Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 02 Core Pure Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
    - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$
    - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$
- Formula
    - Attempt to use the correct formula (with values for *a*, *b*and *c*).
- Completing the square
    - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

- Differentiation
    - Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)
- Integration
    - Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks	AOs
1(a)	$\lvert z\rvert=\left(\sqrt{2^2+\left(-2\sqrt{3}\right)^2}=\right)4$ and $\lvert w\rvert=\left(\sqrt{(-1)^2+\left(\sqrt{3}\right)^2}=\right)2$ Alt: $z=-2\left(-1+\sqrt{3}\mathrm{i}\right)=-2w$	B1	1.1b
	$\dfrac{z}{w}=\dfrac{2-2\sqrt{3}\mathrm{i}}{-1+\sqrt{3}\mathrm{i}}=-2\Rightarrow\left\lvert\dfrac{z}{w}\right\rvert=2$	M1	2.1
	$\dfrac{\lvert z\rvert}{\lvert w\rvert}=\dfrac{4}{2}=2$ therefore $\left\lvert\dfrac{z}{w}\right\rvert=\dfrac{\lvert z\rvert}{\lvert w\rvert}$	A1cso	2.4
		(3)	
(b)	$\arg(z)=-\dfrac{\pi}{3}$ or $\dfrac{5\pi}{3}$ and $\arg(w)=\dfrac{2\pi}{3}$ Alt: $\arg(w)=\dfrac{2\pi}{3}$ and $\arg(w^2)=\dfrac{4\pi}{3}$	B1	1.1b
	$zw=\left(2-2\sqrt{3}\mathrm{i}\right)\left(-1+\sqrt{3}\mathrm{i}\right)=4+4\sqrt{3}\mathrm{i}\Rightarrow\arg(zw)=\tan^{-1}\left(\dfrac{4\sqrt{3}}{4}\right)=\dfrac{\pi}{3}$ (or $zw=-2w^2=-2\left(-2-2\sqrt{3}\mathrm{i}\right)\Rightarrow\arg(zw)=\ldots$) Alt: $\arg z=\arg(-2w)=\arg w\pm\pi$, $\arg(zw)=\arg\left(-2w^2\right)=\arg w^2\pm\pi$	M1	2.1
	$\arg(z)+\arg(w)=-\dfrac{\pi}{3}+\dfrac{2\pi}{3}=\dfrac{\pi}{3}$ therefore $\arg(zw)=\arg(z)+\arg(w)$	A1cso	2.4
		(3)	
			(6 marks)

Notes:

(a)

B1: Correct values for $\lvert z\rvert$ and $\lvert w\rvert$. No need for method, may see modulus-argument forms (polar or exponential) used. Alternatively, correctly identifies $z=-2w$

M1: Finds $\dfrac{z}{w}$ and then $\left\lvert\dfrac{z}{w}\right\rvert$. Look for multiplication of numerator and denominator by the conjugate of the denominator for finding $\dfrac{z}{w}$. In the look for alternative factoring out the -2 and cancelling z oe. Allow this mark if modulus-argument forms (polar or exponential) are used to deduce the modulus.

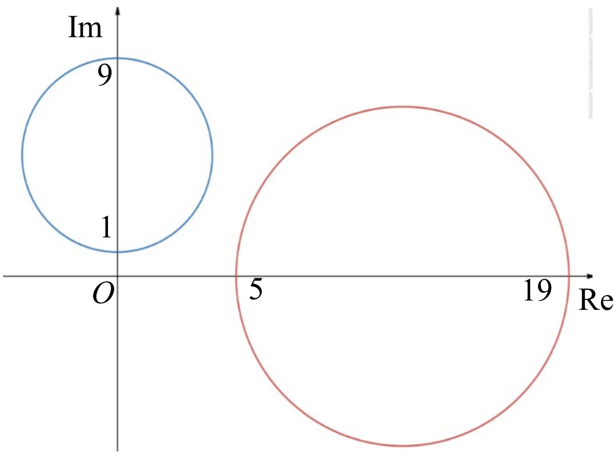
A1: Correctly shows that $\frac{\lvert z\rvert}{\lvert w\rvert} = 2$ and $\left\lvert\frac{z}{w}\right\rvert = 2$ hence state that $\left\lvert\frac{z}{w}\right\rvert = \frac{\lvert z\rvert}{\lvert w\rvert}$ cso. If the result is not stated at the end, we require both $\left\lvert\frac{z}{w}\right\rvert$ and $\frac{\lvert z\rvert}{\lvert w\rvert}$ to have explicitly been seen at some stage in their work and a minimal conclusion (e.g. "LHS=RHS") to be given. Note $\frac{z}{w} = \ldots = -2$ followed by $\lvert -2\rvert = 2$ without ever seeing $\left\lvert\frac{z}{w}\right\rvert$ will be A0. If modulus-argument forms (polar or exponential) were used all working must have been correct.
(b) **B1:** Correct values for $\arg(z)$ and $\arg(w)$. Accept degrees equivalents here and throughout. Alternatively, correct values for $\arg(w)$ and $\arg(w^2)$. **M1:** Finds zw and then $\arg(zw)$. Do not allow attempts via modulus-argument forms (polar or exponential) that use the sum of arguments to prove the sum of arguments of the product. Alternatively, finds both $\arg(z)$ and $\arg(zw)$ in terms of $\arg(w)$ using $\arg(-p) = \arg(p) \pm \pi$. **A1:** Shows that $\arg(z) + \arg(w) = \frac{p}{3}$ and following $\arg(zw) = \frac{p}{3}$ hence that $\arg(zw) = \arg(z) + \arg(w)$ cso If the result is not stated at the end, we require both sides of the result to have been seen at some stage in their work and a minimal conclusion (e.g. "LHS=RHS") to be given. Alternatively, correctly establishes the result using $\arg(zw) = \arg(w^2) \pm \pi = 2\arg(w) \pm \pi$ and $\arg z + \arg w = \arg w \pm \pi + \arg w = 2\arg w \pm \pi$ with $\arg w^2 = 2\arg w$ verified.

Question	Scheme	Marks	AOs
2(a)	Shortest distance $=\dfrac{\lvert 2\times3+4\times(-5)+(-1)\times2-3\rvert}{\sqrt{2^2+4^2+(-1)^2}}=\ldots$	M1	1.1b
	awrt 4.15 or $\dfrac{19}{\sqrt{21}}$ or $\dfrac{19\sqrt{21}}{21}$	A1	1.1b
		(2)	
(b)	$\overrightarrow{AB}=\begin{pmatrix}6\\-2\\1\end{pmatrix}-\begin{pmatrix}3\\-5\\2\end{pmatrix}=\begin{pmatrix}3\\3\\-1\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}3\\-5\\2\end{pmatrix}-\begin{pmatrix}6\\-2\\1\end{pmatrix}=\begin{pmatrix}-3\\-3\\1\end{pmatrix}$	M1	3.4
	$\mathbf{r}=\begin{pmatrix}6\\-2\\1\end{pmatrix}+\lambda\begin{pmatrix}3\\3\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-5\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\3\\-1\end{pmatrix}$ $\mathbf{r}=\begin{pmatrix}6\\-2\\1\end{pmatrix}+\lambda\begin{pmatrix}-3\\-3\\1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-5\\2\end{pmatrix}+\lambda\begin{pmatrix}-3\\-3\\1\end{pmatrix}$	A1	3.3
		(2)	
(c)	$\cos\theta=\dfrac{\begin{pmatrix}3\\3\\-1\end{pmatrix}.\begin{pmatrix}2\\4\\-1\end{pmatrix}}{\sqrt{3^2+3^2+(-1)^2}\times\sqrt{2^2+4^2+(-1)^2}}\Rightarrow\theta=\ldots$ or $90-\theta=\ldots$	M1	3.1b
	$(f=)72^\circ$	A1	1.1b
		(2)	
(d)	Distance $=\sqrt{3^2+3^2+(-1)^2}=\sqrt{19}=$ awrt 4.36	B1	1.1b
		(1)	
(e)	**Not likely to match**, as **unlikely** that the flight path of the arrow will be a **straight line**.	B1	3.2b
		(1)	
			(8 marks)

Notes:
(a) **M1:** Correct method to find the shortest distance. Condone $\pm d$ in the formula. Longer methods are possible. E.g. if done after part (d), then $\sqrt{19}\cos 17.97....$ is a correct method. Another approach is to form the general line $\mathbf{r}=\begin{pmatrix}3\\-5\\2\end{pmatrix}+\mu\begin{pmatrix}2\\4\\-1\end{pmatrix}$, substitute into plane equation $\Rightarrow \mu=\frac{19}{21}$ then scale the normal vectors magnitude: $\frac{19}{21}\times\sqrt{2^2+4^2+(-1)^2}=...$ **A1:** Correct exact distance or awrt 4.15
(b) **M1:** Finds the direction of the line. Accept either direction and implied by 2 out of three coordinates correct if no method shown. **A1:** Correct model for the flight path of the arrow. Must be an equation ie, including the **r** = … (but condone *l* = … or other letter).
(c) **M1:** A complete method to find the required angle or angle between normal and plane. Uses the dot product of the direction vector of the line and normal vector of the plane to find the angle (need not subtract from 90 for this mark). Allow if a (single) minor slip is made if the method is clearly attempting to use the correct vectors. Note may use $\sin\phi=\frac{\begin{pmatrix}3\\3\\-1\end{pmatrix}.\begin{pmatrix}2\\4\\-1\end{pmatrix}}{\sqrt{3^2+3^2+(-1)^2}\times\sqrt{2^2+4^2+(-1)^2}}\Rightarrow\phi=...$ directly. **A1:** Correct final angle.
(d) **B1:** $\sqrt{19}$ or awrt 4.36
(e) **B1:** States not likely to match (or equivalent wording) **and** gives a suitable reason why. This will most commonly be that the arrow will not travel in a straight line, but e.g. accept “the arrow is not a particle so the tip may travel into the target, not stop at the plane”. Do not accept answers that suggest the arrow will travel less far than the answer to (d). Do not accept “air resistance” arguments unless they specifically refer to the arrow not travelling in a straight line.

Question	Scheme	Marks	AOs
3(a)	$n=1, \text{LHS}=\begin{pmatrix}1&5\\0&2\end{pmatrix}^1=\begin{pmatrix}1&5\\0&2\end{pmatrix}, \text{RHS}=\begin{pmatrix}1&5(2^1-1)\\0&2^1\end{pmatrix}=\begin{pmatrix}1&5\\0&2\end{pmatrix}$ So the result is true for $n=1$	B1	2.2a
	Assume true for $n=k$, $\begin{pmatrix}1&5\\0&2\end{pmatrix}^k=\begin{pmatrix}1&5(2^k-1)\\0&2^k\end{pmatrix}$ then for $n=k+1$ $\begin{pmatrix}1&5\\0&2\end{pmatrix}^{k+1}=\begin{pmatrix}1&5(2^k-1)\\0&2^k\end{pmatrix}\begin{pmatrix}1&5\\0&2\end{pmatrix}$ or $\begin{pmatrix}1&5\\0&2\end{pmatrix}^{k+1}=\begin{pmatrix}1&5\\0&2\end{pmatrix}\begin{pmatrix}1&5(2^k-1)\\0&2^k\end{pmatrix}$	M1	2.4
	$=\begin{pmatrix}1&5+2\times5(2^k-1)\\0&2(2^k)\end{pmatrix}$ or $\begin{pmatrix}1&5(2^k-1)+5(2^k)\\0&2(2^k)\end{pmatrix}$	M1 A1	1.1b 1.1b
	$\begin{pmatrix}1&10(2^k)-5\\0&2^{k+1}\end{pmatrix}=\begin{pmatrix}1&5[2(2^k)-1]\\0&2^{k+1}\end{pmatrix}=\begin{pmatrix}1&5(2^{k+1}-1)\\0&2^{k+1}\end{pmatrix}$	A1	2.1
	"If true for $n=k$ then true for $n=k+1$" and as it is "**true for $n=1$**" the statement is "**true for all** (positive integers) ***n***"	A1	2.4
		(6)	
(b)	Reflection	B1	1.1b
	Reflection in the y-axis or line $x=0$	B1	1.1b
		(2)	
(c)	$\begin{pmatrix}1&5(2^n-1)\\0&2^n\end{pmatrix}\begin{pmatrix}-1&0\\0&1\end{pmatrix}=\begin{pmatrix}-1&5(2^n-1)\\0&2^n\end{pmatrix}$	B1	1.1b
		(1)	
(d)	$\begin{pmatrix}-1&5(2^n-1)\\0&2^n\end{pmatrix}\begin{pmatrix}27\\1\end{pmatrix}=\begin{pmatrix}-27+5(2^n-1)\\2^n\end{pmatrix}$ Sets $-27+5(2^n-1)=2^n \Rightarrow 2^n=...$	M1	3.1a
	$(\mathbf{A}^n=)\begin{pmatrix}1&5('8'-1)\\0&'8'\end{pmatrix}=\begin{pmatrix}1&35\\0&8\end{pmatrix}$	M1 A1	1.1b 1.1b
		(3)	
(12 marks)			

Notes:
(a) **B1:** Shows that the result holds for $n = 1$. Must see substitution in the RHS minimum required is $\begin{pmatrix} 1 & 5(2-1) \\ 0 & 2 \end{pmatrix}$ and reaches $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$ No need to state "true for $n = 1$" for this mark. **M1:** Assumes the result is true for some value of $n = k$, and sets up a matrix multiplication of the assumed result multiplied by the original matrix, either way round (no need to carry out for this mark, it is for essentially explaining the procedure). "Assume (true for) $n = k$" (oe) is sufficient for the assumption and may even be part of the conclusion and accept any alternative wording that indicates the assumption has been made. Allow for setting up the multiplication in reverse (ie working from $n = k+1$ towards $n = k$, $\mathbf{A}^k = \mathbf{A}^{k+1}\mathbf{A}^{-1}$ oe) **M1:** Carries out the multiplication. (Allow the reverse case.) **A1:** Achieves a correct un-simplified matrix. Accept in working in reverse. **A1:** Reaches a correct simplified matrix with **no errors, the correct un-simplified matrix seen previously and at least one intermediate line which must be correct.** If working from both sides all steps in showing the two sides are equal must be seen. If working in reverse they must return to $\mathbf{A}^{k+1} = ...$ for this mark. **A1:** Correct formal conclusion. This mark is dependent on the M and previous A marks having been scored and an attempt at the check for $n = 1$ (if e.g. they didn't show sufficient detail). It is gained by conveying the ideas of **all three bold** points at **the end of their solution**. Note: Some cases may use $n = 0$ as the base case. These can score full marks if dealt with correctly, but the conclusion must be consistent with their initial check to score the final A. For the B mark minimum $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (or $\mathbf{I}$) and $\begin{pmatrix} 1 & 5(1-1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ should be seen. (No need to state true for $n = 0$ for this mark – but must have consistent conclusion for the final A as noted above). If unsure send to review.
(b) **B1:** Identifies the transformation as a reflection. Accept stretch with scale factor −1. **B1:** Identifies reflection and the correct line of reflection. Condone phrasing such as "across the *y*-axis". Must be a single transformation – B0 if they state something else also happens. NB Allow B1B0 for answers such as "flip in the y-axis" that convey the correct transformation in imprecise language.
(c) **B1:** Correct matrix
(d) **M1:** A complete method to find a value for 2^n (or *n*). Multiplies the coordinates $(27, 1)$ by their matrix **C** and sets the *x* and *y* coordinates equal to reach a value for 2^n or *n* (condone negative values for 2^n). Note you may allow this for reach a value of *a* if $a = 2^n$ is stated or clearly implied by their working. **M1:** Uses their value of 2^n or *n*, to find the matrix $\mathbf{A}^n$. Note substituting into **C** is M0 without further work to find $\mathbf{A}^n$. **A1**: Correct matrix.

Question	Scheme	Marks	AOs
4(i)	$z_1 + z_2 = a + bi + c + di$ leading to $b + d = 0$ (oe) or $z_2 - z_1 = (c + di) - (a + bi)$ leading to $c - a = 2$ (oe)	B1	1.1b
	Forms equations using modulus information $\|z_1\| = \sqrt{13} \Rightarrow a^2 + b^2 = 13$ and $\|z_2\| = 5 \Rightarrow c^2 + d^2 = 25$	M1	3.1a
	Uses their equations to solve simultaneously to find at least one value **<u>e.g Method 1</u>** $c = 2 + a$, $d = -b$ leading to $(2 + a)^2 + (-b)^2 = 25 \Rightarrow a^2 + 4a + 4 + b^2 = 25$ Solve with $a^2 + b^2 = 13$ gives $13 + 4a + 4 = 25 \Rightarrow 4a = 8 \Rightarrow a = ...$ **<u>e.g. Method 2</u>** $\left.\begin{array}{l} a^2 + b^2 = 13 \\ c^2 + d^2 = 25 \end{array}\right\} \Rightarrow a^2 - c^2 = -12$ and solves simultaneous with $c - a = 2$ to find a value for a or c.	M1	3.1a
	Uses their equations to find values for all the constants	ddM1	2.1
	$a = 2, b = 3, c = 4, d = -3$	A1	2.3
		(5)	
(ii)(a)	 	B1 B1	1.1b 1.1b
		(2)	
(ii)(b)	Distance between centres $= \sqrt{5^2 + 12^2} = 13$	M1	1.1b
	$13 \pm (7 + 4)$	dM1	3.1a
	$2 \leqslant \|z - w\| \leqslant 24$	A1	1.1b
		(3)	
			(10 marks)

Notes:
(i) **B1:** States either $b+d=0$ or $c-a=2$ (oe) **M1:** Uses the modulus information to write down two more equations (need not be squared). Accept if they forget to square one of the moduli but the sum of squares must be correct. **M1:** Uses their equations to solve simultaneously to find at least one value. If they assume $a=2$, you may allow this mark for finding at least one of b or d. **ddM1:** Dependent on both previous method marks. Uses their equations to find values for all of the constants. This mark is not available if $a=2$ is assumed. All must be found from algebra. **A1:** Correct values from correct work.
(ii)(a) **B1:** One circle drawn with correct intercepts OR both circles drawn in correct position, not overlapping, but with no intercepts shown. Be tolerant on the shape but must be a clear attempt at a circle – a closed loop with no obvious kinks. Labels on imaginary axis may be just numbers or i and 9i. **B1:** Both circles drawn with correct intercepts and the circles does not intersect each other. Again be tolerant as per first B mark.
(ii)(b) **M1:** Finds the distance between the centres of the circles. **dM1**: Dependent on previous method mark. Full method to find the nearest and furthest points of circles. E.g. Uses their distance +/- sum of radii. If their distance between circles is less than 11, there must a check seen to show this has been considered. If by error they show the circles touch (not cross) then note that finding twice the sum of radii is a valid method for the greatest distance. **A1**: Correct answer. Allow if the centres were on the negative axes. Alt: There may be attempts via finding the line through the centres. **M1:** Full method to find the equation of the line through the two centres: $m=\frac{0-5}{12-0}=-\frac{5}{12}\Rightarrow y=-\frac{5}{12}(x-12)$ (oe) **dM1:** Full method to find the nearest and furthest points of circles, so finds the intersection points of this line with the circles and finds distances between the relevant points. $\left.\begin{array}{l} x^2+\left(-\frac{5}{12}x+5-5\right)^2=16 \\ (x-12)^2+\left(-\frac{5}{12}x+5\right)^2=49 \end{array}\right\}\Rightarrow \begin{array}{l} \frac{169}{144}x^2=16\Rightarrow x=\pm\frac{48}{13}\Rightarrow y=\frac{45}{13},\frac{85}{13} \\ \frac{169}{144}x^2-\frac{169}{6}x+120=0\Rightarrow x=\frac{72}{13},\frac{240}{13}\Rightarrow y=\pm\frac{35}{13} \end{array}$ $\text{nearest}=\sqrt{\left(\frac{72}{13}-\frac{48}{13}\right)^2+\left(\frac{35}{13}-\frac{45}{13}\right)^2}=2,\text{furthest}=\sqrt{\left(\frac{240}{13}+\frac{48}{13}\right)^2+\left(-\frac{35}{13}-\frac{85}{13}\right)^2}=24$ **A1:** Correct answer.

Question	Scheme	Marks	AOs
5(i)	$\begin{vmatrix} 2 & -1 & 1 \\ 1 & p & -3 \\ 3 & 1 & -2 \end{vmatrix} = 2\left(p\times -2-1\times -3\right)+\left(1\times -2-3\times -3\right)+\left(1\times 1-3\times p\right)=0$ $=2(-2p+3)+7+1-3p=0\Rightarrow p=\ldots$	M1	2.1
	$p=2$	A1	1.1b
		(2)	
(ii)	$\left.\begin{array}{r} 2x-y+z=3 \\ x+2y-3z=q \end{array}\right\}\Rightarrow 3x+y-2z=3+q$ compares with $3x+y-2z=4$ leading to $3+q=4\Rightarrow q=\ldots$ **Alternatively** $\left.\begin{array}{r} 2x-y+z=3 \\ 3x+y-2z=4 \end{array}\right\}\Rightarrow 5x-z=7$ $\left.\begin{array}{r} 4x-2y+2z=6 \\ x+2y-3z=q \end{array}\right\}\Rightarrow 5x-z=6+q\Rightarrow 6+q=7\Rightarrow q=\ldots$	M1	3.1a
	$q=1$	A1	1.1b
		(2)	
	(4 marks)		

Notes:

(i)

M1: Finds the determinant of the matrix, sets = 0 and solves to find a value of p. Allow for any recognisable attempt at finding the determinant (may be slips in coefficients or signs). Accept if det $= 14-7p$ appears with no working shown.

A1: $p=2$ from correct work.

Alt I Using normal

M1: Attempts the cross product between sets of pairs of planes and scales (if appropriate) and equates directions to solve for p.

A1: Correct p

FYI

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 1 & p & -3 \end{vmatrix}=(3-p)\mathbf{i}+7\mathbf{j}+(2p+1)\mathbf{k},\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix}=\mathbf{i}+7\mathbf{j}+5\mathbf{k},\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & p & -3 \\ 3 & 1 & -2 \end{vmatrix}=(3-2p)\mathbf{i}-7\mathbf{j}+(1-3p)\mathbf{k}$$

(ii)

M1: A complete method to find the value of q. May be implied by correct value if no incorrect working is shown.

E.g. Adds together equations 1 and 2 and compares with equation 3 to find a value for q.

Alternatively: Uses equations $2x - y + z = 3$ and $3x + y - 2z = 4$ to eliminate one variable. Uses equation $x + 2y - 3z = q$ and one of the other equations to eliminate the same variable and compare to find a value for *q*. Another possible method is to identify a point on both planes 1 and 3 (e.g. let *x* = 1 and solve for *y* and *z* to get $(1,-3,-2)$ then substitute into middle equation to find *q*) **A1:** $q = 1$
Alternatively II the question may be done as a whole. Eg. $\left.\begin{matrix}2x-y+z=3\\3x+y-2z=4\end{matrix}\right\} \Rightarrow x+2y-3z=1 \Rightarrow p=2, q=1$ is the most direct method. Variations are possible, e.g. $\left.\begin{matrix}2x-y+z=3\\x+py-3z=q\\3x+y-2z=4\end{matrix}\right\} \Rightarrow \begin{matrix}3(1)+(2): 7x+(p-3)y=9+q\\2(1)+(3): \quad 7x-y=10\end{matrix} \Rightarrow \begin{matrix}p-3=-1 \Rightarrow p=...\\9+q=10 \Rightarrow q=...\end{matrix}$ Score as follows: **M1:** Attempts to solve at least two of the equations simultaneously to eliminate one variable or match coefficients of the third equation and uses the equation to identify one of the unknowns. Condone slips as long as the method is clear. **A1:** Correct *p* or correct *q*. **M1:** Full method to use the linear dependence of the equations to find both variables. **A1:** Correct *p* and *q*. Alt III: Using points on the common line. **M1:** Finds two separate points on each of the first and third plane. **A1:** Two correct points. **M1:** Forms and solves two equations in *p* and *q* using their points. **A1:** Correct *p* and *q*. E.g. $\left.\begin{matrix}x=0 \Rightarrow \begin{cases}-y+z=3\\y-2z=4\end{cases} \Rightarrow y=-10, z=-7\\x=1 \Rightarrow \begin{cases}-y+z=1\\y-2z=1\end{cases} \Rightarrow y=-3, z=-2\end{matrix}\right\} \Rightarrow \begin{matrix}-10p+21=q\\1-3p+6=q\end{matrix} \Rightarrow p=..., q=...$ Some useful equations: Eliminating $x: \begin{cases}(2p+1)y-7z=2q-3\\5y-7z=-1\\(3p-1)y-7z=3q-4\end{cases}$ Eliminating $y: \begin{cases}(2p+1)x+(p-3)z=3p+q\\5x-z=7\\(3p-1)x+(3-2p)z=4p-q\end{cases}$ Eliminating $z: \begin{cases}7x+(p-3)y=9+q\\7x-y=10\\7x+(3-2p)y=12-2q\end{cases}$

Question	Scheme	Marks	AOs
6(a)	$2x^4 + Ax^3 - Ax^2 - 5x + 6 = 0 \Rightarrow x^4 + \frac{A}{2}x^3 - \frac{A}{2}x^2 - \frac{5}{2}x + \frac{6}{2} = 0$ $\alpha\beta\gamma\delta = \frac{6}{2}$ $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{5}{2}$	B1	3.1a
	$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta} = \frac{3(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)}{\alpha\beta\gamma\delta} = \frac{3\times\left(\frac{5}{2}\right)}{\left(\frac{6}{2}\right)} = \ldots$	M1	1.1b
	$= \frac{5}{2}$	A1	1.1b
		(3)	
(b)	$\alpha + \beta + \gamma + \delta = -\frac{A}{2} \quad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = -\frac{A}{2}$	B1	1.1b
	$(\alpha + \beta + \gamma + \delta)^2 =$ $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$	M1 A1	3.1a 1.1b
	$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta} = \frac{3(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)}{\alpha\beta\gamma\delta} = \frac{3\times\left(\frac{5}{2}\right)}{\left(\frac{6}{2}\right)} = \ldots$	dM1	1.1b
	$A = -1, -3$	A1	1.1b
		(5)	
		(8 marks)	

Notes:

(a)

B1: Identifies the correct values for the product and triple pair sum. If seen these must be correct, but they may be implied if not explicitly extracted, e.g. $\sum \frac{3}{\alpha_i} = 3\frac{\sum \alpha_i\alpha_j\alpha_k}{\alpha\beta\gamma\delta} = 3\times\frac{5}{6} = \frac{5}{2}$ does not technically contain an incorrect step if one realises the 2's will cancel. A good indication will be what happens in part (b) – if there they realise the 2's are involved then allow b.o.d. for a calculation such as here given in (a).

Note $\sum \alpha\beta\gamma\delta$ is a correct notation for the product of roots.

M1: Uses the correct identity and their values of the product and triple sum. (The values need not be correct for this mark.)

A1: Correct value following correct identity with no indication of incorrect values for triple sum and product (see comment on the B mark).

(b) **B1:** Identifies the correct values for the sum and pair sum. Allow when first seen – some will list all these in part (a), which is fine for this mark. **M1:** Attempts to find the identity for $(a+b+g+d)^2$ in terms of the sum of squares and pair sum (seen or implied). Allow attempts where the "2" is incorrect with no other method shown. **A1:** Correct identity (seen or implied). **dM1:** Dependent on previous method mark. Substitutes the sum and pair sum of the roots into their identity and forms and solves a 3TQ for A (usual rules). **A1:** Correct values of A			

6(a) Alt	$x=\frac{3}{w}\Rightarrow 2\left(\frac{3}{w}\right)^4+\ldots-5\left(\frac{3}{w}\right)+6=0$ or $x=\frac{1}{w}\Rightarrow 2\left(\frac{1}{w}\right)^4+\ldots-5\left(\frac{1}{w}\right)+6=0$	B1	3.1a
	$\Rightarrow \ldots+\ldots w+\ldots-15w^3+6w^4=0$ $\Rightarrow \sum\frac{3}{\alpha_i}=-\left(\frac{-15}{6}\right)$ or $\Rightarrow 2+\ldots-5w^3+6w^4=0$ $\Rightarrow 3\sum\frac{1}{\alpha_i}=3\times-\left(\frac{-5}{6}\right)$	M1	1.1b
	$=\frac{5}{2}$	A1	1.1b
		(3)	

Alt (a): Some may use a transformation. These can be scored as

B1: Makes a correct substitution into the equation to solve the problem. This will probably be $x=\frac{3}{w}$ but note that $x=\frac{1}{w}$ can also be used.

M1: Multiplies through by w^4 and extracts the correct sum of roots from the new equation. If using $x=\frac{1}{w}$ they must also multiply through by the 3 to gain this mark.

A1: Correct answer from correct work on the relevant coefficients (the others need not be seen).

Question	Scheme	Marks	AOs
7(a)	$x = r\cos\theta = a(1+\sin\theta)\cos\theta = a(\cos\theta + \sin\theta\cos\theta)$ $\frac{dx}{d\theta} = A\cos\theta\cos\theta + B(1+\sin\theta)\sin\theta$ or $A\sin\theta + B\cos^2\theta + C\sin^2\theta$ or $x = r\cos\theta = a(1+\sin\theta)\cos\theta = a(\cos\theta + 0.5\sin 2\theta)$ $\frac{dx}{d\theta} = A\sin\theta + B\cos 2\theta$	M1	3.1a
	$\frac{dx}{d\theta} = a(-\sin\theta + \cos 2\theta)$ or $a(\cos^2\theta - (1+\sin\theta)\sin\theta)$ or $a(-\sin\theta + \cos^2\theta - \sin^2\theta)$	A1	1.1b
	$\frac{dx}{d\theta} = a(-\sin\theta + \cos 2\theta) = 0 \Rightarrow -\sin\theta + 1 - 2\sin^2\theta = 0$ $2\sin^2\theta + \sin\theta - 1 \Rightarrow \sin\theta = ...$ or $\frac{dx}{d\theta} = a(-\sin\theta + \cos^2\theta - \sin^2\theta) = 0 \Rightarrow -\sin\theta + 1 - \sin^2\theta - \sin^2\theta = 0$ $2\sin^2\theta + \sin\theta - 1 = 0 \Rightarrow \sin\theta = ...$	M1	3.1a
	$\sin\theta = \frac{1}{2}\{-1\}$	A1	1.1b
	$r = a\left(1+\frac{1}{2}\right)$ and $\theta = \sin^{-1}\left(\frac{1}{2}\right)$	M1	1.1b
	$\left(\frac{3}{2}a, \frac{\pi}{6}\right)$ and $\left(\frac{3}{2}a, \frac{5\pi}{6}\right)$	A1	2.2a
		(6)	
(b)	E.g. $x = \frac{3}{2}a\cos\left(\frac{\pi}{6}\right) \Rightarrow 2 \times \frac{3\sqrt{3}}{4}a = 10 \Rightarrow a = ...$ or $\sin\frac{\pi}{3} = \frac{5}{3a/2} \Rightarrow a = ...$ or $\Rightarrow 10^2 = \left(\frac{3a}{2}\right)^2 + \left(\frac{3a}{2}\right)^2 - 2\left(\frac{3a}{2}\right)^2\cos\frac{2\pi}{3} \Rightarrow a = ...$ or $\frac{10}{\sin\frac{2\pi}{3}} = \frac{\frac{3a}{2}}{\sin\frac{\pi}{6}} \Rightarrow a = ...$ or $y = \frac{3}{2}a\sin\frac{\pi}{6} \Rightarrow 5^2 + \left(\frac{3a}{4}\right)^2 = \left(\frac{3a}{2}\right)^2 \Rightarrow a = ...$	M1	3.3
	$a = \frac{20\sqrt{3}}{9}$ *	A1*	2.1
		(2)	

(c)	$\left(\frac{1}{2}\right)\int r^2\ \mathrm{d}\theta = K\int\ \left(1+2\sin\theta+\sin^2\theta\right)\mathrm{d}\theta$ $= K\int\ \left(1+2\sin\theta+\left[\frac{1}{2}-\frac{1}{2}\cos 2\theta\right]\right)\mathrm{d}\theta = K\left[p\theta \pm q\cos\theta \pm r\sin 2\theta\right]$	M1	3.4
	$=\left(\frac{1}{2}\times\right)\frac{400}{27}\left[\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]$	A1	1.1b
	Area bounded by the curve $=\frac{1}{2}\int_{-\pi}^{\pi}\left[\frac{20\sqrt{3}}{9}(1+\sin\theta)\right]^2\ \mathrm{d}\theta$ or $=\frac{1}{2}\int_{0}^{2\pi}\left[\frac{20\sqrt{3}}{9}(1+\sin\theta)\right]^2\ \mathrm{d}\theta$ $\frac{200}{27}\left[\left(\frac{3}{2}(2\pi)-2\cos(2\pi)-\frac{1}{4}\sin(4\pi)\right)-\left(\frac{3}{2}(0)-2\cos(0)-\frac{1}{4}\sin(0)\right)\right]$ or $\frac{200}{27}\left[\left(\frac{3}{2}(\pi)-2\cos(\pi)-\frac{1}{4}\sin(2\pi)\right)-\left(\frac{3}{2}(-\pi)-2\cos(-\pi)-\frac{1}{4}\sin(-\pi)\right)\right]$	dM1	3.4
	$\frac{200}{9}p$ or awrt 69.8 (m^2)	A1	1.1b
		(4)	
			(12 marks)

Notes:

(a)

M1: Substitutes the equation of C into $x = r\cos q$ and differentiates to the required form. The $\frac{\mathrm{d}x}{\mathrm{d}q}$ need not be seen – if it is a clear attempt at differentiation allow the marks.

A1: Fully correct differentiation.

M1: Uses correct trig work to solve $\frac{\mathrm{d}x}{\mathrm{d}q} = 0$ to achieve a value for $\sin q$ via a 3 TQ in sin (oe)

A1: Correct values for $\sin q$. The -1 need not be stated but any other values found will be A0.

M1: Uses $r = a(1+\sin q)$ and their $q = \sin^{-1}(...)$ (or $\cos^{-1}$ if by error they ended up with a value for cos θ – and may have come from $\frac{\mathrm{d}y}{\mathrm{d}q} = 0$) to find a polar coordinate. May be implied by a correct coordinate following $\sin\theta = \frac{1}{2}$ having been found.

A1: Deduces the correct polar coordinates from correct work for A and B with no incorrect extras. May be listed, or may given as (θ, r). You may ignore values from $\sin\theta = -1$ that are not used.

NB: Allow the final two marks in (a) if it is initially forgotten but the correct work is done as part of answering part (b).

(b) **M1:** Uses $2x = 10$ where $x = r\cos\theta$ to find a value for *a* by any valid method. Several are shown in the scheme, there may be variations on these. **A1**: Correct value achieved with no errors cso. Must see a correct unsimplified equation in *a* with trig terms evaluated before the final answer.
(c) **M1:** Attempts to use the model and area $=\left(\frac{1}{2}\right)\int r^2 \, d\theta,$ multiplies out, uses the identity $\sin^2\theta = \frac{1}{2}(\pm 1 \pm \cos 2\theta)$ to get into an integrable form and integrates. Limits are not required and the $\frac{1}{2}$ may be missing for this mark. Condone the *a* not being squared. NB: There may be less direct methods attempted (e.g. integration by parts). Such attempts must be complete attempts reaching an integral of the correct form. If you are unsure in any cases then use the review system. **A1:** Correct integration of $\left(\frac{1}{2}\right)\int r^2 \, d\theta$ condoning the $\frac{1}{2}$ not being present at this stage. Allow with a^2 used instead of the value. **dM1**: Dependent on the first method mark. Full method for the area, with correct $\frac{1}{2}\int_\alpha^\beta r^2 \, d\theta$ and applies correct limits over 2π ($\theta = 0$ and $\theta = 2\pi$ or $\theta = -\pi$ and $\theta = \pi$), or applies $\int_\alpha^\beta r^2 \, d\theta$ with limits $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. Accept equivalent combinations of limits split as two integrals and summed, but must be a full method. If no evidence of the substitution of limits is seen accept answers following the integral as long as suitable limits were linked to the integral. Condone if just -0 is shown for a lower limit of 0 substituted. **A1:** Correct area. Must have substituted for the *a*.

Question	Scheme	Marks	AOs
8(a)	$\frac{dy}{dx} = A\sin x\sinh x + B\cos x\cosh x$	M1	1.1b
	$\frac{dy}{dx} = -\sin x\sinh x + \cos x\cosh x$ and $\frac{d^2y}{dx^2} = -2\sin x\cosh x$	A1	1.1b
	$\frac{d^2y}{dx^2} = C\sin x\cosh x \quad \frac{d^3y}{dx^3} = D\cos x\cosh x + E\sin x\sinh x$ $\frac{d^4y}{dx^4} = F\cos x\sinh x$ (unsimplified)	M1	2.1
	$\frac{d^3y}{dx^3} = -2\cos x\cosh x - 2\sin x\sinh x$ and $\frac{d^4y}{dx^4} = -4\cos x\sinh x$	A1	1.1b
	$\frac{d^4y}{dx^4} = -4y$	A1	2.1
		(5)	
(b)	$\frac{d^5y}{dx^5} = "-4"\frac{dy}{dx}$ or $\frac{d^5y}{dx^5} = -"4"\cos x\cosh x + "4"\sin x\sinh x$ or $y^{(5)} = "-4" \times$ their y'	B1ft	2.2a
	When $x = 0$ $y = 0,\ y' = 1,\ y'' = 0,\ y^{(3)} = -2,\ y^{(4)} = 0,\ y^{(5)} = -4$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \frac{x^5}{5!}y^{(5)}(0) + ...$	M1	1.1b
	$(y =)\,x - \frac{x^3}{3} - \frac{x^5}{30} + ...$	A1	2.5
		(3)	
		(8 marks)	

Notes:

(a)

M1: Uses the product rule to find the correct form for at least the first derivative.

A1: Achieves correct first and second derivatives. Need not be simplified.

M1: Uses the product rule to find the correct form for the second, third and fourth derivatives.

A1: Correct (unsimplified) expressions for the third and fourth derivatives.

A1: Correct fourth derivative in terms of y from correct work. Note do not accept just $k = -4$, we must see the $\frac{d^4y}{dx^4} = -4y$

(b)

B1ft: Correct fifth derivative or value for the fifth derivative deduced following through on their k from part (a). If they have stated y' incorrectly allow for $y^{(5)} = \text{"}-4\text{"} \times \text{their } y'$ provided their y' is non-zero

M1: Attempts the evaluation of all the derivatives at $x = 0$ and applies the Maclaurin formula correctly with their values to achieve at least three non-zero terms.

A1: Correct simplified expansion. Accept with f(x) = … or with nothing before – mark the expression. Accept as a list of terms.

8(a) Alt	$u = \cos x \quad v = \sinh x$ $u' = -\sin x \quad v' = \cosh x$ $u'' = -\cos x \quad v'' = \sinh x$ $u''' = \sin x \quad v''' = \cosh x$ $u^{iv} = \cos x \quad v^{iv} = \sinh x$	M1 A1	1.1b 1.1b
	$\dfrac{d^4y}{dx^4} = uv^{iv} + 4u'v''' + 6u''v'' + 4u'''v' + u^{iv}v = ...$	M1	2.1
	$= \cos x\sinh x - 4\sin x\cosh x - 6\cos x\sinh x + 4\sin x\cosh x + \cos x\sinh x$	A1	1.1b
	$\dfrac{d^4y}{dx^4} = (-4\cos x\sinh x =) - 4y$	A1	2.1
		(5)	

There may be attempts via Leibnitz theorem.

M1: Attempts first four derivatives for each of cos x and sinh x

A1: Correct derivatives.

M1: Applies Leibnitz theorem correctly with their derivatives.

A1: Correct unsimplified expression.

A1: Correct fourth derivative in terms of y.

There may be rare cases where exponential forms are used in part (a). For reference here are the derivatives they should get.

$$y = \frac{1}{2}\left(e^x - e^{-x}\right)\cos x$$

$$\frac{dy}{dx} = \frac{1}{2}\left(e^x + e^{-x}\right)\cos x - \frac{1}{2}\left(e^x - e^{-x}\right)\sin x = \frac{1}{2}e^x\left(\cos x - \sin x\right) + \frac{1}{2}e^{-x}\left(\cos x + \sin x\right)$$

$$\frac{d^2y}{dx^2} = -\left(e^x + e^{-x}\right)\sin x \quad \left[= \frac{1}{2}e^x\left(\cos x - \sin x\right) + \frac{1}{2}e^x\left(-\sin x - \cos x\right) - \frac{1}{2}e^{-x}\left(\cos x + \sin x\right) + \frac{1}{2}e^{-x}\left(-\sin x + \cos x\right)\right]$$

$$\frac{d^3y}{dx^3} = -\left(e^x - e^{-x}\right)\sin x - \left(e^x + e^{-x}\right)\cos x$$

$$\frac{d^4y}{dx^4} = -\left(e^x + e^{-x}\right)\sin x - \left(e^x - e^{-x}\right)\cos x - \left(e^x - e^{-x}\right)\cos x + \left(e^x + e^{-x}\right)\sin x = -2\left(e^x - e^{-x}\right)\cos x = -4\sinh x\cos x$$

NB Some are misreading the function as $y = \cosh x \sinh x$ or $y = \cos x \sin x$. These can be score method marks as long as work is equivalent (ie via product rule), but reduction of the problem to triviality via an identity is less demand and so will score M0. Allow SC A1 for the first A if the equivalent form is reached via the misread to that shown in the scheme.

Question	Scheme	Marks	AOs
9(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\lambda}{\sqrt{1-9x^2}}$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{1-9x^2}}$	A1	1.1b
		(2)	
Alt	$\frac{1}{3}\sin y = x \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \lambda \cos y = \lambda \sqrt{1-9x^2}$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{1-9x^2}}$	A1	1.1b
		(2)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots\sin\left(\arcsin 3x\right) \times \text{their}\left(\frac{3}{\sqrt{1-9x^2}}\right)$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9x}{\sqrt{1-9x^2}}$	A1	2.1
		(2)	
Alt	$\arccos y = \arcsin 3x \left[\Rightarrow -\frac{1}{\sqrt{1-y^2}}\frac{\mathrm{d}y}{\mathrm{d}x} = \text{their}\left(\frac{3}{\sqrt{1-9x^2}}\right)\right]$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots\sqrt{1-y^2} \times \text{their}\left(\frac{3}{\sqrt{1-9x^2}}\right)$	**M1**	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9x}{\sqrt{1-9x^2}}$	**A1**	2.1
		(2)	
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9x}{\sqrt{1-9x^2}} = 4 \Rightarrow -9x = 4\sqrt{1-9x^2} \Rightarrow 81x^2 = 16\left(1-9x^2\right)$ Leading to a value for x or x^2	M1	3.1a
	$x^2 = \frac{16}{225}$	A1	1.1b
	$x = -\frac{4}{15}$	A1	2.3
		(3)	
			(7 marks)

Notes:
(a) **M1:** Uses the chain rule to differentiate to the correct form. (λ can be 1). Condone a missing bracket on $3x^2$ for this mark. **A1:** Correct answer. Accept $\frac{dy}{dx} = \frac{3}{\sqrt{1-(3x)^2}}$ oe **<u>Alternatively</u>** **M1:** Uses implicit differentiation and trig identities to achieve the correct form for $\frac{dx}{dy}$ in terms of x **A1:** Correct answer. Accept $\frac{dy}{dx} = \frac{3}{\sqrt{1-(3x)^2}}$ oe Accept $\frac{dy}{dx} = \frac{3}{\cos(\arcsin 3x)}$
(b) **M1:** Uses the chain rule and their answer to part (a) to differentiate to the correct form. May use arccos y = arcsin $3x$ – see scheme for form but must get to $\frac{dy}{dx} = \ldots$ **A1:** Correct simplified answer. If using the Alt they must replace the y and simplify to achieve the same answer. Must be their answer to (b). Do not accept forms in trig functions such as $\frac{dy}{dx} = -3\tan\left(\arcsin 3x\right)$ as these are not simplest form. Do not allow recovery if they simplify the sin(arcsin..) as part of their working in part (c).
(c) **M1:** A complete method to find a value for at least x^2. Sets their answer to part (b) equal to 4, rearranges, squares both sides to reach a quadratic in x before achieving a value for x or x^2. Note that if they never simplify the sin(arcsin $3x$) in the derivative they will not be able to solve for x^2 and so will not gain this mark. **A1:** Correct value for x^2 from a correct derivative in (b) (may be implied). **A1:** Selects the correct value of x. Must have come from a correct derivative in (b). **Caution:** Watch out for those who get $\frac{dy}{dx} = \frac{9x}{\sqrt{1-9x^2}}$ in (b) which will give the correct values for part (c) but should score (b) M1A0 (c) M1A0A0. **Alt** **M1:** $\frac{dy}{dx} = -3\tan\left(\arcsin 3x\right) = 4 \Rightarrow \tan\left(\arcsin 3x\right) = -\frac{4}{3} \Rightarrow \sin\left(\arcsin 3x\right) = -\frac{4}{5} \Rightarrow 3x = \ldots$ Uses the alt form in trig functions, set equal to 4 and proceeds to use right angle triangle or trig identities to proceed to a value for $3x$ **A1:** Correct value for $3x$ **A1:** Correct value for x

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