9.4 Resolution

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Course	DP IB Physics
Section	9. Wave Phenomena (HL only)
Topic	9.4 Resolution
Difficulty	Medium

To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

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The correct answer is **A** because:

- The ability to resolve two objects through a circular aperture depends on the angular separation, θ, and the Rayleigh criterion
  - The smaller the value of θ according to the Rayleigh criterion, the greater the resolution
- The Rayleigh criterion is equal to the angular separation when the images can just be resolved
- The Rayleigh criterion is determined by
  - $\theta = 1.22\frac{\lambda}{b}$
  - Where θ = angular separation, λ = wavelength of light with b = lens diameter
- Therefore, $\theta \propto \lambda$
- Blue light has a shorter wavelength than red light
  - Therefore, θ will be smaller and the resolution improved

B is incorrect as	lens diameter, b, and angular separation, θ, are related by $\theta \propto \frac{1}{b}$, therefore, if the diameter, b, decreases then θ increases. An increase in θ means a decrease in the ability to resolve two objects
C is incorrect as	angular separation, θ, and distance between the objects, d, are related by $\theta \propto \frac{1}{d}$, therefore, a decrease in d leads to an increase in θ. Hence, it is harder to resolve the two objects
D is incorrect as	a change in lens material will not change the resolving power, as long as the refractive index is constant

2

The correct answer is **C** because:

- The angle at which the two stars can just be resolved is determined by the Rayleigh criterion, θ
  - $\theta = 1.22\frac{\lambda}{b}$
- List the known quantities:
  - Wavelength of light, $\lambda = 3\text{ mm} = 3 \times 10^{-3}\text{ m}$
  - Diameter of the telescope collection dish, $b = 30\text{ cm} = 30 \times 10^{-2}\text{ m}$
- Substitute in values:
  - $\theta = 1.22\frac{(3 \times 10^{-3})}{(30 \times 10^{-2})} = 1.22 \times \frac{1}{100}$
  - Therefore, $\theta = 0.0122$ rad

A is incorrect as	$\frac{\lambda}{b} = 0.01$ has not been multiplied by 1.22 which is required for a circular aperture
B is incorrect as	the diameter of the telescope collection dish has not been converted from cm to m
D is incorrect as	$\frac{\lambda}{b} = 0.01$ has been divided by 1.22 rather than multiplied by 1.22 as required by the equation for the Rayleigh criterion

3

The correct answer is **C** because:

- The resolution of two objects is possible when the angular separation is greater than or equal to the Raleigh criterion, θ
- $\theta \geq 1.22\frac{\lambda}{b}$
  - Where λ is the wavelength of light and b is the diameter of the aperture

- The diameter of a circle is equal to twice the radius
  - $b = 2r$
- Substituting in 2r for b gives:
  - $\theta \geq 1.22\frac{\lambda}{2r}$
- Therefore, $\theta \geq 0.61\frac{\lambda}{r}$

A is incorrect as	the radius, r, has not been multiplied by 2
B is incorrect as	the radius, r, has been squared rather than multiplied by 2
D is incorrect as	the operator ≤ has been used rather than ≥

4

The correct answer is **B** because:

- The magnitude of the Rayleigh criterion will equal the angular separation, θ at the point the galaxies can just be resolved
- The Rayleigh criterion is given by $\theta = 1.22\frac{\lambda}{b}$
- List the known quantities:
  - Wavelength of light, $\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{ m}$
  - Angular separation, $\theta = 1 \times 10^{-6}\text{ rad}$
  - Diameter of the telescope's collection dish = b
- Rearrange to make b, the subject
  - $b = 1.22\frac{\lambda}{\theta}$
- Substitute in values:
  - $b = 1.22\frac{(500 \times 10^{-9})}{(1 \times 10^{-6})} = 1.22 \times 0.5$
  - Therefore, $b = 0.61\text{ m}$

A, **C** & **D** are incorrect as	the wrong orders of magnitude have been used for the wavelength of light

Calculation errors involving orders of magnitude are very easy to make. Make sure you check prefixes and powers of 10 very carefully!

5

The correct answer is **A** because:

- The resolvance, R, of the diffraction grating can be calculated using $R = \frac{\lambda}{\Delta\lambda}$
- λ is the average wavelength
  - $\lambda = \frac{(500 + 400)}{2} = 450 \text{ nm} = 450 \times 10^{-9} \text{ m}$
- $\Delta\lambda$ is the change in wavelength
  - $\Delta\lambda = 500 - 400 = 100 \text{ nm} = 100 \times 10^{-9} \text{ m}$
- Therefore, $R = \frac{\lambda}{\Delta\lambda} = \frac{(450 \times 10^{-9})}{(100 \times 10^{-9})} = 4.5$
- R is also given by $R = N \times m$, where:
  - N = number of lines illuminated
  - m = order of diffraction
- Rearranging for N:
  - $N = \frac{R}{m}$
- Substituting in values for R and m gives:
  - $N = \frac{4.5}{2}$
- Therefore, number of lines illuminated = 2.25

B is incorrect as	the values of wavelength have been added together, λ = 500 + 400 = 900 nm, rather than subtracted, λ = 500 – 400 = 100 nm **and** 1 has been used as the the order of diffraction, rather than 2

C is incorrect as	the values of wavelength have been added together, $\lambda = 500 + 400 = 900$ nm, rather than subtracted, $\lambda = 500 - 400 = 100$ nm
D is incorrect as	1 has been used as the the order of diffraction, rather than 2

6

The correct answer is **D** because:

- According to the Rayleigh criterion the two sources of wavelength, λ through a single slit, b are just resolvable when the angle is equal to that of the first diffraction minimum, θ:
  - $\theta = \frac{\lambda}{b}$
- For a circular aperture this value is multiplied by 1.22:
  - $\theta = 1.22\frac{\lambda}{b}$
- Therefore, replacing the circular aperture with a narrow slit will enable the beams to be resolved

A is incorrect as	moving the laser beams further away from the aperture reduces the angular separation. Since resolution requires the angular separation to be greater than or equal to the Rayleigh criterion this change makes the sources harder to resolve
B is incorrect as	using red light will make the sources harder to resolve since an increase in wavelength will result in an increase in the magnitude of the Rayleigh criterion
C is incorrect as	decreasing the size of the aperture will result in an increase in the magnitude of the Rayleigh criterion, and so the sources will be harder to resolve

To answer questions with qualitative answers, it is often necessary to use the relevant equations. In this case these equations are:

- $\theta = \frac{s}{d}$, where s is the separation between the objects and d is the distance between the objects and the observer
- $\theta = 1.22\frac{\lambda}{b}$, where λ is the wavelength of light and b is the diameter of the aperture

By considering the relationship between θ and the other variables, it is possible to consider the effect of changing each variable.

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The correct answer is **C** because:

- The images can just be resolved, therefore the Rayleigh criterion applies
  - $\theta = 1.22\frac{\lambda}{b}$
  - $\theta = 1.22 \times \frac{(600 \times 10^{-9})}{(2.0 \times 10^{-3})}$
  - $\theta = 1.22 \times (300 \times 10^{-6}) = 3.66 \times 10^{-4}$ rad
- When the images can just be resolved, the Rayleigh criterion is equal to the angular separation, θ
  - $\theta = \frac{s}{d}$
    - $d = 2$ km $= 2000$ m
    - $s =$ distance between two bushes
  - $s = \theta \times d$
  - $s = (3.66 \times 10^{-4}) \times 2000 = 0.732$ m

A is incorrect as	3.66×10^{-4} m is the *magnitude* of the angular separation, θ, not the separation of the two bushes, s
B is incorrect as	km has not been converted to m

D is incorrect as	nm have not been converted to m for the wavelength **nor** mm to m for the diameter of the person's pupils **and** then the magnitude of the angular separation, θ, has been given, not the separation between the two bushes, s

8

The correct answer is **C** because:

- The number of lines, N, illuminated by the beam can be found by:
  - N = width of beam in mm × number of lines per mm
  - $N = 0.2 \times 100 = 20$
- Resolving power, R is found by:
  - $R = N \times m$
- Resolving power is also related to the wavelength:
  - $R = \frac{\lambda}{\Delta\lambda}$
    - where λ = average wavelength and $\Delta\lambda$ = change in wavelength
- These two equations can be combined:
  - $N \times m = \frac{\lambda}{\Delta\lambda}$
  - $\Delta\lambda \times (N \times m) = \lambda$
  - $\Delta\lambda = \frac{\lambda}{(N \times m)} = \frac{(600 \times 10^{-9})}{(20 \times 3)} = \frac{(600 \times 10^{-9})}{60}$
  - $\Delta\lambda = 10 \times 10^{-9} = 10$ nm
- The two wavelengths can be determined by $\lambda \pm \frac{\Delta\lambda}{2}$
  - The average wavelength is 600 nm and $\Delta\lambda$ is 10 nm
  - $\lambda_1 = 600 + \frac{10}{2} = 605$ nm
  - $\lambda_2 = 600 - \frac{10}{2} = 595$ nm

A is incorrect as	$\lambda \pm \Delta\lambda$, has been used, rather than $\lambda \pm \frac{\Delta\lambda}{2}$
B is incorrect as	the first order spectrum, $m = 1$, has been used. This then gives $\Delta\lambda = \frac{(600 \times 10^{-9})}{20} = 30$ nm
D is incorrect as	the first order spectrum has been used **and** $\lambda \pm \Delta\lambda$, has been used, rather than $\lambda \pm \frac{\Delta\lambda}{2}$

9

The correct answer is **D** because:

- Resolvance, *R*, is given by the product of the number of lines, *N*, illuminated by the beam and the order of the spectrum
- The order of the spectrum in this example is constant
  - Therefore only *N* will affect the resolvance

- *N* = width of the beam in mm × number of lines per mm
  - Therefore, the maximum resolvance is found when *W* × *L* is **largest**
- The values for line D give the largest product:
  - 1.5 × 500 = 750 lines illuminated

A is incorrect as	the product of W × L = 3.0 × 200 = 600 lines illuminated. This is fewer than 750 lines, therefore A does not have the highest resolvance.
B is incorrect as	the product of W × L = 4.0 × 100 = 400 lines illuminated. This is fewer than 750 lines, therefore B does not have the highest resolvance.
C is incorrect as	the product of W × L = 1.0 × 350 = 350 lines illuminated. This is fewer than 750 lines, therefore C does not have the highest resolvance.

10

The correct answer is **A** because:

- Resolution is determined by the magnitude of the Rayleigh criterion, θ:
  - $\theta = 1.22\frac{\lambda}{b}$ where λ = wavelength of light (m) and b = diameter of slit (m)
- A higher resolution is possible when the magnitude of θ is as small as possible
- The equation tells us that $\theta \propto \frac{1}{b}$
  - Therefore an increased diameter leads to greater resolution
  - This eliminates options **C** and **D**
- The equation tells us that $\theta \propto \lambda$
  - Since $\lambda \propto \frac{1}{f}$, $\theta \propto \frac{1}{f}$
  - Therefore an increase in frequency leads to a greater resolution, hence **A** is correct