Exam Papers Practice

# 9.1 Simple Harmonic Motion

## Mark Schemes

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| Course | DP IB Physics |
| Section | 9. Wave Phenomena (HL only) |
| Topic | 9.1 Simple Harmonic Motion |
| Difficulty | Medium |



To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

1

The correct answer is **B** because:

- Acceleration is at a maximum at the point of **maximum** displacement from the equilibrium
- Velocity is also zero at the point of maximum displacement from the equilibrium position
  - The point of maximum displacement from the equilibrium position is Y
- Therefore, the only answer that fits these criteria is **B**

Position Y is the negative maximum displacement of string S. Acceleration acts in the opposite direction to displacement, so position Y is the point of positive maximum acceleration. Velocity is a maximum at the equilibrium (position Z) and is zero at the points of maximum displacement (position Y) (both in the negative and positive direction).

2

The correct answer is **D** because:

- The equation relating the period *T* to the mass-spring system is $T = 2\pi\sqrt{\frac{m}{k}}$
  - Therefore, the only factors that affect the period of a mass-spring system are the mass *m* and the spring constant *k*
  - The change in acceleration of free fall has no effect on the period of the mass-spring system
  - Hence, the period of the mass-spring system will be the same on Mars as it was on Earth
  - This eliminates options **A** and **C**
- The equation relating the period *T* to the simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$
  - The factors that affect the period of a simple pendulum are the length *l* and the acceleration of free fall *g*
  - Therefore, the period of the simple pendulum will be different on Mars than it is on Earth

- This eliminates option **B**
- Therefore, the only answer that fits the criteria is **D**

In order to answer this question, you do not need to know by what factor the period of the simple pendulum would change, as the only options given are that it stays the same or that it increases. But if you would like a reminder to show you how to work it out, here is one below.

$$T = 2\pi\sqrt{\frac{l}{g}}$$ Begin with the equation

$$= 2\pi\sqrt{l}\,\frac{1}{\sqrt{g}}$$ Isolate the constants

$$T \propto \frac{1}{\sqrt{g}}$$ Inverse proportion

Increasing g will cause a decrease in T

Decreasing g will cause an increase in T

Halving g using IB proportional reasoning method

$$T_1 = 2\pi\sqrt{l}\,\frac{1}{\sqrt{0.5g}}$$ Half of g is 0.5g

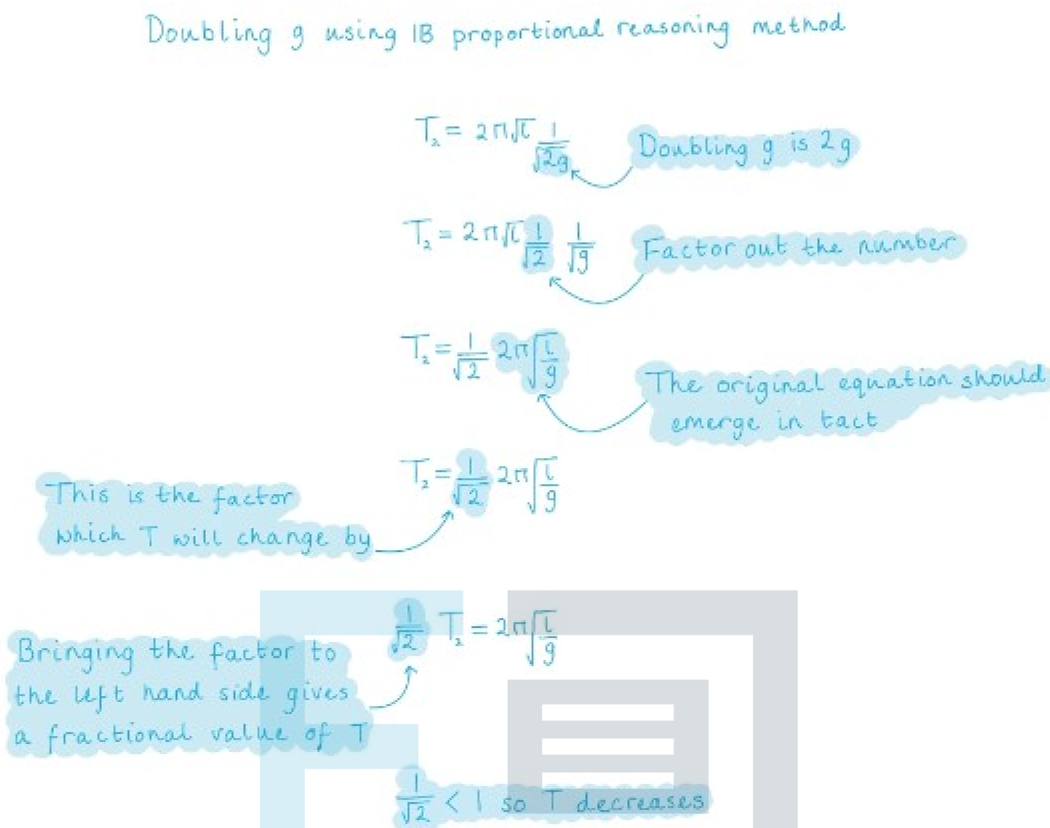
$$T_1 = 2\pi\sqrt{l}\,\frac{1}{\sqrt{0.5}}\,\frac{1}{\sqrt{g}}$$ Factor out the number

$$T_1 = \frac{1}{\sqrt{0.5}}\,2\pi\sqrt{\frac{l}{g}}$$ The original equation should emerge in tact

$$T_1 = \frac{1}{\sqrt{0.5}}\,2\pi\sqrt{\frac{l}{g}}$$ This is the factor which T will change by

$$\frac{1}{\sqrt{0.5}}\,T_1 = 2\pi\sqrt{\frac{l}{g}}$$ Bringing the factor to the left hand side gives a fractional value of T

$$\frac{1}{\sqrt{0.5}} > 1$$ so T increases




3

The correct answer is **B** because:

- The period of oscillation of an object in SHM is constant
  - This is one of the conditions for SHM
  - The oscillation is isochronous
- The frequency of the oscillation is the inverse of the period
  - Therefore, if period is constant, frequency is also constant
- Option **B** is the only answer that meets these criteria

| | |
|---|---|
| **A** is incorrect as | if the period is constant, then the frequency must also be constant |
| **C** is incorrect as | if the period is constant, then the frequency must also be constant.<br><br>The spring constant defines how stiff a spring is, it is not a condition of SHM |

| | |
|---|---|
| **D** is incorrect as | the spring constant defines how stiff a spring is, it is not a condition of SHM.<br><br>The acceleration of freefall describes the gravitational field strength of the environment, but it is not a condition of SHM |

Isochronous means occurring at the same time.

4

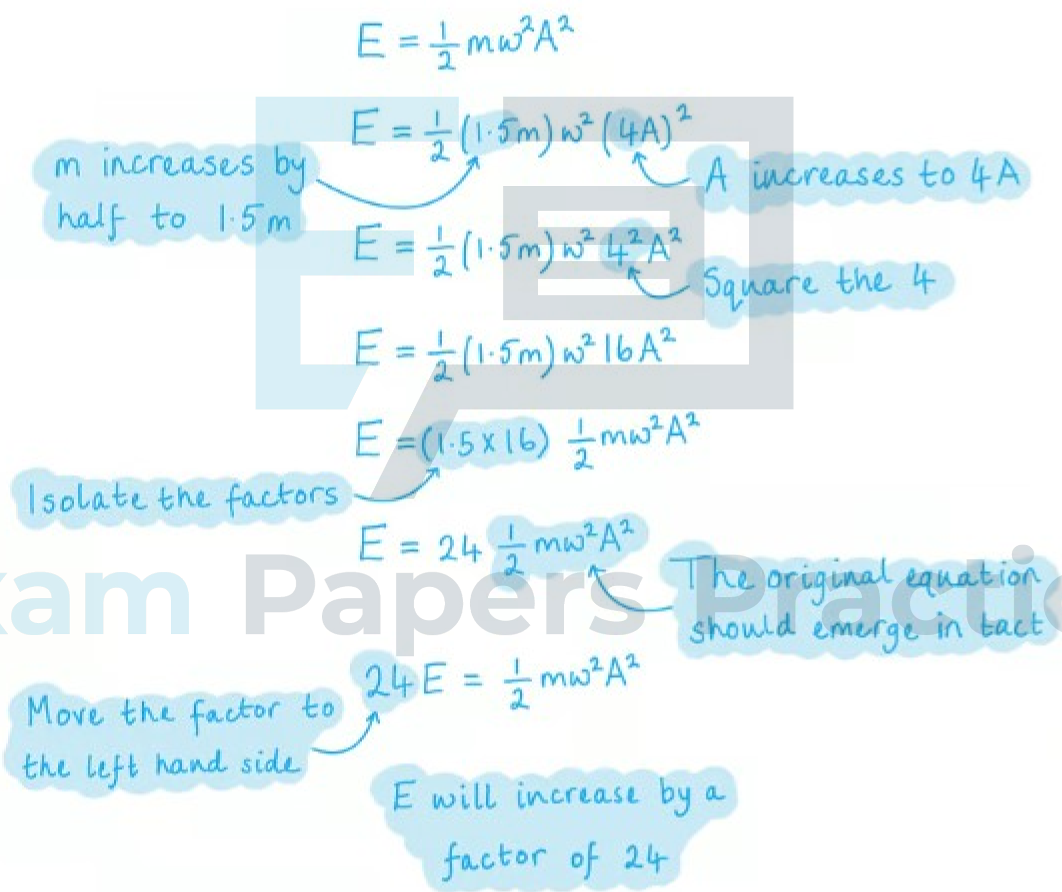
The correct answer is **A** because:

- The total energy equation can be written as $E = \frac{1}{2} m\omega^2 A^2$
- The mass $m$ increases by half becoming $1.5m$
- Amplitude $A$ increases to $4A$
- Therefore:
  - $E = \frac{1}{2}(1.5m)\,\omega^2(4A)^2$
  - $E = \frac{1}{2} 1.5m\omega^2 16A^2$
- Using proportional reasoning and separating out the factors gives:
  - $E = (1.5 \times 16)\frac{1}{2} m\,\omega^2 A^2$
  - $E = 24\frac{1}{2} m\,\omega^2 A^2$
  - Hence, $24E = \frac{1}{2} m\,\omega^2 A^2$
- Increasing the mass $m$ by half and increasing the amplitude to $4A$ would cause the energy to increase by a factor of 24

| | |
|---|---|
| **B** is incorrect as | factor 24 was multiplied by the $\frac{1}{2}$ from $\frac{1}{2} m\omega^2 A^2$.<br><br>Remember that when using the proportional reasoning technique, the equation needs to emerge intact once you have removed the factors. |

| | |
|---|---|
| **C** is incorrect as | the question has been misinterpreted. This calculation was performed with 0.5*m*. The question reads that *m* has **increased** by half, not that *m* has been halved |
| **D** is incorrect as | the 4 from 4*A* has not been squared |

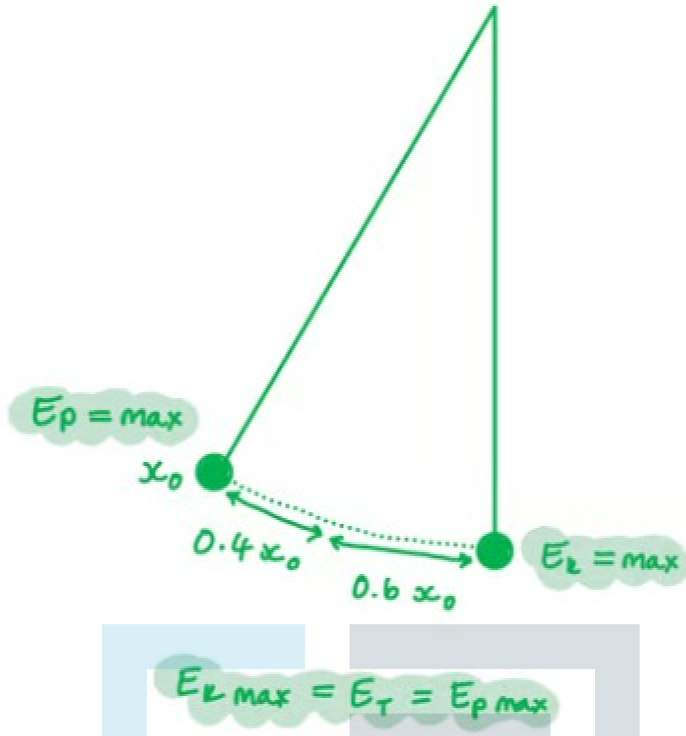
If you need a reminder on how to use proportional reasoning to find the factor:






5

The correct answer is **A** because:

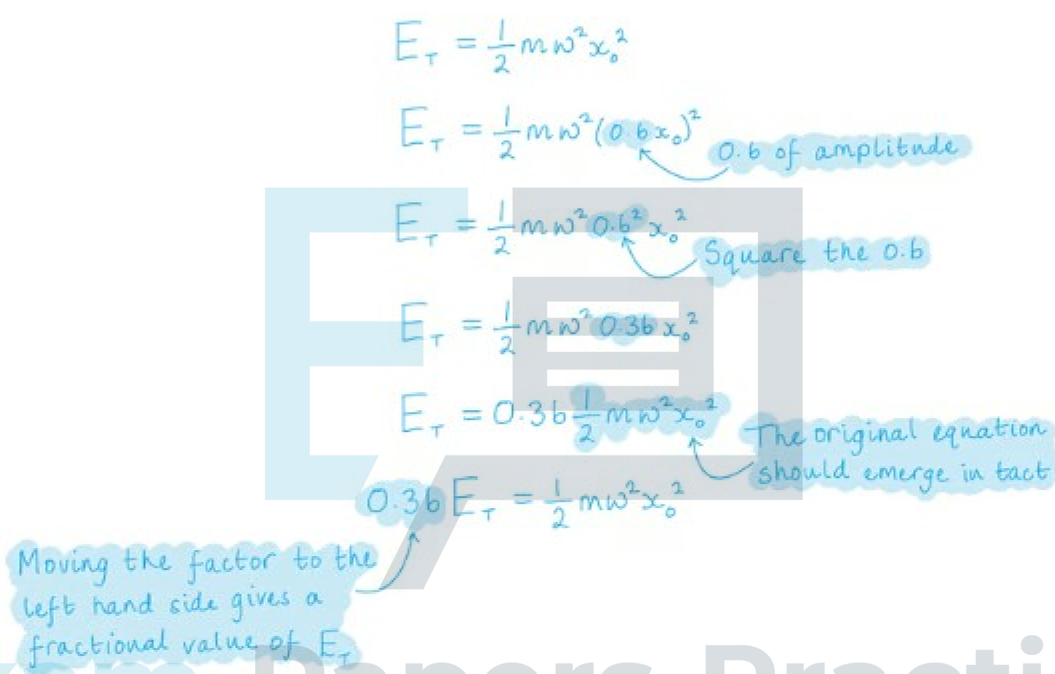



- At $0.4x_0$ from the maximum displacement, this is 0.6 of the amplitude $x_0$
- Maximum kinetic energy is the same as total energy at equilibrium
- The equation for total energy is $E_T = \frac{1}{2} m\omega^2 x_0^2$
  - Adding in the value 0.6 *A*, it becomes $E_T = \frac{1}{2} m\omega^2 (0.6x_0)^2 = \frac{1}{2} m\omega^2 0.36x_0^2$
  - Using the proportional reasoning method to isolate the factor, $E_T = 0.36\, \frac{1}{2} m\omega^2 x_0^2$
  - So, $0.36E_T = \frac{1}{2} m\omega^2 x_0^2$
- The potential energy of the system at a distance of $0.4x_0$ from maximum displacement will be 0.36 of the maximum kinetic energy, $0.36\ E_k$

Drawing a diagram can be crucial to understanding which values to use in the calculation. If you are at all unsure of the scenario being presented, sketch a quick diagram to visualise what's going on.

The wording of the question is difficult because it uses the term maximum kinetic energy to mean total energy at equilibrium which gives you the opportunity to use the wrong equation. It also gives the distance from the amplitude as a fraction of the amplitude inviting you to use the wrong value. **Make sure you don't fall into these traps**! Sketching a diagram is the best way to avoid them.

If you need a reminder of how to find the factor using the proportional reasoning method:






6

The correct answer is **D** because:

- The defining equation for SHM is $a = -\omega^2 x$
  - Maximum displacement means that $x = x_0$ and acceleration is at a maximum at amplitude $x_0$
  - $a_{max} = -\omega^2 x_0$
- Rearranging to solve for $x_0$ gives:
  - $x_0 = -\frac{a_{max}}{\omega^2}$
- Angular velocity is $\omega = \frac{2\pi}{T}$

- Combining these equations gives:
  - $x_0 = -\frac{a_{max}}{(\frac{2\pi}{T})^2}$
  - $x_0 = -\frac{a_{max}T^2}{4\pi^2}$

$\omega$ can also be expressed as $2\pi f$, since $T = \frac{1}{f}$. The correct arrangement using frequency would be $x_0 = -\frac{a_{max}}{4\pi^2 f^2}$, which is close to, but not exactly options **A** or **B**.

There are lots of equations to keep track of in SHM. If you familiarise yourself with the interchangeable expressions as part of your revision, this will greatly reduce the amount of time you have to spend looking up equations and rearranging equations in the exam!

7

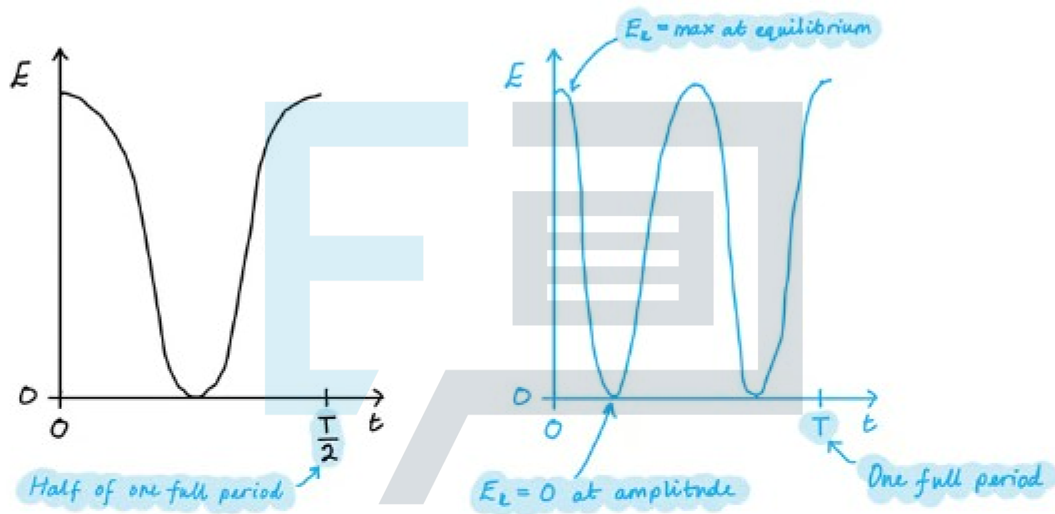
The correct answer is **B** because:

- The value for energy is always positive
  - This eliminates option **A**
- Kinetic energy is at a maximum at the point of equilibrium
  - $t = 0$ at the point of equilibrium
  - At $t = 0$, kinetic energy is at a maximum
  - This eliminates option **D**
- The time axis shows that the graph is for half on one period
  - The ion begins at equilibrium, passes equilibrium at $\frac{T}{2}$, and returns to equilibrium in one full period
  - The graph for half of one period would begin at equilibrium and end at equilibrium at $\frac{T}{2}$
  - This eliminates option **C**

- The only answer that fits these criteria is option **B**

This question is only concerned with half a period, so the kinetic energy will go from maximum at equilibrium, to zero at amplitude, and back to maximum as it passes equilibrium again.

For a full period, the kinetic energy would go from maximum at equilibrium, to zero at positive amplitude, to maximum as it passes through equilibrium, to zero at negative amplitude, and back to maximum as it approaches the equilibrium again.




8

The correct answer is **B** because :

- The equation describing the period of a pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$
  - Changing the mass of the pendulum bob will have no effect on the period
- $T \propto \sqrt{l}$
  - Decreasing the length will therefore decrease the period
  - This eliminates option **A**

- The length of the pendulum is halved so $T = 2\pi\frac{\sqrt{0.5l}}{\sqrt{g}}$
    - $T$ will decrease by a factor of $\sqrt{0.5}$
    - Using an approximation, $\sqrt{0.5} \approx 0.7$
        - 0.50 is close to 0.49, therefore $\sqrt{0.5} \approx \sqrt{0.49}$
        - $7^2 = 49$, therefore $\sqrt{0.49} = 0.7$
    - This eliminates options **C** and **D**
- The only answer that fits these criteria is option **B**

Make sure you practice approximations for proportionality questions as part of your revision. These type of questions come up a lot and you only have a very limited time to perform the calculations in the exam. Approximations can be made using the nearest square numbers and will be accurate enough to lead you to the correct answer. Remember that approximations are not entirely accurate, so the answer you get may not be exactly the same as the answer options given. In these instances, you are looking for the closest answer option to your approximation.

If you need a more detailed look at how to get to the answer:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{0.5l}{g}}$$ Half the length is 0.5l

Isolate the factor $$T = 2\pi\sqrt{0.5}\sqrt{\frac{l}{g}}$$

$$T = \sqrt{0.5}\,2\pi\sqrt{\frac{l}{g}}$$ The equation should emerge in tact

Move the factor to the left hand side $$\sqrt{0.5}\,T = 2\pi\sqrt{\frac{l}{g}}$$

$\sqrt{0.5} < 1$ so T will decrease

$$\sqrt{0.5} = \sqrt{0.50}$$

To approximate $\sqrt{0.5}$ use square numbers close to 50

$$\sqrt{0.5} \approx \sqrt{0.49}$$

| $x$ | $x^2$ |
|---|---|
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |

So, $\sqrt{0.5} \approx 0.7$

Therefore, $\sqrt{0.5}\,T \approx 0.7T$



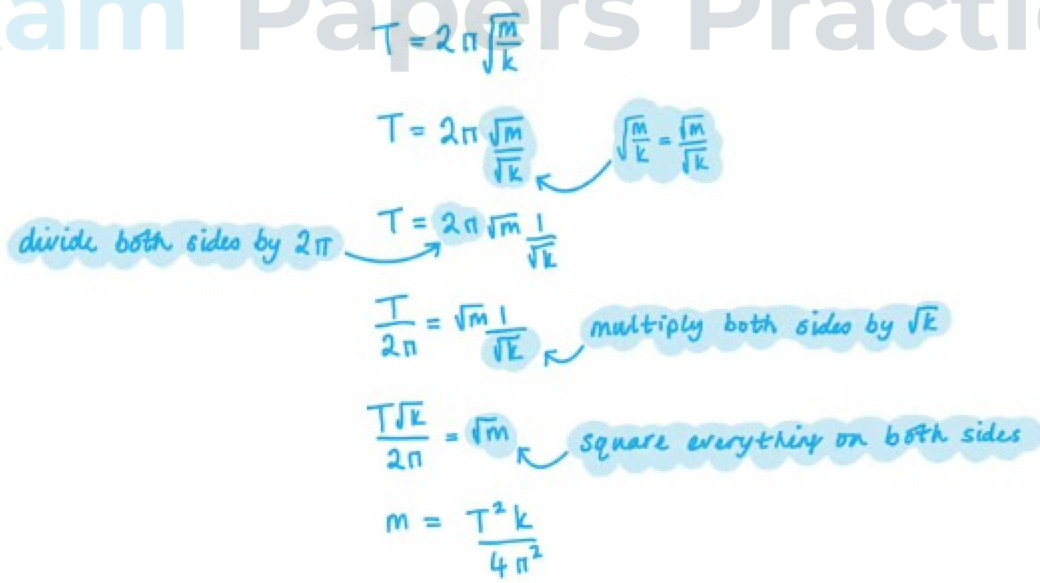
9

The correct answer is D because:

- The equation linking the period $T$ and mass $m$ of a mass-spring system is $T = 2\pi\sqrt{\frac{m}{k}}$
- Rearranging the equation to solve for mass gives:
  - $m = \frac{T^2k}{4\pi^2}$
- Doubling the period gives:
  - $m = \frac{(2T)^2k}{4\pi^2}$
  - $m = \frac{4T^2k}{4\pi^2}$
  - $m = 4\frac{T^2k}{4\pi^2}$
- The mass would need to be **4** times larger to double the period
  - $4m = \frac{T^2k}{4\pi^2}$

If you need a reminder on how to rearrange the original equation to solve for m:

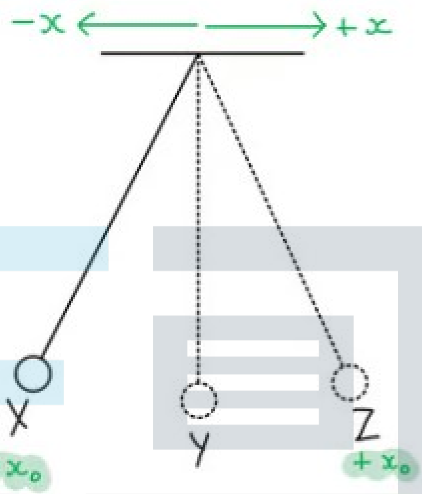




10

The correct answer is **B** because:

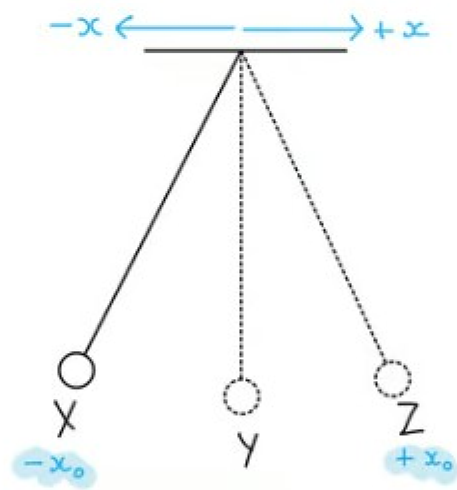
- Since the diagram shows the pendulum being released from position X, it can be assumed that the forward direction is positive therefore:




- Acceleration is zero at the point of equilibrium, position Y
    - This discounts options **A** and **C**
- Displacement is at a maximum at amplitude, so the maximum negative displacement will be at $-x_0$
    - $-x_0$ is position X
    - Both options **B** and **D** still apply
- Velocity is at a maximum at equilibrium, Position Y
    - This discounts option **D**
- The only answer that fits these criteria is option **B**

The best way to tackle a question like this is by the process of elimination. Take one variable at a time and determine what it must be, then discount the answer options that do not apply. Working methodically in this way will save time and avoid confusion.

If you need a reminder of when the displacement, velocity, acceleration and Force are at their maximums:




| Position | x | v | a | F |
|---|---|---|---|---|
| X | − max | zero | + max | + max |
| Y | zero | ± max | zero | zero |
| Z | + max | zero | − max | − max |