

# CAMBRIDGE INTERNATIONAL EXAMINATIONS

## Cambridge International AS & A Level Mathematics

**Paper Details:** Paper 1 Pure Mathematics 1 (9709/12)

**Series:** February/March 2026

**Document Type:** Published Mark Scheme (Confidential)

**Total Marks:** 75 Marks

### Mark Scheme Notes & Abbreviations:

- **M1**: Method mark, awarded for a valid method applied to the problem.
- **A1**: Accuracy mark, dependent on a previous M mark.
- **B1**: Independent mark for a correct statement or step.
- **FT**: Follow through of a previous incorrect result.
- All non-exact numerical answers must be given to 3 significant figures unless specified otherwise.

## DETAILED SOLUTIONS AND MARK ALLOCATIONS

Q	ANSWERING DETAILS / SCHEME	MARKS	GUIDANCE
1	Differentiate with respect to $x$ : $dy/dx = 3x^2 - 6x$	<b>B1</b>	Correct derivative.
	Set $dy/dx = 0 \Rightarrow 3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$ . Find coordinates: <ul style="list-style-type: none"><li>• When <math>x = 0</math>, <math>y = 9 \Rightarrow (0, 9)</math></li><li>• When <math>x = 2</math>, <math>y = 2^3 - 3(2)^2 + 9 = 5 \Rightarrow (2, 5)</math></li></ul> Determine nature using the second derivative: $d^2y/dx^2 = 6x - 6$ <ul style="list-style-type: none"><li>• At <math>x = 0</math>, <math>d^2y/dx^2 = -6 &lt; 0 \Rightarrow (0, 9)</math> is a <b>Local Maximum</b>.</li><li>• At <math>x = 2</math>, <math>d^2y/dx^2 = 6 &gt; 0 \Rightarrow (2, 5)</math> is a <b>Local Minimum</b>.</li></ul>	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>	Set to 0 and solve. Both coordinates correct. Second derivative test. Both natures correct.
2	<b>(a) Perimeter of shaded region:</b> <ul style="list-style-type: none"><li>• Arc Length = <math>r\theta = 9 \times 0.8 = 7.2</math> cm.</li><li>• Remaining segment <math>AB = 12 - 9 = 3</math> cm.</li><li>• Calculate length <math>BC</math> using the cosine rule: <math>BC^2 = 12^2 + 9^2 - 2(12)(9) \cos(0.8)</math> <math>BC^2 = 145 + 81 - 216 \times 0.6967 = 74.511</math></li></ul>	<b>B1</b> <b>M1</b> <b>A1</b> <b>A1</b>	Arc length correct. Cosine rule for $BC$ . $BC$ correct. Final answer to 3 sf.

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	$BC = \sqrt{74.511} \approx 8.632$ cm. • Total Perimeter = $7.2 + 3 + 8.632 = 18.832 \approx 18.8$ cm.		
	<b>(b) Area of shaded region:</b> • Area of triangle $ABC = 0.5 \times 12 \times 9 \times \sin(0.8)$ $Area_{\Delta} = 54 \times 0.71736 \approx 38.737$ cm <sup>2</sup> . • Area of sector = $0.5 \times r^2 \theta = 0.5 \times 9^2 \times 0.8 = 32.4$ cm <sup>2</sup> . • Shaded Area = $38.737 - 32.4 = 6.337 \approx 6.34$ cm <sup>2</sup> .	<b>M1</b> <b>B1</b> <b>A1</b>	Triangle area formula. Sector area correct. Final answer to 3 sf.
3	<b>(a) Find constants:</b> Using vertex form: $y = a(x - 3)^2 - 5$ Substitute $(0, 31) \Rightarrow 31 = a(0 - 3)^2 - 5$ $36 = 9a \Rightarrow a = 4$ . Expand to standard form: $y = 4(x^2 - 6x + 9) - 5 = 4x^2 - 24x + 31$ . Thus, $a = 4, b = -24, c = 31$ .	<b>M1</b> <b>A1</b> <b>A1</b> <b>A1</b>	Use vertex form. Find a = 4. Find b = -24. Find c = 31.
	<b>(b) Translation by &amp;binom;-2;7:</b> New vertex is $(3 - 2, -5 + 7) = (1, 2)$ . Equation: $y = 4(x - 1)^2 + 2 \Rightarrow y = 4x^2 - 8x + 6$ .	<b>M1</b> <b>A1</b>	Apply shift to vertex. Correct final form.
4	<b>(a) Find inverse function:</b> Let $y = (2x + 1)/(x - 3) \Rightarrow y(x - 3) = 2x + 1$ $xy - 3y = 2x + 1 \Rightarrow xy - 2x = 3y + 1$ $x(y - 2) = 3y + 1 \Rightarrow x = (3y + 1)/(y - 2)$ $f^{-1}(x) = (3x + 1)/(x - 2)$	<b>M1</b> <b>A1</b> <b>A1</b>	Rearrange to isolate x. Correct algebra steps. Correct expression.
	<b>(b) Composite function ff(x):</b> $ff(x) = [2((2x+1)/(x-3)) + 1] / [((2x+1)/(x-3)) - 3]$ Multiply numerator and denominator by $(x - 3)$ : Numerator: $2(2x + 1) + 1(x - 3) = 4x + 2 + x - 3 = 5x - 1$ Denominator: $(2x + 1) - 3(x - 3) = 2x + 1 - 3x + 9 = 10 - x$ $ff(x) = (5x - 1)/(10 - x) \therefore p = 5, q = -1, r = 10, s = -1$ .	<b>M1</b> <b>A1</b> <b>A1</b>	Substitute f(x) into f. Simplify algebraic fraction. Correct integers values.
5	<b>(a) Identity Proof:</b> $LHS = 1/(1 - \sin\theta) + 1/(1 + \sin\theta)$ Combine over common denominator: $LHS = [(1 + \sin\theta) + (1 - \sin\theta)] / [(1 - \sin\theta)(1 + \sin\theta)]$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>	Common denominator. Correct numerator

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	<p>LHS = <math>2 / (1 - \sin^2\theta)</math>            Since <math>1 - \sin^2\theta = \cos^2\theta</math>:            LHS = <math>2 / \cos^2\theta = \text{RHS}</math>.</p> <p><b>(b) Solve equation:</b>  <math>3 [1/(1-\sin\theta) + 1/(1+\sin\theta)] = 12 \Rightarrow 3 (2/\cos^2\theta) = 12</math>  <math>6 / \cos^2\theta = 12 \Rightarrow \cos^2\theta = 0.5</math>  <math>\cos\theta = \pm 1/\sqrt{2}</math>            For <math>0^\circ \leq \theta \leq 360^\circ</math>:            • <math>\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ</math></p>		<p>expansion.            Use of trig identity.            Completed proof.</p>
6	<p><b>(a) Circle Properties:</b></p> <ul style="list-style-type: none"> <li>• Centre = <math>(0, 0)</math></li> <li>• Radius = <math>\sqrt{(45/4)} = 3\sqrt{5} / 2</math></li> </ul> <p><b>(b) Intersection distance:</b>            Substitute <math>y = 1.5 - x</math> into circle equation:  <math>x^2 + (1.5 - x)^2 = 11.25</math>  <math>x^2 + 2.25 - 3x + x^2 = 11.25</math>  <math>2x^2 - 3x - 9 = 0 \Rightarrow (2x + 3)(x - 3) = 0</math></p> <ul style="list-style-type: none"> <li>• Points: <math>x = 3 \Rightarrow y = -1.5 \Rightarrow A(3, -1.5)</math></li> <li>• Points: <math>x = -1.5 \Rightarrow y = 3 \Rightarrow B(-1.5, 3)</math></li> </ul> <p>Distance <math>AB = \sqrt{[(3 - (-1.5))]^2 + (-1.5 - 3)^2}</math>  <math>AB = \sqrt{[4.5^2 + (-4.5)^2]} = \sqrt{[20.25 + 20.25]} = \sqrt{40.5} = 9\sqrt{2} / 2</math></p>	<p><b>B1</b> <b>B1</b></p> <p><b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b></p>	<p>Correct centre.            Correct exact radius.</p> <p>Substitution strategy.            Correct quadratic form.            Solve for x coordinates.            Find y coordinates.            Distance formula used.            Exact value fully simplified.</p>
7	<p><b>(a) Find y in terms of x:</b>            Expand numerator: <math>dy/dx = (x^2 - 6x + 9) / x^{1/2} = x^{3/2} - 6x^{1/2} + 9x^{-1/2}</math>            Integrate term by term:  <math>y = (2/5)x^{5/2} - 4x^{3/2} + 18x^{1/2} + C</math>            Substitute <math>(4, 7)</math>:  <math>7 = (2/5)(32) - 4(8) + 18(2) + C</math>  <math>7 = 12.8 - 32 + 36 + C \Rightarrow 7 = 16.8 + C \Rightarrow C = -9.8</math> (or <math>-49/5</math>)  <math>y = 0.4x^{5/2} - 4x^{3/2} + 18x^{1/2} - 9.8</math></p>	<p><b>M1</b> <b>A1</b> <b>A1</b> <b>M1</b> <b>A1</b></p>	<p>Split terms properly.            At least two terms correct.            All integration correct.            Substitute point to find C.            Correct equation.</p>

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	<p><b>(b) Transformation:</b></p> <p>The graph is translated vertically. Curve D passes through (4, k) while C passes through (4, 7).</p> <p><b>Translation by vector:</b> <math>\begin{pmatrix} 0 \\ k - 7 \end{pmatrix}</math></p>	<p><b>M1</b></p> <p><b>A1</b></p>	<p>Recognize vertical translation.</p> <p>State vector or description.</p>
8	<p><b>(a) Binomial Expansion:</b></p> $(3 - 2x)^4 = 3^4 + 4(3^3)(-2x) + 6(3^2)(-2x)^2 + 4(3)(-2x)^3 + (-2x)^4$ $= 81 - 216x + 216x^2 - 96x^3 + 16x^4$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Apply binomial theorem.</p> <p>First three terms correct.</p> <p>Fully correct simplified form.</p>
	<p><b>(b) Substitute <math>x = \sqrt{2}</math>:</b></p> $= 81 - 216\sqrt{2} + 216(\sqrt{2})^2 - 96(\sqrt{2})^3 + 16(\sqrt{2})^4$ $= 81 - 216\sqrt{2} + 216(2) - 96(2\sqrt{2}) + 16(4)$ $= 81 - 216\sqrt{2} + 432 - 192\sqrt{2} + 64$ <p>Combine terms: <math>577 - 408\sqrt{2} \therefore p = 577, q = -408</math>.</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Substitute <math>x = \sqrt{2}</math>.</p> <p>Simplify coefficients.</p> <p>Correct exact integers values.</p>
9	<p>Rewrite the equation: <math>x^{2/3} + 2x^{1/3} - 24 = 0</math></p> <p>Let <math>u = x^{1/3} \Rightarrow u^2 + 2u - 24 = 0</math></p> <p>Factorize: <math>(u + 6)(u - 4) = 0 \Rightarrow u = 4</math> or <math>u = -6</math></p> <p>Solve for x:</p> <ul style="list-style-type: none"> <li><math>x^{1/3} = 4 \Rightarrow x = 4^3 = 64</math></li> <li><math>x^{1/3} = -6 \Rightarrow x = (-6)^3 = -216</math></li> </ul>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Recognize as quadratic in <math>x^{1/3}</math>.</p> <p>Obtain <math>u = 4, -6</math>.</p> <p>Cube to find x values.</p> <p>Both answers correct.</p>
10	<p><b>(a) Find 20th term of AP:</b></p> $u_7 = a + 6d = 77$ $S_{11} = (11/2)(2a + 10d) = 11(a + 5d) = 880 \Rightarrow a + 5d = 80$ <p>Subtract the two simultaneous equations:</p> $(a + 6d) - (a + 5d) = 77 - 80 \Rightarrow d = -3$ <p>Find a: <math>a + 5(-3) = 80 \Rightarrow a = 95</math>.</p> <p>Calculate 20th term: <math>u_{20} = a + 19d = 95 + 19(-3) = 38</math>.</p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Correct equation for u7.</p> <p>Use of sum formula to get <math>a + 5d = 80</math>.</p> <p>Find a and d values.</p> <p>Correct 20th term.</p>
	<p><b>(b) Smallest N such that <math>S_N &lt; 50</math>:</b></p> $S_N = (N/2)[2(95) + (N - 1)(-3)] < 50$	<p><b>M1</b></p> <p><b>A1</b></p>	<p>Set up correct inequality.</p> <p>Obtain standard</p>

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	$N [190 - 3N + 3] < 100 \Rightarrow N(193 - 3N) < 100$ $3N^2 - 193N + 100 > 0$ Solving quadratic boundary equation: $N \approx 63.8$ or $0.52$ Test discrete integer values: <ul style="list-style-type: none"> <li>• When <math>N = 63</math>, <math>S_{63} = 31.5 \times (193 - 189) = 126</math></li> <li>• When <math>N = 64</math>, <math>S_{64} = 32 \times (193 - 192) = 32 &lt; 50</math></li> </ul> Therefore, the smallest number of terms is $N = 64$ .	<b>M1</b> <b>A1</b>	quadratic form. Test values or solve boundary. State $N = 64$ .
11	<b>(a) Equation of straight line:</b> $dy/dx = -3x^2 + 18x - 15 = 0 \Rightarrow x^2 - 6x + 5 = 0 \Rightarrow (x - 1)(x - 5) = 0$ . Stationary points at $x = 1$ and $x = 5$ . <ul style="list-style-type: none"> <li>• At <math>x = 1</math>, <math>y = -1 + 9 - 15 - 3 = -10 \Rightarrow (1, -10)</math></li> <li>• At <math>x = 5</math>, <math>y = -125 + 225 - 75 - 3 = 22 \Rightarrow (5, 22)</math></li> </ul> Gradient $m = (22 - (-10)) / (5 - 1) = 32 / 4 = 8$ . Equation of line: $y - 22 = 8(x - 5) \Rightarrow y = 8x - 18$ . (Shown)	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>A1</b>	Differentiate and set to 0. Find stationary $x$ values. Find corresponding $y$ values. Find gradient $m = 8$ . Fully shown correct equation.
	<b>(b) Verification:</b> At $x = 3$ : <ul style="list-style-type: none"> <li>• Line: <math>y = 8(3) - 18 = 6</math></li> <li>• Curve: <math>y = -(3)^3 + 9(3)^2 - 15(3) - 3 = -27 + 81 - 45 - 3 = 6</math></li> </ul> Since both give $y = 6$ , they meet at $x = 3$ .	<b>B1</b>	Valid verification.
	<b>(c) Shaded Area:</b> The shaded area is bounded between $x = 3$ and $x = 5$ where the curve is above the straight line. $\text{Area} = \int_3^5 [(-x^3 + 9x^2 - 15x - 3) - (8x - 18)] dx$ $= \int_3^5 (-x^3 + 9x^2 - 23x + 15) dx$ Integrate: $[-x^4/4 + 3x^3 - 23x^2/2 + 15x]_3^5$ <ul style="list-style-type: none"> <li>• Upper Limit (5): <math>-625/4 + 375 - 575/2 + 75 = 6.25</math></li> <li>• Lower Limit (3): <math>-81/4 + 81 - 207/2 + 45 = 2.25</math></li> </ul> $\text{Area} = 6.25 - 2.25 = 4$	<b>M1</b> <b>A1</b> <b>A1</b> <b>M1</b> <b>A1</b>	Set up correct integral limits. Correct combined function. Correct term-by-term integration. Substitute limits accurately. Correct final area value = 4.