EXAM PAPERS PRACTICE

9.1 Telescopes

ASTROPHYSICS

AQA A Level Revision Notes

A Level Physics AQA

9.1 Telescopes

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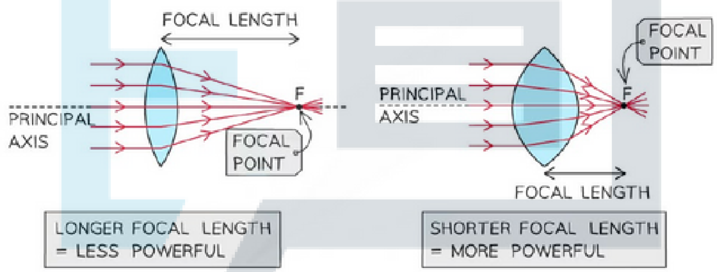
9.1.1 Lenses & Ray Diagrams for Telescopes

Lenses & Ray Diagrams for Telescopes

- A lens is a piece of equipment that forms an image by **refracting** light
- There are two types of lenses:
  - A convex, or **converging** lens
  - A concave, or **diverging** lens
- **Note:** in the Astrophysics module, only converging lenses will be discussed

- In a converging lens, parallel rays of light are brought to a focus along the principal axis
  - This point is called the focal point

- The distance from the centre of the lens to the focal point is called the **focal length**
  - This length depends on how **curved**, or how **thick**, the lens is
  - The **more** curved (**thicker**) the lens, the **shorter** the focal length
  - The **shorter** the focal length, the more **powerful** the lens

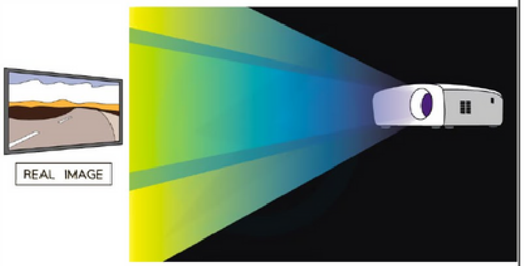


The focal length is shorter in a lens that is thicker and more curved. This makes for a more powerful lens

Real & Virtual Images

- Images produced by lenses can be either **real** or **virtual**

Real image	Virtual image
light converges towards a focal point	light diverges away from a focal point
always inverted	always upright
can be projected onto a screen	cannot be projected onto a screen
intersection of two solid lines	intersection of two dashed lines (or a dashed and a solid line)



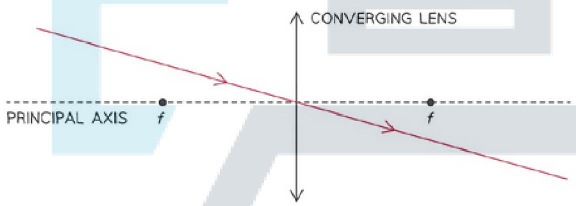
example: image from a projector onto a screen

example: image in a mirror

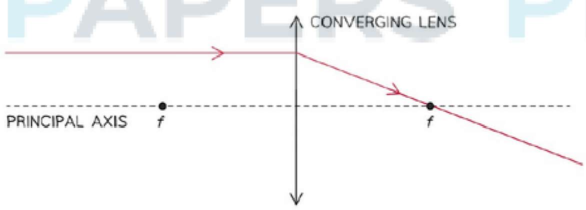
Constructing Ray Diagrams

- When constructing ray diagrams of refractors, it is generally assumed that the lenses used are **very thin**
    - This simplifies the situation by reducing the amount the incident rays of light refract
- As a result, the three main rules for constructing ray diagrams are as follows:

1. Rays passing through the principal axis will pass through the optical centre of the lens undeviated



2. Rays that are parallel to the principal axis will be refracted and pass through the focal point *f*



3. Rays passing through the focal point *f* will emerge parallel to the principal axis

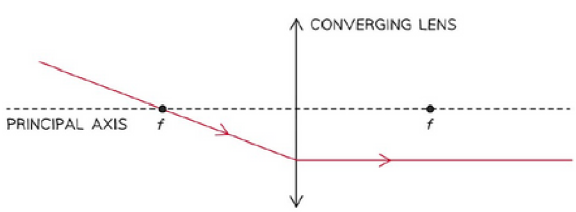
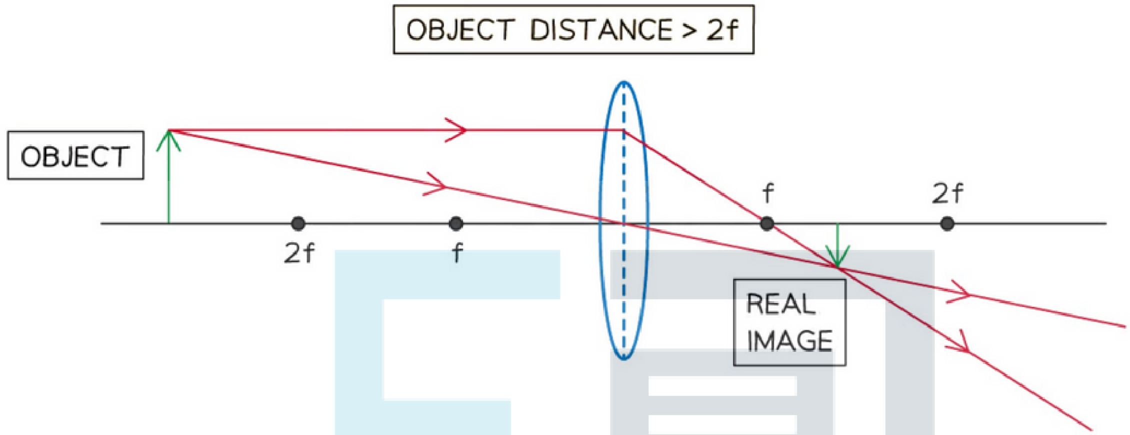


Image Formation by a Converging Lens

- Images formed by lenses can be described by their
  - **Nature:** Real or virtual
  - **Orientation:** Inverted or upright (compared to the object)
  - **Size:** Magnified (larger), diminished (smaller), or the same size (compared to the object)

Drawing ray diagrams of real images

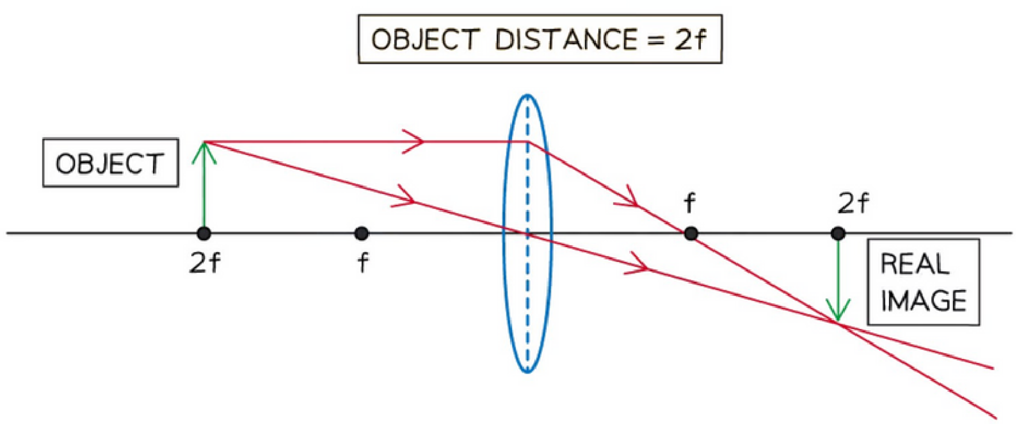
- For an object placed at a distance **greater than** 2 focal lengths...



- The image that forms will have the following properties:

The image forms...	between *f* and 2*f*
The nature of the image is...	real
The orientation of the image is...	inverted
The size of the image is...	diminished

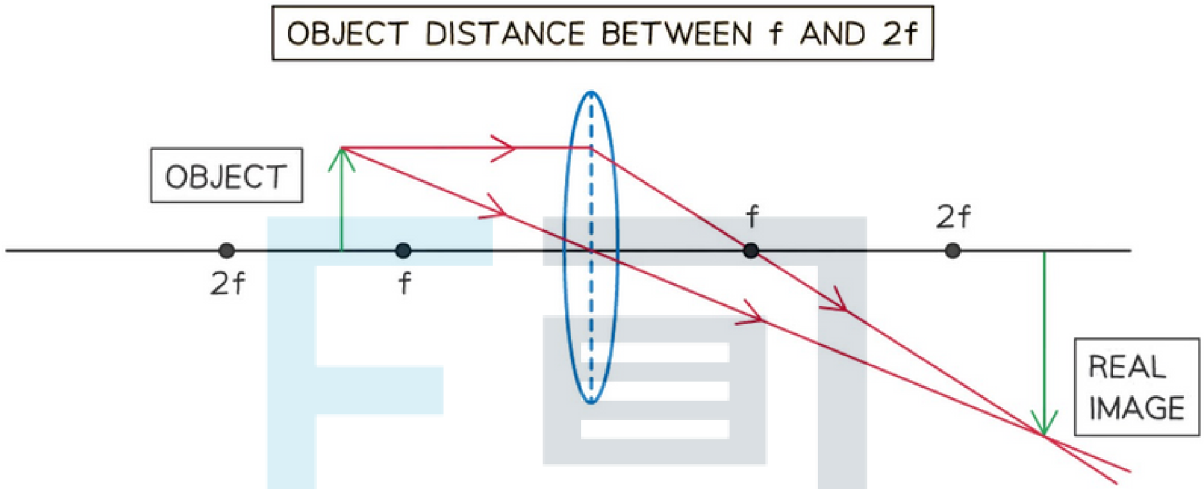
- For an object placed at a distance **equal to** 2 focal lengths...



- The image that forms will have the following properties:

The image forms…	at 2*f*
The nature of the image is…	real
The orientation of the image is…	inverted
The size of the image is…	the same

- For an object placed at a distance **between** 1 and 2 focal lengths

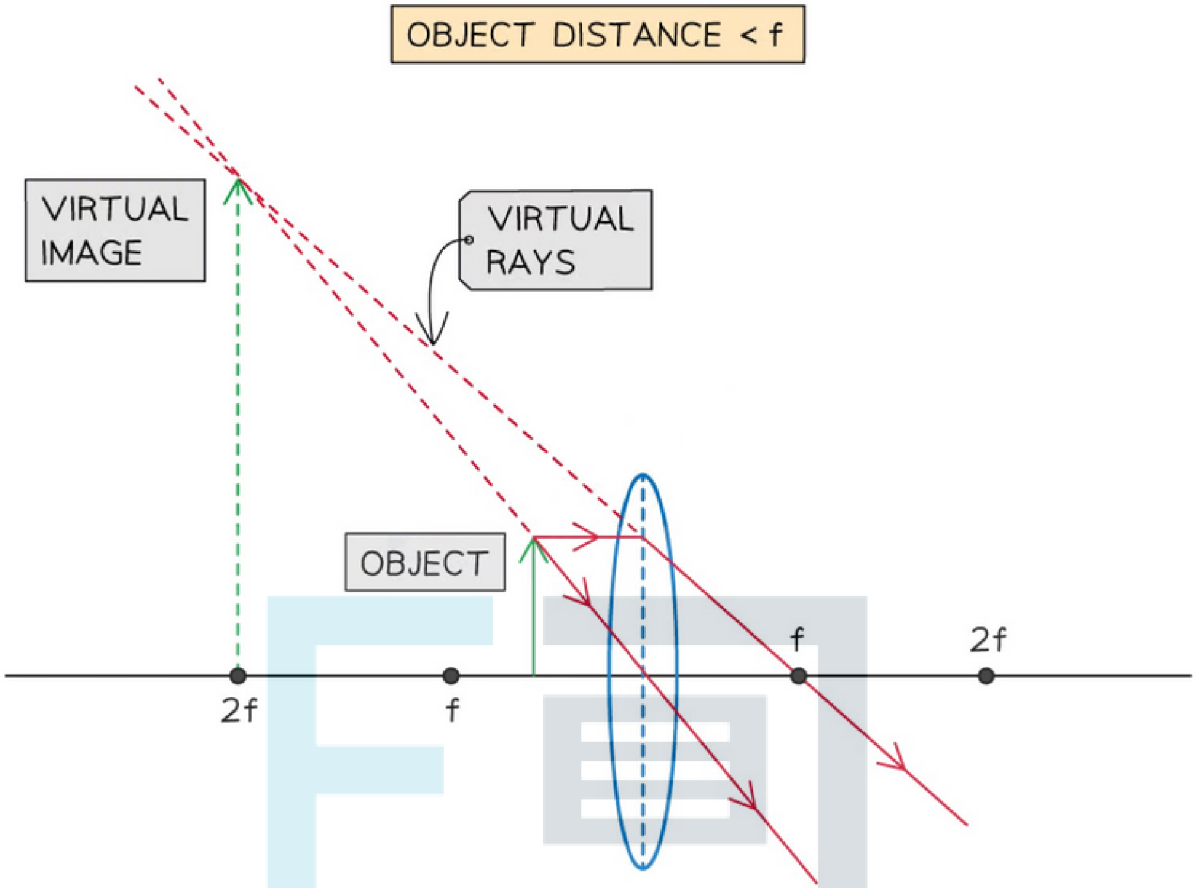


- The image that forms will have the following properties:

The image forms…	beyond 2*f*
The nature of the image is…	real
The orientation of the image is…	inverted
The size of the image is…	magnified

Drawing ray diagrams of virtual images

- For an object placed at a distance **less than** the focal length (i.e. a **magnifying glass**):



- The image that forms will have the following properties:

The image forms...	at 2*f* (on the same side as the object)
The nature of the image is...	virtual
The orientation of the image is...	upright
The size of the image is...	magnified

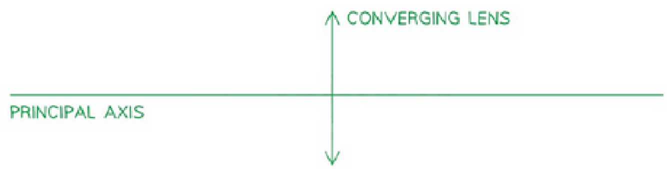
Worked Example

Draw a ray diagram to show how a converging lens can be used to form a diminished image of a real object.

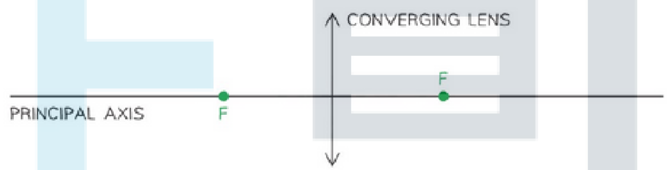
Label the object, image and principal foci of the lens on your diagram.

Answer:

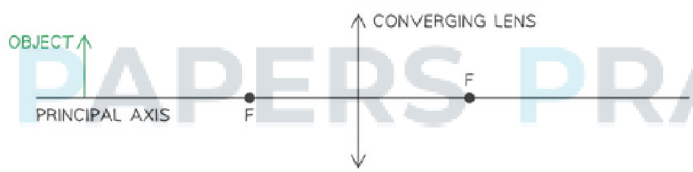
Step 1: Start by drawing and labelling a principal axis and the lens as a line or a very thin ellipse



Step 2: Mark and label the focal points on each side of the lens

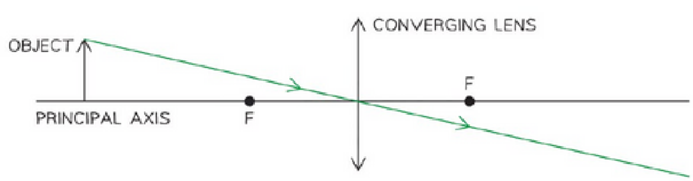


Step 3: Draw and label the object at a distance greater than the focal length on the left side of the lens

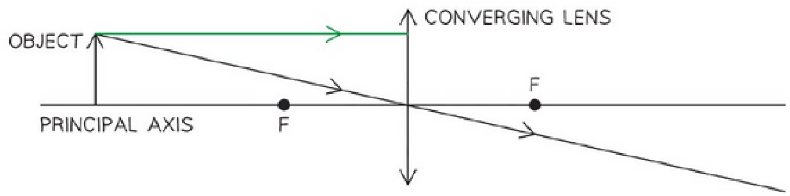


- Tip: the object should be placed a distance of at least 2F away from the lens

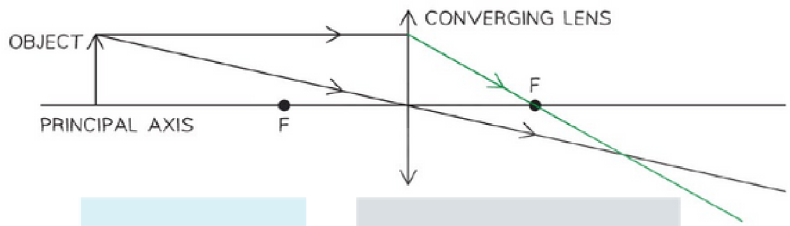
Step 4: Draw a ray through the optical centre of the lens



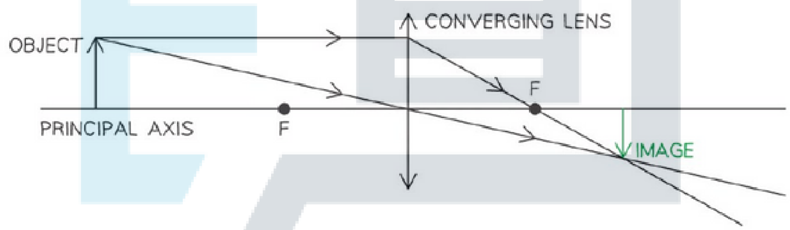
Step 5: Draw a second ray from the object to the lens which is parallel to the principal axis



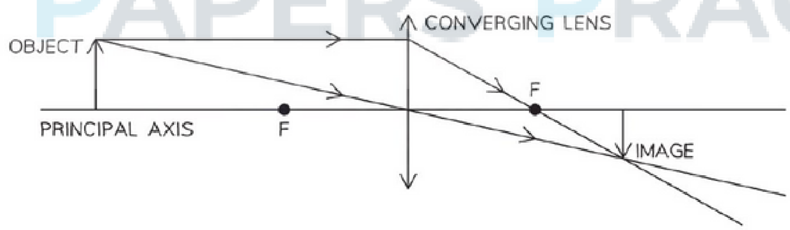
Step 6: Draw the continuation of the ray passing through the focal point on the right side of the lens



Step 7: Draw and label the image at the point where the rays meet



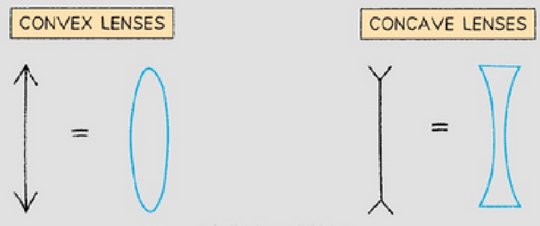
Step 8: Check your final image and make sure everything is included to gain the marks



- For a three-mark question, examiners will be looking for:
  - One ray drawn through the optical centre of the lens
  - A second ray drawn which produces a diminished (smaller) image (which must pass through a labelled focal point)
  - Both the object and the image must be drawn and labelled correctly

Exam Tip

When drawing ray diagrams, convex (converging) and concave (diverging) lenses can be simplified using the following symbols:



The Lens Equation & Magnification

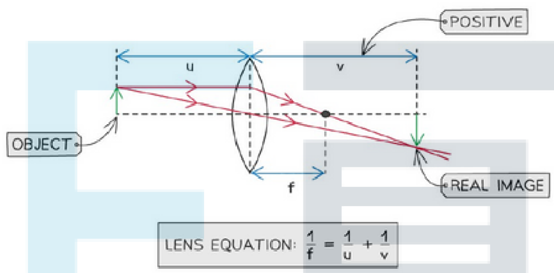
The Lens Equation

- The lens equation relates the **focal length** of a lens to the **distances** between the **lens** and the **image** and the **object**

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- Where:
  - f = focal length (m)
  - v = distance of the image from lens (m)
  - u = distance of the object from lens (m)

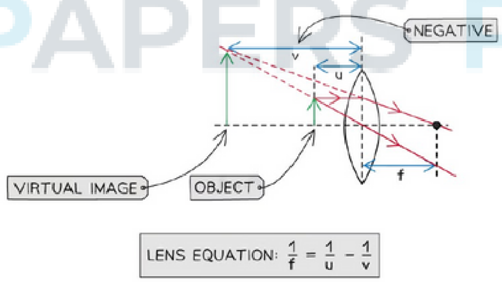
Lens Equation for a Real Image



All values are positive for a real image

- This equation only works for **thin** converging (or diverging) lenses
  - If the image is real, the value of v is **positive**
  - If the image is virtual, the value of v is **negative**

Lens Equation for a Virtual Image



A virtual image forms on the same side as the object making the value of v negative

Magnification as a Ratio of Heights

- Magnification, M, is the ratio of the image height to the object height

$$M = \frac{h_i}{h_o}$$

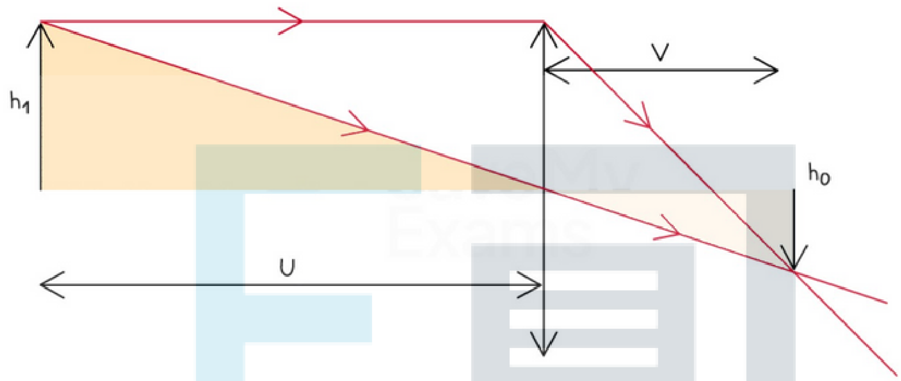
- Where:

- M = magnification
- h_i = image height (m)
- h_o = object height (m)

Magnification as a Ratio of Distances

- A diagram of an object and its real image will produce similar triangles
- Therefore, magnification can be determined by comparing the distances from the lens to the object and the image

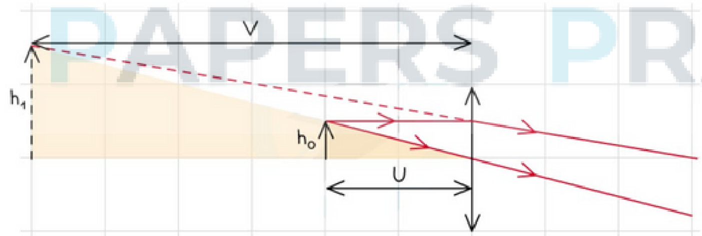
Magnification Ray Diagram for a Real Image



Magnification for a real image can be derived using similar triangles

- This also works for virtual images

Magnification Ray Diagram for a Virtual Image



Magnification for a virtual image can also be derived using similar triangles

- As magnification can be described using the ratio of the two opposite sides h_i and h_o, the same ratio must apply to the two adjacent sides v and u
- Therefore, magnification can also be written as:

$$M = \frac{v}{u}$$

- Where:
  - M = magnification

  - v = distance from lens to image (m)
  - u = distance from lens to object (m)

- Since magnification is a **ratio**, it has **no units**

Worked Example

A magnifying glass has a focal length of 15 cm. It is held 5 cm away from a component which is being examined.

Determine the magnification of the image.

Answer:

Step 1: Write the known values

- Focal length, f = 15 cm
- Distance between object and lens, u = 5 cm

Step 2: Use the lens formula and rearrange to make v the subject

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$v = \left(\frac{1}{f} - \frac{1}{u}\right)^{-1}$$

$$v = \left(\frac{1}{15} - \frac{1}{5}\right)^{-1} = -7.5 \text{ cm}$$

- The negative sign indicates a virtual image is formed (expected for a magnifying glass) and can be ignored for the next step

Step 3: Use the magnification formula to find the magnification of the image

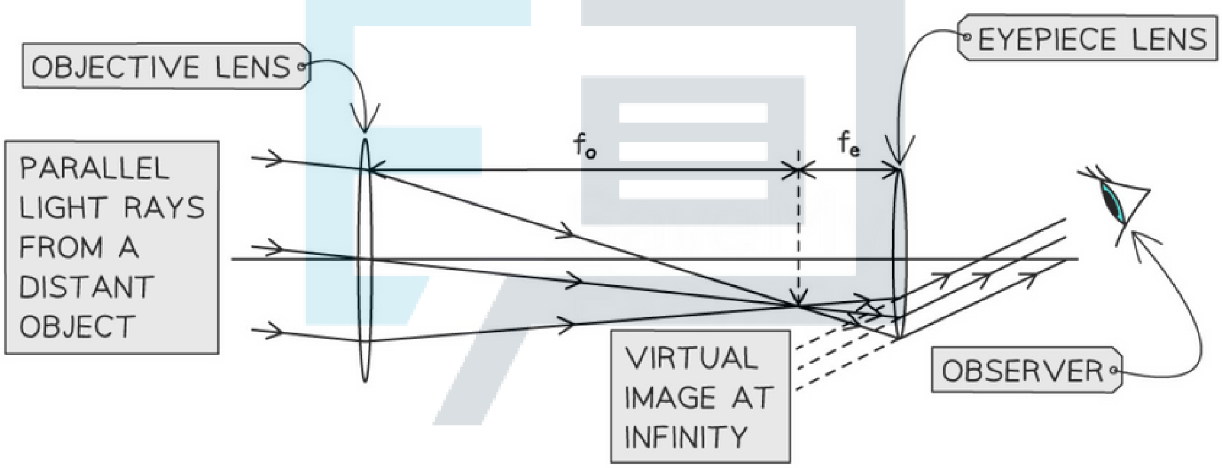
$$M = \frac{v}{u} = \frac{7.5}{5} = 1.5$$

9.1.2 Refracting Telescopes

Ray Diagram for a Refracting Telescope

- A refracting telescope, or **refractor**, utilises **two converging lenses** to project images of distant objects
- One of the lenses is called an **objective lens**
  - This lens collects the light from stars and brings it to a focus at its focal length f_o
- The other lens is called an **eyepiece lens**
  - This lens is placed at a distance of its focal length f_e away from the image and produces parallel rays of light to be analysed

Ray Diagram for a Refractor



Ray diagram of a refracting telescope in normal adjustment showing axial and non-axial rays

- A simple refractor is usually adjusted so that the final image is at **infinity**
  - This is known as **normal adjustment**
- For a refractor to be in normal adjustment
  - Both lenses must be arranged so that their focal points **meet** in the **same** place
  - The focal length of the objective lens must be **longer** than the focal length of the eyepiece lens, i.e. $f_o > f_e$
- **Note:** in the exam, you will be expected to draw this ray diagram with at least 3 non-axial rays - see the worked example below

Worked Example

Draw a ray diagram for an astronomical refracting telescope in normal adjustment.

Your diagram should show the paths of three non-axial rays passing through both lenses.

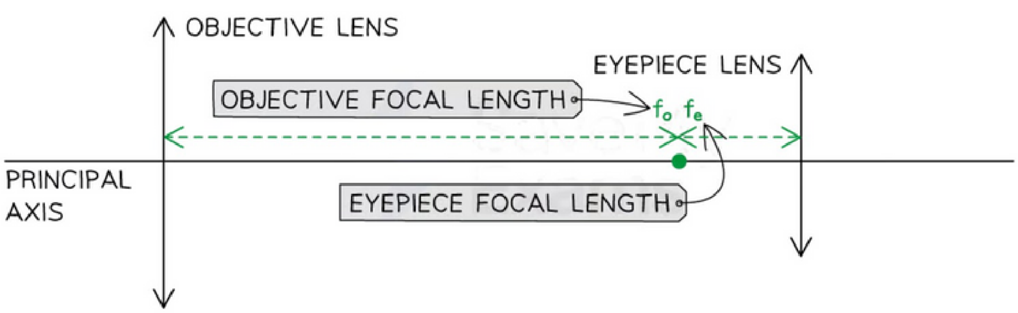
Label the principal foci of the two lenses.

Answer:

Step 1: Start by drawing and labelling the two lenses

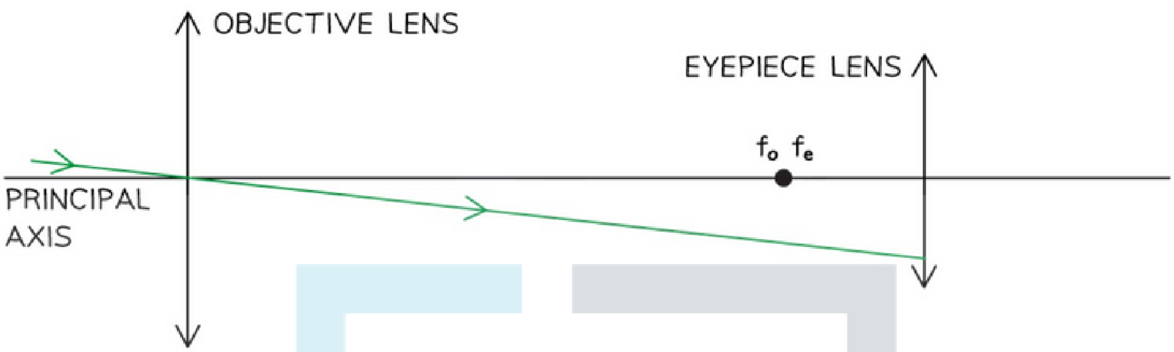
- If the question does not include a principal axis, make sure to draw this first!
- You can draw the lenses as lines with arrows or very thin ellipses

Step 2: Mark and label the common principal foci



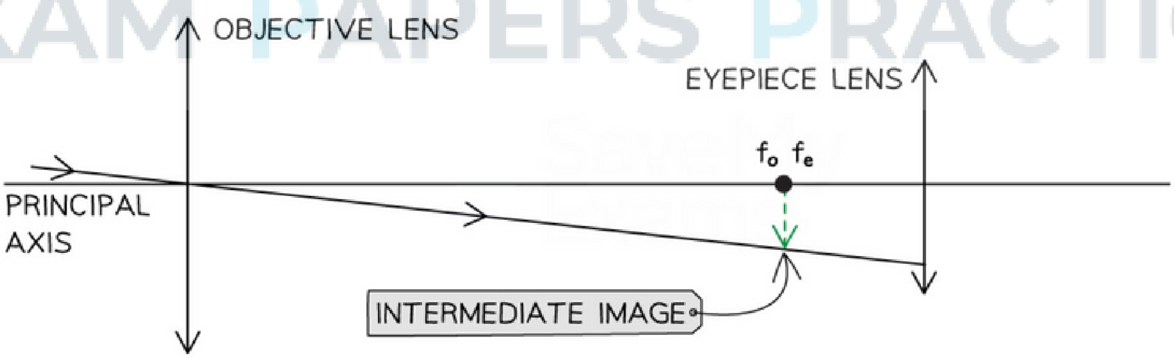
- The objective focal length **must** be longer than the eyepiece focal length $(f_o > f_e)$
- You could also mark a single point and label it F

Step 3: Draw an off-axis ray through the centre of the objective to the eyepiece



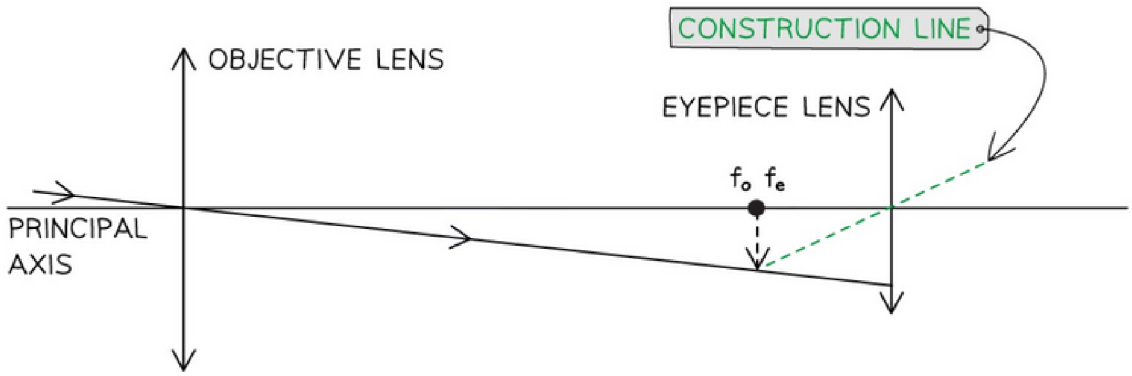
- The rays **must** be off-axis (non-axial), meaning drawn at an angle to the principal axis

Step 4: Draw an arrow to show the intermediate image from the common principal foci to this ray



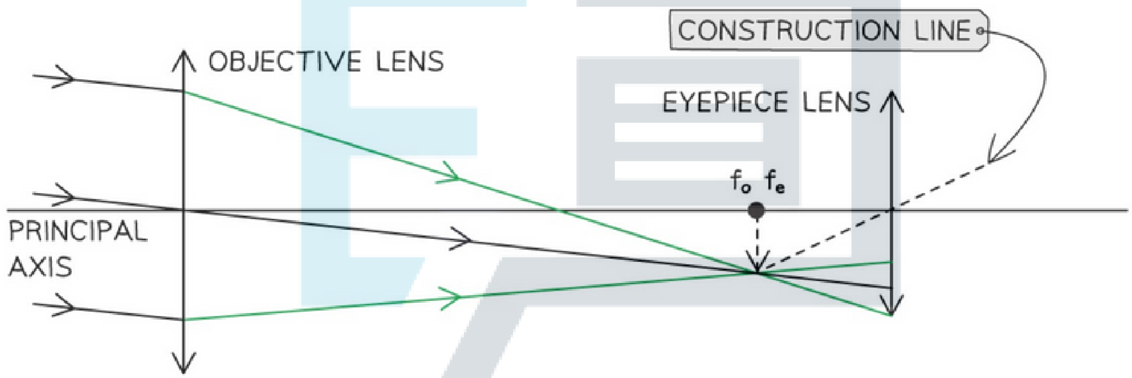
- You don't have to do this step, but it will help the examiner to clearly see your working

Step 5: Draw a construction line from the end of the intermediate image through the centre of the eyepiece

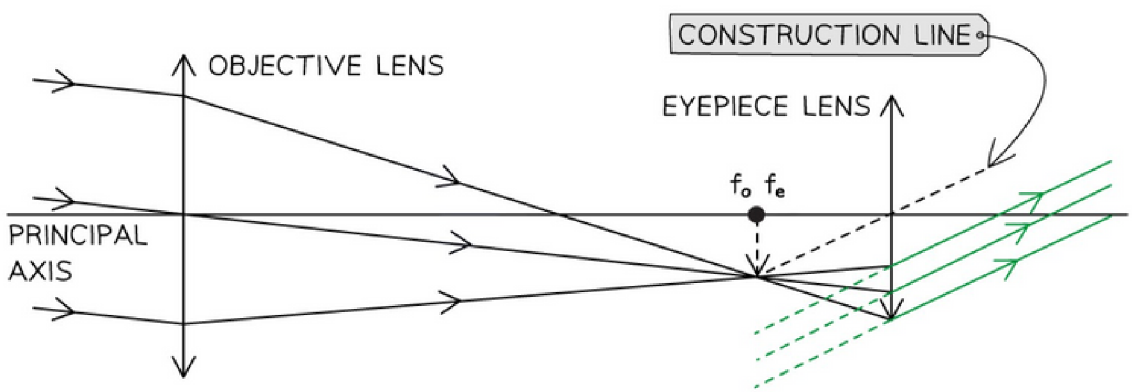


- You don't have to do this either, but it will help the examiner to clearly see your working

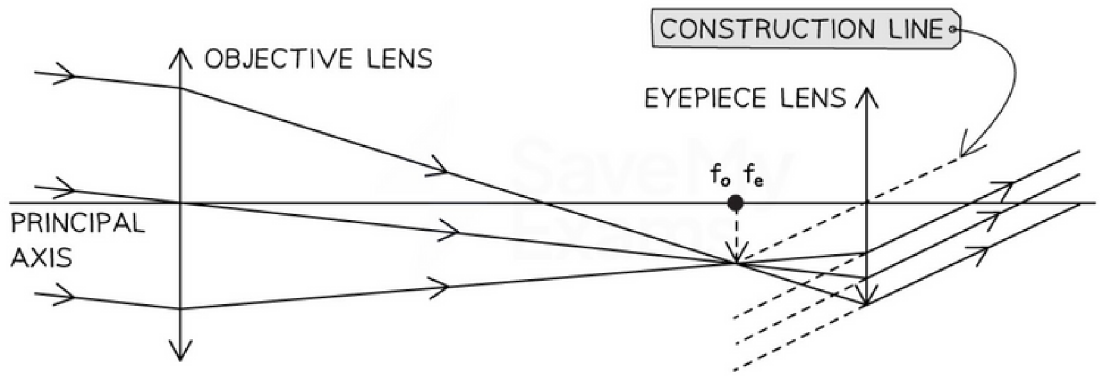
Step 6: Draw two rays to the eyepiece, crossing at where the focal lengths meet



Step 7: Draw the continuation of the three rays from the eyepiece, parallel to the construction line



Step 8: Check your final image and make sure everything is included to gain the marks



- For a three-mark question, examiners will be looking for:
  - Both focal points are marked and labelled at the same point on the principal axis with $f_o > f_e$
  - Three off-axis rays drawn through the objective lens
  - Three rays drawn through the eyepiece lens parallel to a construction line

Exam Tip

It is important that you get lots of practice drawing this diagram, it's a common exam question, and examiners can be extremely meticulous when it comes to marking them

Make sure to **avoid** these common problems when drawing the diagram:

- Drawing axial rays (parallel to the principal axis) rather than non-axial rays (at an angle to the principal axis)
- Bending the central ray at the objective lens
- Bending the rays at the intermediate image (i.e. at f_o, f_e)
- Not drawing the rays parallel to each other, or the construction line, at the eyepiece lens
- Labelling the principal foci where the rays cross, rather than on the principal axis

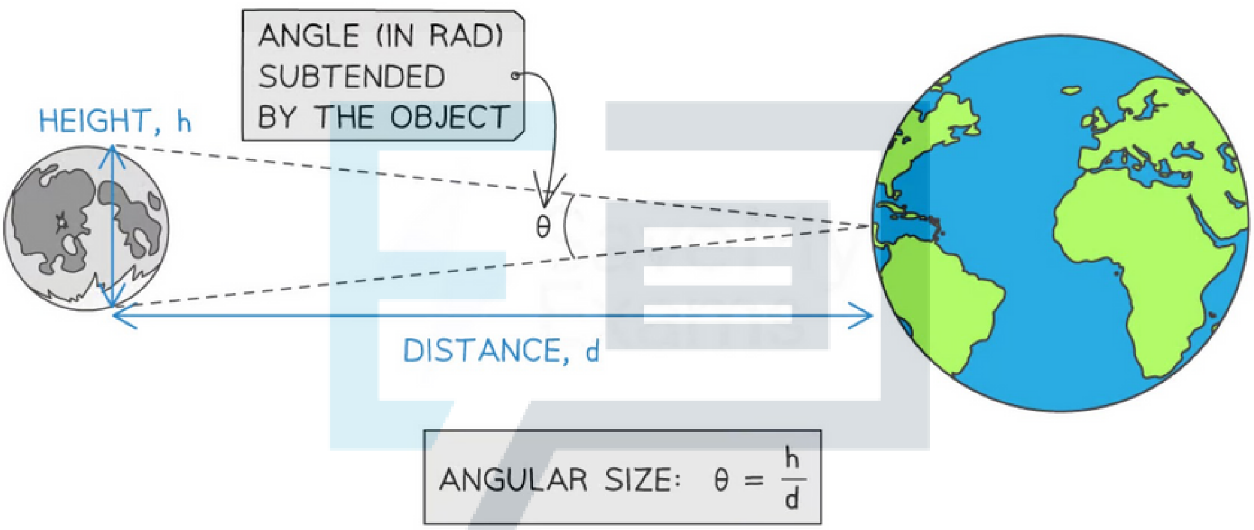
Angular Magnification

- The angle, in radians, subtended by an object of height *h*, at a distance *d* away is given by:

$$\theta = \frac{h}{d}$$

- This is also known as the **angular size** of an object
- **Note:** this is the angle between the rays seen at the extremities of the object to the eyes or telescope lens

Angular Size of the Moon



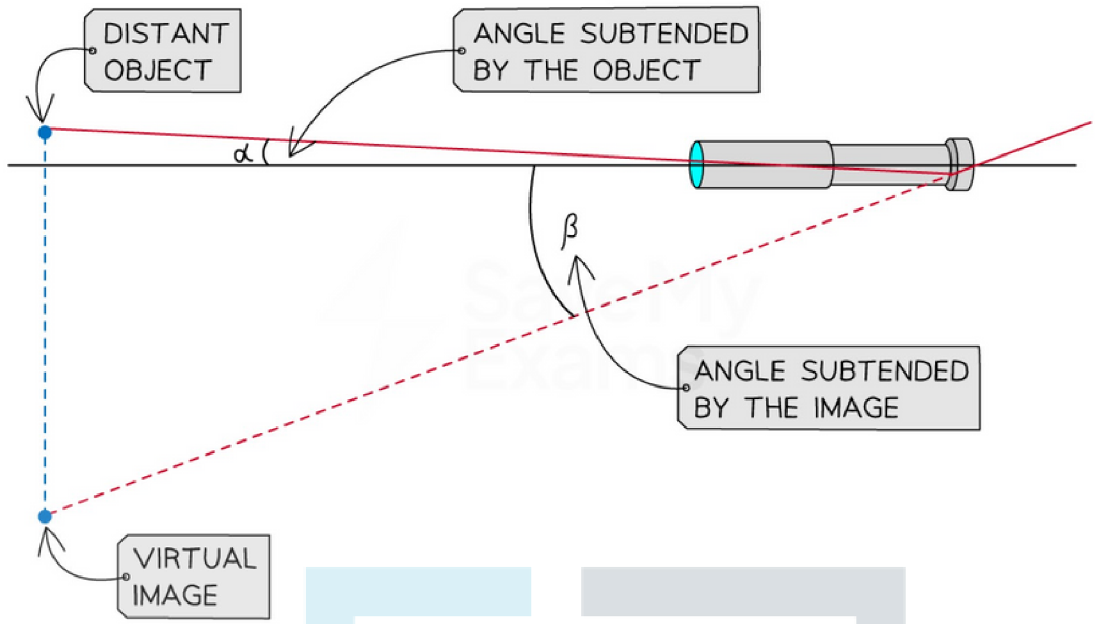
How to calculate the angle subtended by an astronomical object

- For astronomical objects that are very distant, essentially at **infinity**, it isn't easy to directly measure their size or the distance to them
- Therefore, it makes more sense to use **angular magnification**, which is defined as:

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}} = \frac{\beta}{\alpha}$$

- Like magnification, angular magnification is a **ratio** and has **no units**

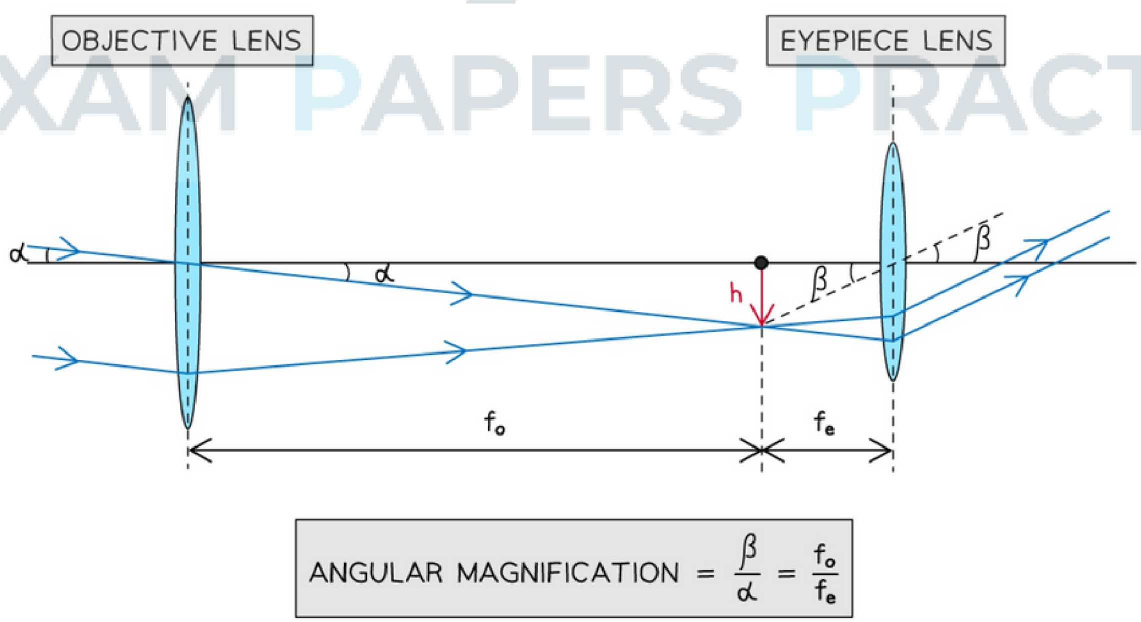
Angles Subtended by the Object & the Image



The angle subtended by an astronomical object and the angle subtended by the image produced by the telescope

- Telescopes **magnify** the angular size of distant objects, this means that:
  - The telescope produces an image which subtends a **larger angle** than the object
  - When viewed by the **naked eye**, the angle subtended by the object α is **much less** than the angle subtended by the image β when viewed through a **telescope**

Angular Magnification of a Refractor



How to determine the angular magnification of a refracting telescope in normal adjustment

- When looking at the ray diagram of a refractor in normal adjustment, the rays of light inside the telescope can be seen to form **similar triangles**
- The angle subtended by the object (α) can be described as:

$$\tan \alpha = \frac{h}{f_o} \Rightarrow \alpha = \frac{h}{f_o}$$

- The angle subtended by the image (β) can be described as:

$$\tan \beta = \frac{h}{f_e} \Rightarrow \beta = \frac{h}{f_e}$$

- Where:
  - h = the height of the image (m)
  - f_o = focal length of the objective lens (m)
  - f_e = focal length of the eyepiece lens (m)
  - $\tan \theta \approx \theta$ when the angle is very small (rad)
- Combining these equations leads to the following expression for angular magnification:

$$M = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$$

- This equation tells us that:
  - To achieve greater magnifications, **longer** objective focal lengths (f_o) and **shorter** eyepiece focal lengths (f_e) are required
  - To achieve this, refractors must be **very long**, as the length is equal to $f_o + f_e$

Worked Example

A student is looking for suitable lenses to use in a simple refracting astronomical telescope.

The student sets out to measure the focal length of a converging lens by placing an object and screen at a fixed distance of 300 cm apart.

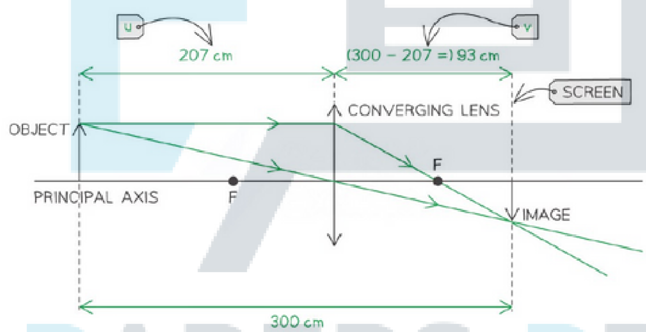
A sharp image is observed on the screen when the lens is placed between the object and the screen at a distance of 207 cm from the object.

(a)
Calculate the focal length of the lens.
(b)
State whether the lens formed the eyepiece or objective, giving reasons for your answer.

Answer:

(a)

Step 1: Sketch a quick diagram to visualise the scenario



Step 2: Write down the lens equation

$$\text{lens equation: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- Where u = 207 cm and v = 93 cm

Step 3: Calculate the focal length of the lens

$$\frac{1}{f} = \frac{1}{207} + \frac{1}{93}$$

$$f = \left(\frac{1}{207} + \frac{1}{93}\right)^{-1} = 64 \text{ cm}$$

(b)

You could either argue...

- The eyepiece forms a virtual, inverted, magnified image
- The objective lens forms a real, inverted, diminished image of a distant object very near the focal point of the eyepiece

- The image formed by the lens in **(a)** is real, inverted, and diminished
- Therefore, the lens must be an objective lens

Or, you could argue...

- In a refracting telescope, angular magnification is given by: $M = \frac{f_o}{f_e}$
- Hence, the eyepiece must have a shorter focal length and the objective must have a longer focal length: $f_o > f_e$
- For the best possible magnification, the image due to the objective lens, which acts as the object for the eyepiece lens, must be located at the focal point of the eyepiece
- Hence, the two lenses must be separated by a distance $f_o + f_e$ (i.e. the telescope length)
- Therefore, the lens must be an objective lens

Worked Example

An astronomical refracting telescope which is 1.23 m long and has an angular magnification of 200 is used to observe Neptune at its closest approach to Earth.

- distance from Earth to Neptune at closest approach = 4.3×10^9 km
- diameter of Neptune = 4.9×10^4 km

(a)
Calculate the focal lengths of the objective lens and eyepiece lens.
(b)
Calculate the angle subtended by the image of Neptune when viewed through this telescope.

Answer:

(a)

Step 1: List the relevant equations and known quantities

Telescope length: $f_o + f_e = 1.23 \text{ m}$

Angular magnification: $M = \frac{f_o}{f_e} = 200$

Step 2: Write an expression for one of the focal lengths in terms of the other

$$f_o = 200 f_e$$

$$200 f_e + f_e = 1.23 \text{ m}$$

$$201 f_e = 1.23 \text{ m}$$

Step 3: Solve to determine the focal lengths of the lens

- Focal length of the eyepiece: $f_e = \frac{1.23}{201} = 6.1 \times 10^{-3} \text{ m} \approx 0.01 \text{ m}$
- Focal length of the objective: $f_o = 200 \times (6.1 \times 10^{-3}) = 1.22 \text{ m}$

(b)

Step 1: Recall the equations for angular size and angular magnification

Angular side: $\theta = \frac{h}{d}$

- Where diameter, $h = 4.9 \times 10^4$ km and distance, $d = 4.3 \times 10^9$ km

Angular magnification: $M = \frac{\beta}{\alpha}$

- Where α = angle subtended by Neptune at unaided eye and β = angle subtended by the image of Neptune at eye

Step 2: Determine the angle subtended by Neptune at unaided eye

$$\theta = \frac{4.9 \times 10^4}{4.3 \times 10^9} = 1.14 \times 10^{-5} \text{ rad}$$

- The angle subtended by Neptune with the unaided eye: $\alpha = 1.14 \times 10^{-5}$ rad

Step 3: Determine the angle subtended by the image of Neptune at the eyepiece

$$M = \frac{\beta}{\alpha} \Rightarrow \beta = \alpha M$$

$$\beta = (1.14 \times 10^{-5}) \times 200 = 2.28 \times 10^{-3} \text{ rad}$$

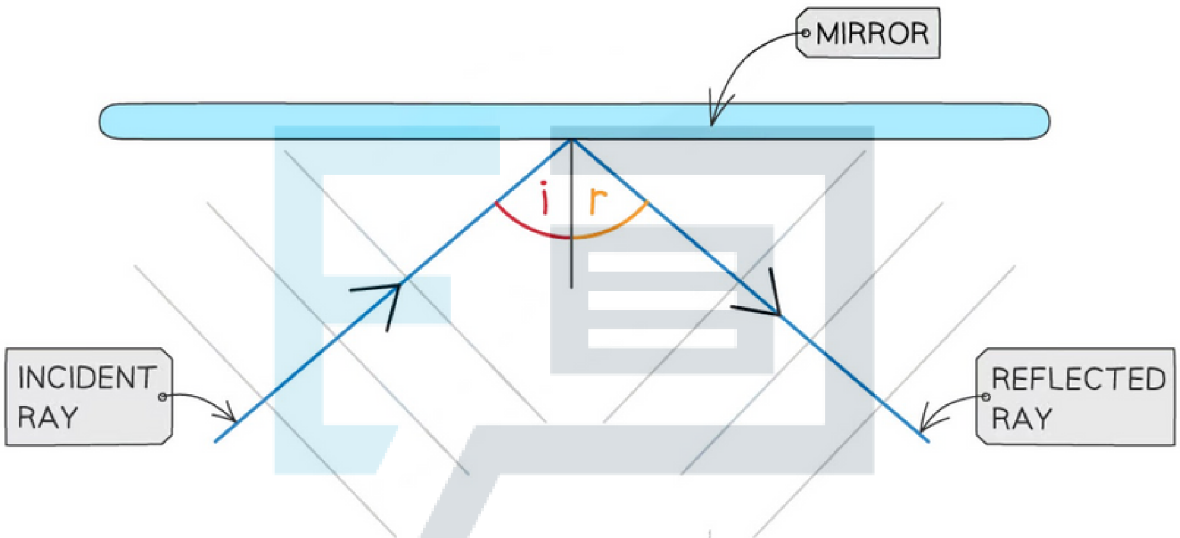
- Angle subtended by the image of Neptune at eyepiece: $\beta = 2.28 \times 10^{-3}$ rad

9.1.3 Reflecting Telescopes

The Cassegrain Telescope

- Reflecting telescopes, or reflectors, utilise parabolic (curved) mirrors to collect and focus light from distant objects
  - The most common type of reflector is known as a **Cassegrain telescope**
- Reflectors work because of the law of reflection where the angle of incidence, *i* is **equal** to the angle of **reflection,** *r*

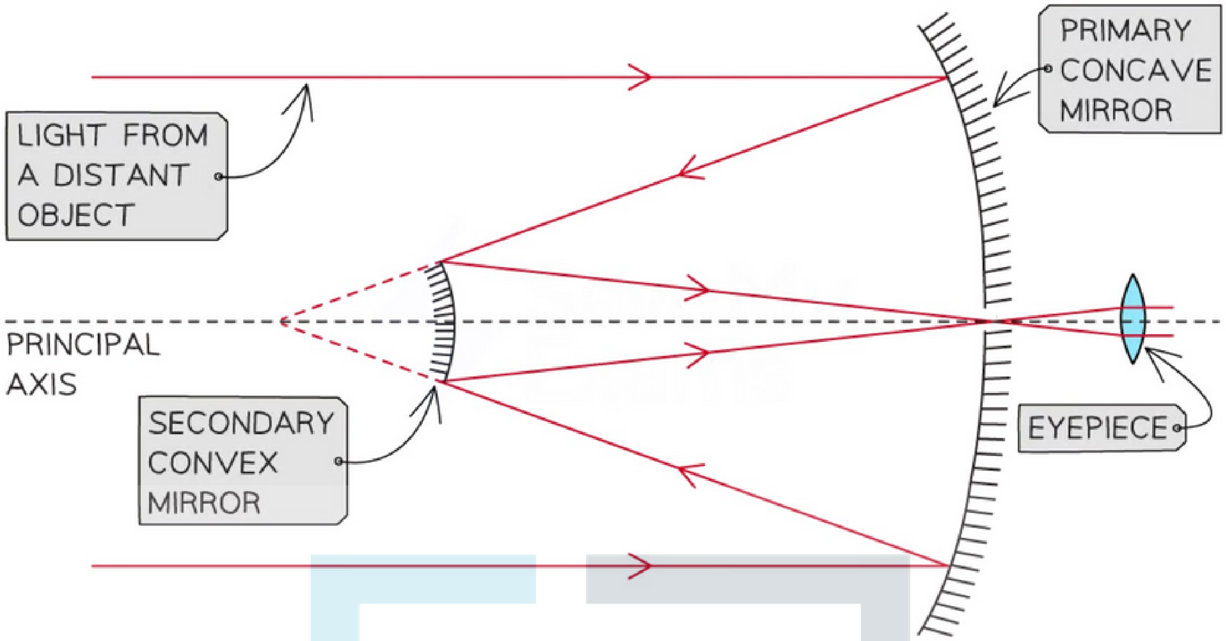
The law of reflection on a ray diagram



Reflecting telescopes work through the law of reflection

- Instead of two converging lenses, a reflector uses a **primary mirror** and a **secondary mirror**
- The primary mirror is **large** and **concave**
  - Here, incident light reflects towards a focal point which is behind the secondary mirror
- The secondary mirror is **smaller** and **convex**
  - Here, light reflects again to form a real, magnified image at the eyepiece
- The rays are directed through an aperture towards an **eyepiece lens** which is located behind the primary mirror

Ray Diagram of a Cassegrain Reflecting Telescope



Ray diagram of a reflecting telescope in a Cassegrain arrangement

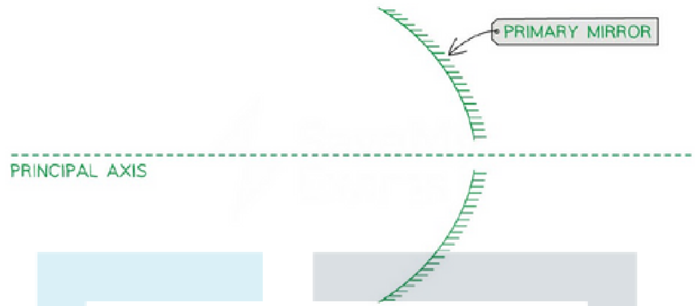
- The important features of the ray diagram for a Cassegrain telescope are:
  - The rays enter the telescope **parallel** to the principal axis
  - The curvature of the mirrors does **not** have to be the same
  - The rays do **not** cross before the secondary mirror, they only cross **in** the aperture of the primary mirror
  - Shading (or the lines on the mirror) indicates the **non**-reflective side of the mirrors

Worked Example

Draw a ray diagram to show the path of two rays, parallel to the axis, through a Cassegrain telescope, as far as the eyepiece.

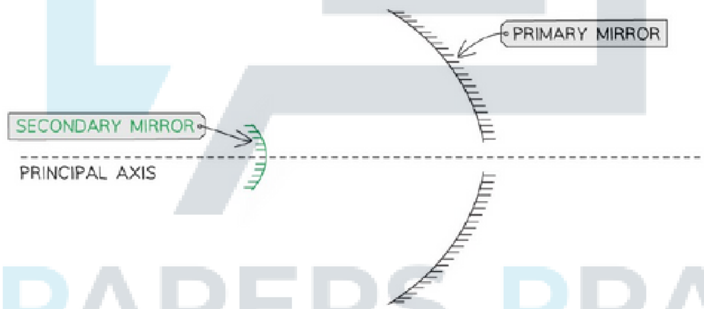
Answer:

Step 1: Start by drawing and labelling a principal axis, a primary concave mirror and an eyepiece lens



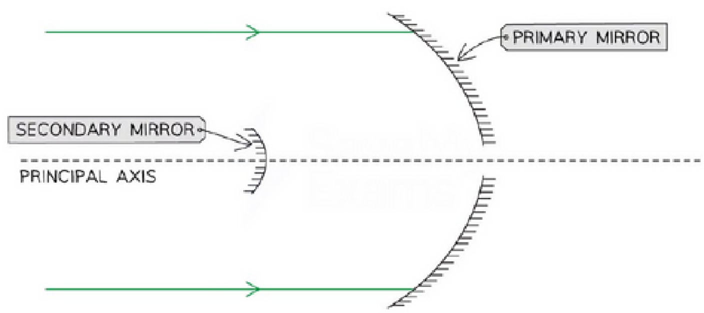
- Make sure to include the aperture (gap) at the centre of the primary mirror

Step 2: Draw and label a secondary convex mirror

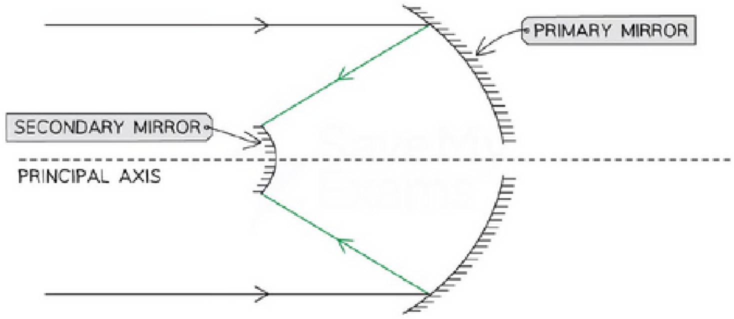


- Use lines or shading to indicate the non-reflective side of each mirror

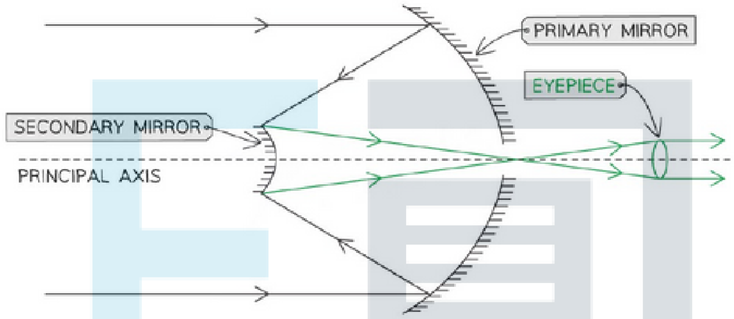
Step 3: Draw two rays parallel to the principal axis



Step 4: Draw the reflection of the rays by the primary mirror onto the secondary mirror

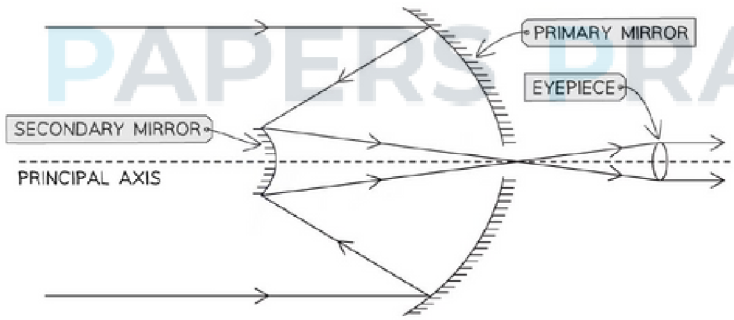


Step 5: Draw the reflection of the rays by the secondary mirror towards the eyepiece



- Make sure the rays do not cross **until** they reach the aperture

Step 6: Check your final image and make sure everything is included to gain the marks



- For a two-mark question, examiners will be looking for:
  - The correct curvature of the mirrors, i.e. a concave primary mirror **and** a convex secondary mirror
  - The primary to look like **one** mirror i.e. it should look like a continuous parabola
  - Two rays entering the telescope **parallel** to the principal axis
  - The rays crossing in the **aperture** of the primary mirror

Exam Tip

Make sure to **avoid** these common problems when drawing the ray diagram of a Cassegrain telescope:

- Drawing the two halves of the primary mirror with different curvatures so it looks like two separate mirrors
  - Make sure to draw these with a pencil. You could always draw the full curvature of the mirror then just rub out a gap in the centre of the drawing
- Drawing the secondary mirror as a straight line, or even concave
- Forgetting to include the eyepiece lens
- Drawing the rays as crossing before hitting the secondary mirror

The Cassegrain arrangement is the **only** reflector you must learn about in this course.

You will notice that light rays from distant astronomical objects are always drawn as **parallel**. This is because the object is so far away, that by the time it reaches the lens, the rays are parallel, even if it's an irregularly shaped object.

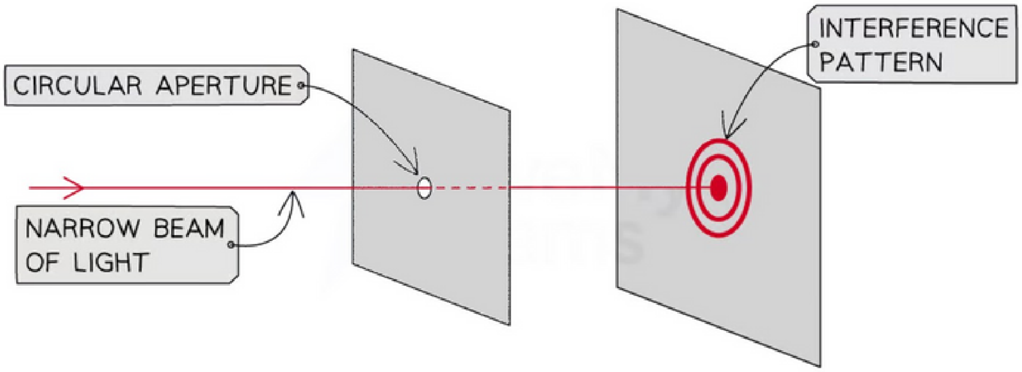
There's a common misconception associated with the secondary mirror is that it blocks the light (which it does) resulting in the central portion of the image being missing (which is not the case). There will be a slight reduction in the amount of light, but the light rays from a distant source are parallel so that even light from a source on the mirror axis will reach the secondary and be collected.

9.1.5 Resolving Power of Telescopes

Minimum Angular Resolution

- A circular aperture, such as a lens in a telescope, is designed so that a cone of light can enter into a region behind it
  - This allows light to **act like a point source** once it passes through
- When two point sources are placed **near each other**, or viewed from a large distance, they will appear to be a single **unresolved** source of light
  - For example, two distant car headlights may initially appear as a single point source until the car moves close enough for your eyes to resolve them into two individual headlights
- Light from any object passing through a circular aperture, including the human eye, will **diffract** and create interference fringes upon the **detector** inside
  - The pattern is **circular** and is an approximate pattern for a circular aperture
- The large **central maximum** is called an **Airy disc** and is twice as wide as the further maxima in the pattern

Diffraction of Light using a Circular Aperture

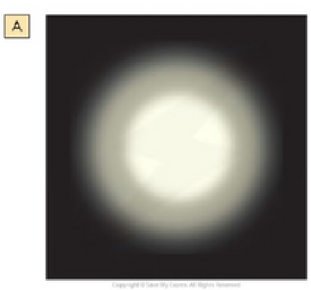
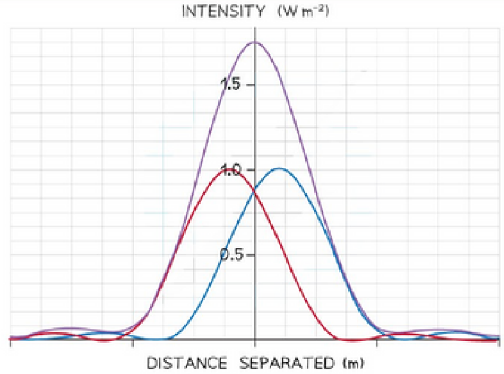


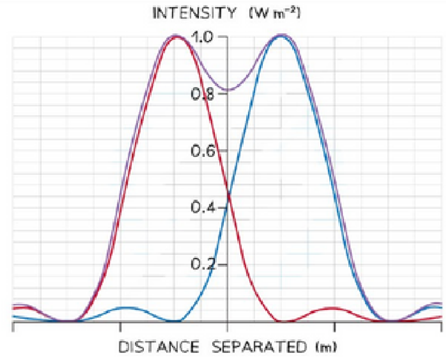
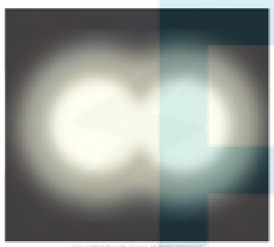
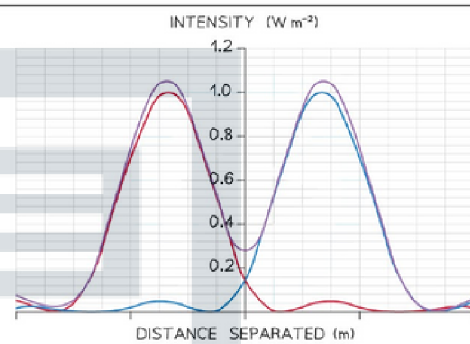
A circular interference pattern can be seen when light is diffracted through a circular aperture instead of a rectangular slit

- Diffraction also affects how well a telescope can resolve fine detail
- The **resolving power** or **minimum angular resolution** of a telescope can be determined using the **Rayleigh criterion**
- The Rayleigh criterion states that:

  Two sources will be resolved if the central maximum of one diffraction pattern coincides with the first minimum of the other

- The resolution, or resolving power, of a telescope can be increased by reducing the amount the light diffracts, for example, by:
  - **Increasing** the **diameter** of the aperture
  - Operating at a **shorter** wavelength of light

Visual diffraction pattern	Variation of intensity with separation
	
A. Two sources that cannot be resolved	Two sources (red & blue curves) that are too close together appear as one single source (purple curve)

B 	
B. Two sources that can only just be resolved, as defined by the Rayleigh Criterion	Two sources (red & blue curves) which do not overlap significantly, but just about appear as two sources (purple curve)
C 	
C. Two sources that are clearly resolved	Two sources (red & blue curves) which are far apart enough to appear as two distinct sources (purple curve)

Exam Tip

The terms 'resolution' and 'resolving power' are often both used interchangeably to describe the quality of a telescope in terms of the minimum angular separation it can achieve

For example, saying

- A telescope has a resolution (or resolving power) of 0.005 degrees

Is the same as saying

- A telescope can resolve two stars which have an angular separation of at least 0.005 degrees

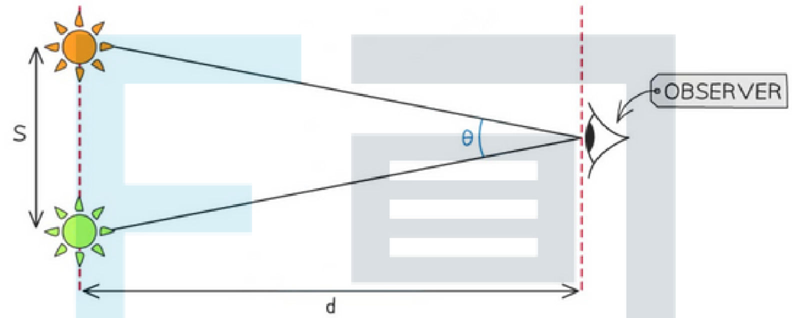
Remember that the 'Airy disc' is just the **central maximum** of the interference pattern, not the name of the whole pattern itself.

The Rayleigh Criterion

- The Rayleigh Criterion can be mathematically described by considering **angular separation** and **single-slit diffraction** through a **circular aperture**
- Angular separation can be calculated using the equation:

$$\theta = \frac{s}{d}$$

- Where:
  - θ = angular separation (rad)
  - s = distance between the two sources (m)
  - d = distance between the sources and the observer (m)



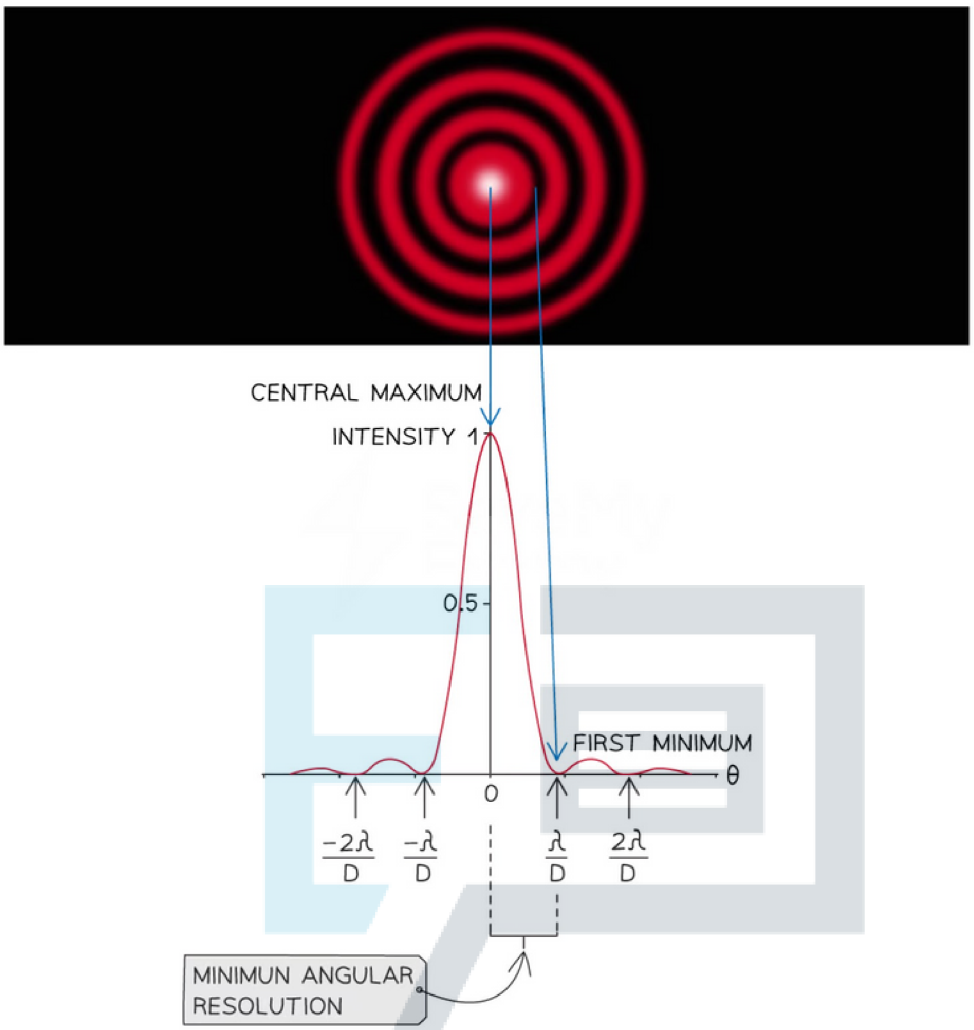
Angular separation, θ, is equal to the separation, s, of two sources divided by the distance, d, between the sources and the observer

- In single-slit diffraction, minima in the pattern appear at angles given by:

$$\sin\theta = \frac{n\lambda}{D}$$

- Where:
  - θ = the angle of diffraction (rad)
  - n = the order of the minimum (1,2,3 etc)
  - λ = the wavelength of the light (m)
  - D = the slit width (m)

Intensity Pattern from a Circular Aperture



The minimum angular resolution of a telescope can be determined using the angula separation between a source's central maximum and the first minimum

- For a telescope, the first minimum (when $n = 1$) occurs when the angle of diffraction is:

$$\sin\theta = \frac{\lambda}{D}$$

- Using the small-angle approximation $(\sin\theta \approx \theta)$, we obtain an expression for the minimum angular resolution of the telescope (The Rayleigh Criterion)

$$\theta \approx \frac{\lambda}{D}$$

- Where:
  - θ = minimum angular resolution of the telescope (rad)
  - λ = operating wavelength of the telescope (m)
  - D = diameter of the telescope's aperture (m)

- The Rayleigh criterion can therefore be written mathematically as follows:

1. Sources are **resolvable** when

$$\theta > \frac{\lambda}{D}$$

2. Sources are **just resolvable** when

$$\theta \approx \frac{\lambda}{D}$$

3. Sources are **not resolvable** when

$$\theta < \frac{\lambda}{D}$$

- With the **circular aperture**, the value is multiplied by a **factor** of **1.22**

$$\theta = \frac{1.22\lambda}{D}$$

which explains the "approximately equals" sign (≈)

Worked Example

The supermassive black hole at the centre of the Milky Way galaxy is 25,000 light years from the Earth. It has a Schwarzchild radius of 1.2×10^{10} m and emits radio waves at a frequency of 230 GHz.

The Event Horizon Telescope (EHT) is an array of radio telescopes which has the same resolution as a single radio telescope with a diameter of 8000 km.

(a)
Calculate the minimum angular separation which could be resolved by the EHT.

(b)
Deduce whether the resolution of the EHT is suitable to obtain a detailed view of the supermassive black hole. Support your answer with a calculation.

Answer:

(a)

Step 1: List the relevant quantities

- Frequency of the radio waves, f = 230 GHz = 230×10^9 Hz
- Speed of light, $c = 3 \times 10^8$ m s^{-1}
- Diameter of EHT, D = 8000 km = 8000×10^3 m

Step 2: Write down the equation for the minimum angular separation and substitute in the wave equation

- Wave equation: $\lambda = \frac{c}{f}$
- Minimum angular separation: $\theta = \frac{\lambda}{D} = \frac{c}{fD}$

Step 3: Calculate the minimum angular separation the EHT can resolve

$$\theta = \frac{3 \times 10^8}{(230 \times 10^9) \times (8000 \times 10^3)} = 1.63 \times 10^{-10} \text{ rad}$$

- This means the EHT can obtain a detailed view of objects down to an angular size of 1.63×10^{-10} rad

(b)

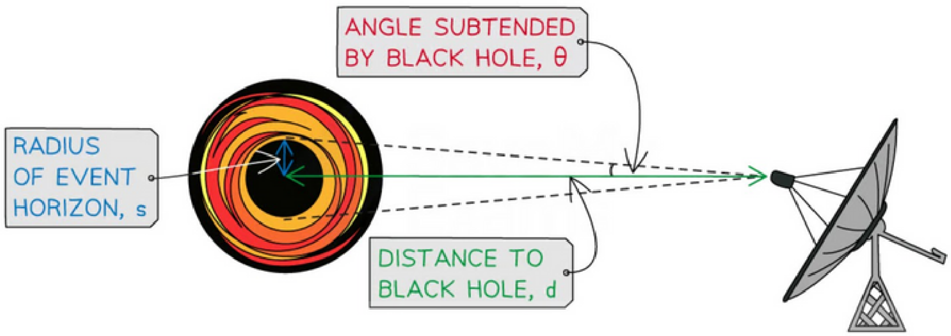
Step 1: List the relevant quantities

- Schwarzschild radius of the black hole, $s = 1.2 \times 10^{10}$ m
- Light year, 1 ly = 9.46×10^{15} m (included in the data booklet)
- Distance to black hole, d = 25 000 ly

Step 2: Write down the equation for angular size

- Angular size = angle subtended by the black hole (i.e. its Schwarzchild / event horizon radius):

$$s = d\theta \quad \Rightarrow \quad \theta = \frac{s}{d}$$



Step 3: Carry out a supporting calculation and write a conclusion

You could use...

Method 1: Calculate the angle subtended by the black hole

- At a distance of 25 000 ly, the black hole subtends an angular size of:

$$\theta = \frac{s}{d}$$

$$\theta = \frac{1.2 \times 10^{10}}{25\ 000 \times (9.46 \times 10^{15})} = 5.07 \times 10^{-11} \text{ rad}$$

- Compare with the resolution of the EHT:

$$\frac{1.63 \times 10^{-10}}{5.07 \times 10^{-11}} = 3.2 \approx 3$$

- Conclusion:

  The angle subtended by the black hole, 5.07×10^{-11} rad, is approximately 3 times greater than the limit of the resolution of the EHT, 1.63×10^{-10} rad.

  Therefore, it will not be able to suitably resolve detail in an image of the black hole.

Or, you could use...

Method 2: Calculate the size of an object that could just be resolved at this distance

- At a distance of 25 000 ly, the smallest object the EHT could resolve in detail is:

$$s = d\theta$$

$$s = 25\,000 \times (9.46 \times 10^{15}) \times (1.63 \times 10^{-10}) = 3.85 \times 10^{10} \text{ m}$$

- Compare with the radius of the black hole:

$$\frac{3.85 \times 10^{10}}{1.2 \times 10^{10}} = 3.2 \approx 3$$

- Conclusion:

  The smallest object the EHT could just resolve at this distance, 3.85×10^{10} m, is approximately 3 times greater than the Schwarzchild radius of the black hole, 1.2×10^{10} m.

  Therefore, it will not be able to suitably resolve detail in an image of the black hole.

Exam Tip

It is better to say that θ is the 'minimum angular resolution' of the telescope instead of 'resolving power', as this implies that θ is a power (in Watts) instead of an angle. However, if you are asked for the resolving power in the exam, it means to **calculate θ.**

Remember the wavelength and diameter must be in the **same** units.

9.1.6 Collecting Power of Telescopes

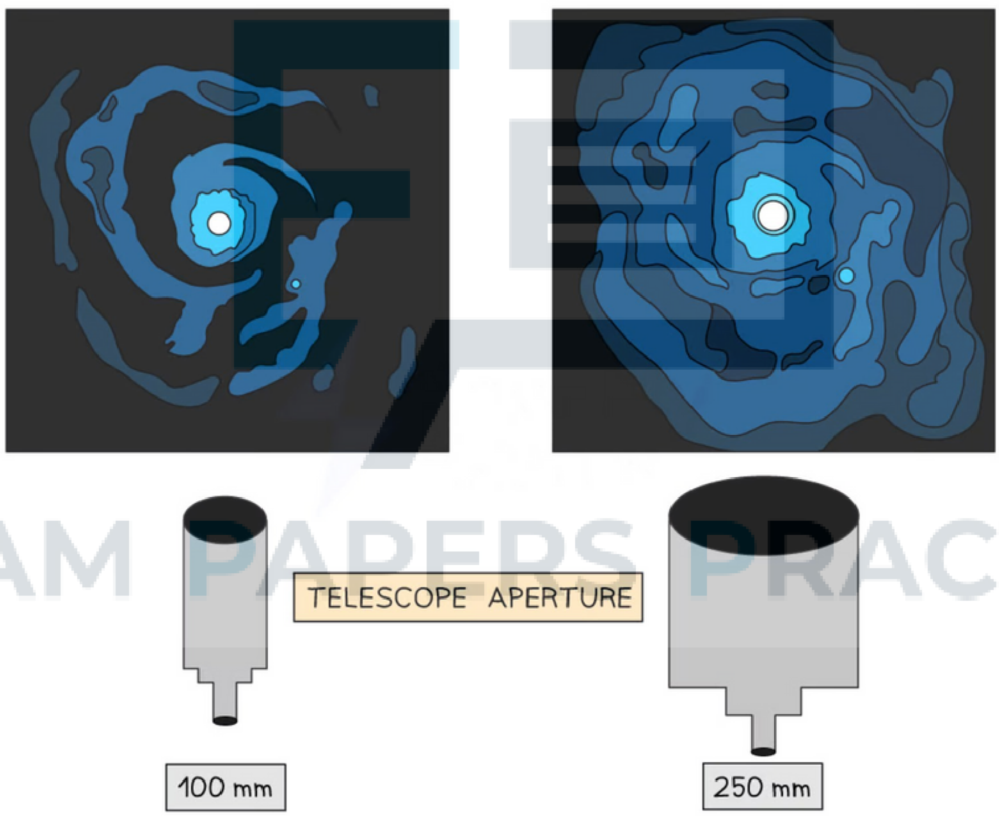
Collecting Power

- Telescopes are designed to gather as much light as possible
  - The more light energy a telescope can gather, the brighter the images it will be able to produce
  - This can be measured by a telescope's **collecting power**
- The collecting power of a telescope is defined as:

A measure of the amount of light energy it collects per second

- This is equivalent to the power per unit area, or intensity of the incident radiation collected

Differences in Collecting Power



The telescope with the larger aperture is able to produce a brighter image

- The collecting power of a telescope is **directly proportional** to the **square** of the diameter of its objective

$$collecting\ power \propto D^2$$

- This is because:
  - Intensity is **proportional** to surface **area**

- The surface area of a circular object of diameter D is equal to $\frac{\pi D^2}{4}$

- This means that objects at **greater** distances can also be seen
  - The intensity of light from a point source decreases **inversely** with the **square** of the **distance** from the inverse square law

Large Diameter Telescopes

- Larger aperture diameter telescopes are **advantageous** for two main reasons:
  - They have a greater collecting power so images are **brighter**
  - They have a greater resolving power so images are **clearer**

- The collecting power of two telescopes can be calculated using the ratio

$$\frac{collecting\ power\ of\ telescope\ 1}{collecting\ power\ of\ telescope\ 2} = \left(\frac{D_1}{D_2}\right)^2$$

- The resolving power of two telescopes operating at the same wavelength can be calculated using the ratio

$$\frac{resolving\ power\ of\ telescope\ 1}{resolving\ power\ of\ telescope\ 2} = \frac{\theta_1}{\theta_2} = \frac{\frac{\lambda}{D_1}}{\frac{\lambda}{D_2}} = \frac{D_2}{D_1}$$

Worked Example

The largest refracting telescope still in operation is the Yerkes refractor. The construction of this telescope later paved the way for the Otto Struve reflector to be built. Both telescopes detect optical wavelengths of light.

The table below summarises some of the properties of the two optical telescopes.

Telescope	Type	Objective diameter / cm
Yerkes	refractor	102
Otto-Struve	reflector	208

Compare the two telescopes in terms of their collecting power and resolving power.

Answer:

Step 1: Compare the collecting power of the telescopes

- Since collecting power, or intensity of light collected ∝ area ∝ (diameter)^2

$$\frac{collecting\ power\ of\ reflector}{collecting\ power\ of\ refractor} = \left(\frac{D_{reflector}}{D_{refractor}}\right)^2$$

$$\frac{collecting\ power\ of\ reflector}{collecting\ power\ of\ refractor} = \left(\frac{208}{102}\right)^2 = 4.16 \approx 4$$

- The collecting power of the Otto-Struve reflector is 4 times greater than the Yerkes refractor, meaning it will produce brighter images

Step 2: Compare the resolving power of the telescopes

- Resolving power, or minimum angular resolution: $\theta \propto \frac{1}{D}$ (for the same wavelength)

$$\frac{\theta_{reflector}}{\theta_{refractor}} = \frac{D_{refractor}}{D_{reflector}}$$

$$\frac{\theta_{reflector}}{\theta_{refractor}} = \frac{102}{208} = 0.49 \approx 0.5$$

- The Yerkes refractor can resolve detail half as well as the Otto-Struve reflector
- Therefore, the resolving power of the Otto-Struve reflector is twice as great as the Yerkes refractor, meaning it will produce clearer images

Exam Tip

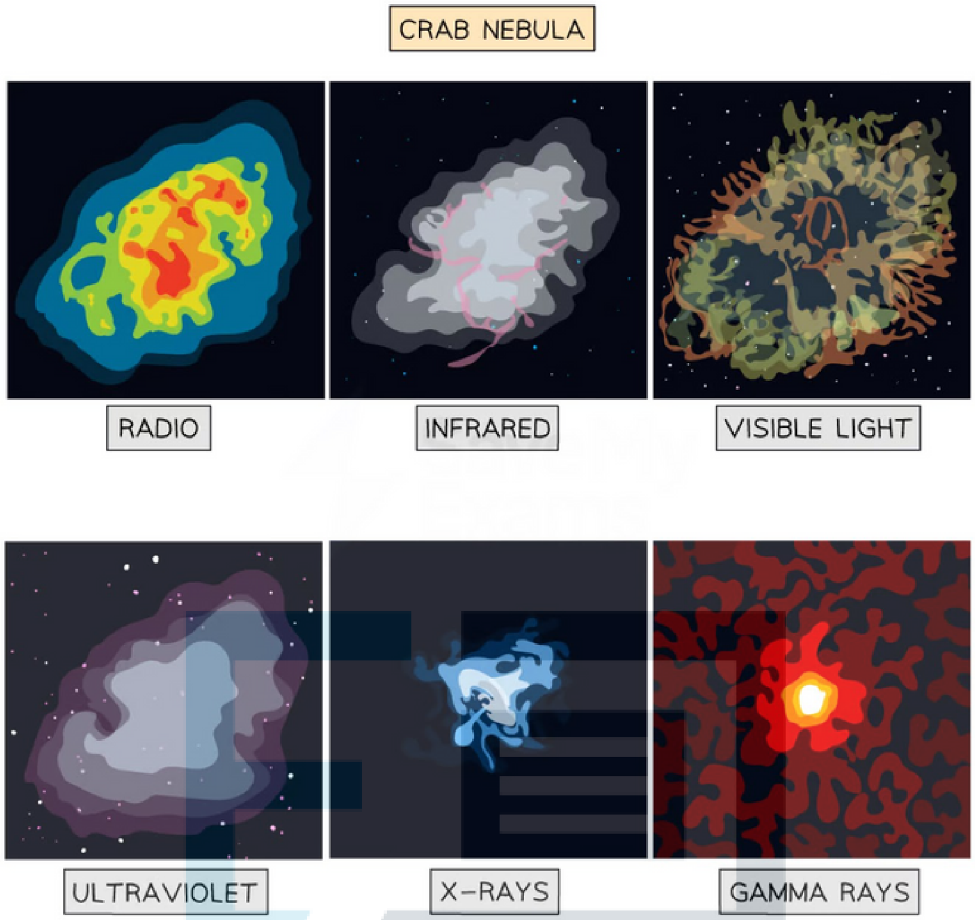
Remember: the **smaller** the value of θ, the **greater** the resolving power

9.1.7 Radio, IR, UV & X-Ray Telescopes

Non-Optical Telescopes

- An **optical** telescope is one which detects wavelengths of light from the **visible** part of the electromagnetic spectrum
- Telescopes that look at other parts of the electromagnetic spectrum are known as **non-optical** telescopes, such as
  - **Radio** telescopes
  - **Infrared** (IR) telescopes
  - **Ultraviolet** (UV) telescopes
  - **X-ray** telescopes
- Being able to collect radiation from all parts of the electromagnetic spectrum opens up a whole world of new information for astronomers
  - For example, different areas of a supernova remnant (the Crab Nebula) are found to emit strongly at all wavelengths
  - In particular, radio waves, X-rays and gamma rays all appear to originate from the **neutron star** at its centre, whilst the infrared, visible and ultraviolet wavelengths appear to come from the **nebula** that surrounds it
- **Note:** images of astronomical objects are often given 'false colour' to help us visualise wavelengths the human eye cannot see

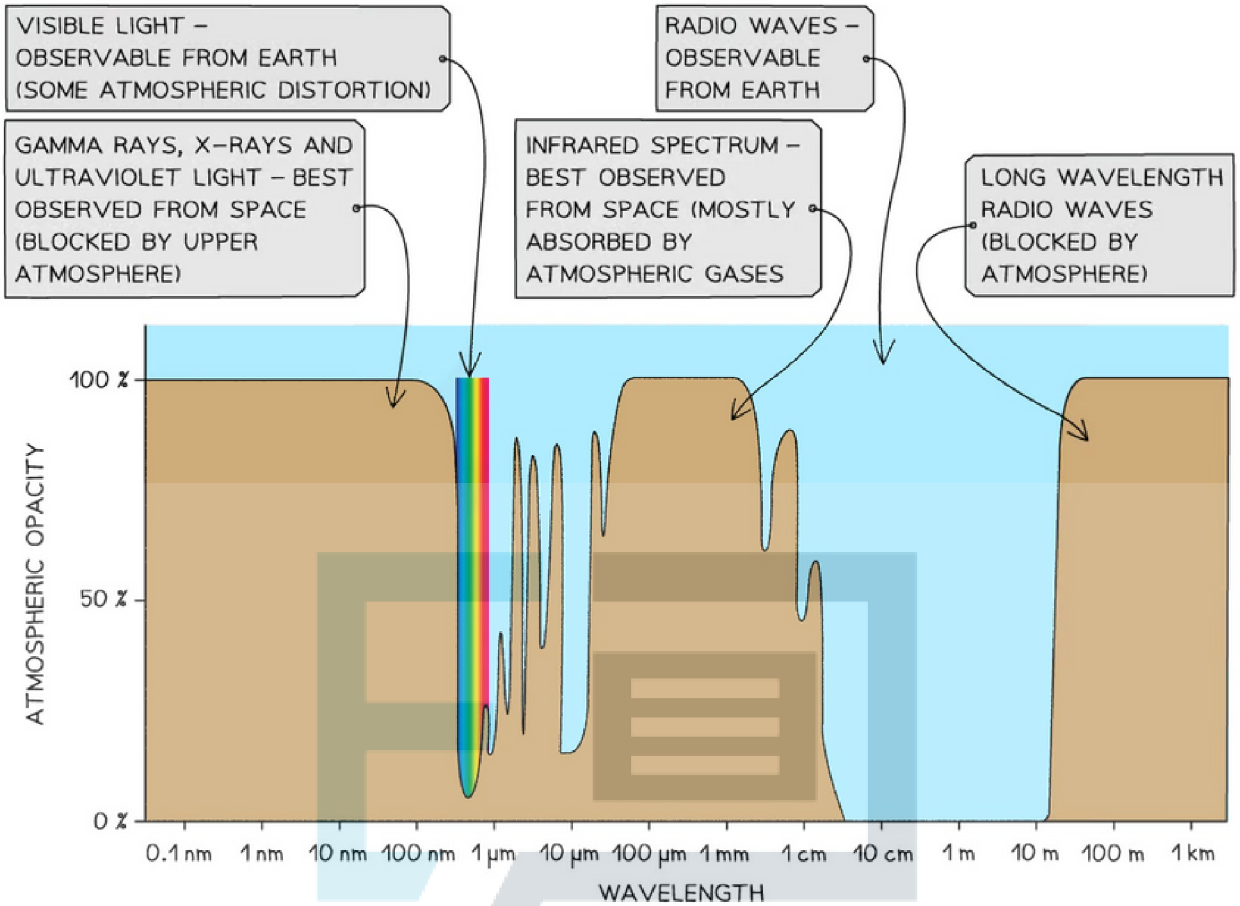
Crab Nebula at Different Wavelengths



The different wavelengths detected in the Crab Nebula tell astronomers a lot about the final stages of a massive star's life cycle

- Many telescopes are designed to detect a range of wavelengths that span multiple regions of the electromagnetic spectrum
- However, the operating wavelength range of a telescope is greatly limited by the **absorption** of certain wavelengths by the Earth's **atmosphere**

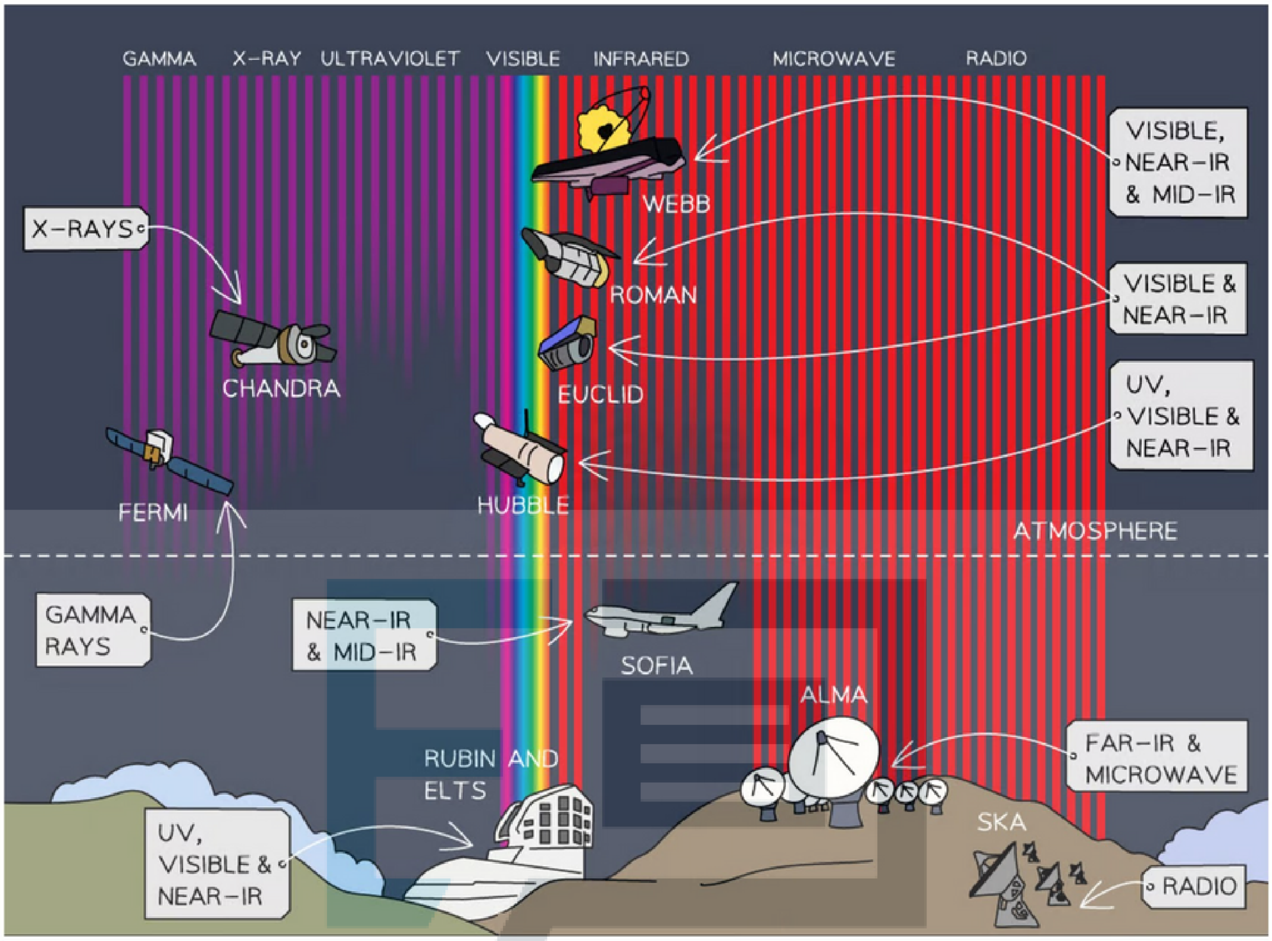
Absorption of Wavelengths by the Atmosphere



Gamma-rays, X-rays, ultraviolet and infrared wavelengths are best observed from space

- The graph of atmospheric opacity against wavelength shows that large ranges of wavelengths are partially, or completely, **absorbed** by our atmosphere
- This means that **ground-based** telescopes are able to observe:
  - All **visible** wavelengths (although there is often some distortion)
  - Very narrow ranges of **infrared** wavelengths
  - Most **microwave** & **radio** wavelengths
- Whereas, above the atmosphere, space-based telescopes are able to detect **all** wavelengths, making it possible to access:
  - **Gamma** rays, **X-rays** & **ultraviolet** rays
  - All **infrared** wavelengths (usually split into near-IR, mid-IR and far-IR)

Ground & Space-Based Telescopes



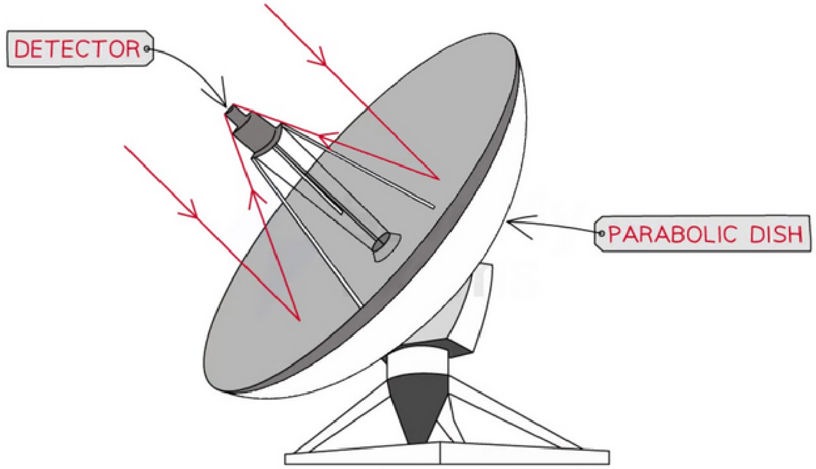
Some of the ground-based and space-based telescopes currently in operation

- The main benefits of putting telescopes into **space** are to **eliminate**:
  - The absorption of electromagnetic waves by the **atmosphere**
  - **Light pollution** and other interference at ground level
  - **Atmospheric effects**, such as scattering or scintillation (i.e. twinkling) of light

Radio Telescopes

- Location: ground-based
- Wavelength range: 1 mm to 10 m
- Typical resolution: 10^{-3} rad

Structure of a Radio Telescope



The radio telescope is made up of a detector and parabolic dish

	Similarities with optical telescopes	Differences with optical telescopes
Structure	• Both use parabolic surfaces to reflect waves	• Radio uses a single primary reflector, optical uses two mirrors • Radio dish does not need to be as smooth as optical mirrors
Positioning	• Both can be ground-based as the atmosphere is transparent to most radio and optical wavelengths	• Optical must be placed high up (to avoid atmospheric distortions) and away from cities (to avoid light pollution) • Radio must be located remotely (away from radio sources)
Uses	• Both are used to detect hydrogen emission lines (radio at 21 cm, visible at 410 nm, 434 nm, 486 nm and 656 nm)	• Radio waves are not absorbed by dust, whereas optical waves are, so radio telescopes are used to map the Milky Way
Resolving power	• Radio waves are longer than optical waves, so radio has a much lower resolving power (~10^{-3} rad) • Optical more likely to produce detailed images	
Collecting power	• Radio are larger in diameter, so they have a greater collecting power than optical telescopes • Therefore, radio is more likely to produce brighter images (although many radio sources are weak)	

Infrared Telescopes

- Location: predominantly space-based, but some ground-based observatories exist
- Wavelength range: 700 nm to 1 mm

- Typical resolution: 10^{-6} rad (ground) to 10^{-7} rad (space)

	Similarities with optical telescopes	Differences with optical telescopes
Structure	• Both IR and optical telescopes are constructed using a primary concave mirror and a secondary convex mirror	• Mirrors in IR telescopes must be kept very cold to avoid interference from surrounding heat
Positioning	• Many ground-based telescopes are able to detect both optical and near-IR wavelengths as long as they are positioned away from cities and are high above the ground	• IR radiation is strongly absorbed by water vapour in the atmosphere, so they must be built in dry, high-altitude locations, or above the atmosphere • The atmosphere is transparent to most optical wavelengths but blocks most IR wavelengths, so space-based IR telescopes are preferable
Uses	• Most objects that emit visible light also emit IR radiation, so valuable information can be obtained from both	• IR telescopes can detect warm objects that do not emit visible light, such as dust in nebulae and brown dwarfs
Resolving power	• IR telescopes have a lower resolving power than optical telescopes of the same size due to having a longer wavelength	
Collecting power	• The collecting power of IR and optical telescopes are similar as their diameters are similar	

Ultraviolet Telescopes

- Location: space
- Wavelength range: 10 to 400 nm
- Typical resolution: 10^{-7} rad

	Similarities with optical telescopes	Differences with optical telescopes
Structure	• Both UV and optical telescopes are constructed using a primary concave mirror and a secondary convex mirror	• Mirrors in UV telescopes must be smoother than those used in optical telescopes

Positioning	• Many space-based telescopes are able to detect both optical and UV wavelengths	• All UV wavelengths are strongly absorbed by the atmosphere (ozone) so UV telescopes must be located in space • Space-based UV telescopes can be inconvenient to maintain
Uses	• Both can be used to determine the chemical composition and temperatures of objects • Many objects that emit visible light also emit UV radiation, so valuable information can be obtained from both	• UV telescopes can detect objects not visible at other wavelengths, such as hot gas clouds near stars, supernovae and quasars
Resolving power	• UV telescopes have a higher resolving power than optical telescopes of the same size due to having a shorter wavelength	
Collecting power	• The collecting power of UV and optical telescopes are similar as their diameters are similar	

X-Ray & Gamma Telescopes

- Location: space
- Wavelength range: X-rays = 0.01 to 10 nm, gamma < 10 nm
- Typical resolution: 10^{-6} rad

	Similarities with optical telescopes	Differences with optical telescopes
Structure	• X-ray & optical telescopes both use parabolic mirrors to reflect and focus waves	• X-ray telescopes are made from a combination of parabolic and hyperbolic mirrors, all of which must be extremely smooth • Gamma telescopes don't use mirrors at all, they use specialised detectors
Positioning	• X-ray, gamma and optical telescopes all perform best when positioned in space, away from the restrictions imposed by the Earth's atmosphere	• All X-ray & gamma wavelengths are strongly absorbed by the atmosphere so these telescopes must be located in space • Space-based telescopes can be inconvenient to maintain
Uses	• X-ray & gamma can provide valuable additional information about visible objects, such as supernova remnants	• X-ray & gamma telescopes can observe otherwise non-visible objects and energetic events, such as neutron stars, black holes, pulsars and gamma-ray bursts (GRBs)

Resolving power	• X-ray & gamma telescopes have a much higher resolving power than optical telescopes of the same size due to their shorter wavelengths
Collecting power	• The collecting power of X-ray & gamma telescopes is much lower than optical telescopes are they have smaller objective diameters • However, X-ray and gamma sources tend to be extremely bright

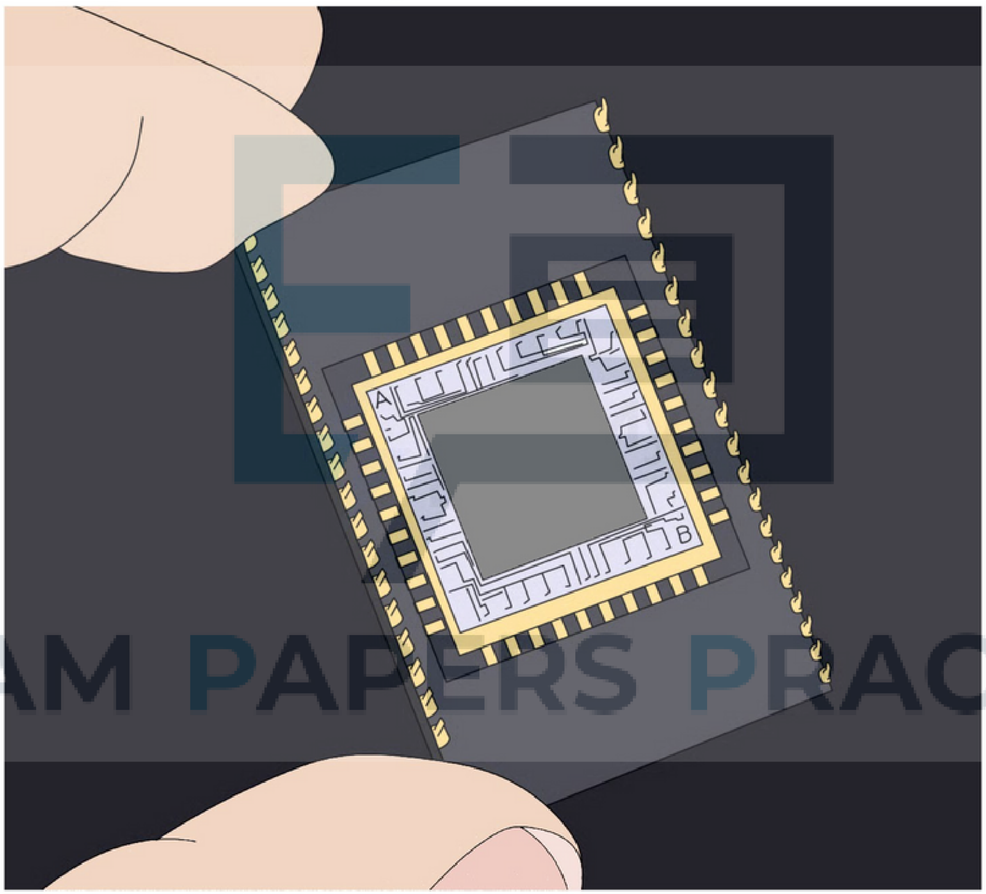
Exam Tip

You need to learn the key points for each type of telescope, so you can back up your arguments for comparisons between them. This could be useful information for a 6 mark question.

Charge-Coupled Devices (CCDs)

- A charge-coupled device (CCD) is a detector which is **highly sensitive** to photons, making it ideal for use in the detection system of modern telescopes
  - Incident photons cause **electrons** to be released
  - The number of electrons released is proportional to the **intensity** of the incident light
  - An image is formed on the CCD, which can be processed electronically to give a **digital image**

A charged-coupled device



A charged-couple device has had a huge impact on modern astronomy

Quantum Efficiency

- Quantum efficiency (QE) is defined as

The percentage of incident photons which cause an electron to be released

- It can also be written as

$$\text{quantum efficiency (QE)} = \frac{\textit{number of electrons produced per second}}{\textit{number of photons absorbed per second}} \times 100\%$$

- In a perfect device, the quantum efficiency will be 100% if every photon generates a photoelectron
    - However, in practice, the quantum efficiency will be **less** than 100% since there will usually always be unavoidable losses
- Some values of QE for different devices are shown in the table

Device	Quantum efficiency (%)
human eye	1–4%
photographic film	4–10%
CCD	70–90%

- Comparison of the eye and a CCD:
    - CCDs are renowned for achieving high values of quantum efficiency, generally upwards of 80%, whereas a human eye is only capable of achieving around 1%

Resolution of a CCD

- The resolution of a CCD is related to the total **number** of pixels per unit area, and their **size**
    - The **smaller** the size of the pixel, the **better** the resolution, hence the **clearer** the image will be
- The typical resolution of a CCD is about 10 μm
- In comparison, the typical resolution of the human eye is about 100 μm, but it can vary widely

- In most cases, the overall resolution of a telescope is **limited** by the **diameter** of the objective
    - Hence, the resolution of the CCD (or the eye) is not likely to make a difference to the final image observed

Convenience of a CCD

- CCDs have an edge over the eye in terms of convenience because:
    - The number of images captured in a time period and exposure time can be **easily adjusted**
    - The information stored on a CCD can be accessed **remotely**
    - The generated images can be stored and analysed **digitally**
    - They can detect a **larger range** of wavelengths, including beyond the visible spectrum

Comparison of a CCD with the human eye

- The main comparisons between the eye and a CCD are summarised in the table

Device	Quantum efficiency (%)	Resolution	Convenience of use
human eye	• Very low ~1%	• Typical resolution ~100 µm	• No additional equipment required
CCD	• Very high 70%+ • Able to detect much fainter objects	• Typical resolution ~10 µm • Resolution can be increased by using smaller pixels	• Remote viewing • Images can be stored and analysed digitally • Long exposure times • Can detect a wider range of wavelengths

Exam Tip

You may see past exam questions on the operation and structure of the CCD, but this knowledge is no longer required - the focus is now on the comparison between the CCD and the eye.