9.2 Classifications of Star

ASTROPHYSICS

AQA A Level Revision Notes

A Level Physics AQA

9.2 Classification of Stars

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9.2.1 Brightness & Apparent Magnitude

Brightness & Apparent Magnitude

- Apparent magnitude *m* is defined as

  The perceived brightness of a star as seen from Earth

- The value of *m* is a number with no unit
- The **Hipparcos scale** of apparent magnitude initially classified brightness by assigning values from 1.0 to 6.0, where
  - Magnitude 1.0 represented the **brightest** stars that could be seen with the naked eye
  - Magnitude 6.0 represented dimmer stars that were **only just visible** to the naked eye

- As astronomy progressed, the Hipparcos scale was able to be defined more precisely as a **logarithmic** scale
- This allows stars to be compared meaningfully in terms of their brightness, or **intensity**, such that:

  A magnitude 1 star is 100 times brighter than a magnitude 6 star

- This means that a change of 5 orders of magnitude corresponds to:
  - m = 6 → m = 1: an **increase** in brightness by a factor of 100
  - m = 1 → m = 6: a **decrease** in brightness by a factor of 100

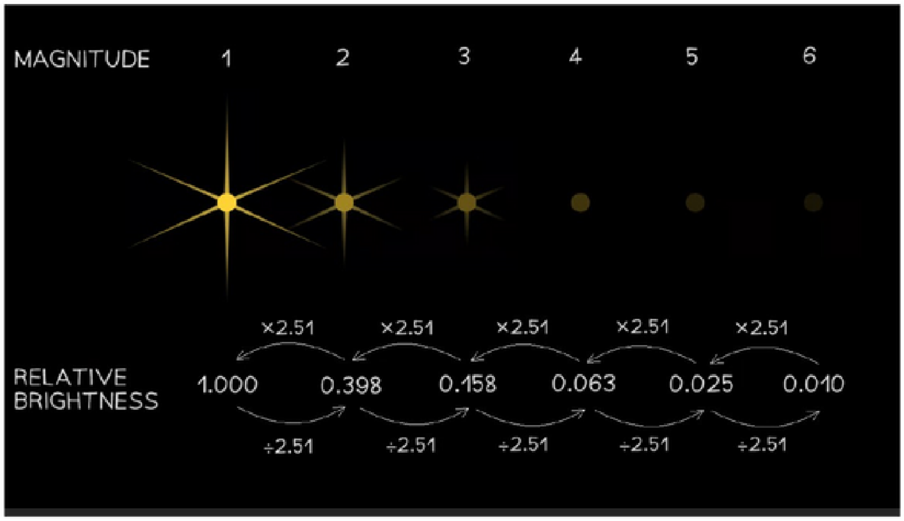
- Therefore, for a change of 1 order of magnitude:
  - Magnitude decreases by 1 (e.g. m = 6 → m = 5): brightness **increases** by a factor of $100^{\frac{1}{5}} \approx 2.51$
  - Magnitude increases by 1 (e.g. m = 1 → m = 2): brightness **decreases** by a factor of $100^{\frac{1}{5}} \approx 2.51$

- To compare the brightness of two objects, A and B, we can write an expression in terms of the ratio of their intensities:

$$\frac{I_A}{I_B} = 2.51^{(m_B - m_A)}$$

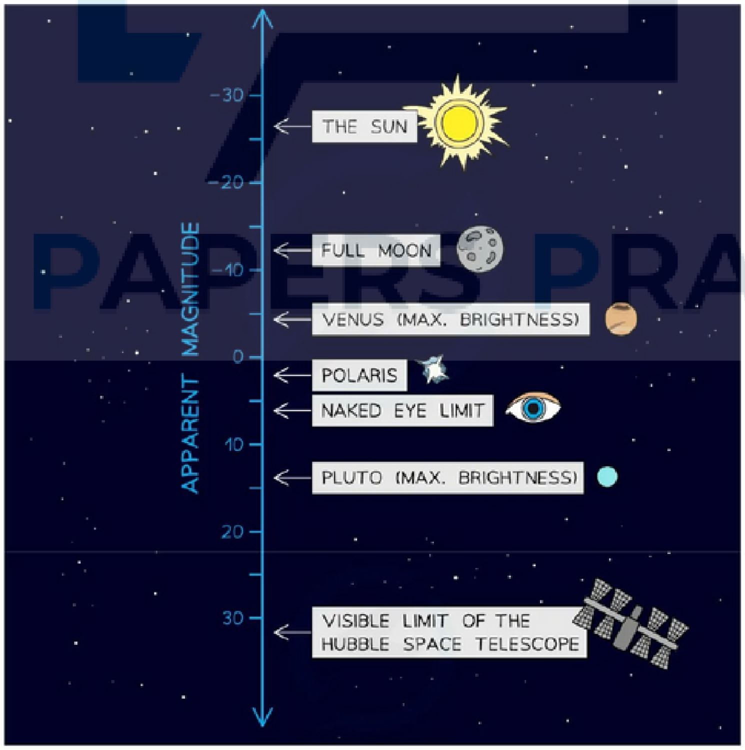
- Where:
  - I_A = intensity of object A (W m^{-2})
  - I_B = intensity of object B (W m^{-2})
  - m_A = apparent magnitude of object A
  - m_B = apparent magnitude of object B

The Relationship between Apparent Magnitude & Brightness



- Since the invention of the telescope, the scale has been expanded, so that
    - The **more negative** the apparent magnitude, the **brighter** an object appears e.g. the Sun has an apparent magnitude of –26
    - The **more positive** the apparent magnitude, the **fainter** an object appears e.g. Pluto has an apparent magnitude of 15

Apparent Magnitudes of Common Astronomical Objects



Worked Example

The constellation of Centaurus contains two triple star systems, Alpha Centauri and Beta Centauri.

Alpha Centauri is made up of the binary star pair Alpha Centauri A and B along with Proxima Centauri, the closest star to the Earth after the Sun.

Beta Centauri also contains a binary star pair Beta Centauri Aa and Ab, along with Beta Centauri B.

To the naked eye, some of these stars appear to be a single star. The combined apparent magnitudes are shown in the table.

Star	Apparent magnitude
Alpha Centauri A	−0.27
Alpha Centauri B	
Proxima Centauri	11.1
Beta Centauri Aa	0.61
Beta Centauri Ab	
Beta Centauri B	

Compare the stars as seen by a naked-eye observer on Earth. Support your answer with suitable calculations.

Answer:

Step 1: Identify the brightest and dimmest objects

- The **higher** the magnitude, the **dimmer** the star
    - Proxima Centauri has the highest apparent magnitude
    - Therefore, Proxima Centauri appears the dimmest
- The **lower** the magnitude, the **brighter** the star
    - The binary pair Alpha Centauri A and B have the lowest apparent magnitude
    - Therefore, Alpha Centauri A and B appear the brightest

Step 2: Identify how many stars will be visible to the naked eye

- Only <u>two</u> points of light will be visible:
    - Alpha Centauri A and B appear as a single star
    - All three stars in Beta Centauri appear as a single star
- According to the Hipparcos scale, magnitude 6 is the dimmest star that can be observed by the naked eye
    - Therefore, Proxima Centauri will <u>not</u> be visible

Step 3: Make comparisons between the intensities of the stars

- The ratio of the brightness, or intensities, of two objects is given by

$$\frac{I_A}{I_B} = 2.51^{(m_B - m_A)}$$

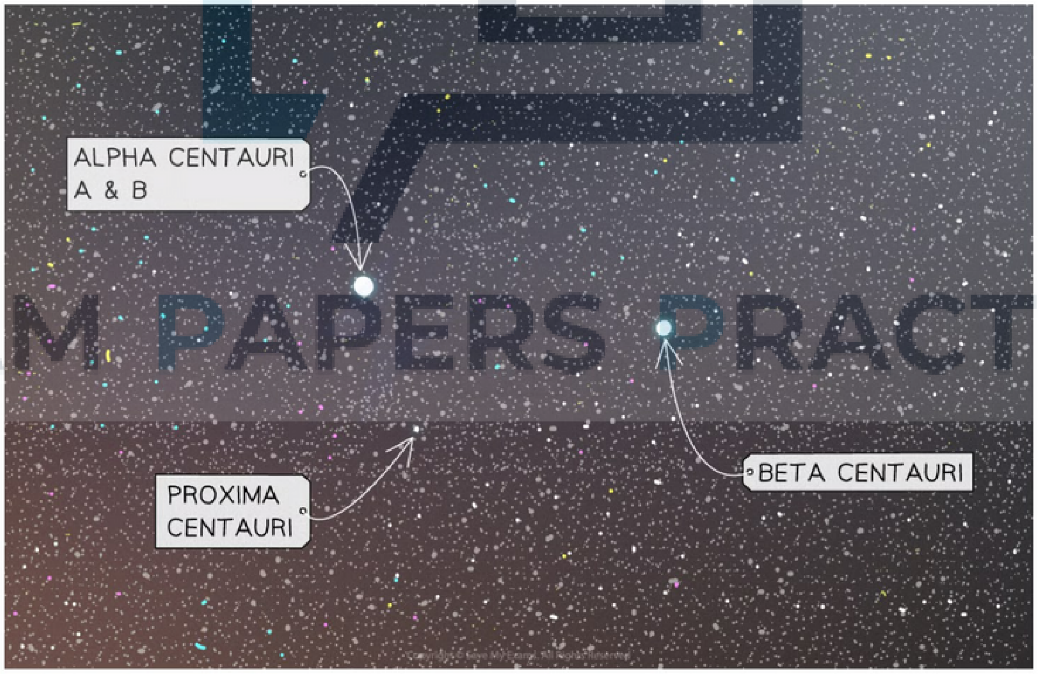
- Where object **A** = Alpha Centauri A and B (appearing as one star), and object **B** = Beta Centauri
- The difference in magnitude between A and B is

$$m_B - m_A = 0.61 - (-0.27) = 0.88$$

- The difference in brightness is therefore:

$$\frac{I_A}{I_B} = 2.51^{0.88} = 2.25$$

- This means that Alpha Centauri AB appears 2.25 times brighter than Beta Centauri



Exam Tip

The change in intensity equation is **not** given on your data sheet, so you must remember this.

9.2.2 Inverse Square Law of Radiation

Brightness & Luminosity

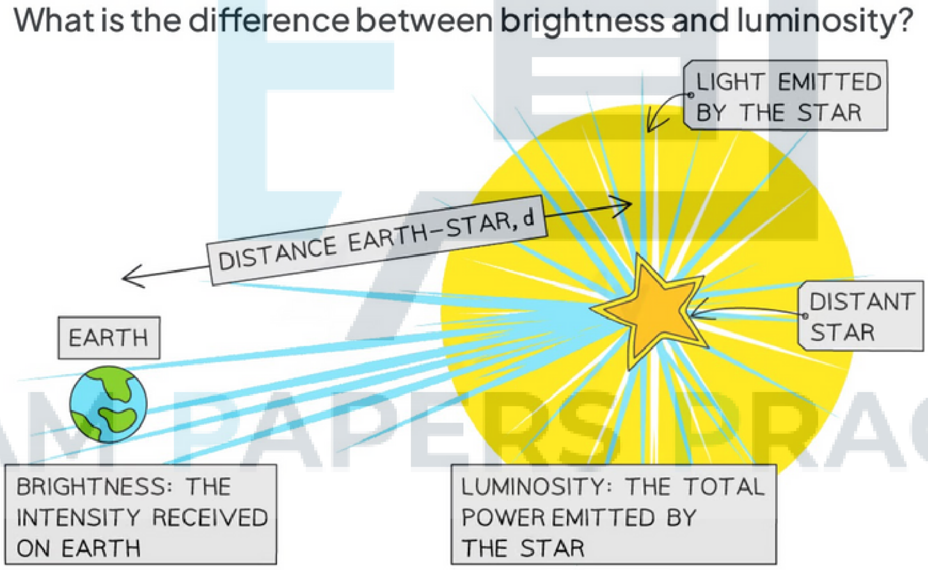
- How much light the star emits is given by its luminosity *L*, which is defined as:

The total power output of radiation emitted by a star

- Luminosity is measured in units of **watts (W)**
- The brightness of a star is defined as:

The intensity of radiation received on Earth from a star

- Brightness is equivalent to power per unit area, or light intensity, and is measured in **watts per metre squared (W m^{-2})**
- The brightness of a star depends on two main factors:
  - How much light the star emits (i.e. its **luminosity**)
  - How **far away** the star is (more distant stars are usually fainter than nearby stars)



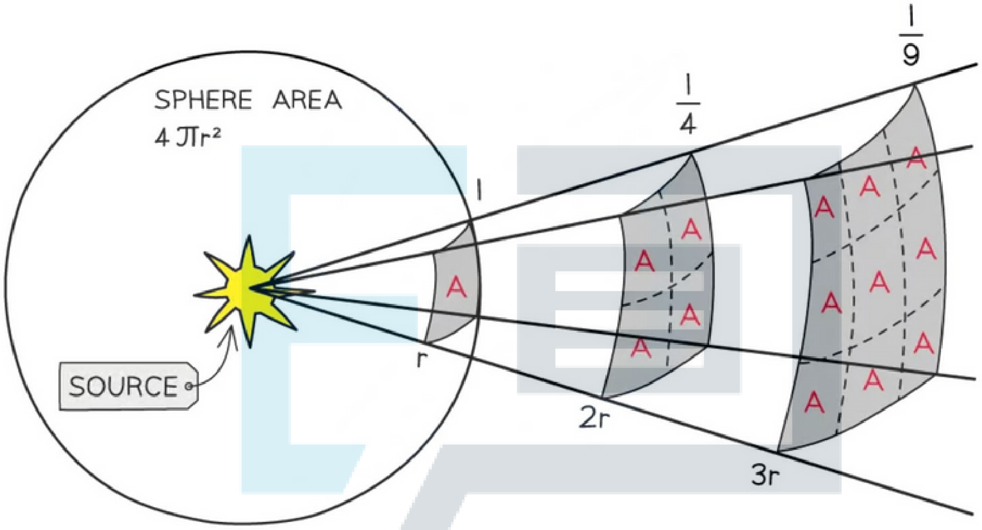
The luminosity is the total power output of the star, whereas the brightness is the power as measured on Earth

- Knowing the luminosity and brightness of a star is useful because it allows us to determine how **far away** it is from the Earth, as
  - Luminosity tells us how bright the star is at its **surface**
  - Brightness tells us how bright the star is as observed from the **Earth**
- Therefore, by the time the radiation from the distant star reaches the Earth, it will have spread out over a **very large area**
  - This means the intensity of the radiation detected on Earth will only be a **fraction** of the value of the star's luminosity

Inverse Square Law of Radiation

- Light sources which are **farther** away appear **fainter** because the light it emits is **spread out** over a greater area
- The moment the light leaves the surface of the star, it begins to spread out uniformly through a **spherical** shell
  - The surface area of a sphere is equal to $4\pi r^2$
- The radius *r* of this sphere is equal to the distance *d* between the star and the Earth
  - By the time the radiation reaches the Earth, it has been spread over an area of $4\pi d^2$

Inverse square law of radiation



When the light is twice as far away, it has spread over four times the area, hence the intensity is four times smaller

- The **inverse square law of radiation** can be calculated using:

$$I = \frac{L}{4\pi d^2}$$

- Where:
  - I = apparent brightness, or observed intensity on Earth (W m^{-2})
  - L = luminosity of the source (W)
  - d = distance between the star and the Earth (m)
- This equation assumes:
  - The source can be treated as a **point**
  - The power from the source radiates **uniformly** through space
  - No radiation is **absorbed** or **scattered** between the star and the Earth
- This equation tells us:
  - For a given star, the luminosity is **constant**
  - The intensity of the emitted light follows an inverse square law

  - For stars with the **same luminosity**, the star with the **greater** apparent brightness is **closer** to the Earth

Worked Example

A star has a known luminosity of 9.7×10^{27} W. Observations of the star show that the intensity of light received on Earth from the star is 114 nW m^{-2}.

Determine the distance of the star from Earth.

Answer:

Step 1: Write down the known quantities

- Luminosity, $L = 9.7 \times 10^{27}$ W
- Intensity, $I = 114 \text{ nW m}^{-2} = 114 \times 10^{-9} \text{ W m}^{-2}$

Step 2: Write down the inverse square law of radiation and rearrange for distance d

$$I = \frac{L}{4\pi d^2}$$

$$d = \sqrt{\frac{L}{4\pi I}}$$

Step 3: Substitute in the values and calculate the distance d

$$d = \sqrt{\frac{9.7 \times 10^{27}}{4\pi \times (114 \times 10^{-9})}}$$

distance, $d = 8.2 \times 10^{16}$ m

Exam Tip

Don't forget to **square** the distance in any calculations, or take the **square root** if you need to calculate the distance, like in the worked example.

Astronomical Distances

- Astronomical distances are very large and as a result, are usually measured using:
  - Astronomical Units (AU)
  - Light-years (ly)
  - Parsecs (pc)

Astronomical Unit (AU)

- The astronomical unit (AU) is defined as

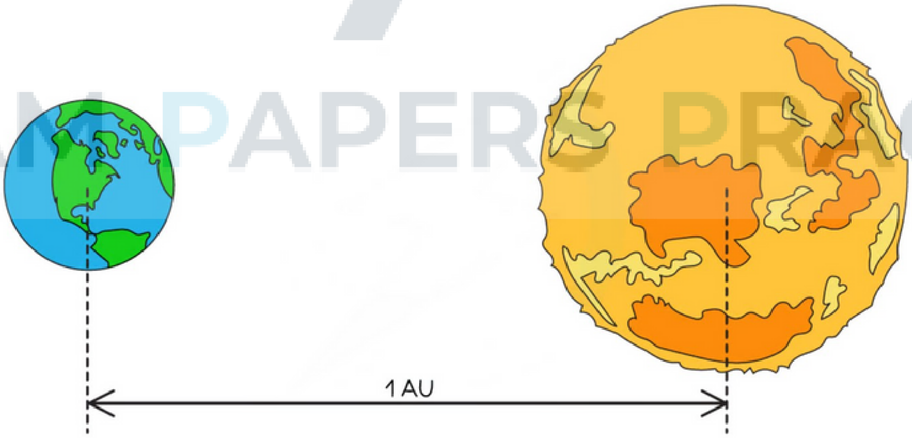
The mean distance from the centre of the Earth to the centre of the Sun

- As the Earth's orbit around the Sun is **elliptical** it will be
  - Slightly closer to the Sun in January (1.471×10^{11} m)
  - Slightly further away from the Sun in July (1.521×10^{11} m)
- Calculating the mean of these two values gives:

$$\frac{(1.471 \times 10^{11}) + (1.521 \times 10^{11})}{2} = 1.496 \times 10^{11} \text{ m}$$

- Therefore, 1 astronomical unit (AU) = 1.496×10^{11} m = 1.5×10^{11} m
- The astronomical unit is useful for studying distances on the scale of the **solar system**

How Far is an Astronomical Unit?



An astronomical unit (AU) is the mean distance between the Earth and the Sun

Light-year (ly)

- A light-year is defined as:

The distance travelled by light in one year

- This can be calculated using:

Distance = speed × time

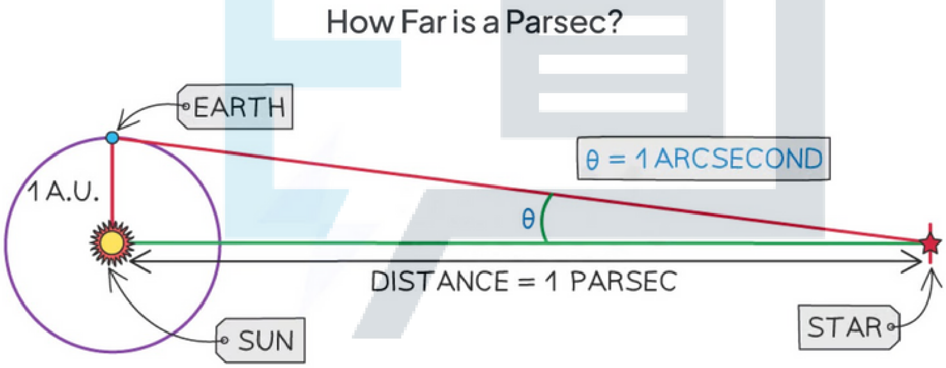
- Where:
  - The speed of light is 3×10^8 m s^{-1}
  - 1 year = 60 × 60 × 24 × 365 = 3.15×10^7 s
- Hence, the distance travelled by light in one year = $(3 \times 10^8) \times (3.15 \times 10^7) = 9.46 \times 10^{15}$ m
  - Therefore, 1 light-year ≈ 9.5×10^{15} m

Parsec (pc)

- Angles smaller than 1 degree can be measured in **arcminutes** or **arcseconds**
  - 1 degree = 60 arcminutes
  - 1 arcminute = 60 arcseconds
  - Therefore, 1 degree = 60 × 60 = 3600 arcseconds
  - 1 arcsecond = 1/3600 degree

- The parsec is defined as

> **The distance at which the radius of the Earth's orbit (1 AU) around the Sun subtends at an angle of 1 arcsecond**



A parsec is defined using parallax angles

- Given that 1 AU = 1.496×10^{11} m, trigonometry can be used to express 1 parsec in metres:

$$\tan\theta = \frac{opp}{adj} = \frac{1\ AU}{1\ pc}$$

$$\tan\left(\frac{1}{3600}\right) = \frac{1\ AU}{1\ pc}$$

$$1\,\text{pc} = \frac{1\ AU}{\tan\left(\frac{1}{3600}\right)} = \frac{1.496 \times 10^{11}}{\tan\left(\frac{1}{3600}\right)} = 3.09 \times 10^{16}\ \text{m}$$

- Therefore, 1 parsec ≈ 3.1×10^{16} m

- The parsec (1 pc = 3.1×10^{16} m) and the light-year (1 ly = 9.5×10^{15} m) are much **greater** in size than the astronomical unit (1 AU = 1.496×10^{11} m)
- This makes them useful for studying **interstellar distances**

- For example, on the scale of distances between the Earth and stars, or neighbouring galaxies

Worked Example

The closest star to Earth is a triple-star system called Alpha Centauri, which is approximately 4.35 light-years from Earth.

Calculate the distance between the Earth and Alpha Centauri in:

(a)
astronomical units (AU)
(b)
parsecs (pc)

1 astronomical unit (AU) = 1.496×10^{11} m.

Answer:

List the known quantities:

- Distance to Alpha Centauri = 4.35 ly
- 1 AU = 1.496×10^{11} m
- 1 light-year ≈ 9.5×10^{15} m (from data booklet)
- 1 parsec ≈ 3.1×10^{16} m (from data booklet)

- Convert 4.35 light-years into metres:

$$\text{distance} = 4.35\ \text{ly} = 4.35 \times (9.5 \times 10^{15})$$

$$\text{distance} = 4.13 \times 10^{16}\ \text{m}$$

(a) Convert from metres into AU:

$$\text{distance (AU)} = \frac{4.13 \times 10^{16}}{1.496 \times 10^{11}}$$

$$\text{distance (AU)} = 2.8 \times 10^{5}\ \text{AU (2 s.f)}$$

(b) Convert from metres into parsecs:

$$\text{distance (pc)} = \frac{4.13 \times 10^{16}}{3.1 \times 10^{16}}$$

$$\text{distance (pc)} = 1.3\ \text{pc (2 s.f)}$$

Exam Tip

You do not need to learn these conversion factors for astronomical distances, you just need to know how to use them. The following are given in the data booklet:

1 astronomical unit = 1.50×10^{11} m

1 light year = 9.46×10^{15} m

1 parsec = 2.06×10^{5} AU = 3.08×10^{16} m = 3.26 ly

9.2.4 Absolute Magnitude

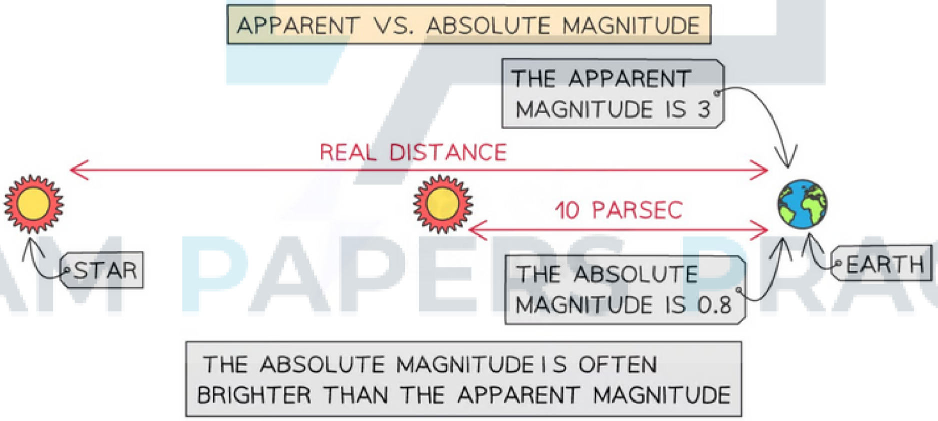
Absolute Magnitude

- The inherent brightness, or **intensity** of a star, as seen to the naked eye on Earth, depends on its:
  - **Luminosity**
  - **Distance** from Earth
- If two different stars have the **same** apparent magnitude it does not necessarily mean they emit the same amount of light or are the same size
  - Therefore, it's useful to compare how bright they would appear to be if they were **exactly** the **same distance** from the Earth
  - This is where the concept of **absolute magnitude** comes in
- The absolute magnitude of a star is defined as:

> **The apparent magnitude it would have if it were observed from a distance of 10 parsecs away from Earth**

- Since most stars are much further than 10 parsecs away, they would appear **brighter** if observed at a distance of 10 parsecs

Apparent vs. absolute magnitude



The absolute magnitude is often brighter than the apparent magnitude

- For example, a real bright star very far away would have the same **apparent magnitude** as a dim star close by
  - However, their **absolute** magnitudes will be different
- This means that:
  - Absolute and apparent magnitudes are measured on the same **logarithmic** scale
  - Values of absolute magnitudes are **more negative** than their associated apparent magnitudes
- The relationship between the apparent magnitude, absolute magnitude and distance of a star from Earth is:

$$m - M = 5 \log \left(\frac{d}{10} \right)$$

- Where:
  - M = absolute magnitude
  - m = apparent magnitude
  - d = distance of the star from Earth (measured in parsecs)

- The difference between apparent and absolute magnitude $(m - M)$ is known as the **distance modulus**
- This is useful for quickly determining the relative distance of a star
  - Distance modulus is **negative** for stars **closer** than 10 pc
  - Distance modulus is **positive** for stars **further** away than 10 pc

Worked Example

The table shows some information about four stars in the constellation of Pegasus.

Star	Apparent magnitude	distance / ly
Algenib	2.84	390
Enif	2.39	690
Markab	2.49	140
Scheat	2.42	200

(a) State which of the stars in the table is the brightest on the absolute magnitude scale.

(b) State which of the stars in the table is the dimmest on the absolute magnitude scale.

(c) Calculate the absolute magnitude of Algenib.

Answer:

(a) The brightest on the absolute magnitude scale is...

- All have similar values of apparent magnitude
- Therefore, furthest = brightest = <u>Enif</u>

(b) The dimmest on the absolute magnitude scale is...

- All have similar values of apparent magnitude
- Therefore, closest = dimmest = <u>Markab</u>

(c) Calculate the absolute magnitude of Algenib:

Step 1: List the known quantities

- Apparent magnitude of Algenib, m = 2.84
- Distance to Algenib, d = 390 ly
- 1 pc = 3.26 ly (from data booklet)

Step 2: Convert the distance into parsecs

$$\text{Distance to Algenib: } d = \frac{390}{3.26} = 119.6 \text{ pc}$$

Step 3: Rearrange the magnitude equation and calculate the absolute magnitude

$$m - M = 5\log\left(\frac{d}{10}\right) \Rightarrow M = m - 5\log\left(\frac{d}{10}\right)$$

$$M = 2.84 - 5\log\left(\frac{119.6}{10}\right)$$

Absolute magnitude: $M = -2.55$

Exam Tip

Be specific in the language you use when comparing magnitudes - a 'bigger' magnitude could either mean brighter (greater intensity) or dimmer (bigger number)

To avoid confusion, make sure to say 'brighter' or 'dimmer' magnitudes rather than larger or smaller.

You must be comfortable with working with **logs** to manipulate this equation.

9.2.5 Wien's Displacement Law

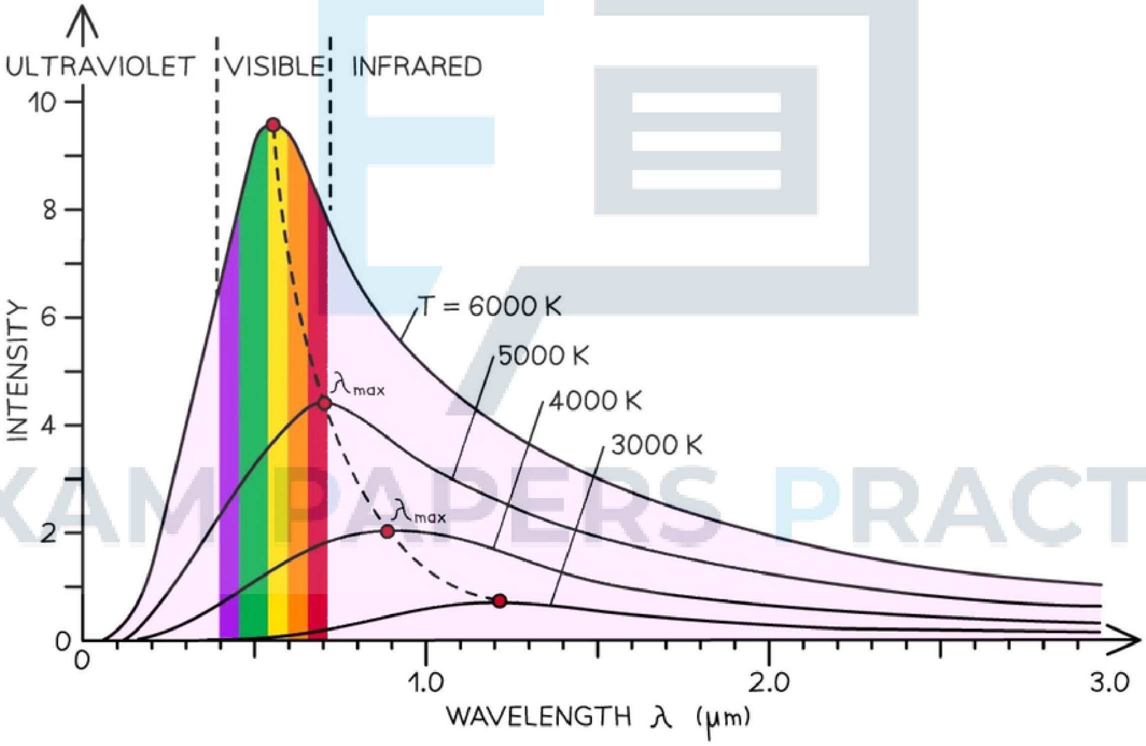
Wien's Displacement Law

- All bodies emit a spectrum of thermal radiation in the form of electromagnetic waves

- An ideal black-body radiator is one that **absorbs and emits all wavelengths**
  - A perfect black-body radiator is a theoretical object
  - However, **stars** are the best approximation there is

- The radiation emitted from a black body has a **characteristic spectrum** that is determined by the temperature alone
- This can be represented on a black-body radiation curve of **intensity** against **wavelength**
  - As the temperature increases, the peak of the curve moves
  - This moves to a **lower** wavelength and a **higher** intensity

Black-body radiators at different temperatures



The intensity-wavelength graph shows how thermodynamic temperature links to the peak wavelength for four different bodies

- Wien's displacement law relates the observed wavelength of light from an object to its surface temperature, it states:

  The black body radiation curve for different temperatures peaks at a wavelength that is inversely proportional to the temperature

- This relation can be written as:

$$\lambda_{max} \propto \frac{1}{T}$$

- Where:
  - λ_{max} = the maximum wavelength emitted by an object at the peak intensity (m)
  - T = the surface temperature of an object (K)
- Wien's displacement law can be written as:

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

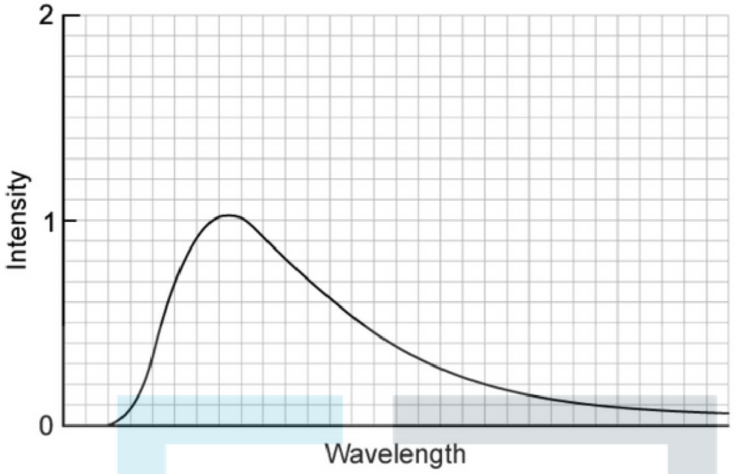
- This equation shows that the **higher** the temperature of a body...
  - The **shorter** the wavelength at the peak intensity (i.e. hotter objects tend to be **white** or **blue**, and cooler objects tend to be **red** or **yellow**)
  - The greater the **intensity** of the radiation at each wavelength

Table to compare surface temperature and star colour

Star Colour	Temperature / K
blue	>33 000
blue-white	10 000 – 30 000
white	7500 – 10 000
yellow-white	6000 – 7500
yellow	5000 – 6000
orange	3500 – 5000
red	<3500

Worked Example

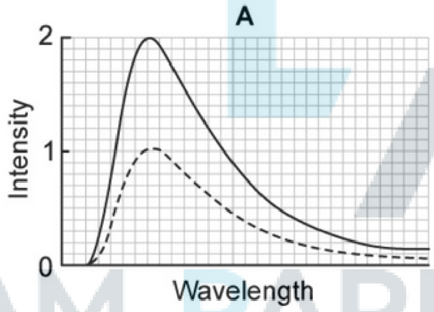
The black-body radiation curve of an object at 900 K is shown in the diagram below.



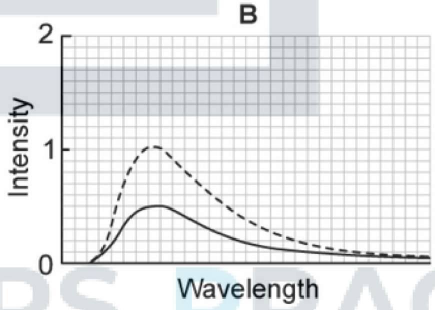
Which of the following shows the black-body radiation curve of an object at 650 K?

The dashed line represents the curve of the object at 900 K.

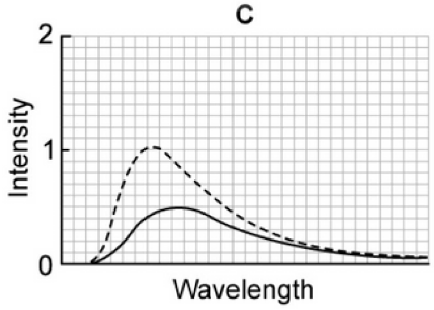
A



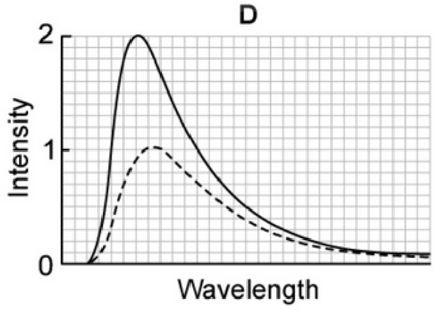
B



C

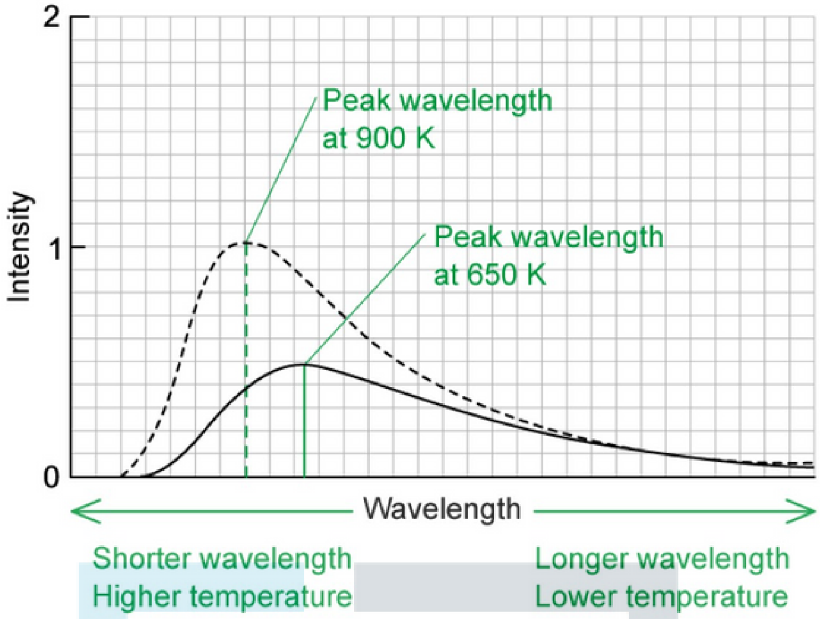


D



Answer: C

- From Wien's displacement law: $\lambda_{max} \propto \frac{1}{T}$
- Therefore, a curve with a **longer peak wavelength** will correspond to a **lower temperature**


2
1
0
Intensity
Peak wavelength at 900 K
Peak wavelength at 650 K
Wavelength
Shorter wavelength
Higher temperature
Longer wavelength
Lower temperature

Worked Example

The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm.

Determine which of these two stars, Betelgeuse or Rigel, is cooler.

Answer:

Step 1: List the known quantities

- Maximum emission wavelength of Rigel = 263 nm = 263×10^{-9} m
- Maximum emission wavelength of Betelgeuse = 828×10^{-9} m

Step 2: Write down Wien's displacement law and rearrange for temperature T

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}}$$

Step 3: Calculate the surface temperature of each star

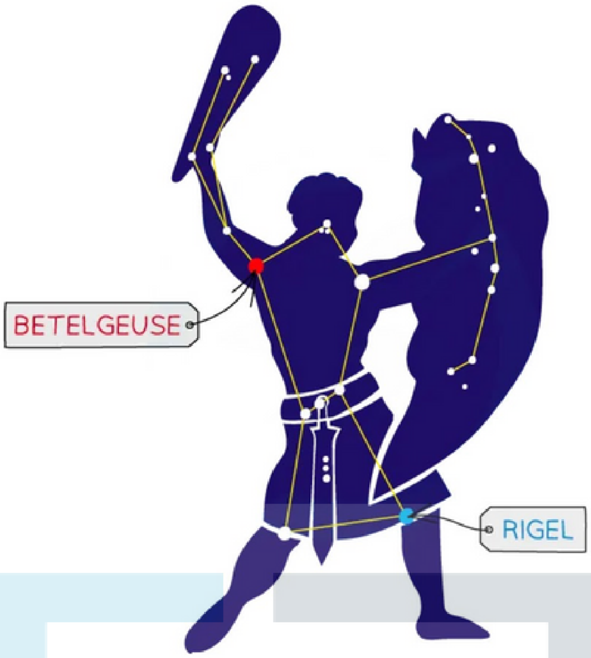
$$\text{Rigel: } T = \frac{2.9 \times 10^{-3}}{263 \times 10^{-9}} = 11\,027 = 11\,000 \text{ K}$$

$$\text{Betelgeuse: } T = \frac{2.9 \times 10^{-3}}{828 \times 10^{-9}} = 3502 = 3500 \text{ K}$$

Step 4: Write a concluding sentence

- Betelgeuse has a surface temperature of 3500 K, therefore, it is much cooler than Rigel

Wien's law and stars in the Orion constellation



The Orion Constellation; cooler stars, such as Betelgeuse, appear red or yellow, while hotter stars, such as Rigel, appear white or blue

Exam Tip

Note that the temperature used in Wien's Law is in **Kelvin** (K). Remember to convert from °C if the temperature is given in degrees in the question before using the Wien's Law equation.

If you're asked to draw a black-body curve in the exam, make sure to avoid these common errors:

- Drawing the right side of the curve steeper than the left side (remember it increases sharply and decreases more steadily)
- Drawing the right side of the curve so that it decreases to zero (it should not meet zero as it decreases)
- Drawing the curve through the intensity axis (there are no negative wavelengths!)

9.2.6 Stefan's Law

Stefan's Law

- The total power P radiated by a perfect black body depends on **two** factors:
    - It's absolute temperature
    - It's surface area

- The relationship between these is known as **Stefan's Law** or the **Stefan-Boltzmann Law**, which states:

> **The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body**

- The Stefan-Boltzmann Law can be calculated using:

$$P = \sigma A T^4$$

- Where:
    - P = total power emitted across all wavelengths (W)
    - σ = the Stefan-Boltzmann constant
    - A = surface area of the body (m)
    - T = absolute temperature of the body (K)

- The Stefan-Boltzmann law is often used to calculate the luminosity of celestial objects, such as stars
- The surface area of a star (or other spherical object) is equal to $\boldsymbol{A = 4\pi r^2}$
    - Where r = radius of the star

- The Stefan-Boltzmann equation then becomes:

$$L = 4\pi r^2 \sigma T^4$$

- Where:
    - L = luminosity of the star (W)
    - r = radius of the star (m)
    - σ = the Stefan-Boltzmann constant
    - T = surface temperature of the star (K)

Worked Example

The surface temperature of Proxima Centauri, the nearest star to Earth, is 3000 K and its luminosity is 6.506×10^{23} W.

Calculate the radius of Proxima Centauri in solar radii and show your working clearly.

Solar radius $R_\odot = 6.96 \times 10^8$ m

Answer:

Step 1: List the known quantities:

- Surface temperature, $T = 3000$ K
- Luminosity, $L = 6.506 \times 10^{23}$ W
- Stefan's constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- Radius of the Sun, $R_\odot = 6.96 \times 10^8$ m

Step 2: Write down the Stefan-Boltzmann equation and rearrange for radius r

$$L = 4\pi R^2 \sigma T^4$$

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Step 3: Substitute the values into the equation

$$R = \sqrt{\frac{6.506 \times 10^{23}}{4\pi \times (5.67 \times 10^{-8}) \times 3000^4}}$$

Radius of Proxima Centauri: $R = 1.061 \times 10^8$ m

Step 4: Find the ratio of the radii of Proxima Centauri and the Sun

$$\frac{R}{R_\odot} = \frac{1.061 \times 10^8}{6.96 \times 10^8} = 0.152\ R_\odot$$

- Proxima Centauri has a radius which is about 0.152 times smaller than the Sun

9.2.7 Emission & Absorption Spectra in Stars

Emission & Absorption Spectra in Stars

- There are three types of light spectra:
  - Continuous emission spectra
  - Emission line spectra
  - Absorption line spectra

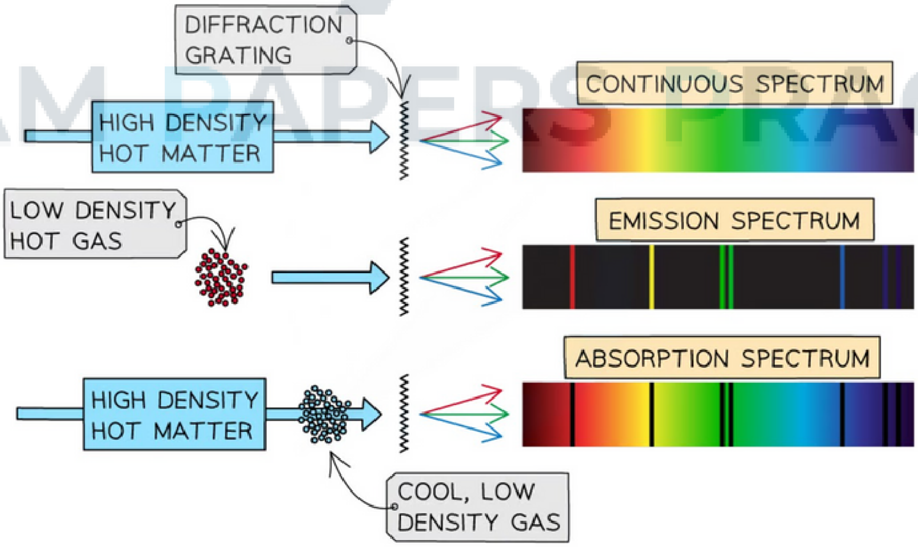
- **Continuous spectrum**: created when photons of all wavelengths are emitted
  - Appearance: a broad range of colours (depending on a star's temperature)
  - Produced by: **hot, dense sources**, such as the cores of stars

- **Emission spectrum**: created when photons are emitted by excited electrons in a hot gas
  - Appearance: discrete wavelengths represented by coloured lines on a black background
  - Produced by: **hot, low-pressure gases**, such as a nebula surrounding a star

- **Absorption spectrum**: created when photons are absorbed by electrons in a cool gas
  - Appearance: discrete wavelengths represented by dark lines on a continuous spectrum
  - Produced by: light passing through **cool, low-pressure gases**, such as the photosphere of a star

- **Note**: the lines in an absorption spectrum correspond to the same lines in the emission spectrum of the same element

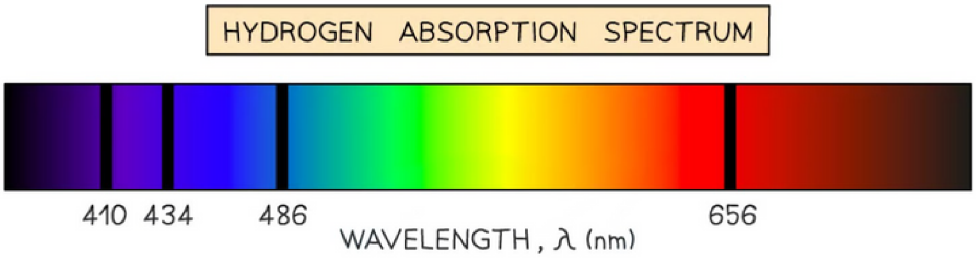
The difference between continuous, emission & absorption spectra

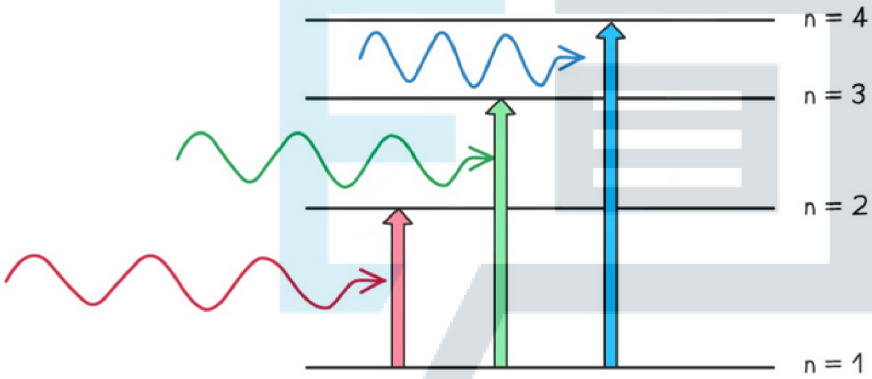


An absorption spectrum is the combination of an emission spectrum on top of a continuous spectrum

Chemical Composition of a Star

- Stellar spectral lines are caused by the interactions between **photons** and the **atoms** present in **gaseous layers** of stars
- Photons produced by fusion reactions in a star's core move towards the layers of gas in the outer atmosphere of the star
    - The photons produced in the core form a **continuous spectrum**
    - Photons are absorbed by the gas atoms, which excite and re-emit other photons of various frequencies in random directions



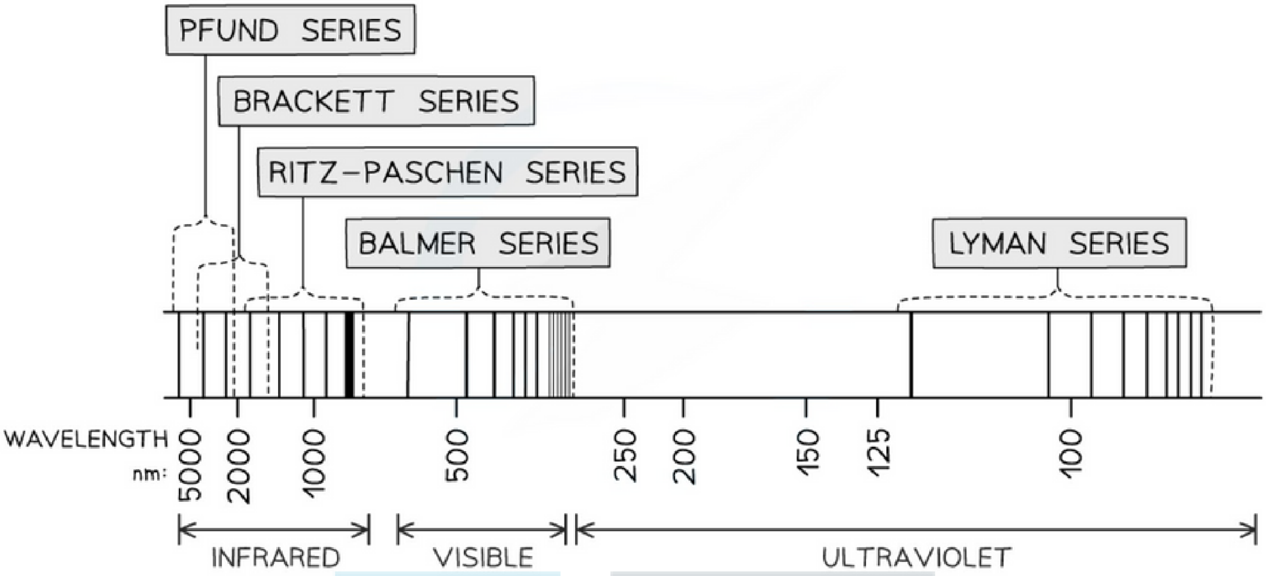


An absorption line will appear in a spectrum if an absorbing material is placed between a source and the observer

- Each gas produces a **unique pattern** of spectral lines due to the specific transition between the element's energy levels
    - The presence of absorption lines in a star's spectrum act as **fingerprints**
    - They can be used to determine the presence of a **certain element** within the star
- The **chemical composition** of a star can be investigated even when extremely distant
    - If the element is present in the star, its characteristic pattern of spectral lines will appear as dark lines in the absorption line spectrum of the star
- The Sun is predominantly made up of **hydrogen** and **helium** gas
    - This can be verified by comparing the **emission line spectra** of hydrogen and helium with the **absorption line spectrum** of the Sun

The Hydrogen Spectrum

- Each element produces its own unique set of lines corresponding to specific energy level transitions
- The spectrum of hydrogen was the first to be studied in great detail

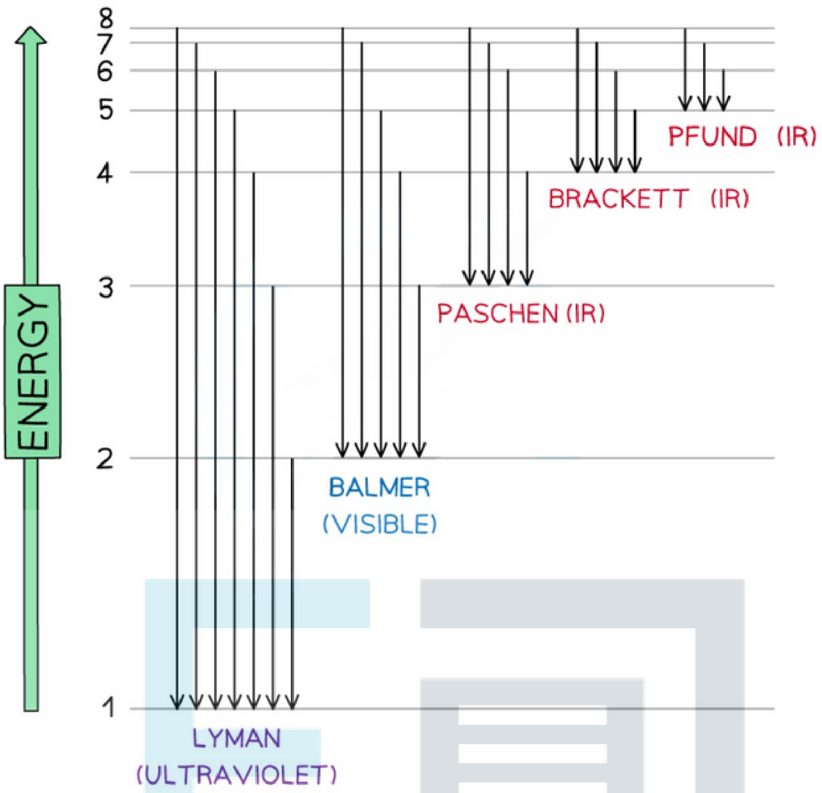


The full hydrogen spectrum

- In spectra of hydrogen:
  - The **Lyman** series converges on the ground state $n = 1$
  - The **Balmer** series converges on the second energy level $n = 2$
  - The **Ritz-Paschen** converges on the third energy level $n = 3$, and so on

- The Lyman series photons will have the **most** energy since they have the **shortest** wavelength
- The Pfund series photons will have the **least** energy since they have the **longest** wavelength

- **Note:** in this course, you only need to remember the Balmer series, the others are only mentioned here for context



Electron transitions in the hydrogen spectrum

- The discovery of these electron transitions has enabled astronomers to study the nature and chemical composition of objects in the Universe

Worked Example

Which of the following electron transitions in a hydrogen atom would result in the emission of visible light?

A. $n = 1$ to $n = 2$

B. $n = 2$ to $n = 3$

C. $n = 2$ to $n = 1$

D. $n = 3$ to $n = 2$

Answer: **D**

- A photon is emitted when an electron moves from a higher energy level to a lower energy level
    - This eliminates options **A** & **B**
- Emission in the visible region occurs for an electron transitioning from any higher energy level to ***n* = 2**
- Therefore, the transition $n = 3$ to $n = 2$ would result in the emission of visible light

Worked Example

Explain why:

(a)

Hot, dense sources produce continuous spectra

(b)

Hot, low pressure gases produce emission spectra

(c)

Hot, dense sources observed through cool, low pressure gases produce absorption spectra

Answer:

(a) Hot, dense sources, such as the cores of stars, produce continuous spectra because:

- In a hot, dense material, the atoms or molecules are so close together that they interact with one another
- This leads to a spread of energy states that are not clearly defined
- Therefore, photons of all frequencies are emitted leading to an uninterrupted band of colour

(b) Hot, low pressure gases produce emission line spectra, because:

- Hot gases produce emission line spectra when photons are emitted due to the transition of electrons
  between discrete energy levels in atoms of the gas
- The line spectrum has certain, fixed frequencies related to the differences in energy between the various energy levels of the atoms of the gas
- In a low pressure gas, the atoms or molecules are not close together
- This means the energy levels of the gas atoms or molecules are clearly quantised and well-defined
- Therefore, only photons which correspond to the differences in energy between the energy levels of a bound electron are seen

(c) Hot, dense sources observed through cold gases produce absorption spectra because:

- Atoms of different elements in the cold gas absorb energy emitted from the hot source but only at particular energy values
- These particular energy values correspond to the differences in energy between the energy levels of a bound electron
- This means that particular frequencies of light are absorbed, creating black lines in the continuous emission spectrum

Exam Tip

Given an absorption line spectrum for a specific star, you can be asked to identify a star of similar chemical composition. It is important to pay attention to the spacing between the lines to be able to correctly identify the most similar star to the given one.

Stellar Spectral Classes

- The spectral classes used today were categorised by the astronomer Annie Jump-Cannon
- She reordered the original alphabetical system into seven temperature classes:

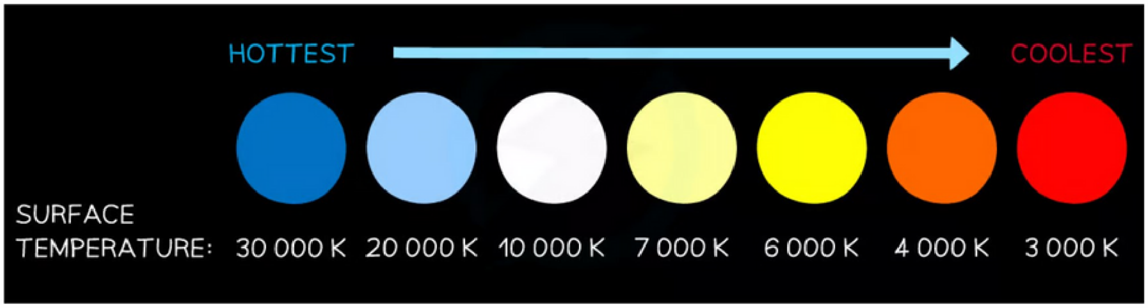
OBAFGKM

- The table shows how the spectra of stars can be categorised based on their
    - Intrinsic colour
    - Temperature
    - Prominent absorption lines

Spectral class	Intrinsic colour	Temperature / K	Prominent absorption lines
O	blue	25 000 – 50 000	He^+, He, H
B	blue	11 000 – 25 000	He, H
A	blue-white	7500 – 11 000	H (strongest), ionised metals
F	white	6000 – 7500	ionised metals
G	yellow-white	5000 – 6000	ionised and neutral metals
K	orange	3500 – 5000	neutral metals
M	red	< 3500	neutral atoms, TiO

- The intrinsic colour of a star is related to its peak emission wavelength which is attributed to its temperature, as described by Wien's law

Relationship between colour and temperature of stars



The colour of a star correlates to its temperature

- The relationship between temperature and absorption spectra is related to the effect of the energy on the state of the atoms or molecules present in the atmospheres of stars
- At **low** temperatures:
    - There may not be enough energy to excite atoms or break molecular bonds

  - This results in the TiO and neutral atoms, as seen in classes K and M

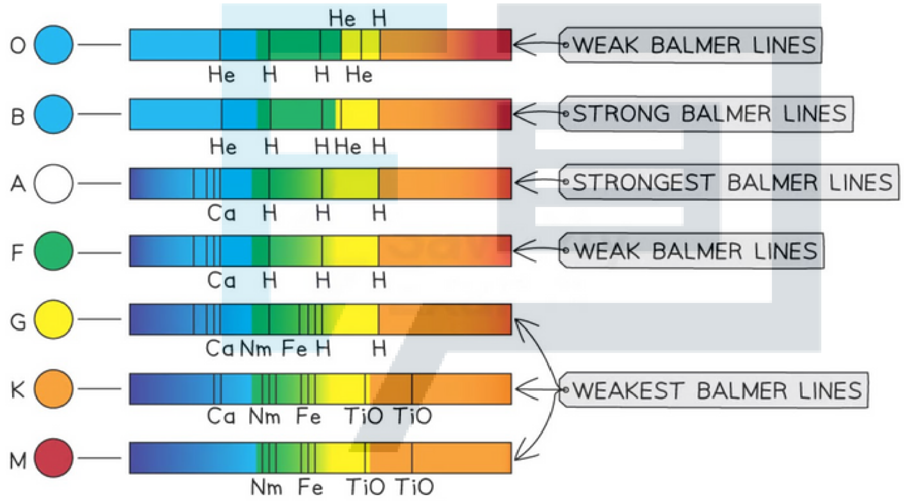
- At **higher** temperatures:
  - Atoms have too much energy to form molecules
  - As a result, ionisation can take place, as seen in classes F and G

- At the **hottest** temperatures:
  - Hydrogen and helium are found to be in higher abundance in the atmospheres of the hottest stars
  - This means that their spectral lines start to dominate, as seen in classes O, B and A

The Balmer Series of Hydrogen

- The absorption and emission spectra of hydrogen and helium are of particular importance to astronomers due to their abundance in the universe

Spectra of Different Spectral Classes



Spectral classes in terms of the prominence of their Balmer lines

- There are many series of spectra, but the most important is the Balmer series, which involves

Electron transitions either to or from the second energy level $(n = 2)$

- The Balmer series is of great importance because the wavelengths of photons created are in the visible spectrum
- The prominence of Balmer lines in a star's atmosphere varies depending on the surface temperature, as shown in the table:

Spectral class	Prominence of Balmer lines	Explanation
O	weak	star's atmosphere too hot hydrogen likely to be ionised
B	slightly stronger	

A	strongest	high abundance of hydrogen in n = 2 state
F	weak	star's atmosphere too cool hydrogen unlikely to be excited
G	very weak / none	too little atomic hydrogen far too cool to be excited
K		
M		

Worked Example

The Winter Triangle consists of three stars, Procyon, Betelgeuse and Sirius.

Procyon and Sirius are binary systems both containing a main sequence star (denoted by A) and a white dwarf (denoted by B)

The surface temperature of these stars is shown in the table.

Star	Surface temperature / K
Betelgeuse	3500
Procyon A	6500
Sirius A	9900

(a)

State and explain the spectral classes that each star belongs to.

(b)

State and explain which star has the most prominent Hydrogen Balmer absorption lines.

Answer:

(a)

- Spectral class is related to a star's surface temperature, so:
- Betelgeuse is a type M star
    - The temperature range for class M is < 3500 K
- Procyon A is a type F star
    - The temperature range for class F is 6000 – 7500 K
- Sirius A is a type A star
    - The temperature range for class A is 7500 – 11 000 K

(b)

- Sirius A has the strongest Balmer absorption lines
- This is because A class stars have hot enough atmospheres for electrons in hydrogen atoms to be excited to the *n* = 2 state
- Whereas, the atmospheres of class F and M stars are cooler, so may not be hot enough for electrons in hydrogen atoms to be excited to the *n* = 2 state

Exam Tip

A common mnemonic for remembering the order of the spectral classes, developed by Annie Jump Cannon herself, is

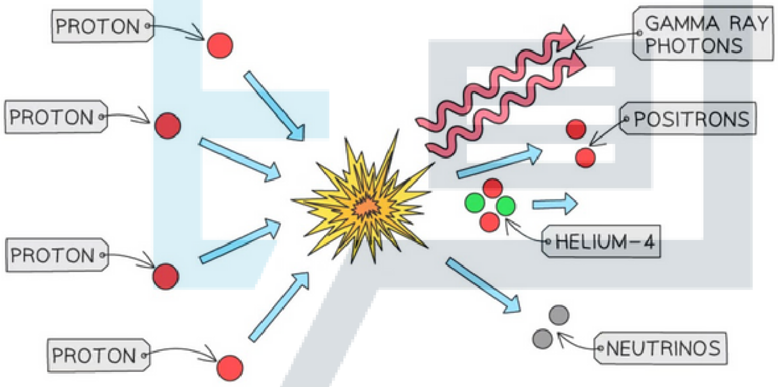
'Oh be a fine girl, kiss me!'

Star Formation

Conditions for Fusion

- For nuclear fusion to occur, both nuclei must have sufficiently **high kinetic energy** to overcome the electrostatic repulsion between protons
- The conditions required to achieve this are:
  - **Very high temperature** (on the scale of 100 million Kelvin)
  - **Very high pressure and density**

- Four hydrogen nuclei (protons) are fused into one helium nucleus, producing two gamma-ray photons, two neutrinos and two positrons
  - Massive amounts of energy are released
  - The momentum of the gamma-ray photons results in an outward acting pressure called radiation pressure

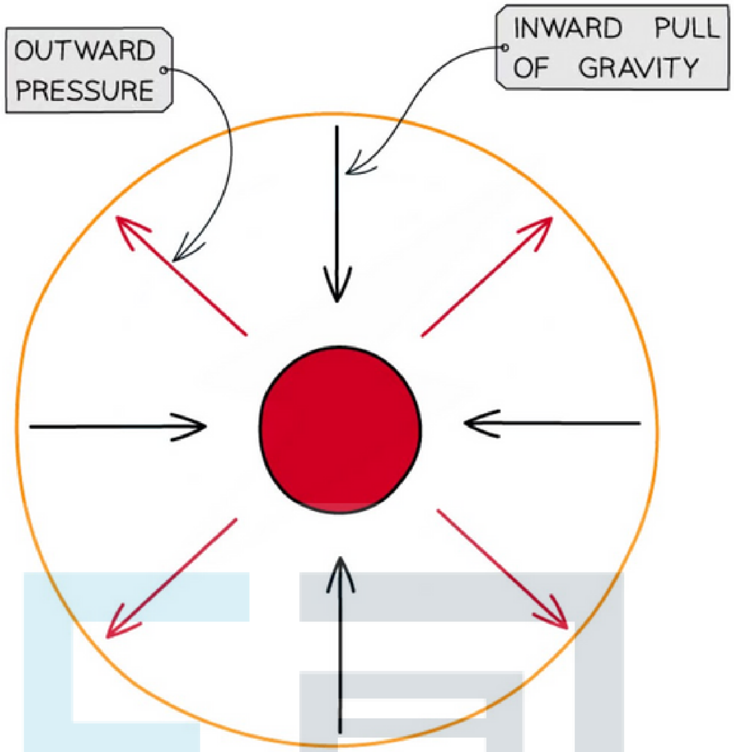


Nuclear fusion of hydrogen nuclei to form helium nuclei

Equilibrium in Stars

- Once the core temperature of a star reaches millions of degrees kelvin and the fusion of hydrogen nuclei to helium nuclei begins
  - The protostar's gravitational field continues to attract more gas and dust, increasing the temperature and pressure of the core
  - With more frequent collisions, the kinetic energy of the particles increases, increasing the probability that fusion will occur
  - Eventually, when the core becomes **hot** enough and fusion reactions can occur, they will begin to produce an **outward radiation pressure** which balances the inward pull of gravity

- The star reaches a **stable state** where the inward and outward **forces** are in **equilibrium**
  - As the temperature of the star increases and its volume decreases due to gravitational collapse, the gas pressure increases
  - The gas pressure and the radiation pressure act **outwards** to balance the gravitational force (weight, $F = mg$) acting **inwards**



Equilibrium in stars occurs when the outward radiation pressure is balanced with the inward gravitational force

- If the **temperature** of a star **increases**, the **outward pressure** will also **increase**
    - If outward pressure > gravitational force, the star will **expand**
- If the **temperature drops** the **outward pressure** will also **decrease**
    - If outward pressure < gravitational force, the star will **contract**
- As long as these two forces are **balanced**, the star will remain **stable**

9.2.10 Evolution of a Low Mass Star

Evolution of a Low Mass Star

- The life cycle of a star follows predictable stages
- The exact route a star's development takes depends on its initial mass

Initial stages for all masses

- The first four stages in the life cycle of stars are the same for stars of all masses
- After these stages, the life-cycle branches depending on whether the star is:
  - **Low mass**: stars with a **mass** of **less** than about 8 times the mass of the Sun ($< 8M_{Sun}$)
    - The Sun is assumed to be a low mass star and follows this evolution
  - **High mass**: stars with a **mass** of **more** than about 8 times the mass of the Sun ($> 8M_{Sun}$)

1. Nebula

- All stars form from a giant cloud of **hydrogen gas** and **dust** called a **nebula**
  - **Gravitational attraction** between individual atoms forms denser clumps of matter
  - This inward movement of matter is called gravitational collapse

2. Protostar

- The gravitational collapse causes the gas to heat up and glow, forming a **protostar**
  - Work done on the particles of gas and dust by collisions between the particles causes an increase in their kinetic energy, resulting in an increase in **temperature**
  - Protostars can be detected by telescopes that can observe **infrared radiation**

- Eventually, the temperature will reach millions of **degrees Kelvin** and the fusion of hydrogen nuclei to helium nuclei begins
  - The protostar's gravitational field continues to attract more gas and dust, increasing the temperature and pressure of the core
  - With more frequent collisions, the kinetic energy of the particles increases, increasing the probability that fusion will occur

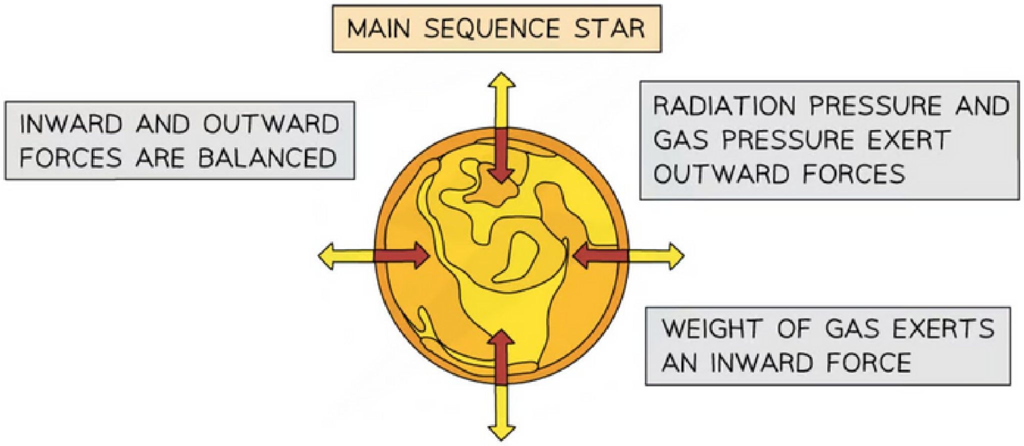
3. Main Sequence Star

- The star reaches a **stable state** when the inward and outward **forces** are in **equilibrium**
  - As the temperature of the star increases and its volume decreases due to gravitational collapse, the **gas pressure** increases

- The star joins the **main sequence** when fusion reactions begin in the star's core

- A main sequence star is one in which radiation pressure is produced by the thermonuclear fusion of hydrogen nuclei into helium nuclei

Forces acting on a main sequence star



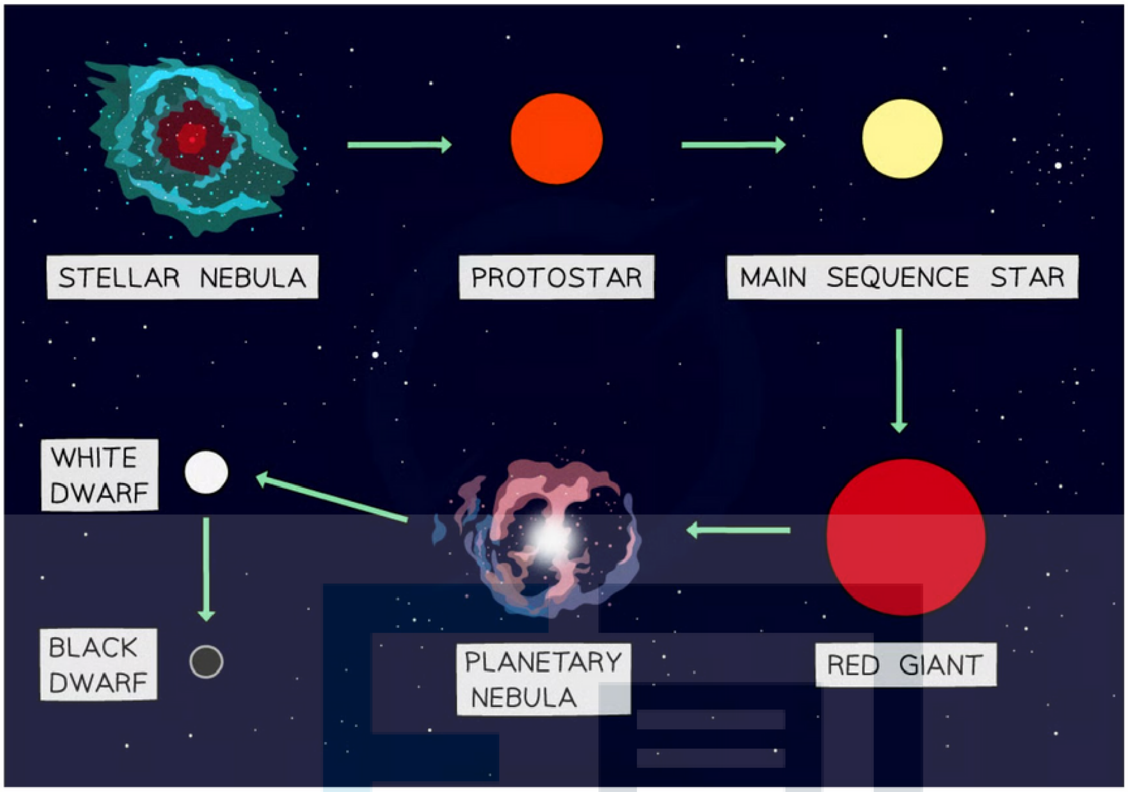
The balanced inward and outward forces will remain that way for millions, or even billions of years

- A star will spend most of its life on the **main sequence**
    - 90% of stars are on the main sequence
    - Main sequence stars can vary in mass from ~10% of the mass of the Sun to 200 times the mass of the Sun
    - The Sun has been on the main sequence for 4.6 billion years and will remain there for an estimated 6.5 billion years

Next Stages for Low Mass Stars

- The fate of a star beyond the main sequence depends on its **mass**
    - The cut-off point for a low-mass star is less than about **8 times** the mass of the Sun
    - A low-mass star will become a **red giant** before turning into a **white dwarf**

Evolution of a Low-Mass Star



The lifecycle of a low mass star

4. Red Giant

- Hydrogen fuelling the star begins to run out
  - Most of the hydrogen nuclei in the core of the star have been fused into helium
  - Nuclear fusion **slows**
  - The energy released by fusion reactions decreases
- The star initially shrinks and compresses the core until fusion can continue in the **shell** around the core
- Once fusion reactions start again, the outer layers expand and cool as a **red giant** forms
- A red giant is a large, low-temperature, luminous star in which helium nuclei are fused into more massive nuclei such as beryllium, carbon and oxygen

5. Planetary Nebula

- The **outer layers** of the star are **released**
- Core helium burning releases massive amounts of energy in fusion reactions

6. White Dwarf

- The solid **core collapses** under its own mass, leaving the remnant of the core called a **white dwarf**
- A white dwarf is an extremely **dense**, **hot** star powered by the gravitational potential energy released as it contracts, rather than by nuclear fusion

Worked Example

Stars less massive than our Sun will leave the main sequence and become red giants.

Describe and explain the next stages of evolution for such stars.

Answer:

Step 1: Plan your answer

- Make a list of the remaining stages in the evolution of a low-mass star adding any important points or keywords

Red giant	Planetary nebula	White dwarf
• Fuel runs out • Forces no longer balanced • Expands and cools • Fusion continues in shell	• Carbon-oxygen core not hot enough for further fusion • Outer layers released	• Hot, dense remnant of the core

Step 2: Use the plan to keep the answer concise and logically sequenced

Low-mass stars leave the main sequence and become red giants when the hydrogen in the core runs out. Reduced energy released by fusion leads to radiation pressure decreasing

Radiation pressure and gas pressure no longer balance the gravitational pressure and the core collapses. Fusion no longer takes place inside the core

The outer layers expand and cool to form a red giant. Temperatures generated by the collapsing core are high enough for fusion to occur in the shell around the core.

Contraction of the core produces temperatures great enough for the fusion of helium into carbon and oxygen. The carbon-oxygen core is not hot enough for further fusion, so the core collapses

The outer layers are ejected forming a planetary nebula.

The remnant core remains intact leaving a hot, dense, solid core called a white dwarf.

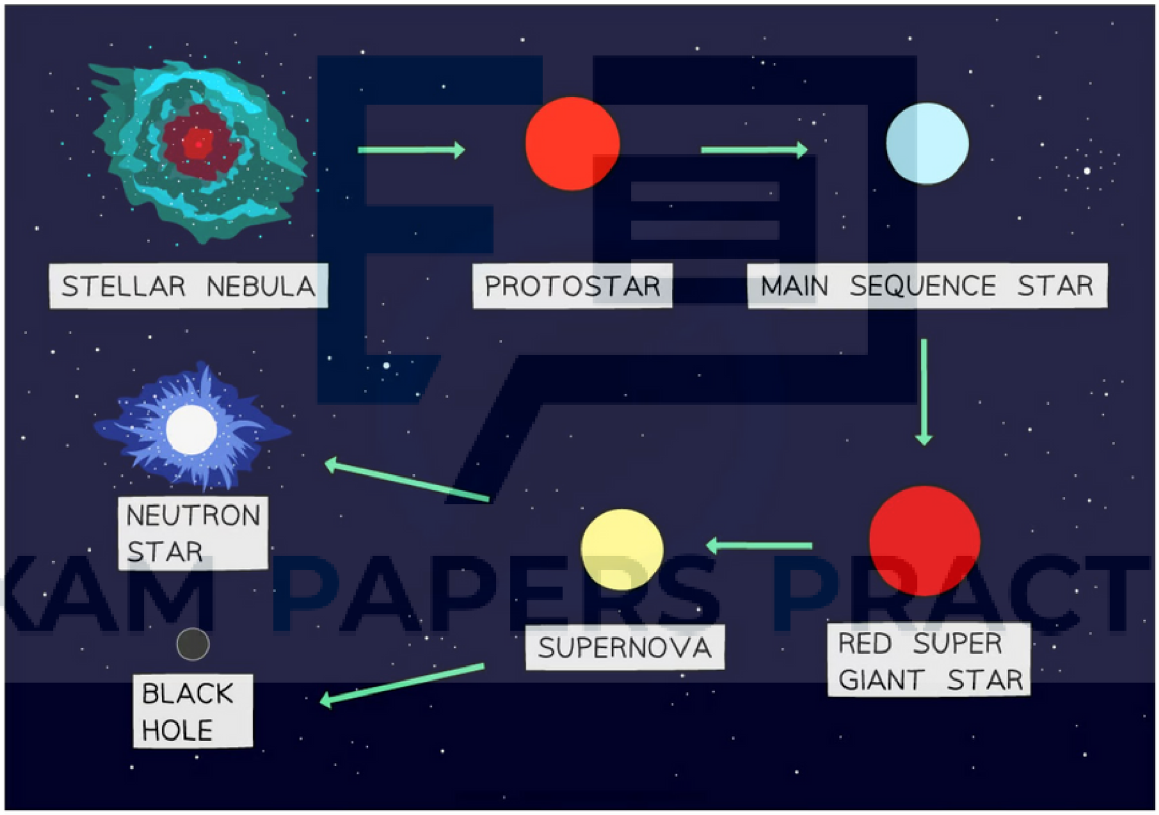
9.2.11 Evolution of a Massive Star

Evolution of a Massive Star

Next Stages for High Mass Stars

- The fate of a star beyond the main sequence depends on its **mass**
  - A star is classed as a high-mass star if it has a mass **greater than 8 times** the mass of the Sun
- A high-mass star will become a **red supergiant** before exploding as a **supernova**
  - Moderately massive stars eventually become **neutron stars**
  - The most massive stars in the Universe become **black holes**

Evolution of a High-Mass Star



Lifecycle of massive stars

4. Red Super Giant

- The star follows the same process as the formation of a red giant
  - The **shell-burning** and **core-burning** cycle in massive stars goes beyond that of low-mass stars, fusing elements up to **iron**

5. Supernova

- The iron core collapses
- The outer shell is blown out in an explosive **supernova**

6. Neutron Star (or Black Hole)

- After the supernova explosion, the collapsed **neutron core** can remain intact having formed a **neutron star**
    - If the neutron core mass is greater than 3 times the solar mass, the pressure on the core becomes so great that the core collapses and produces a **black hole**

Worked Example

Describe the evolution of a star much more massive than our Sun from its formation to its eventual death.

Answer:

Step 1: Plan your answer

- List the stages that a massive star goes through, this will help you form your answer in a logical sequence of events

Nebula	Protostar	Main sequence	Red supergiant	Supernova	Neutron star/black hole
• gravitational collapse	• heats up and glows	• H to He generates energy • stable, forces balanced	• expands and cools • fusion up to iron	• iron core collapses • shockwave explosion	• super dense remnants

Step 2: Use the plan to keep the answer concise and logically sequenced

A star more massive than our Sun will form from clouds of gas and dust called a nebula. The gravitational collapse of matter increases the temperature of the cloud causing it to glow - this is a protostar.

Nuclear fusion of hydrogen nuclei to helium nuclei generates massive amounts of energy. The outward radiation and gas pressure balance the inward gravitational pressure allowing the star to become stable as it enters the main sequence stage.

When the hydrogen runs out, the outer layers of the star expand and cool to form a red supergiant. The core becomes hot enough for helium fusion. Once helium fusion ends, successive cycles of expansion and collapse occur as heavier elements are fused in the core, up to iron.

Eventually, once iron has formed in the core and fusion reactions can no longer continue, the outward layers of the star collapse and the star undergoes a shockwave explosion known as a supernova.

The remnant of the core collapses further and forms either a neutron star or a black hole.

9.2.12 Supernovae & Gamma Ray Bursts (GRBs)

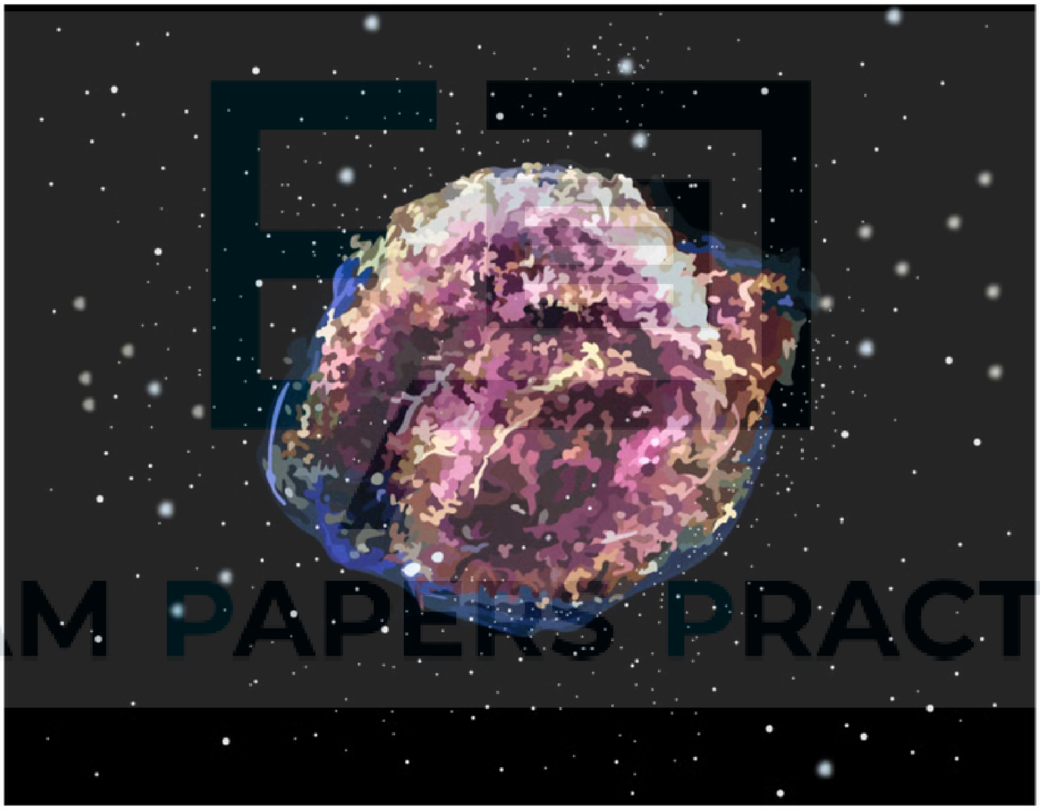
Supernovae

- A supernova is defined as

An object which exhibits a rapid and enormous increase in absolute magnitude

- Supernovae are found to occur when:
  - Either, a supergiant star collapses and then explodes - a **Type II** supernova
  - Or, a white dwarf accrues matter and explodes - a **Type 1a** supernova

A supernova



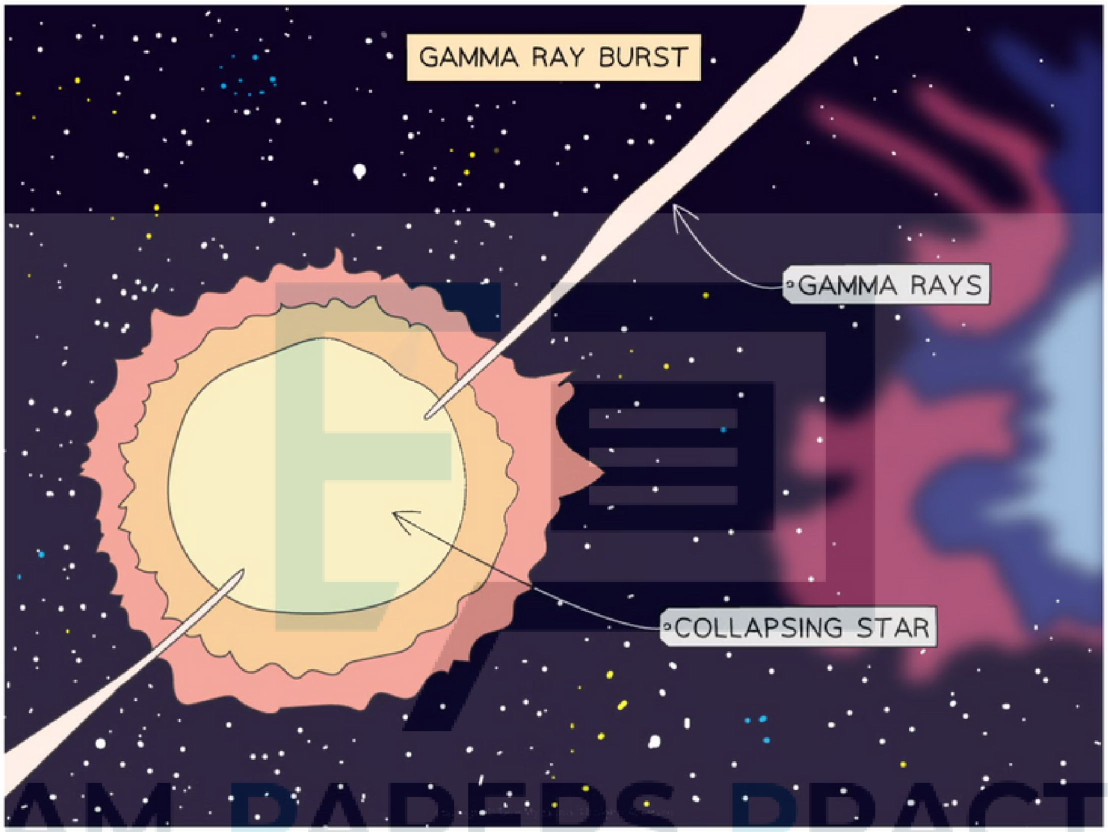
A supernova explosion

Gamma Ray Bursts

- A gamma-ray burst (GRB) is defined as:

> **A short, extremely high energy burst of gamma radiation emitted by a collapsing supergiant star**

Gamma ray burst



Gamma ray burst from a collapsing star

- These bright gamma sources can produce, in a very short period of time, as much energy as the **total energy output of the Sun** over its 10 billion-year lifespan
  - They typically emit energies in the region of 10^{44} to 10^{47} J
- This energy is usually highly focused, or **collimated**, as narrow beams which are ejected from the poles of the exploding star
  - Because of this, some astronomers have **concerns** that the evolution of a supergiant star in the local part of our galaxy could pose a danger to life on Earth
- However, so far, gamma-ray bursts have only ever been detected at great distances, and never in the Milky Way

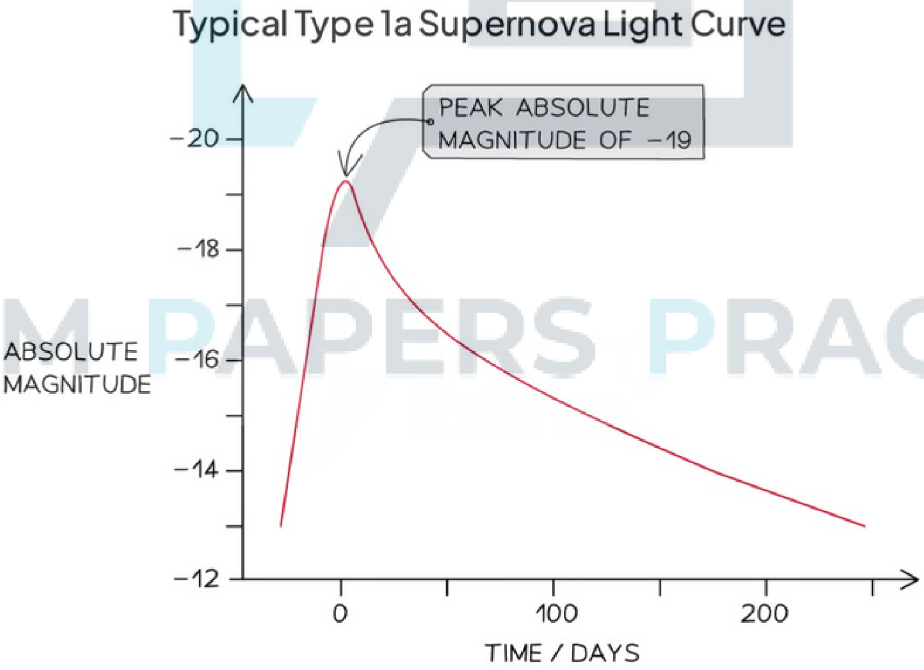
9.2.13 Supernovae as Standard Candles

Type 1a Supernovae as Standard Candles

- A standard candle is defined as:

> **An astronomical object of known brightness that can be used to calculate galactic distances**

- The most common examples of standard candles are:
    - Cepheid variable stars
    - Type 1a supernovae
- Type 1a supernovae can be used as standard candles because they reach the **same peak value of absolute magnitude** each time
    - This type of supernova involves an exploding white dwarf in a binary star system
    - The white dwarf **increases in mass** as it **attracts material from its binary pair**
    - Eventually the white dwarf reaches a critical mass, known as the **Chandrasekhar Limit**
    - This critical mass means the explosion **is the same each** time, hence it produces a very **consistent light curve**



The light curve of a type 1a supernova always has the same characteristic shape and reaches the same peak absolute magnitude

- Another advantage of using Type 1a supernovae as standard candles is that they are **extremely bright**
    - This means they can be used to measure the distance to the furthest galaxies

Worked Example

An astronomer observes a type 1a supernova in a distant galaxy with an apparent magnitude of +11 at its peak of brightness.

At its peak, it is known that a type 1a supernova has an absolute magnitude of about −19.

Show that the galaxy is about 10 Mpc away.

Answer:

Step 1: Write down the magnitude equation

$$m - M = 5\log\left(\frac{d}{10}\right)$$

- Where
  - Apparent magnitude, $m = 11$
  - Absolute magnitude, $M = -19$

Step 2: Substitute in the values of m and M and rearrange

$$5\log\left(\frac{d}{10}\right) = 11 - (-19) = 30$$

$$\log\left(\frac{d}{10}\right) = \frac{30}{5} = 6$$

Step 3: Take logs of both sides to determine the distance d

$$\frac{d}{10} = 10^6$$

$$d = 10 \times 10^6 \text{ pc} = 10 \text{ Mpc}$$

9.2.14 Neutron Stars & Black Holes

Neutron Stars

- Neutron stars are objects which form after a supernova has ejected the outer layers of a star into space
  - A core which has a mass between 1.4 and 3 solar masses will become a **neutron star**
- A neutron star is defined as:

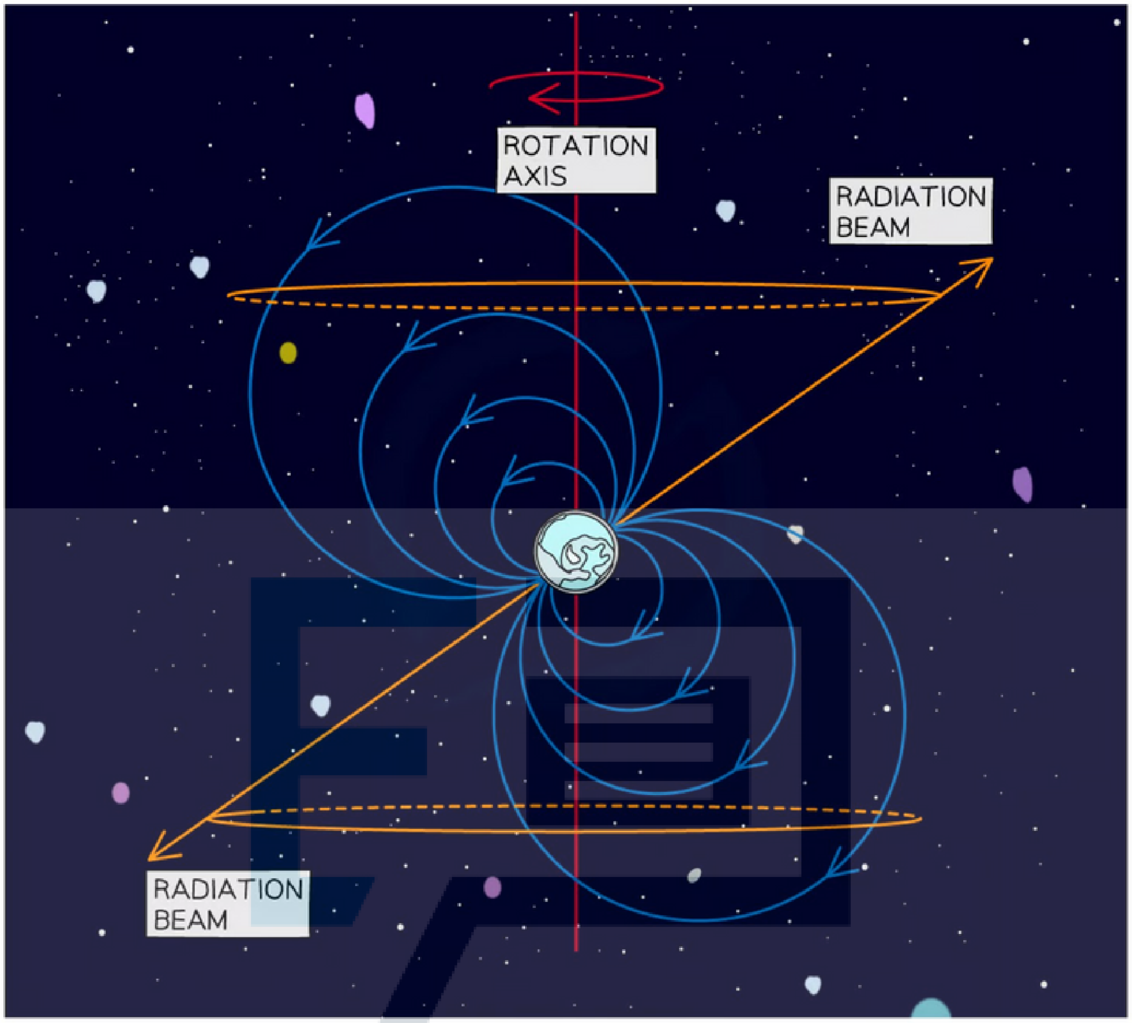
An extremely dense collapsed star made up of neutrons

- Neutron stars are extremely **small** and **dense** (~10^{17} kg m^{-3})
  - A neutron star with the mass of the Sun would have a diameter of about 30 km
  - A teaspoon of neutron star would have a mass of about 100 million tonnes
- The immense gravitational forces acting on the core crush the electrons and protons until they combine into **neutrons**, via reverse **beta decay**:

$$p + e^- \rightarrow n + \nu$$

- Further collapse is prevented by neutron degeneracy pressure
- Some neutron stars **rotate rapidly** (up to 600 times per second) emitting bursts of highly directional electromagnetic radiation
  - These stars are called **pulsars**

What is a pulsar?



A fast-rotating neutron star is called a pulsar

- Pulsars are much easier to identify than slow, or non-rotating, neutron stars
  - This is because they **emit radiation periodically** which makes them easier to detect
  - In particular, they emit radio waves strongly, and sometimes X-rays and gamma rays

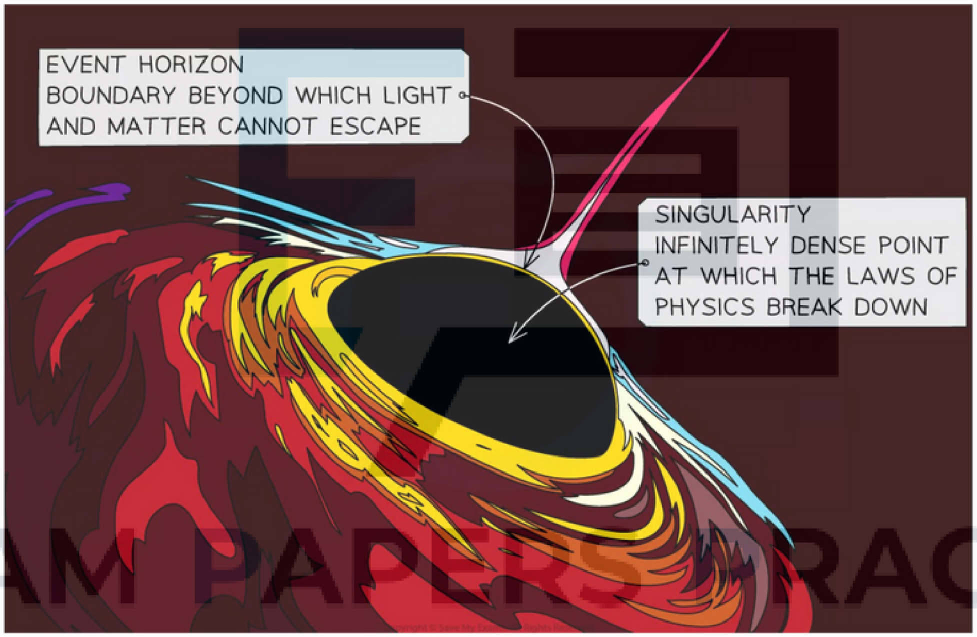
Black Holes

- After a supernova has ejected the outer layers of a star into space, the most massive cores can collapse into an infinitely dense point called a **singularity**
  - A core which has a mass greater than 3 solar masses will become a **black hole**

- The **gravitational field** around a black hole is so strong that nothing, not even light, can escape it

- The boundary at which light and matter cannot escape the gravitation pull of the black hole is called the **event horizon**
- The escape velocity beyond the event horizon is **greater than the speed of light**
  - This is why black holes cannot be seen directly, as photons cannot escape beyond the event horizon

What is a black hole?



A black hole is an object which is so dense that its escape velocity is greater than the speed of light

Schwarzchild Radius of a Black Hole

- The radius of a black hole's event horizon is called the **Schwarzschild radius** and is given by:

$$R_s \approx \frac{2GM}{c^2}$$

- Where:
  - R_s = the Schwarzschild radius (m)
  - G = gravitational constant
  - M = mass of the black hole
  - c = speed of light

Supermassive Black Holes

- Observations of stars at the centre of the Milky Way suggest that a mass equivalent to millions of stars is contained in a very small volume
- Astronomers determined that the mass at the galactic centre is, in fact, a **supermassive black hole**
    - Sagitarrius A*, the one in our galactic centre, has a mass of 4 million solar masses
- Since this discovery, over 150 supermassive black holes have been identified at the centres of other galaxies similar to the Milky Way
    - This is thought to be strong evidence that supermassive black holes exist at the centres of nearly all large galaxies

Worked Example

Some galaxies, known as Seyfert galaxies, have very active galactic centres. They are believed to host supermassive black holes at their centres.

The black hole at the centre of the galaxy NGC 5252 is found to have a mass 9.5×10^9 times that of the Sun.

(a)
Explain what is meant by the 'event horizon' of a black hole.
(b)
Calculate the radius of the event horizon in terms of solar radii.
(c)
Calculate the average density of matter inside the event horizon.
(d)
Compare your answer to (c) with the density of a black hole which has the same mass as the Sun.

Answer:

(a) An event horizon is:

- The boundary of the region around a black hole inside of which light cannot escape

(b)

Step 1: List the known quantities

- Mass of the black hole, $M = 9.5 \times 10^9\ M_\odot$
- Mass of the Sun, $M_\odot = 1.99 \times 10^{30}$ kg (from the data booklet)
- Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} (from the data booklet)
- Speed of light, $c = 3.00 \times 10^8$ m s^{-1} (from the data booklet)
- Radius of the Sun, $R_\odot = 6.96 \times 10^8$ m (from the data booklet)

Step 2: Write down the equation for the Schwarzschild radius

$$R_s = \frac{2GM}{c^2}$$

- **Note:** this equation is included in the data booklet

Step 3: Calculate the radius of the event horizon

$$R_s = \frac{2 \times (6.67 \times 10^{-11}) \times (9.5 \times 10^9) \times (1.99 \times 10^{30})}{(3.00 \times 10^8)^2}$$

Schwarzschild radius: $R_S = 2.8 \times 10^{13}$ m

Step 4: Convert to solar radii

$$\text{Schwarzschild radius: } R_S = \frac{2.8 \times 10^{13}}{6.96 \times 10^{8}} = 40\,000\ R_\odot$$

- This means the size of the black hole is the equivalent length of 40,000 Suns placed side-by-side

(c)

Step 1: Recall the equations for density and volume

- We can treat the volume of the black hole as a sphere:

$$V = \frac{4}{3}\pi R_S^{\ 3}$$

- The density of the black hole is therefore given by

$$\rho = \frac{M}{V} = \frac{3M}{4\pi R_S^{\ 3}}$$

Step 2: Calculate the average density of the matter within the event horizon

$$\rho = \frac{3 \times (9.5 \times 10^{9}) \times (1.99 \times 10^{30})}{4\pi \times (2.8 \times 10^{13})^{3}} = 0.21\ \text{kg m}^{-3}$$

(d)

Step 1: Write expressions for the densities and Schwarzschild radii of the black holes

- Density of a solar-mass black hole:

$$\rho_1 = \frac{M_\odot}{V_1} = \frac{3M_\odot}{4\pi R_1^{\ 3}}$$

- Where $R_1 = \dfrac{2GM_\odot}{c^2}$

- Density of a supermassive black hole:

$$\rho_2 = \frac{(9.5 \times 10^{9})M_\odot}{V_2} = (9.5 \times 10^{9})\frac{3M_\odot}{4\pi R_2^{\ 3}}$$

- Where $R_2 = (9.5 \times 10^{9})\dfrac{2GM_\odot}{c^2}$

Step 2: Determine the ratio of the densities and make a comparison

$$\frac{\rho_1}{\rho_2} = \frac{3M_\odot}{4\pi R_1{}^3} \times \frac{1}{9.5\times10^9}\frac{4\pi R_2{}^3}{3M_\odot}$$

$$\frac{\rho_1}{\rho_2} = \frac{1}{9.5\times10^9}\left(\frac{R_2}{R_1}\right)^3$$

- Substituting $\frac{R_2}{R_1} = 9.5 \times 10^9$:

$$\frac{\rho_1}{\rho_2} = \frac{1}{9.5\times10^9}(9.5\times10^9)^3 = (9.5\times10^9)^2 = 9\times10^{19}$$

- This means that the solar-mass black hole is **~10^{20} times denser** than the supermassive black hole
- The density of a black hole must be proportional to the inverse square of its mass, i.e. the more massive the black hole, the less dense it is

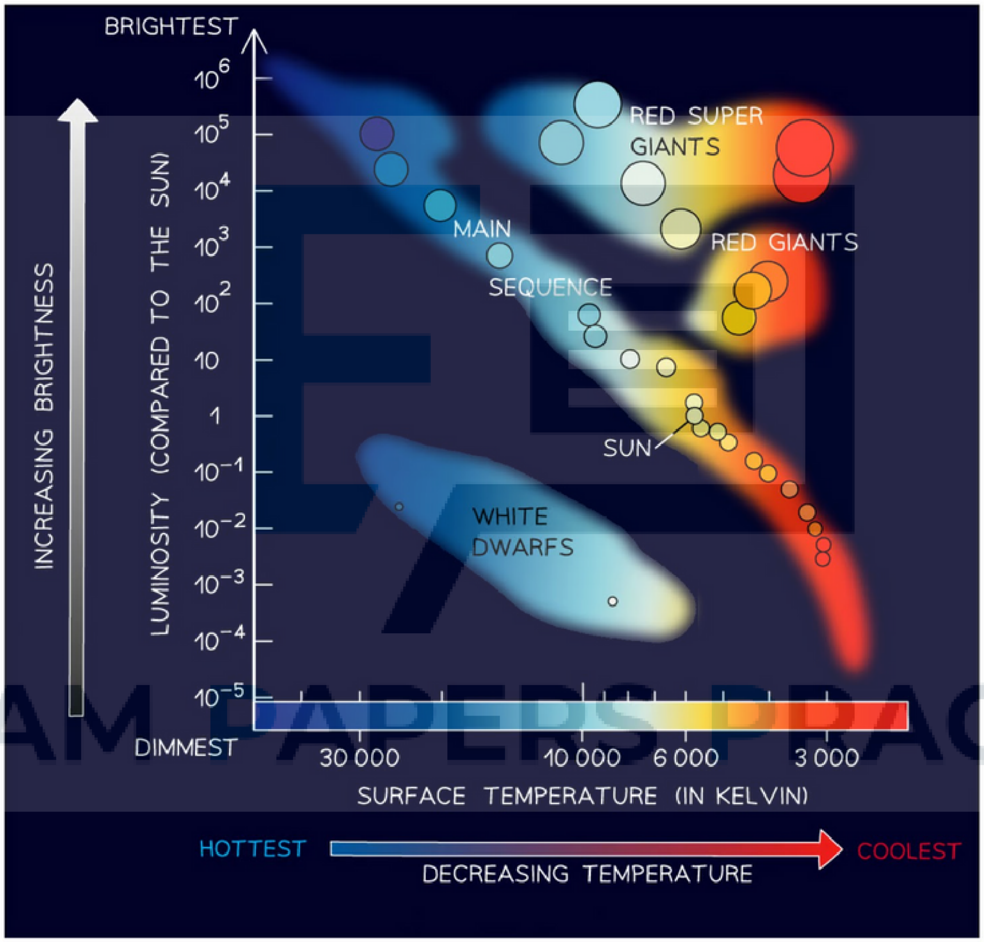
Exam Tip

When writing the definition for the event horizon of a black hole, make sure to be clear that it is the **boundary where the escape velocity = c**

Avoid definitions that describe the event horizon as a point or a distance, as this is not correct.

The Hertzsprung–Russell (HR) Diagram

- Danish astronomer Ejnar Hertzsprung, and American astronomer Henry Noris Russell, independently plotted the luminosity of different stars against their **temperature**
  - **Luminosity**, relative to the Sun, on the y-axis, goes from dim (at the bottom) to bright (at the top)
  - **Temperature**, in degrees Kelvin, on the x-axis, goes from **hot** (on the left) **to cool** (on the right)



The Hertzsprung–Russell Diagram depicts the luminosity of stars against their temperature

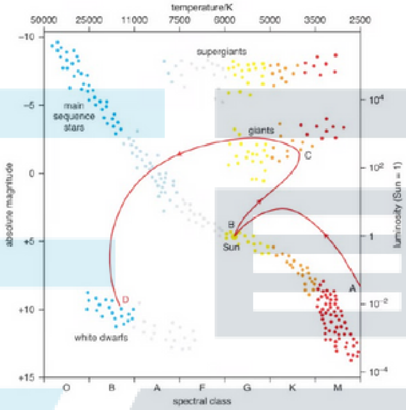
- Hertzsprung and Russel found that the stars clustered in **distinct areas**
- Most stars are clustered in a band called the **main sequence**
  - For main sequence stars, luminosity **increases** with surface temperature
- A smaller number of stars clustered above the main sequence in two areas, **red giants**, and **red supergiants**
  - These stars show an increase in luminosity at **cooler** temperatures
  - The only explanation for this is that these stars are much **larger** than main sequence stars

- Below and to the left of the main sequence are the **white dwarf stars**
  - These stars are **hot**, but not very luminous
  - Therefore, they must be much **smaller** than main sequence stars
- The Hertzsprung-Russell Diagram only shows stars that are in **stable phases**
  - Transitory phases, such as supernovae, happen quickly in relation to the lifetime of a star
  - Black holes cannot be seen since they emit no light

Evolutionary Path of Sun-Like Stars

- The evolutionary path of stars similar to the Sun can be described using a H-R diagram

Evolutionary path of a solar mass star



Protostar to Main Sequence (A to B):

- The protostar collapses from a cold cloud of gas
- Initially, it is visible as a very dim cool star as it moves onto a fixed position on the main sequence
- Its position on the main sequence is determined by the star's mass

Main Sequence to Red Giant (B to C):

- On the main sequence, the star is stable while it fuses hydrogen into helium nuclei
- Once hydrogen fusion stops, the star begins to collapse under gravity
- This heats up the core until further nuclear reactions reignite the star
- The massive increase in temperature causes the star to expand into a red giant, which could be 100 times the current diameter of the Sun
- As the outer layers move further from the core, its surface temperature will be lower, at about 3000 K, and the extremely large surface area causes it to be much more luminous

Red Giant to White Dwarf (C to D):

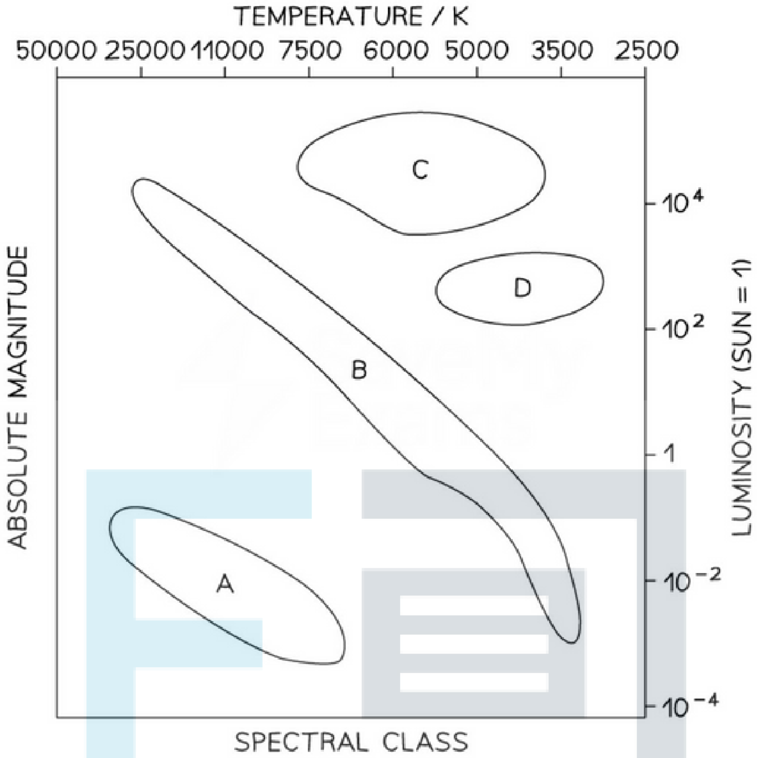
- When the supply of helium runs out in the star, nuclear fusion stops and the star collapses into a white dwarf
- The surface temperature of a white dwarf is generally very hot ~10 000K
- Due to the small surface area of a white dwarf, its luminosity is very low

Lifetimes of Stars

- The brightest stars have very short lifetimes (a few million years)
  - These stars use up nuclear fuel at a much higher rate
- The dimmest stars have extremely long lifetimes in comparison (~10^{12} years)
  - These stars use up nuclear fuel at a much slower rate
- Stars on the main sequence with high luminosities are **massive** and very **bright**
  - A star that is 10^6 times brighter than the Sun will use up its nuclear fuel 10^6 times faster than the Sun
  - A star that has a mass 100 times that of the Sun will live about $\frac{100}{10^6}$ or 10^{-4} times as long

Worked Example

Stars can be classified using a Hertzsprung-Russell (HR) diagram.



(a)

Label the spectral class and absolute magnitude axes with suitable scales.

(b)

State the types of stars found in areas A, B, C and D.

(c)

Label with an **S** the position of the Sun, and draw a line to show the evolution of a star similar to the Sun.

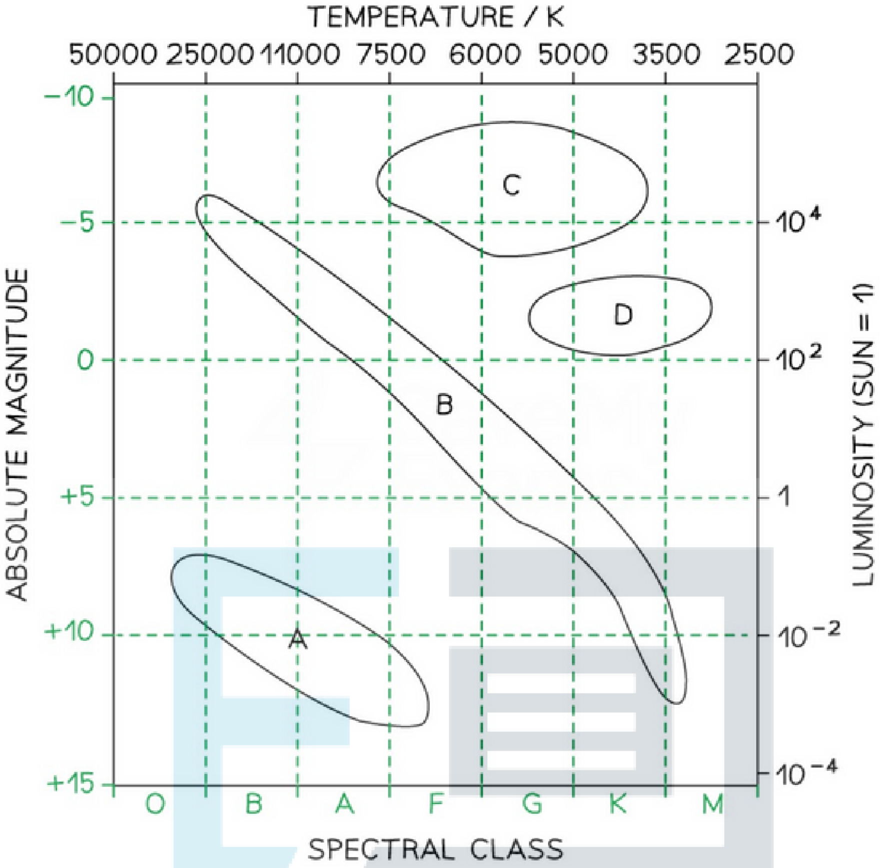
(d)

On the H-R diagram, plot the star with a surface temperature of 20 000 K and a luminosity 10 000 times greater than the Sun and label it Star **X**.

Answer:

(a)

- Use the luminosity scale as a guide for the absolute magnitude scale
- Absolute magnitude scale should be from +15 to –10 (but +15 to –15 would be allowed)
- Use the temperature scale as a guide to label the spectral classes
- Spectral classes must be in the correct order OBAFGKM

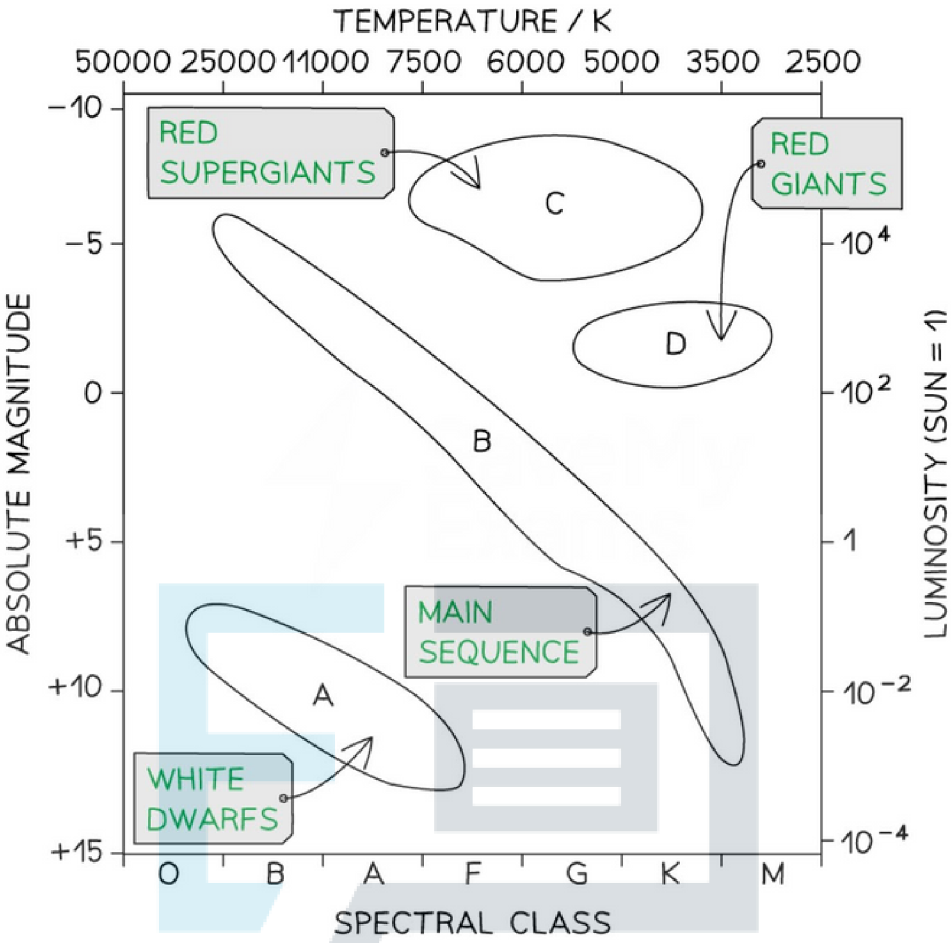


(b)

- The main sequence is the easiest to recognise as it is the long band diagonally central to the diagram where the majority of stars are found
    - Region **B** = main sequence

- White dwarf stars are hot, but not very luminous
- Therefore, they will be in the region below and to the left of the main sequence
    - Region **A** = white dwarfs

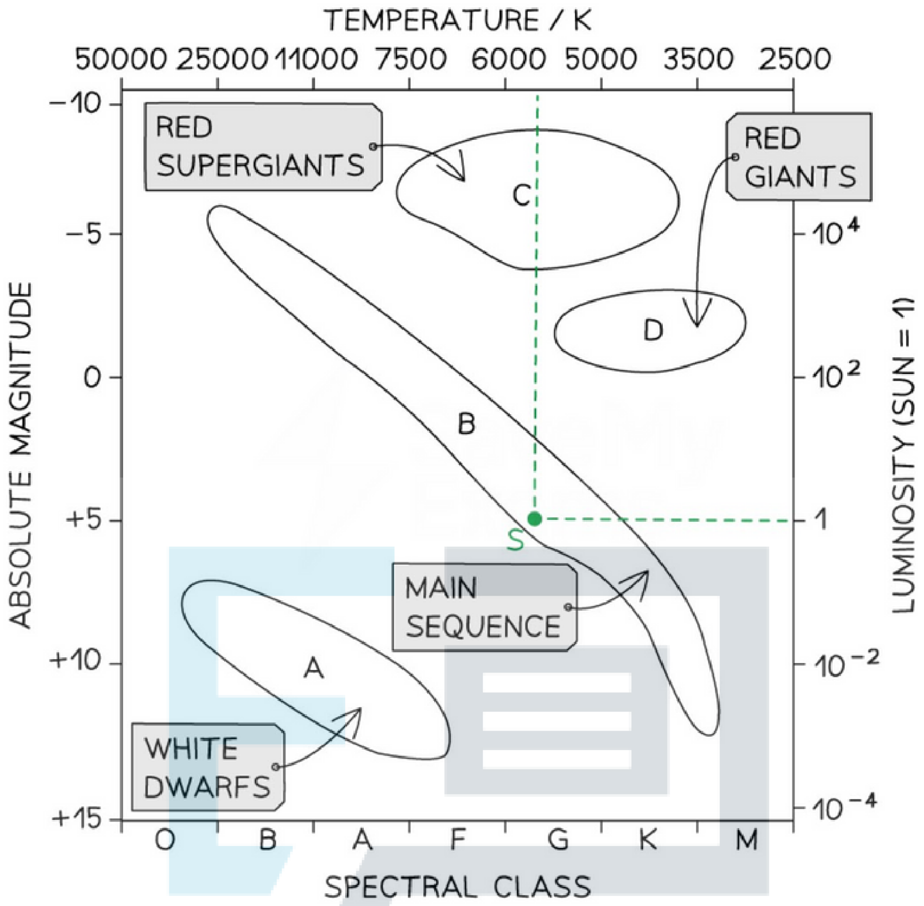
- Red giants and red supergiants have a greater luminosity than main sequence stars and a lower temperature
- Therefore, they will be in the region above and to the right of the main sequence
- Red supergiants are **more luminous** than the red giants, hence, they will appear above the red giants on the graph
    - Region **C** = red supergiants
    - Region **D** = red giants



(c)

Step 1: Identify the position of the Sun on the HR diagram

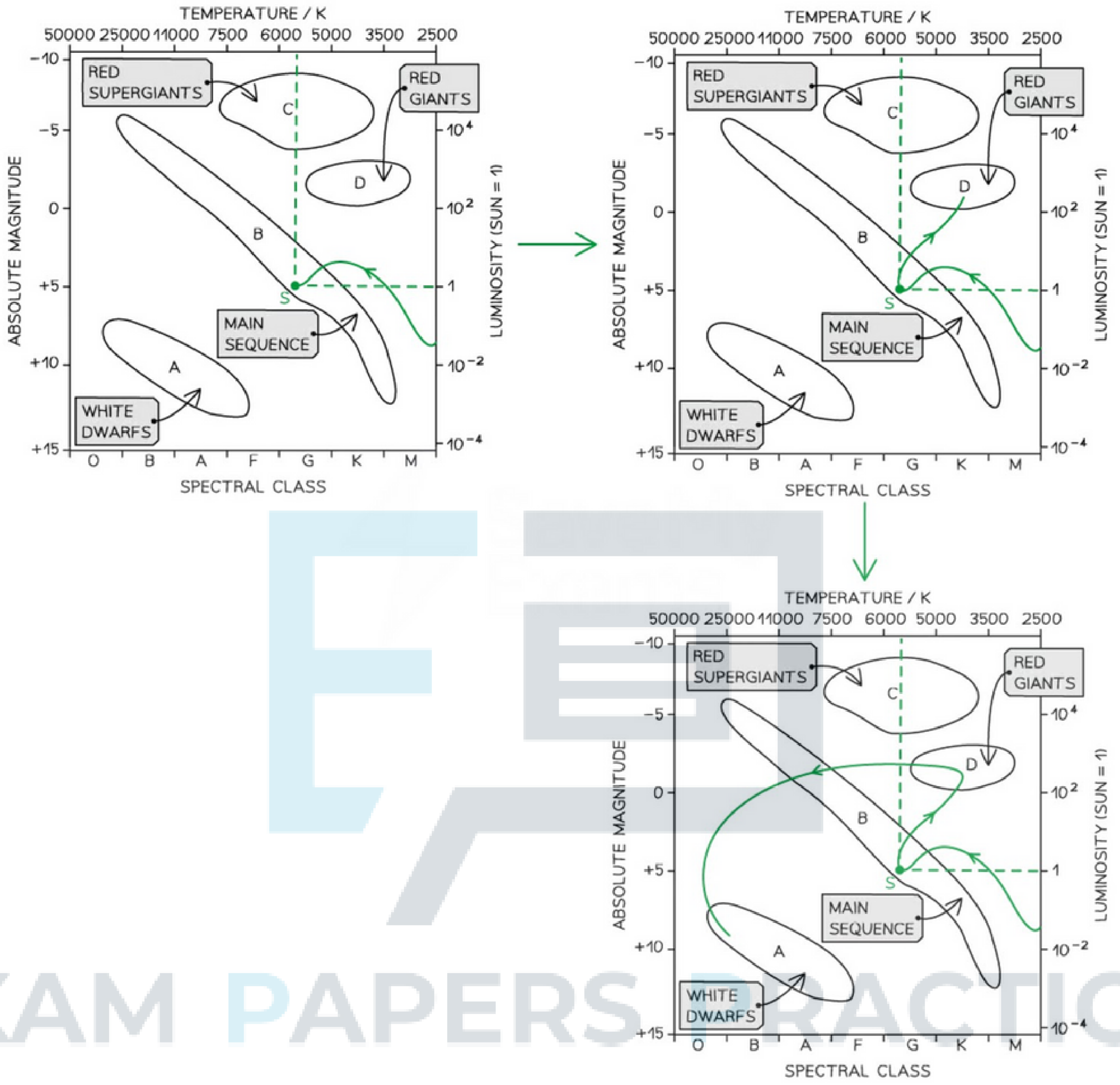
- The luminosity of the Sun is 1 (as it's a relative scale), or abs mag +5
- The temperature of the Sun is 5800 K (between 5500–6000 K is allowed)



- **Tip:** Use a ruler and pencil to draw a line from the position of the Sun to the luminosity axis (y-axis)

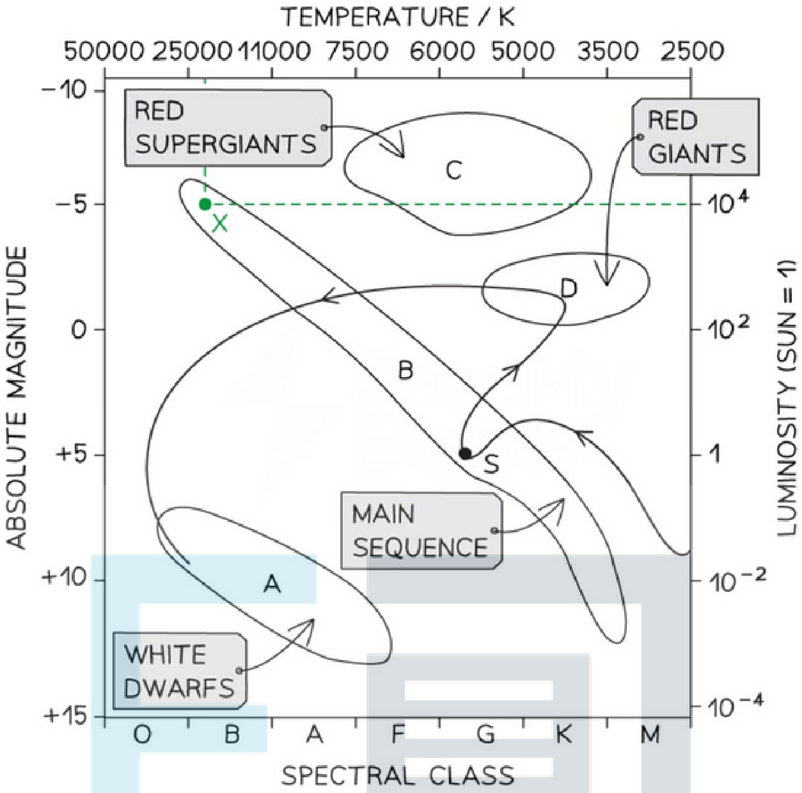
Step 2: Draw the evolutionary path of the Sun

- Start the line from the right to S to represent the transition from protostar to the main sequence
- From S, continue the line up and to the right into the red giant region
- Curve the line around to the left and down into the white dwarf region



(d)

- Luminosity of Star **X** = 10 000 (or 10^4) times that of the Sun
- Surface temperature of Star **X** = 20 000 K



Exam Tip

Drawing an HR diagram on a blank pair of axes is a common exam question, including labelling the axes with suitable scales, so make sure you get plenty of practice with this!

Some key points to remember when drawing this diagram are:

- Make sure to put absolute magnitude on the y-axis, starting at +15 at the bottom and up to −10 at the top
- Make sure to put temperature on the x-axis, starting from 50 000 K on the left to 2500 K on the right
- Always draw the main sequence as a band, not a line, and give it some curvature, don't use a ruler here (for once!)
- Make sure each region is distinctive and not touching one another
- The giants should have absolute magnitudes **less than 0**
- The dwarfs should have absolute magnitudes **greater than 10**