$ they can score the first M1M1 for differentiating u twice and v three times. Then will score M0A0A0. (b) M1: Substitutes $k = 9$ into their result from part (a) and solves the resulting 3TQ which leads to real roots A1: These values only following full marks in (a)

Question	Scheme	Marks	AOs
7(a)	Examples: $x=\frac{1}{P^2}\Rightarrow\frac{dx}{dt}=-\frac{2}{P^3}\frac{dP}{dt}$ or $\frac{dP}{dt}=\frac{dP}{dx}\times\frac{dx}{dt}=-\frac{P^3}{2}\frac{dx}{dt}$ or $P=x^{-\frac{1}{2}}\Rightarrow\frac{dP}{dt}=-\frac{1}{2}x^{-\frac{3}{2}}\frac{dx}{dt}$ or $\frac{dP}{dx}=-\frac{1}{2}x^{-\frac{3}{2}}\Rightarrow\frac{dx}{dt}=\frac{dx}{dP}\times\frac{dP}{dt}=-2x^{\frac{3}{2}}\times\left(4tP^3-P\right)$ or $x=\frac{1}{P^2}\Rightarrow xP^2=1\Rightarrow P^2\frac{dx}{dt}+2Px\frac{dP}{dt}=0$ or $x=\frac{1}{P^2}\Rightarrow P^2=\frac{1}{x}\Rightarrow 2P\frac{dP}{dt}=-\frac{1}{x^2}\frac{dx}{dt}$	M1	3.1a
	$\frac{1}{P}\times-\frac{P^3}{2}\frac{dx}{dt}=\frac{4t}{x}-1\Rightarrow-\frac{1}{2x}\frac{dx}{dt}=\frac{4t}{x}-1$ or $x^{\frac{1}{2}}\times-\frac{1}{2}x^{-\frac{3}{2}}\frac{dx}{dt}=\frac{4t}{x}-1\Rightarrow-\frac{1}{2x}\frac{dx}{dt}=\frac{4t}{x}-1$ or $\frac{dx}{dt}=-2x^{\frac{3}{2}}\times\left(4tP^3-P\right)=-2x^{\frac{3}{2}}\times\left(\frac{4t}{x^{\frac{3}{2}}}-\frac{1}{x^{\frac{1}{2}}}\right)$ or $\frac{1}{P}\times-\frac{P}{2x}\frac{dx}{dt}=\frac{4t}{x}-1\Rightarrow-\frac{1}{2x}\frac{dx}{dt}=\frac{4t}{x}-1$ or $\frac{1}{P}\times-\frac{1}{2Px^2}\frac{dx}{dt}=\frac{4t}{x}-1\Rightarrow-\frac{1}{2x}\frac{dx}{dt}=\frac{4t}{x}-1$	M1	1.1b
	$\frac{dx}{dt}-2x=-8t$ * cso	A1*	2.1
		(3)	
(b)	$I=e^{\int-2dt}=e^{-2t}$	B1	2.2a
	$xe^{-2t}=\int-8te^{-2t}\{dt\}$	M1	1.1b
	$=4te^{-2t}-\int4e^{-2t}\{dt\}$	M1	3.1a
	$=4te^{-2t}+2e^{-2t}+k$	A1	1.1b
	$xe^{-2t}=4te^{-2t}+2e^{-2t}+k\Rightarrow\frac{1}{0.5^2}=2+k\Rightarrow k=...\{2\}$ or	M1	3.4

	$xe^{-2t}=4te^{-2t}+2e^{-2t}+k\Rightarrow x=4t+2+ke^{2t}\Rightarrow\dfrac{1}{P^2}=4t+2+ke^{2t}$ $P=0.5,t=0\Rightarrow\dfrac{1}{0.5^2}=2+k\Rightarrow k=...\{2\}$		
	$\dfrac{1}{P^2}=4t+2+2e^{2t}\Rightarrow P^2=\dfrac{1}{4t+2+2e^{2t}}$ cso	A1	2.1
		(6)	
(c)	$t=3\Rightarrow P^2=\dfrac{1}{14+2e^6}\Rightarrow P=0.03490326...$	M1	3.4
	This value is close to 0.034 so the model is reliable. • Between 0.0323 and 0.0357 good model • Between 0.0306 and 0.0323 or 0.0357 and 0.0374 either a good or bad model with reasoning • Less than 0.0306 or more than 0.0374 bad model	A1ft	3.5a
		(2)	
			(11 marks)

Notes

(a)

M1: Identifies and applies a correct strategy for the differentiation. This may be seen when $\frac{dP}{dt}$ is substituted into the equation.

M1: Substitutes into the given differential equation and proceeds to an equation in x and t only.

A1*: Correct proof with sufficient working shown and no errors, cso

(b)

B1: Deduces the correct integrating factor

M1: Applies their integrating factor I to obtain $Ix=\int \pm 8It\{dt\}$ condone missing dt

M1: Recognises that integration by parts is required and applies this correctly to reach the form $=Ate^{-2t}-\int Be^{-2t}\{dt\}$

A1: Correct integration including an arbitrary constant

M1: Uses the conditions given in the model to find the constant of integration. This may be seen before rearranging to get $P^2=...$, condone a slip

A1: Correct equation with no errors seen, cso, missing dt during working would lose this mark

Note that the first 4 marks in (b) can also be obtained as follows:

B1: AE: $m-2=0\Rightarrow m=2\Rightarrow x=Ae^{2t}$ (Correct CF)

M1: PI: $x=at+b\Rightarrow\frac{dx}{dt}=a$ (Selects the correct PI form and differentiates)

M1: $a-2at-2b=-8t\Rightarrow a=...(4),b=...(2)$ (Substitutes and compares coefficients to find a and b)

A1: $x=4t+2+Ae^{2t}$ (Correct expression)

(c)

M1: Uses the model with t = 3 to find P, where $P^2>0$ or compares their value of P^2 with $0.001156 = 0.034^2$

A1ft: Compares their value of P with 0.034 and makes a suitable conclusion, making any comparison with P^2 is A0

Question	Scheme	Marks	AOs
8(a)	$\frac{x^2}{64}+\frac{y^2}{36}=1,\ y=mx+c$ $\Rightarrow \frac{x^2}{64}+\frac{(mx+c)^2}{36}=1 \Rightarrow \frac{x^2}{64}+\frac{m^2x^2+2cmx+c^2}{36}=1$	M1	1.1b
	$\Rightarrow 36x^2+64m^2x^2+128cmx+64c^2=2304$ $\Rightarrow (16m^2+9)x^2+32cmx+16c^2-576=0$	A1	2.1
		(2)	
(b)	$b^2-4ac=0 \Rightarrow (32cm)^2-4(16m^2+9)(16c^2-576)=0$ $(\Rightarrow 36864m^2-576c^2+20736=0, 64m^2-c^2+36=0)$	M1	3.1a
	$x=10, y=20 \Rightarrow 20=10m+c$ $\Rightarrow 64m^2-(20-10m)^2+36=0$ or $c^2-64\left(\frac{20-c}{10}\right)^2-36=0$	M1	3.1a
	$36m^2-400m+364=0$ $324m^2-3600m+3276=0$ o.e. or $\Rightarrow 36c^2+2560c-29200=0$ o.e.	A1	1.1b
	$36m^2-400m+364=0 \Rightarrow m=1, \frac{91}{9} \Rightarrow c=...$ or $\Rightarrow 36c^2+2560c-29200=0 \Rightarrow c=10, -\frac{730}{9} \Rightarrow m=...$	dM1	1.1b
	$y=x+10, \quad y=\frac{91}{9}x-\frac{730}{9}$ o.e.	A1	1.1b
		(5)	
			(7 marks)

Notes
(a) M1: Attempts to solve simultaneously by substituting $y = mx + c$ into $\frac{x^2}{64} + \frac{y^2}{36} = 1$ and attempts to multiply out $(mx + c)^2$ to obtain an equation in *x*, *m* and *c* A1: Proceeds to obtain the equation in the required form with the correct value of *k* (b) M1: Realises that for *l* to be a tangent, the discriminant must be zero and applies this to their equation correctly, may contain *k* M1: Uses the other condition that *l* passes through (10, 20) to eliminate *c* or *m* to obtain an equation in *c* or *m* only following an attempt at setting their discriminant =0 A1: Correct 3TQ in *c* or *m* dM1: Dependent on previous M. For a complete method to obtain at least one set of values for *c* and *m*. I.e. Finds values for *m* or *c* from a 3TQ (no need to check the roots) and attempts to find the value of the other constant A1: Both correct equations in any form

Question	Scheme	Marks	AOs
9(a)	e.g. $\overrightarrow{EF}=(7-10)\mathbf{i}+(-2+1)\mathbf{j}+(7+6)\mathbf{k}$ $\overrightarrow{EH}=(13-10)\mathbf{i}+(3+1)\mathbf{j}+(-4+6)\mathbf{k}$	M1	3.1a
	For example, $\left\|\overrightarrow{EF}\times\overrightarrow{EH}\right\|=\left\|\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -3 & -1 & 13\\ 3 & 4 & 2\end{vmatrix}\right\|=\left\|-54\mathbf{i}+45\mathbf{j}-9\mathbf{k}\right\|=\sqrt{54^2+45^2+9^2}$	M1	2.1
	$9\sqrt{62}$ cso	A1	1.1b
		(3)	
(b)	$(\mathbf{r}-(10\mathbf{i}-\mathbf{j}-6\mathbf{k}))\times(6\mathbf{i}-5\mathbf{j}+\mathbf{k})=\mathbf{0}$ or $\left(\mathbf{r}-\begin{pmatrix}10\\-1\\-6\end{pmatrix}\right)\times\begin{pmatrix}6\\-5\\1\end{pmatrix}=\mathbf{0}$ o.e. Or $(\mathbf{r}-(10\mathbf{i}-\mathbf{j}-6\mathbf{k}))\times(54\mathbf{i}-45\mathbf{j}+9\mathbf{k})=\mathbf{0}$ or $\left(\mathbf{r}-\begin{pmatrix}10\\-1\\-6\end{pmatrix}\right)\times\begin{pmatrix}54\\-45\\9\end{pmatrix}=\mathbf{0}$ o.e.	M1 A1ft	1.1b 2.2a
		(2)	
(c)	$6x-5y+z=d\rightarrow d=6\times25-5\times(-4)+13$ $54x-45y+9z=d\rightarrow d=54\times25-45\times(-4)+9\times13$	M1	3.1a
	$6x-5y+z=183$ o.e. $54x-45y+9z=1647$	A1	1.1b
	$6(10+6\lambda)-5(-1-5\lambda)+\lambda-6=183\Rightarrow\lambda=...(2)$ $54(10+6\lambda)-45(-1-5\lambda)+9(\lambda-6)=1647\Rightarrow\lambda=...(18)$ Or $6x-5y+z=d\rightarrow d=6\times10-5\times(-1)-6\Rightarrow 6x-5y+z=59$ $\frac{183}{\sqrt{6^2+5^2+1^2}}-\frac{59}{\sqrt{6^2+5^2+1^2}}=...$	M1	3.1a
	$\lambda=2\Rightarrow V=9\sqrt{62}\times\left\|2(6\mathbf{i}-5\mathbf{j}+\mathbf{k})\right\|=9\sqrt{62}\times2\sqrt{6^2+5^2+1}=...$ or $\lambda=2\Rightarrow V=\left\|(-54\mathbf{i}+45\mathbf{j}-9\mathbf{k})\square 2(6\mathbf{i}-5\mathbf{j}+\mathbf{k})\right\|=...$	dM1	3.1a
	$=1116$	A1	1.1b
		(5)	
	Alternative 1 $6x-5y+z=d\rightarrow d=6\times10-5\times(-1)-6$	M1	3.1a

	$54x-45y+9z=d \rightarrow d=54\times10-45\times(-1)+9(-6)$		
	$6x-5y+z=59$ o.e. $54x-45y+9z=-531$	A1	1.1b
	$\dfrac{\left\|(25\times6)+(-4\times-5)+(13\times1)\pm59\right\|}{\sqrt{6^2+5^2+1^2}}=\ldots\left\{\dfrac{124}{\sqrt{62}}\right\}$ $\dfrac{\left\|(25\times54)+(-4\times-45)+(13\times9)\pm531\right\|}{\sqrt{54^2+45^2+9^2}}=\ldots\left\{\dfrac{124}{\sqrt{62}}\right\}$	M1	3.1a
	$V=9\sqrt{62}\times\dfrac{124}{\sqrt{62}}$	dM1	3.1a
	$=1116$	A1	1.1b
		(5)	
	Alternative 2 $\begin{pmatrix}25\\-4\\13\end{pmatrix}-\begin{pmatrix}10\\-1\\-6\end{pmatrix}=\begin{pmatrix}15\\-3\\19\end{pmatrix}$	M1 A1	3.1a 1.1b
	$\dfrac{\left\|\begin{pmatrix}15\\-3\\19\end{pmatrix}\bullet\begin{pmatrix}6\\-5\\1\end{pmatrix}\right\|}{\sqrt{6^2+5^2+1^2}}=\ldots$ or $\begin{vmatrix}3&4&2\\-3&-1&13\\15&-3&19\end{vmatrix}$ or $\begin{pmatrix}15\\-3\\19\end{pmatrix}\bullet\begin{pmatrix}-54\\45\\-9\end{pmatrix}$	M1	3.1a
	$V=9\sqrt{62}\times\dfrac{124}{\sqrt{62}}$ Or $\begin{vmatrix}3&4&2\\-3&-1&13\\15&-3&19\end{vmatrix}=3(-19+39)-4(-57-195)+2(9+15)$ Or $\begin{pmatrix}15\\-3\\19\end{pmatrix}\bullet\begin{pmatrix}-54\\45\\-9\end{pmatrix}=15\times-54-3\times45+19\times-9$	dM1	3.1a
	$=1116$	A1	1.1b
			(10 marks)

Notes

(a)
M1: Adopts a correct strategy by finding 2 appropriate vectors that will enable the area to be calculated, not two parallel vectors. If no method seen then 2 correct values will imply this mark.

$\overrightarrow{EF}=\overrightarrow{HG}=\begin{pmatrix}-3\\-1\\13\end{pmatrix},\overrightarrow{EH}=\overrightarrow{FG}=\begin{pmatrix}3\\4\\2\end{pmatrix},\overrightarrow{EG}=\begin{pmatrix}0\\3\\15\end{pmatrix},\overrightarrow{FH}=\begin{pmatrix}6\\5\\-11\end{pmatrix},$

M1: Forms the cross product of 2 non-parallel vectors and calculates the magnitude to find that required area. May split into two triangles and then add the areas. If uses the cross product of the diagonal vectors they would need to divide their answer by 2 to score this mark.
A1: Correct area cso, look out for correct signs
(b)
M1: Substitutes the position and direction vectors into the correct positions, condone the lack of brackets, and a sign slip and missing = 0
A1ft: Deduces the correct equation in the required form, follow through on their normal vector as long as both method marks scored in (a), must have correct bracketing and = 0
(c)
M1: Recognises the requirement to find the equation of Π by forming the necessary scalar product
A1: Correct equation for Π
M1: Performs the key step of finding the value of the parameter that determines the required distance or vector that will allow the volume to be calculated
Alternatively find the equation of the plane containing P and then finds the shortest distance between the two planes
dM1: Completes the strategy by attempting the product of their answer to part (a) with the length of the relevant vector or attempts the magnitude of the scalar triple product using the relevant vectors
A1: Correct volume
Alternative 1
M1: Recognises the requirement to find the equation of the plane containing the parallelogram P by forming the necessary scalar product
A1: Correct equation for the plane
M1: Finds the shortest distance between the point $(25,-4,13)$ and the plane
dM1: Dependent on previous method. Finds the required volume by multiplying their answer to (a) multiplied by the shortest distance
A1: Correct volume
Alternative 2
M1: Find the vector between E and the position vector on Π
A1: Correct vector
M1: Finds the shortest distance between the planes or states determinant approach of 3 appropriate vectors for example $\overrightarrow{EF},\overrightarrow{EH},\overrightarrow{EA}$ where A is $(25,-4,13)$ or the triple scalar product
dM1: Finds the required volume by multiplying their answer to (a) by the shortest distance or uses the determinant approach of 3 appropriate vectors or attempts the triple scalar product.
If no working is seen the answer must be correct for their vectors.
Using $\frac{1}{6}$ volume is dM0
A1: Correct volume

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