Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
AS Mathematics (8MA0)
Paper 01 Pure Mathematics

Summer 2025
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$ leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

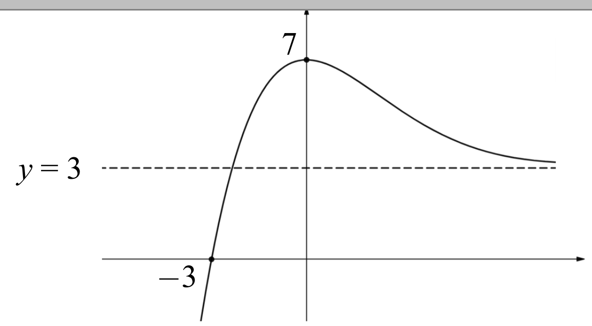
Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks	AOs
1			

For <u>both</u> parts of question 1:

- **There must be a sketch to score any marks.**
- Condone the coordinates of the turning point or intercepts having missing brackets e.g. 0, 7 or (0, 7
- Asymptotes must be labelled as an equation i.e. not just a value on the y-axis and must correspond to the sketch.
- The asymptote does not need to be drawn as a dotted line but the curve must be asymptotic to the correct line.
- Points/asymptote equations may be stated away from the sketch but must be fully correct and correspond to the sketch
- If there is ambiguity, the sketch takes precedence.
- Mark positively regarding both sketches and mark the candidate's intention regarding the shape, position of the turning point, and behaviour at the asymptote so that the curves do not need to be "perfectly" drawn
- Labelling of points as e.g. A and B can be ignored.
- Attempts to do their sketches on Figure 1 should be sent to review.

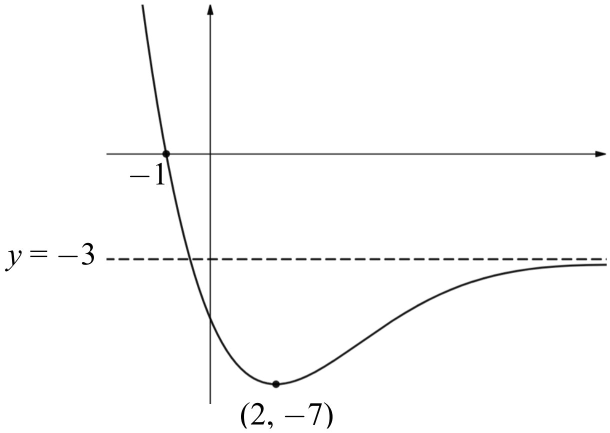
Question	Scheme	Marks	AOs
1(i)			
	Shape for a horizontal translation	B1	1.1b
	Two of $(0,7)$, $(-3,0)$ and $y=3$ correctly labelled.	B1	1.1b
	All three of $(0,7)$, $(-3,0)$ and $y=3$ correctly labelled.	dB1	1.1b
		(3)	

(i) Notes

B1: Requires the shape for a horizontal translation in either direction.
There should be no change in the y coordinate for any point and no change in the equation of the asymptote if drawn.
Evidence can be obtained e.g. by the position of the maximum or the x intercept.
Do not be concerned if they are contradictory but it should not just be the original curve.

B1: Two of $(0, 7)$, $(-3, 0)$ and $y=3$ correctly labelled.
The maximum does not necessarily have to correspond with the (0, 7) for this mark.
Allow 7 and/or -3 marked in the correct place and condone $(7, 0)$ and/or $(0, -3)$ as long as they are in the correct place.
The curve must not clearly cross the asymptote if awarding for this condition.

dB1: All three of $(0,7)$, $(-3,0)$ and $y=3$ correctly labelled.
The maximum should correspond with the (0, 7) for this mark but mark positively if it is not absolutely clear – you may have to use your own judgement.

	Allow 7 and/or -3 marked in the correct place and condone $(7,\ 0)$ and/or $(0,\ -3)$ as long as they are in the correct place. The curve must not clearly cross the asymptote. Must follow both previous B1's.		
1(ii)	 		
	Shape for a reflection in x-axis	B1	1.1b
	Two of $(2,\ -7)$, $(-1, 0)$ and $y = -3$ correctly labelled.	B1	1.1b
	All three of $(2,\ -7)$, $(-1, 0)$ and $y = -3$ correctly labelled.	dB1	1.1b
		(3)	
			(6 marks)

(ii) Notes

B1: Requires the shape for a reflection in the x-axis.
The curve must be in quadrants 2, 3 and 4 only.
There should be a minimum in quadrant 4
The coordinates of the x intercept and the minimum and the position of the asymptote can be ignored for this mark.

B1: Two of $(2,\ -7)$, $(-1, 0)$ and $y = -3$ correctly labelled where the $(2,\ -7)$ is a minimum.
Allow -1 marked in the correct place and condone $(0,\ -1)$ as long as it is the correct place.
Allow the $(2,\ -7)$ to be indicated by the labels 2 and -7 on the appropriate axes
Allow the curve to be asymptotic to $y = -3$ from above.
The curve must not clearly cross the asymptote if awarding for this condition.

dB1: All three of $(2,\ -7)$, $(-1,\ 0)$ and $y = -3$ correctly labelled where the $(2,\ -7)$ is a minimum.
Allow -1 marked in the correct place and condone $(0,\ -1)$ as long as it is the correct place.
Allow the $(2,\ -7)$ to be indicated by the labels 2 and -7 on the appropriate axes.
The curve must be asymptotic to $y = -3$ from below.
The curve must not clearly cross the asymptote.
Must follow both previous B1's.

Question	Scheme	Marks	AOs
2(a)	$A(-3,0)$ and $B(2.5,\ 22)$		
	$m=\dfrac{22-0}{2.5--3}=\dfrac{22}{5.5}\{=4\}$	M1	1.1b
	$y-22="4"(x-2.5)$ or $y-0="4"(x--3)$ or $0="4"\times-3+c\Rightarrow c=...$ or $22="4"\times2.5+c\Rightarrow c=...$	dM1	1.1b
	$y=4x+12$	A1	1.1b
		(3)	

(a) Notes

M1: Attempts $\text{Gradient}=\dfrac{22-0}{2.5--3}$.

Condone 1 sign/copying slip only if a correct formula is seen or implied e.g. $\dfrac{y_2-y_1}{x_2-x_1}$

Alternatively solves two simultaneous equations, e.g. $22=\dfrac{5}{2}m+c$ and $0=-3m+c$ to find a value for m **or** c.

dM1: Uses $y-y_1=m(x-x_1)$ with either $A(-3,0)$ or $B(2.5,\ 22)$ or e.g. $\left(-\dfrac{1}{4},\ 11\right)$ (midpoint) and their $m="4"$ with the values correctly placed.
If $y=mx+c$ is used they must proceed as far as $c=\dots$

Alternatively solves two simultaneous equations, e.g. $22=\dfrac{5}{2}m+c$ and $0=-3m+c$ to find m **and** c.

A1: $y=4x+12$ or $y=12+4x$ only.

If they use e.g. $\dfrac{y}{22-0}=\dfrac{x+3}{2.5--3}$ both M's can be scored together.
Condone 1 sign/copying slip only if a correct formula is seen or implied.

Correct answer only scores full marks.

(b)	$y < 4x + 12$ $y \geqslant 2x^2 + 5x - 3$	B1ft	2.2a
	$2x^2 + 5x - 3 \leqslant y < 4x + 12$	B1ft	1.1b
		(2)	
			(5 marks)

(b) Notes

B1ft: Deduces one of $y < "4x + 12"$ o.e. or $y \geqslant 2x^2 + 5x - 3$ o.e.

They must have an equation for *l* and it must be the equation of a straight line.

B1ft: $y < "4x + 12"$ **and** $y \geqslant 2x^2 + 5x - 3$

o.e. e.g. $2x^2 + 5x - 3 \leqslant y < "4x + 12"$, or $y < "4x + 12", y \geqslant 2x^2 + 5x - 3$

Note: For what is an otherwise correctly defined region, if there are any extra inequalities that are contradictory e.g. $-3 < x < 0$ then withhold the second B mark but note that $-3 \leqslant x \leqslant \frac{5}{2}$ or anything "wider" is fine. (Do not be concerned about strictness)

Condone the use of "or" or "$\cup$" for the first B mark but $y < "4x + 12"$ **or** $y \geqslant 2x^2 + 5x - 3$ or $y < "4x + 12" \cup y \geqslant 2x^2 + 5x - 3$ scores B1B0

They must have an equation for *l* and it must be the equation of a straight line.

Note: Apply isw if the correct inequalities are written separately but are then combined incorrectly.

Note: Inequalities cannot be in terms of *R* e.g. $2x^2 + 5x - 3 \leqslant R < "4x + 12"$

Alternatively, some candidates may express their inequalities involving a boundary for a dashed line with $\leqslant$ or $\geqslant$ and a boundary for a solid line with $<$ or $>$.

It may not always be clear so mark positively.

e.g.

B1ft: Deduces one of $y \leqslant "4x + 12"$ o.e. or $y > 2x^2 + 5x - 3$ o.e.

They must have an equation for *l* and it must be the equation of a straight line.

B1ft: $y \leqslant "4x + 12"$ **and** $y > 2x^2 + 5x - 3$ o.e. e.g. $2x^2 + 5x - 3 < y \leqslant "4x + 12"$

They must have an equation for *l* and it must be the equation of a straight line.

In general look at the <u>direction</u> for the first mark to see if one inequality has the correct direction. Then for the second B mark they need <u>correct direction</u> and <u>consistent 'strict' or 'non-strictness'</u> with the other inequality.

So e.g. $y < "4x + 12"$, $y > 2x^2 + 5x - 3$ scores B1B0

Note that some candidates define *R* as $-3 < x < \dfrac{5}{2}$ and this scores no marks on its own.

Question	Scheme	Marks	AOs
3(a)	$\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(...)^2+(\pm2)^2}$ or $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(\pm6)^2+(...)^2}$ or $\left(\lvert\overrightarrow{OB}\rvert^2=\right)(...)^2+(\pm2)^2$ or $\left(\lvert\overrightarrow{OB}\rvert^2=\right)(\pm6)^2+(...)^2$	M1	1.1b
	$\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(\pm2)^2+(\pm6)^2}$	dM1	1.1b
	$\sqrt{40}$ or $2\sqrt{10}$	A1	1.1b
		(3)	
(b)	$\lvert\overrightarrow{OA}\rvert=\sqrt{73}$ or $\lvert\overrightarrow{AB}\rvert=\sqrt{29}$	B1	1.1b
	$40=73+29-2\sqrt{73}\sqrt{29}\cos OAB \Rightarrow \cos OAB=\frac{73+29-40}{2\sqrt{73}\sqrt{29}}$ $\Rightarrow OAB=\cos^{-1}\left(\frac{73+29-40}{2\sqrt{73}\sqrt{29}}\right)$ or $\cos OAB=\frac{73+29-40}{2\sqrt{73}\sqrt{29}}\Rightarrow OAB=...$	M1	3.1a
	$OAB=47.6°$	A1	1.1b
		(3)	
	(6 marks)		

Notes:

Note that marks in (a) can be scored in (b) as long as they are not contradictory.

Note that they are not asked for $\overrightarrow{OB}$ in (a) but $\lvert\overrightarrow{OB}\rvert$. As such, all they need are the magnitudes of the components of $\overrightarrow{OB}$ to find $\lvert\overrightarrow{OB}\rvert$ so you can ignore if $\overrightarrow{OB}$ is correct or not in both parts and full marks can be awarded even if there are sign errors in their $\overrightarrow{OB}$ if they write it as a vector.

(a)

M1: Attempts $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OB}\rvert^2$ with one component correct and the other component non-zero.

Allow $\sqrt{(\pm2)^2+(...)^2}$ or $\sqrt{(\pm6)^2+(...)^2}$ or $(...)^2+(\pm2)^2$ or $(\pm6)^2+(...)^2$

and condone e.g. -2^2 or -6^2

But it must clearly not be an attempt at e.g. $\lvert\overrightarrow{AB}\rvert$ e.g. $\sqrt{5^2+2^2}$

dM1: Complete and correct method for $\lvert\overrightarrow{OB}\rvert$ i.e. $\lvert\overrightarrow{OB}\rvert=\sqrt{(\pm2)^2+(\pm6)^2}$

A1: $\sqrt{40}$ or $2\sqrt{10}$ only but isw if they then use decimals.

Beware in (b) that assuming *OAB* is right angled can give answers that look approximately correct e.g. $\sin OAB = \frac{OB}{AB} = \frac{\sqrt{40}}{\sqrt{73}} \Rightarrow OAB = \sin^{-1}\frac{\sqrt{40}}{\sqrt{73}} = 47.75...$ **but is an incorrect method.**

In (b) mark the method that is most successful.

(b) **Way 1: Cosine rule**

B1: Finds either of $\left|\overrightarrow{OA}\right| = \sqrt{73}$ or $\left|\overrightarrow{AB}\right| = \sqrt{29}$ allow for sight of these values even if not associated with a vector. They may be seen on a diagram or embedded in an attempt at the cosine rule. May be implied by decimal values (see diagram)

M1: A complete and correct method for finding angle *OAB* with their *OA*, *OB* and *AB*.
Correct attempt at the cosine rule leading to a value for angle *OAB* using arccos.
Following the correct use of the cosine rule, if a value for angle *OAB* is just written down or there is no evidence of arccos, you may need to check.
Following the correct use of the cosine rule, sufficient evidence could be e.g.
$\cos OAB = k \Rightarrow OAB = \cos^{-1} k = ...$

A1: awrt $47.6°$ Condone omission of degrees symbol.

(b) **Way 2: Right angled triangles**

B1: Finds any of $\tan^{-1}\left(\frac{8}{3}\right) = 69.4°$, $\tan^{-1}\left(\frac{2}{5}\right) = 21.8°$, $\tan^{-1}\left(\frac{3}{8}\right) = 20.6°$, $\tan^{-1}\left(\frac{5}{2}\right) = 68.2°$

May be implied.

M1: A complete and correct method for finding angle *OAB*.
e.g. attempts $\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{8}\right)$ or $\tan^{-1}\left(\frac{8}{3}\right) - \tan^{-1}\left(\frac{2}{5}\right)$ or $90° - \tan^{-1}\left(\frac{3}{8}\right) - \tan^{-1}\left(\frac{2}{5}\right)$
leading to a value for angle *OAB*.

A1: awrt $47.6°$ Condone omission of degrees symbol.

(b) **Way 3: Scalar product**

B1: Finds $\overrightarrow{AO} \bullet \overrightarrow{AB} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 2 \end{pmatrix} = 15 + 16 = 31$

Allow $\pm\overrightarrow{AO} \bullet \pm\overrightarrow{AB} = \pm 31$

M1: A complete and correct method for finding angle *OAB* with their *OA*, *OB* and *AB*.
e.g. $31 = \left|\overrightarrow{AO}\right|\left|\overrightarrow{AB}\right| \cos OAB = \sqrt{3^2 + 8^2}\sqrt{5^2 + 2^2} \cos OAB \Rightarrow \cos OAB = \frac{31}{\sqrt{73}\sqrt{29}} \Rightarrow OAB = ...$

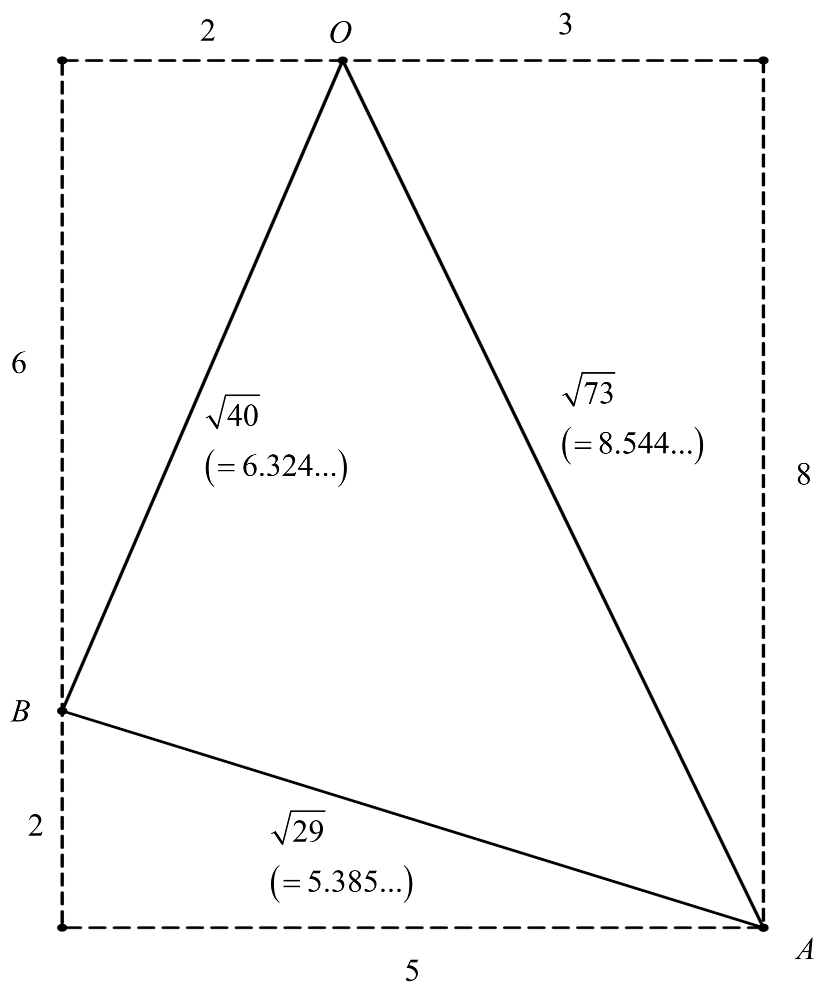
If e.g. $\overrightarrow{OA} \bullet \overrightarrow{AB}$ is attempted then they need to find e.g. $OAB = 180° - \cos^{-1}\frac{-31}{\sqrt{73}\sqrt{29}}$

Following the correct use of the scalar product, if a value for angle *OAB* is just written down or there is no evidence of arccos, you may need to check.

A1: awrt $47.6°$ Condone omission of degrees symbol.

There may be other methods for finding angle *OAB*.

For reference:



Question	Scheme	Marks	AOs
4(a)(i)	$x^2+y^2+10x-4y+1=0$		
	$(x+5)^2+(y-2)^2$	M1	1.1b
	Centre $(-5,\ 2)$	A1	1.1b
(a)(ii)	Radius is $\sqrt{28}$	A1	1.1b
		(3)	
(b)	$2+2\sqrt{7}$ or $2-2\sqrt{7}$ seen	B1	2.2a
	$2-2\sqrt{7}<k<2+2\sqrt{7}$	M1	1.1b
	$\left\{k:2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ oe	A1	2.5
		(3)	
			(6 marks)

Notes:

Condone mislabelled parts e.g. labelled as (a), (b) and (c) not (a)(i), (a)(ii), (b)

(a)

M1: Attempts to complete the square by halving both x and y terms.
Award for sight of $(x\pm5)^2\ldots(y\pm2)^2$ or $(x\pm5)^2$ and $(y\pm2)^2$ seen separately.
May be implied by a centre of $(\pm5,\ \pm2)$

A1: Correct coordinates $(-5,\ 2)$ or e.g. $x=-5,\ y=2$ and condone e.g. $-5,\ 2$

A1: $\sqrt{28}$ $\left(\text{or } 2\sqrt{7}\right)$

(b)

B1: $2+2\sqrt{7}$ **or** $2-2\sqrt{7}$ seen. Accept $\sqrt{28}$ for $2\sqrt{7}$ throughout this question.

M1: $"2-2\sqrt{7}"<k<"2+2\sqrt{7}"$
Selects the inside interval for **their** endpoints which must come from a correct method condoning slips. Condone use of y for k.
May be seen separately e.g. $"2-2\sqrt{7}"<k,\quad k<"2+2\sqrt{7}"$
Condone e.g. $"2-\sqrt{28}"\leqslant k\leqslant"2+\sqrt{28}"$ and allow inexact e.g.
$$"-3.291\ldots"<k<"7.291\ldots"$$
An incorrect method would include e.g. $5-2\sqrt{7}<k<5+2\sqrt{7}$ ie using the x coordinate of the centre rather than the y coordinate.

A1: Correct answer in exact form in set notation or interval notation in terms of k.
Allow: $\left\{k:2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ or $\left\{k:2-2\sqrt{7}<k\right\}\cap\left\{k:k<2+2\sqrt{7}\right\}$ or $\left\{2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ or $\left\{2-2\sqrt{7}<k\right\}\cap\left\{k<2+2\sqrt{7}\right\}$ or $\left(2-2\sqrt{7},\ 2+2\sqrt{7}\right)$ or $\left\{k:2-2\sqrt{7}<k\cap k<2+2\sqrt{7}\right\}$ or $\left\{k:2-2\sqrt{7}<k\right\}$ and $\left\{k:k<2+2\sqrt{7}\right\}$ but **not** $\left\{2-2\sqrt{7}<k\right\}\cup\left\{k<2+2\sqrt{7}\right\}$ and **not** $\left[2-2\sqrt{7},\ 2+2\sqrt{7}\right]$ and **not** $\left\{2-2\sqrt{7}\leqslant k\leqslant 2+2\sqrt{7}\right\}$

(b)	**Alternative 1** (substitutes $y=k$ into the given or their circle equation and uses $b^2-4ac=0$ or $b^2-4ac>0$) Scores in the same way following e.g. $$x^2+10x+\left(k^2-4k+1\right)=0,\ 10^2-4\left(k^2-4k+1\right)>0,\ 24+4k-k^2=0$$
B1:	$2+2\sqrt{7}$ **or** $2-2\sqrt{7}$ seen. Accept $\sqrt{28}$ for $2\sqrt{7}$ throughout this question.
M1:	$"2-2\sqrt{7}"<k<"2+2\sqrt{7}"$ Selects the inside interval for their endpoints which must come from a correct method condoning slips. Condone use of y for k. May be seen separately e.g. $"2-2\sqrt{7}"<k,\quad k<"2+2\sqrt{7}"$ Condone e.g. $"2-\sqrt{28}" \leqslant k \leqslant "2+\sqrt{28}"$ and allow inexact e.g. $"-3.291\ldots" < k < "7.291\ldots"$ If when finding their endpoints they make a sign error and obtain e.g. $k^2-4k-24>0$ and then choose the outside region, this scores M1 but it must be consistent with their inequality.
A1:	Correct answer in exact form in set notation or interval notation in terms of k. Allow: $\left\{k:2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ or $\left\{k:2-2\sqrt{7}<k\right\}\cap\left\{k:k<2+2\sqrt{7}\right\}$ or $\left\{2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ or $\left\{2-2\sqrt{7}<k\right\}\cap\left\{k<2+2\sqrt{7}\right\}$ or $\left(2-2\sqrt{7},\ 2+2\sqrt{7}\right)$ or $\left\{k:2-2\sqrt{7}<k\cap k<2+2\sqrt{7}\right\}$ or $\left\{k:2-2\sqrt{7}<k\right\}$ and $\left\{k:k<2+2\sqrt{7}\right\}$ but **not** $\left\{2-2\sqrt{7}<k\right\}\cup\left\{k<2+2\sqrt{7}\right\}$ and **not** $\left[2-2\sqrt{7},\ 2+2\sqrt{7}\right]$ and **not** $\left\{2-2\sqrt{7}\leqslant k\leqslant 2+2\sqrt{7}\right\}$
(b)	**Alternative 2** (substitutes $x="-5"$ into the given or their circle equation and solves for y) Scores in the same way following e.g. $$25+y^2-50-4y+1-0\Rightarrow y^2-4y-24=0\Rightarrow y=$$
B1:	$2+2\sqrt{7}$ **or** $2-2\sqrt{7}$ seen. Accept $\sqrt{28}$ for $2\sqrt{7}$ throughout this question.
M1:	$"2-2\sqrt{7}"<k<"2+2\sqrt{7}"$ Selects the inside interval for their endpoints which must come from a correct method condoning slips. Condone use of y for k. May be seen separately e.g. $"2-2\sqrt{7}"<k,\quad k<"2+2\sqrt{7}"$ Condone e.g. $"2-\sqrt{28}" \leqslant k \leqslant "2+\sqrt{28}"$ and allow inexact e.g. $"-3.291\ldots" < k < "7.291\ldots"$
A1:	Correct answer in exact form in set notation or interval notation in terms of k. Allow: $\left\{k:2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ or $\left\{k:2-2\sqrt{7}<k\right\}\cap\left\{k:k<2+2\sqrt{7}\right\}$ or $\left\{2-2\sqrt{7}<k<2+2\sqrt{7}\right\}$ or $\left\{2-2\sqrt{7}<k\right\}\cap\left\{k<2+2\sqrt{7}\right\}$ or $\left(2-2\sqrt{7},\ 2+2\sqrt{7}\right)$ or $\left\{k:2-2\sqrt{7}<k\cap k<2+2\sqrt{7}\right\}$ or $\left\{k:2-2\sqrt{7}<k\right\}$ and $\left\{k:k<2+2\sqrt{7}\right\}$ but **not** $\left\{2-2\sqrt{7}<k\right\}\cup\left\{k<2+2\sqrt{7}\right\}$ and **not** $\left[2-2\sqrt{7},\ 2+2\sqrt{7}\right]$ and **not** $\left\{2-2\sqrt{7}\leqslant k\leqslant 2+2\sqrt{7}\right\}$

Question	Scheme	Marks	AOs
5(a)	$\dfrac{x^5-12x^{\frac{1}{2}}}{4x}=...x^4+...$ or $\dfrac{x^5-12x^{\frac{1}{2}}}{4x}=...+...x^{-\frac{1}{2}}$	M1	1.1b
	$\dfrac{1}{4}x^4$ or $-3x^{-\frac{1}{2}}$	A1	1.1b
	$\dfrac{1}{4}x^4-3x^{-\frac{1}{2}}$	A1	1.1b
		(3)	

Note that some candidates are clearly not understanding the demand in part (a) and score no marks there. They do, however, start (b) from scratch and effectively answer part (a) there – in such cases we will allow work in (b) to score in (a).

If there is no labelling of parts, mark in the order presented.

(a) Notes

M1: Attempts to split the fraction up and obtains at least one correct index.
Allow this mark even if they have more than 2 terms and allow the terms to be seen in isolation.

e.g. $\dfrac{x^5-12x^{\frac{1}{2}}}{4x}=...x^4+...$ or $\dfrac{x^5-12x^{\frac{1}{2}}}{4x}=...+...x^{-\frac{1}{2}}$

Allow equivalents for $x^{-\frac{1}{2}}$ e.g. $\dfrac{1}{x^{\frac{1}{2}}}$ or $\dfrac{1}{\sqrt{x}}$. Score as soon as one of these terms is seen.

A1: One correct **simplified** term so either $\dfrac{1}{4}x^4$ or $-3x^{-\frac{1}{2}}$.

Allow equivalent simplified expressions e.g. $0.25x^4$, $-\dfrac{3}{x^{\frac{1}{2}}}$, $-\dfrac{3}{\sqrt{x}}$

Score as soon as one of these simplified terms is seen then isw.

A1: $\dfrac{1}{4}x^4-3x^{-\frac{1}{2}}$ or equivalent as one simplified expression e.g. $0.25x^4-\dfrac{3}{\sqrt{x}}$ or e.g. $\dfrac{1}{4}x^4-\dfrac{3}{x^{\frac{1}{2}}}$

Condone $\dfrac{1}{4}x^4+-3x^{-\frac{1}{2}}$

Do **not** apply isw for this mark. E.g. many candidates are reaching $\dfrac{1}{4}x^4-3x^{-\frac{1}{2}}$ and stating $\dfrac{1}{4}x^4-3x^{-\frac{1}{2}}=x^4-12x^{-\frac{1}{2}}$ and this scores M1A1A0.

But isw can be applied if they simply miscopy their work rather than "change" it.

Apply isw for $\dfrac{1}{4}x^4-3x^{-\frac{1}{2}}=0$

Correct answer only scores full marks in (a)

5(b)	$\frac{1}{4}x^4 \to ...x^5$ or $-3x^{-\frac{1}{2}} \to ...x^{\frac{1}{2}}$	M1	1.1b
	$\frac{1}{20}x^5$ or $-6x^{\frac{1}{2}}$	A1ft	1.1b
	$\frac{1}{20}x^5 - 6x^{\frac{1}{2}} + c$	A1	2.1
		(3)	

(b)

M1: Increases the power of x by one for at least one of their terms.

Look for $x^n \to ...x^{n+1}$ or if they obtain a term $\frac{...}{x}$ and integrate to $...\ln x$ then allow M1 if $x^n \to ...x^{n+1}$ is not seen.

Allow the indices to be unprocessed e.g. $x^4 \to x^{4+1}$

Do not allow this mark if they have differentiated in (a) and then integrate their derivative in (b)

A1ft: One term correct, which may be unsimplified. Allow follow through on their answers to (a). Allow the indices to be unprocessed and condone spurious integral signs e.g.

$$\int \frac{1}{4}x^4 - 3x^{-\frac{1}{2}}\,dx = \int \frac{1}{20}x^5 - 6x^{\frac{1}{2}}$$

A1: cao with **simplified** terms and including the $+c$ but condone spurious integral signs e.g.

$$\int \frac{1}{4}x^4 - 3x^{-\frac{1}{2}}\,dx = \int \frac{1}{20}x^5 - 6x^{\frac{1}{2}} + c$$

Allow e.g. $0.05x^5 - 6\sqrt{x} + c$

Isw **can be applied** here once a correct answer is seen.

Question	Scheme	Marks	AOs
6(a)	$f(x) = ax^3 + bx^2 + 18x + 9$		
	$f(-3) = 0$	M1	1.1b
	$f(-3) = -27a + 9b - 54 + 9 = 0$ $-3a + b = 5$ *	A1*	2.1
		(2)	

(a) Notes

M1: Attempts $f(-3) = 0$ leading to an equation in *a* and *b* only. Condone slips.
The = 0 may be implied by further work for this mark.

A1*: Simplifies and rearranges to the given answer with no errors. There must be at least one **correct** line of working before the given answer and = 0 must be correctly seen at some point in their solution or at the start e.g. stating $f(-3) = 0$.

Isw if they achieve the given answer but then attempt to make *a* or *b* the subject.
Minimum acceptable is e.g. $-27a + 9b - 54 + 9 = 0 \Rightarrow -3a + b = 5$ *
Note: $-27a + 9b - 54 + 9 \Rightarrow -3a + b = 5$ is M1A0* (we do not see = 0 correctly at some point or e.g. $f(-3) = 0$)

Allow invisible brackets to be recovered e.g. $a \times -3^3 = -27a$
Allow attempts to divide algebraically and tabular/grid/inspection methods e.g.

Long division:

$$\begin{array}{r l} & ax^2 + (b-3a)x + (18-3b+9a) \\ \hline x+3 \,\big) & ax^3 + bx^2 + 18x + 9 \\ & ax^3 + 3ax^2 \\ & \quad (b-3a)x^2 + 18x + 9 \\ & \quad (b-3a)x^2 + 3(b-3a)x \\ & \qquad\qquad (18-3b+9a)x + 9 \\ & \qquad\qquad (18-3b+9a)x + 3(18-3b+9a) \\ & \qquad\qquad\qquad\qquad 9 - 54 + 9b - 27a = 0 \end{array}$$

M1: For an attempt to divide algebraically by (*x* + 3). Condone slips but they must have a quadratic quotient of the form $ax^2 + (...a + ...b)x$ and proceed to a remainder of the form $...a + ...b + ...$ which is then set = 0

Grid method:

	ax^2	$(b - 3a)x$	$18 - 3b + 9a$
x	ax^3	$(b - 3a)x^2$	$(18 - 3b + 9a)x$
3	$3ax^2$	$3(b - 3a)x$	$54 - 9b + 27a$

So $9 - (54 - 9b + 27a) = 0$

M1: Condone slips but they must have a quadratic quotient of the form $ax^2 + (...a + ...b)x$ and the bottom right hand cell of the form $...a + ...b + ...$ which is then subtracted from 9 and set = 0

Inspection:

$$ax^3 + bx^2 + 18x + 9 = (x+3)(ax^2 + (b-3a)x + 3) \Rightarrow 3(b-3a) + 3 = 18$$

M1: Condone slips but they must have a quadratic factor of the form $ax^2 + (...a + ...b)x + 3$ and then extract the *x* terms and set = 18

In all cases, A1 is scored for fully correct work leading to the given equation.

(b)	$ax^3 \to ...x^2$, $bx^2 \to ...x$ or $18x \to 18$	M1	1.1b
	$(\mathrm{f}'(x) =)3ax^2 + 2bx + 18$	A1	1.1b
	$(\mathrm{f}'(2) =)3a(2)^2 + 2b(2) + 18 = 14$	dM1	1.1b
	$-3a + b = 5$ and $3a + b = -1$ leading to $a = \ldots$ and $b = \ldots$	ddM1	3.1a
	$a = -1$ and $b = 2$	A1	2.2a
		(5)	

(b)

M1: Decreases the power of x by one for at least one of the terms.
Look for $ax^3 \to ...x^2$, $bx^2 \to ...x$ or $18x \to 18$ but not $9 \to 0$

A1: $(\mathrm{f}'(x) =)3ax^2 + 2bx + 18$

dM1: Substitutes $x = 2$ into their derivative and sets equal to 14 to create a second equation in a and b e.g. $3a(2)^2 + 2b(2) + 18 = 14$

ddM1: Attempts to solve simultaneously their equation in a and b obtained from using $\mathrm{f}'(2) = 14$ and the given equation from part (a) to obtain values for a and b.
Condone copying slips when using the equation from (a) as long as the intention is clear.
You do not need to check their working and this may be done on a calculator.

A1: cao $a = -1$ and $b = 2$ only.
Generally, candidates who use $\mathrm{f}(2) = 14$ or e.g. $\mathrm{f}'(2) = 0$ will score a maximum of M1A1dM0ddM0A0 in (b).

Note that some candidates may use the result in (a) and substitute to eliminate a or b before differentiating e.g.:

$$\mathrm{f}(x) = ax^3 + bx^2 + 18x + 9,\ -3a + b = 5 \Rightarrow \mathrm{f}(x) = ax^3 + (3a + 5)x^2 + 18x + 9$$

$$\Rightarrow \mathrm{f}'(x) = 3ax^2 + 2(3a + 5)x + 18 \Rightarrow 14 = 12a + 12a + 20 + 18 \Rightarrow a = -1$$

Or

$$\mathrm{f}(x) = ax^3 + bx^2 + 18x + 9,\ -3a + b = 5 \Rightarrow \mathrm{f}(x) = \left(\frac{b-5}{3}\right)x^3 + bx^2 + 18x + 9$$

$$\Rightarrow \mathrm{f}'(x) = 3\left(\frac{b-5}{3}\right)x^2 + 2bx + 18 \Rightarrow 14 = 4b - 20 + 4b + 18 \Rightarrow b = 2$$

Score

M1: As main scheme

A1: Correct derivative in terms of a or b

dM1: Substitutes $x = 2$ into their derivative and sets equal to 14 to create an equation in a or b

ddM1: Solves to obtain a value for a or b and then finds the other value.

A1: As main scheme

Question	Scheme	Marks	AOs
7(a)	$P = -175^2 + 260\times175 - 16450$	M1	3.4
	$= -1575 \therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	$-x^2 + 260x - 16450 = 200$ $\Rightarrow x^2 - 260x + 16650 = 0 \Rightarrow x = ...$	M1	3.1b
	$(114.19 <) x < 145.81$	dM1	1.1b
	Maximum Price = £145.81	A1	3.2a
		(3)	
(c)	$-x^2 + 260x - 16450 = -(x \pm k)^2 \pm ...$ **or** $b = -1$	B1	1.1b
	$-x^2 + 260x - 16450 = -(x - 130)^2 \pm ...$ **or** $b = -1$ and $c = -130$	M1	1.1b
	$(P =) 450 - (x - 130)^2$	A1	1.1b
		(3)	
(d)(i)	States maximum profit = £450 000	B1	3.2a
(ii)	States selling price £130	B1	2.2a
		(2)	
		(10 marks)	

Notes:

Look for answers written in the question or on Figure 3

(a)

M1: Substitutes $x = 175$ into $P = -x^2 + 260x - 16450$ to find a value for P.

This is implied by an answer of -1575.

Alternatively attempts to find the solutions of $-x^2 + 260x - 16450 = 0$ usual rules apply so a method must be seen or the correct roots stated (these are 108.78… and 151.21…) and compares with 175 which may be implied.

A1: Finds $P = -1575$ and states that $P < 0$ which may be implied **and** explains that the company would make a loss/negative profit, the profit is negative, the company will not make any money.

Condone suggestions that the loss is £1575 rather than £1 575 000

or obtains the larger root as awrt 151, compares 175 > 151 and states that $P < 0$ which may be implied **and** explains that the company would make a loss/would not make a profit.

Explanations that are incomplete or incorrect score A0 e.g.

- the profit **cannot** be negative (because it can)
- they **might** make a loss (because they will make a loss)
- **it** is negative (without saying what **it** is)

You may ignore any misconceptions or reference to the price of the chair being too cheap for this mark.

Attempts to find the turning point score no marks unless there is some consideration of the profit when $x = 175$

(b)	
M1:	Uses $P \ldots 200$ where … is any inequality or $=$ with $P = -x^2 + 260x - 16\,450$ and proceeds to at least one value for x via a correct method which may be via calculator (you may need to check) and may be inexact.
dM1:	Award for solving to find two **positive** values for x. Allow decimal answers or surds. FYI correct answers are $\dfrac{260 \pm \sqrt{1000}}{2}$ or $130 \pm 5\sqrt{10}$ or 114/146 **or** if only one value of x is seen, this must be the higher value for their quadratic which must have 2 positive solutions (you may need to check). e.g. for the correct quadratic $130 + 5\sqrt{10}$ or 145.81/145.82/145.8/146
A1:	Deduces that the maximum price is £145.81 with units. Allow £145.81p £145.82 is not acceptable and if both values are given score A0. There must be some evidence of the correct equation being solved but condone incorrect inequalities appearing in their working.
(c)	**Allow to score anywhere in the question.** **Note that some candidates use** $200 = -x^2 + 260x - 16\,450$ **in (c) and attempt to complete the square on this. In these cases allow the B mark and the M mark as defined below.**
B1:	Achieves $-(x \pm k)^2 \pm \ldots$ or states that $b = -1$
M1:	Achieves $-(x - 130)^2 \pm \ldots$ or states $b = -1$ and $c = -130$
A1:	$\{P =\} 450 - (x - 130)^2$ or e.g. $\{P =\} -(x - 130)^2 + 450$ or e.g. $\{P =\} 450 - 1(x - 130)^2$ Must be simplified e.g. not $\{P =\} -16450 + 16900 - (x - 130)^2$ Note that this may be done in a variety of ways including equating with the expanded form of $a + b(x + c)^2 = bx^2 + 2bcx + bc^2 + a$ Correct answer only scores full marks. If there is incorrect/muddled working, mark their final answer.
	If there is no labelling in (d), mark in the order presented.
(d)(i)	
B1:	Maximum profit = £450 000 with units. Accept 450 thousand pounds or £450k This is an independent mark so could follow incorrect work in (c).
(d)(ii)	
B1:	Selling price = £130 with units. This is an independent mark so could follow incorrect work in (c).
Note:	Missing units from an otherwise correct answer should only be penalised **once** in (b) or (d) and penalise it the first time it occurs.

Question	Scheme	Marks	AOs
8	$\sin^2 3x = 4\cos^2 3x$		
	$\sin^2 3x = 4\cos^2 3x \Rightarrow \tan^2 3x = 4$ or $\sin^2 3x = 4\cos^2 3x \Rightarrow \sin^2 3x = 4\left(1-\sin^2 3x\right)$ or $\sin^2 3x = 4\cos^2 3x \Rightarrow 1-\cos^2 3x = 4\cos^2 3x$	B1	1.2
	$\tan^2 3x = 4$ or $\sin^2 3x = \frac{4}{5}$ or $\cos^2 3x = \frac{1}{5}$	M1	1.1b
	$\tan^2 3x = 4 \Rightarrow \tan 3x = \sqrt{4} \Rightarrow 3x = \tan^{-1}\sqrt{4} \Rightarrow x = \frac{1}{3}\tan^{-1}\sqrt{4} = ...$ Or $\sin^2 3x = \frac{4}{5} \Rightarrow \sin 3x = \sqrt{\frac{4}{5}} \Rightarrow 3x = \sin^{-1}\sqrt{\frac{4}{5}} \Rightarrow x = \frac{1}{3}\sin^{-1}\sqrt{\frac{4}{5}} = ...$ Or $\cos^2 3x = \frac{1}{5} \Rightarrow \cos 3x = \sqrt{\frac{1}{5}} \Rightarrow 3x = \cos^{-1}\sqrt{\frac{1}{5}} \Rightarrow x = \frac{1}{3}\cos^{-1}\sqrt{\frac{1}{5}} = ...$	dM1	1.1b
	One of awrt $21.1°,\ 38.9°,\ 81.1°$	A1	1.1b
	Awrt $21.1°,\ 38.9°,\ 81.1°$	A1	2.2a
		(5)	
			(5 marks)

Notes:

Allow to work in any variable e.g. θ for all marks.

B1: Accurately recalls and applies either $\frac{\sin^2 3x}{\cos^2 3x} = \tan^2 3x$ or $\cos^2 3x = 1-\sin^2 3x$ or $\sin^2 3x = 1-\cos^2 3x$ to the given equation.

Note that e.g. $\sin^2 3x = 4\cos^2 3x \Rightarrow \sin^2 3x = 4\times 1-\sin^2 3x$ scores B0 unless the missing brackets are recovered.

M1: Reaches $\tan^2 3x = \alpha$ or $\sin^2 3x = \beta$ or $\cos^2 3x = \gamma$ where $\alpha > 0,\ 0 < \beta < 1,\ 0 < \gamma < 1$

Condone the use of $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ for this mark or e.g. a slip in losing the 3 from $3x$ or e.g. $1-\cos^2 3x = 4\cos^2 3x \Rightarrow 3\cos^2 3x = 1 \Rightarrow \cos^2 3x = \frac{1}{3}$ but the process should be essentially correct. May be implied by e.g. $5\cos^2 3x = 1 \Rightarrow \cos 3x = \sqrt{\frac{1}{5}}$.

dM1: Uses the correct order of operations to find a value for x, i.e., attempts to take the square root, inverse sin/cos/tan as appropriate and attempts to divide by 3

You may need to check.

Note that e.g. $\sin^2 3x = \frac{4}{5} \Rightarrow \sin 3x = \sqrt{\frac{4}{5}} \Rightarrow 3x = 23.6 \Rightarrow x = 7.86$ scores M1 but

e.g. $\sin^2 3x = \frac{4}{5} \Rightarrow 3x = 23.6 \Rightarrow x = 7.86$ scores M0

A1: One of awrt $21.1°,\ 38.9°,\ 81.1°$ (Must be in degrees)

A1: All three of awrt $21.1°$, $38.9°$, $81.1°$ (Must be in degrees) and no others between $0°$ and $90°$

A less likely but correct alternative is to use "double" angles e.g.

$$\sin^2 3x = 4\cos^2 3x$$
$$\Rightarrow \frac{1-\cos 6x}{2} = 4\left(\frac{1+\cos 6x}{2}\right)$$
$$\Rightarrow \cos 6x = -\frac{3}{5}$$
$$\Rightarrow x = \frac{1}{6}\cos^{-1}\left(-\frac{3}{5}\right)$$

One of awrt $21.1°,\ 38.9°,\ 81.1°$

Awrt $21.1°,\ 38.9°,\ 81.1°$

B1: Accurately recalls and applies $\sin^2 3x = \dfrac{1-\cos 6x}{2}$ and $\cos^2 3x = \dfrac{1+\cos 6x}{2}$ or equivalent to the given equation.

M1: Reaches $\cos 6x = \alpha$ where $|\alpha| < 1$

Condone the use of $\sin^2 3x = \dfrac{\pm 1 \pm \cos 6x}{2}$ and $\cos^2 3x = \dfrac{\pm 1 \pm \cos 6x}{2}$ for this mark but the process should be essentially correct.

dM1: Uses the correct order of operations to find a value for *x*, i.e., takes inverse cos and attempts to divide by 6

You may need to check.

A1: One of awrt $21.1°,\ 38.9°,\ 81.1°$ (Must be in degrees)

A1: All three of awrt $21.1°,\ 38.9°,\ 81.1°$ and no others between $0°$ and $90°$ (Must be in degrees)

Question	Scheme	Marks	AOs
9	$x^2-3px+5q+4=0$		
	$(3p)^2-4(5q+4)$	B1	1.1b
	$9p^2-20q-16=0$ $(3p+4)(3p-4)=20q$	M1	3.1a
	$q=\frac{1}{20}(3p+4)(3p-4)$ *	A1*	2.1
		(3)	

Note we are marking this B1M1A1 not M1M1A1

B1: **Correct** expression for the discriminant e.g. $(-3p)^2-4(5q+4)$ or $(3p)^2-4(5q+4)$

Do **not** condone missing brackets around the "3*p*" or 5*q* + 4 unless they are recovered **but** missing brackets around the "3*p*" must be recovered before using the difference of 2 squares below. E.g. $3p^2-16=(3p+4)(3p+4)$ scores M0 below.

Allow to be seen embedded in at attempt at the quadratic formula.
Allow for $9p^2-20q-16$ just written down.

M1: Sets their discriminant in terms of p and q $=0$ with the correct use of $\alpha p^2-\beta=\left(\sqrt{\alpha}p+\sqrt{\beta}\right)\left(\sqrt{\alpha}p-\sqrt{\beta}\right)$ so $9p^2-16=20q=(9p+12)(9p-12)$ scores M0 even if "recovered"

A1*: cso Arrives at $q=\frac{1}{20}(3p+4)(3p-4)$ with no errors seen.

Or expands $q=\frac{1}{20}(3p+4)(3p-4)$ and compares with their q and gives a (minimal) conclusion. Any missing brackets must be recovered before reaching the printed answer.

A correct intermediate line must be seen, such as $q=\frac{9p^2-16}{20}$ or $20q=(3p+4)(3p-4)$

	Alternative 1: Completes the square		
	$x^2-3px+5q+4=\left(x-\frac{3p}{2}\right)^2-\frac{9p^2}{4}+5q+4$	B1	1.1b
	Equal roots $\Rightarrow -\frac{9p^2}{4}+5q+4=0$ $\Rightarrow 20q=9p^2-16=(3p+4)(3p-4)$	M1	3.1a
	$q=\frac{1}{20}(3p+4)(3p-4)$*	A1*	2.1

Note we are marking this B1M1A1 not M1M1A1

B1: **Correct** completion of the square $x^2-3px+5q+4=\left(x-\frac{3p}{2}\right)^2-\left(\frac{3p}{2}\right)^2+5q+4$

Do **not** condone missing brackets around the $\frac{3p}{2}$ unless they are recovered.

M1: Sets their $-\left(\frac{3p}{2}\right)^2+5q+4$ which must be in terms of p and q equal to 0 with the correct use of $\alpha p^2-\beta=\left(\sqrt{\alpha}p+\sqrt{\beta}\right)\left(\sqrt{\alpha}p-\sqrt{\beta}\right)$ so $9p^2-16=20q=(9p+12)(9p-12)$ scores M0 even if "recovered"

A1*: cso Arrives at $q=\frac{1}{20}(3p+4)(3p-4)$ with no errors seen.

Any missing brackets must be recovered before reaching the printed answer.

A correct intermediate line must be seen, such as $q=\frac{9p^2-16}{20}$ or $20q=(3p+4)(3p-4)$

	Alternative 2: Calculus		
	$y=x^2-3px+5q+4\Rightarrow\frac{dy}{dx}=2x-3p$	B1	1.1b
	$2x-3p=0\Rightarrow x=\frac{3p}{2}\Rightarrow\frac{9p^2}{4}-\frac{9p^2}{2}+5q+4=0$ $\Rightarrow 20q=9p^2-16=(3p+4)(3p-4)$	M1	3.1a
	$q=\frac{1}{20}(3p+4)(3p-4)$*	A1*	2.1

Note we are marking this B1M1A1 not M1M1A1

B1: **Correct** derivative.

M1: Sets their $\frac{dy}{dx}$ which must be in terms of p and x equal to 0 to find x in terms of p, substitutes into $x^2-3px+5q+4$ and with the correct use of $\alpha p^2-\beta=\left(\sqrt{\alpha}p+\sqrt{\beta}\right)\left(\sqrt{\alpha}p-\sqrt{\beta}\right)$ so $9p^2-16=20q=(9p+12)(9p-12)$ scores M0 even if "recovered"

A1*: cso Arrives at $q=\frac{1}{20}(3p+4)(3p-4)$ with no errors seen.

Any missing brackets must be recovered before reaching the printed answer.

A correct intermediate line must be seen, such as $q=\frac{9p^2-16}{20}$ or $20q=(3p+4)(3p-4)$

	Alternative 3: Working backwards		
	$q=\frac{1}{20}(3p+4)(3p-4)=\frac{1}{20}(9p^2-16)$ $\Rightarrow x^2-3px+5q+4=x^2-3px+\frac{1}{4}(9p^2-16)+4=x^2-3px+\frac{9p^2}{4}$	B1	1.1b
	$x^2-3px+\frac{9p^2}{4}=\left(x-\frac{3p}{2}\right)^2+...$ or $b^2-4ac=(3p)^2-4\times1\times\left(\frac{9p^2}{4}\right)=...$	M1	3.1a
	e.g. $x=\frac{3p}{2}$ only so equal roots or $\left(x-\frac{3p}{2}\right)^2=0$ so equal roots or $b^2-4ac=0$ so equal roots	A1*	2.1

Notes:

Note we are marking this B1M1A1 not M1M1A1

B1: Substitutes the given result into $x^2-3px+5q+4$ and simplifies to obtain $x^2-3px+\frac{9p^2}{4}$ oe e.g. $4x^2-12px+9p^2$

M1: Attempts to complete the square to obtain $x^2-3px+\frac{9p^2}{4}=\left(x-\frac{3p}{2}\right)^2+...$ oe

alternatively attempts the discriminant e.g. $(3p)^2-4\times1\times\left(\frac{9p^2}{4}\right)=...$

A1*: cso Fully correct work with sufficient working shown and no errors seen with a suitable reasoning and a conclusion e.g. "only 1 root so equal roots" or equivalent or obtains a zero discriminant and states e.g. "so equal roots" or equivalent.

Question	Scheme	Marks	AOs
10	$2\log_3(x+1)=1+\log_3(x+7)$		
	$2\log_3(x+1)=\log_3(x+1)^2$	B1	1.1a
	$\log_3\dfrac{(x+1)^2}{x+7}=1$	M1	1.1b
	$\dfrac{(x+1)^2}{x+7}=3$	A1	1.1b
	$(x+1)^2=3(x+7)\Rightarrow x^2-x-20=0\Rightarrow x=...$	M1	1.1b
	$x=5$ only	A1	2.3
		(5)	
			(5 marks)

Notes:

B1: For $2\log_3(x+1)=\log_3(x+1)^2$ oe e.g. $2\log_3(x+1)=\log_3(x^2+2x+1)$

M1: Uses the subtraction law of logs following the above $\log_3\dfrac{(x+1)^2}{x+7}$

Alternatively uses the addition or subtraction law following use of $1=\log_3 3$

i.e., $1+\log_3(x+7)=\log_3(3(x+7))$

They must be combining the given terms and not e.g. $\log_3(x+7)=\log_3 x+\log_3 7=\log_3 7x$

A1: **Correct** equation obtained with logs removed following correct work.

Allow equivalent **correct** equations e.g. $(x+1)^2=3(x+7)$

M1: **Depends on the B mark.**
Simplifies to a 3TQ = 0 and attempts a correct method to solve their quadratic = 0 (usual rules) to obtain a value for x which may be via a calculator (you may need to check) and the "= 0" may be implied.

A1: cso $x=5$ only. If seen, -4 must have been rejected or $x=5$ clearly selected.

There is no requirement to justify why the -4 has been rejected and any comments can be ignored whether correct or otherwise.

See below for some specific examples.

<u>Examples with marking guidelines</u>

$$2\log_3(x+1) = 1 + \log_3(x+7)$$
$$\Rightarrow \log_3(x+1)^2 = 1 + \log_3(x+7)$$
$$\Rightarrow \frac{(x+1)^2}{(x+7)} = 3$$

Scores B1M1A1 then follow the main scheme.

$$2\log_3(x+1) = 1 + \log_3(x+7)$$
$$\Rightarrow \log_3(x+1)^2 = 1 + \log_3(x+7)$$
$$\Rightarrow \frac{(x+1)^2}{(x+7)} = 1$$
$$(x+1)^2 = x+7 \Rightarrow x^2 + x - 6 = 0 \Rightarrow x = \ldots$$

Scores B1M0A0M1A0

$$2\log_3(x+1) = 1 + \log_3(x+7)$$
$$\Rightarrow \log_3(x+1)^2 = 1 + \log_3(x+7)$$
$$\Rightarrow \log_3(x+1)^2 + \log_3 3 = \log_3(x+7)$$
$$\Rightarrow \log_3 3(x+1)^2 = \log_3(x+7)$$
$$3(x+1)^2 = x+7 \Rightarrow$$

Scores B1M1A0 then apply the scheme

Beware incorrect log work which leads to the correct answer:

$$2\log_3(x+1) = 1 + \log_3(x+7)$$
$$\log_3(x+1)^2 = 1 + \log_3(x+7)$$
$$\log_3(x+1)^2 - \log_3(x+7) = 1$$
$$\frac{\log_3(x+1)^2}{\log_3(x+7)} = 1 \Rightarrow \frac{(x+1)^2}{(x+7)} = 3$$
$$(x+1)^2 = 3(x+7) \Rightarrow x^2 - x - 20 = 0 \Rightarrow x = 5$$

Scores B1M0A0M1A0

In Q11 there are a lot of parts so mark positively and condone mislabelling if they are clearly doing the right thing for a particular part.

Question	Scheme	Marks	AOs
11(a)	$(V=)\frac{1}{2}\pi r^2 l = 90000\pi$	B1	1.1b
	$(A=)\pi r^2 + \pi rl$	B1	1.1b
	$l = \frac{180000}{r^2} \Rightarrow A = \pi r^2 + \pi r\left(\frac{180000}{r^2}\right)$ or e.g. $\pi rl = \frac{180000\pi}{r} \Rightarrow A = \pi r^2 + \frac{180000\pi}{r}$ or e.g. $rl = \frac{180000}{r} \Rightarrow A = \pi r^2 + \pi\left(\frac{180000}{r}\right)$	M1	1.1b
	$(A=)\pi r^2 + \frac{180000\pi}{r}$ *	A1*	1.1b
		(4)	

(a) Condone use of e.g. *h* for *l* or e.g. *R* for *r* as long as the intention is clear.

B1: Correct equation seen for volume:

$$\frac{1}{2}\pi r^2 l = 90000\pi \text{ or } \pi r^2 l = 180000\pi \text{ or } \pi r^2 l = 2(90000\pi)$$

or

$$\frac{1}{2}r^2 l = 90000 \text{ or } r^2 l = 180000 \text{ or } r^2 l = 2(90000)$$

May not be implied by subsequent work.

B1: **Correct** expression for the area stated in terms of the radius (or diameter) and length: e.g. $(A=)\pi r^2 + \pi rl$ but allow equivalent correct expression so that the "ends" are any one of:

πr^2	$\frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2$	$\pi\left(\frac{d}{2}\right)^2$	$\frac{2\pi r^2}{2}$	$2\times 0.5\pi r^2$

And the curved surface area is any one of:

πrl	$\frac{1}{2}(2\pi rl)$	$\frac{\pi ld}{2}$

These may be seen separately before being combined at the end.
Condone a missing "$A=$" for this mark. Cannot be implied.

M1: Rearranges their $\frac{1}{2}\pi r^2 l = 90000\pi$ to e.g. $l = ...$ or $\pi rl = ...$ or $rl = ...$ and substitutes in for e.g. l or πrl or rl in their formula for the area which must have at least 2 terms, all of which must be dimensionally correct, to obtain an expression in terms of *r* only. This mark cannot be implied and substitution needs to be shown, not just the printed answer written down. This may be seen combined at the end.

A1*: cso. The "$A=$" does not have to be seen. Ignore any units given.
All previous marks must be scored. Isw once a correct answer is reached from correct work.

(b)	$A=\pi r^2+\frac{180000\pi}{r}\Rightarrow\left(\frac{\mathrm{d}A}{\mathrm{d}r}=\right)2\pi r-180000\pi r^{-2}$	M1 A1	1.1b 1.1b
	$\frac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow r^3=\frac{180000\pi}{2\pi}$	dM1	1.1b
	$\Rightarrow r=\sqrt[3]{90000}$ or $r=\text{awrt }44.8$ (cm)	A1	1.1b
		(4)	
(c)	Finds $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=2\pi+360000\pi r^{-3}$ at $r=44.8$	M1	3.1b
	$\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=(+18.8)>0$ hence minimum (value of A)*	A1	2.4
		(2)	

(b)

M1: Attempts to differentiate $A=\pi r^2+\frac{180000\pi}{r}$ with respect to r.

Award for $\left(\frac{\mathrm{d}A}{\mathrm{d}r}=\right)\ldots r\pm\ldots r^{-2}$

A1: $\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=\right\}2\pi r-180000\pi r^{-2}$ Condone $\frac{\mathrm{d}A}{\mathrm{d}r}$ appearing as $\frac{\mathrm{d}y}{\mathrm{d}x}$ or being absent.

dM1: Sets their $\frac{\mathrm{d}A}{\mathrm{d}r}\ldots 0$ where "…" could be an inequality and arrives at a value for r via $r^{\pm3}\ldots k,\ k>0$ where "…" could be an inequality.

May be implied by e.g. $r=\sqrt[3]{\ldots}$ or $r=\sqrt[-3]{\ldots}$ where "…" > 0

They cannot just go from e.g. $2\pi r-180000\pi r^{-2}=0$ straight to a value for r.

A1: Awrt 44.8 or $\sqrt[3]{90000}$ Condone omission of units or use of incorrect units.

Do not allow e.g. ± 44.8 or e.g. $r>44.8$

(c)

M1: Attempts to find $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ following on from their $\frac{\mathrm{d}A}{\mathrm{d}r}$ which includes $\ldots r\to\text{constant}$ and $\ldots r^{-2}\to\ldots r^{-3}$ and attempts to find its value with their r (may need to check) or considers its sign with reference to their r. Condone slips in substituting as long as the intention is clear.

Alt may consider the value of $\frac{\mathrm{d}A}{\mathrm{d}r}$ either side e.g. $\left(\frac{\mathrm{d}A}{\mathrm{d}r}\right)_{x=44}=-15.6\ldots,\ \left(\frac{\mathrm{d}A}{\mathrm{d}r}\right)_{x=45}=3.49\ldots$

or may consider the value of A either side e.g. $(A)_{x=44.8}=18927,\ (A)_{x=44}=18934,\ (A)_{x=45}=18928$

A1: Fully correct work and conclusion. This requires

- correct work in (b) with the correct value of r
- a correct $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$
- obtaining $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=\text{awrt }19$ (or 6π) (or truncated as 18) which is > 0 or e.g. $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}>0$ as $r>0$
  - a conclusion that it is the minimum

For the alt it would be for

- correct calculations
- e.g. gradient goes from negative to positive so minimum
- e.g. A is larger either side of 44.8 so minimum

(d)	Substitutes $r = 44.8$ into $A = \pi r^2 + \frac{180\,000\pi}{r}$ $\{= \text{awrt } 18900\}$ Or may be seen embedded in a (possibly incorrect) conversion e.g. substitutes $r = 44.8$ into $\frac{30}{100^2}\left(\pi r^2 + \frac{180\,000\pi}{r}\right)$ $\{= \text{awrt } 56.8\}$	M1	3.4
	Minimum cost = £56.78	A1	1.1b
		(2)	
(e)	Accept any sensible assumption, e.g., • The exact amount of metal required can be bought. • No metal goes to waste. • The trough can be made as a perfect (semicircular) cylinder. • It's possible to cut accurately Condone • Need to buy a fixed amount • You can't buy the exact amount of metal • There will be wastage due to the shape • No extra amount of metal is needed • There will be no errors when cutting the metal • The trough will be fully smooth/have no imperfections Do not allow e.g. • The thickness of the sheet is negligible	B1	3.5a
		(1)	
			(13 marks)

(d)

M1: For a correct method of finding A or $\frac{30A}{100^2}$ or e.g. $30A$ from their solution to $\frac{dA}{dr} = 0$

May be implied by e.g. 18928

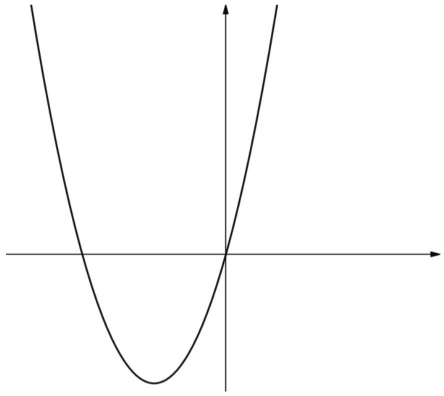
Do not accept attempts using negative values of r.

A1: Minimum cost = £56.78 or £56.79 including units.

(e)

B1: See main scheme.

Do not accept reference to additional work required, such as labour costs, or materials required to weld the metal together.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="5">12(a)</td><td></td><td></td><td></td></tr>
<tr><td>$\cup$ shaped not $\cap$ shaped.</td><td>B1</td><td>1.1b</td></tr>
<tr><td>Crosses the negative x-axis.</td><td>B1</td><td>1.1b</td></tr>
<tr><td>Crosses the x-axis at the origin.</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">Look out for the sketch drawn on Figure 5 but Diagram 1 takes precedence.
(a) Ignore any values given at intersection/turning points.
B1: $\cup$ shaped curve not $\cap$ shaped. Ignore any roots or lack of roots.
B1: At least one intersection with the negative x-axis. (May be a straight line)
Allow their curve/line to touch for this mark.
B1: A curve intersecting the origin. (May be a straight line)
Allow their curve/line to touch for this mark.</td></tr>
</table>

(b)	Correctly identifies $\left(\frac{dy}{dx}=\right)3x^2+9x$ with one correct justification (see below)	B1	2.2a
	It is a positive quadratic with a negative root and a root at the origin.	dB1	2.4

(b)

B1: Deduces that the gradient function is $\left(\frac{dy}{dx}=\right)3x^2+9x$ with one valid justification (see below)

Allow correct calculations for either roots $x=-3$ or $x=0$ or for the turning point at $x=-\frac{3}{2}$ as a valid justification.

dB1: Explanation that includes

- one root being negative or e.g. has a root at $x=-3$ with no implication of there being more than one negative root
- one root being at the origin (or O) e.g. has a root at $x=0$ (condone the y intercept is 0 but **not** "it doesn't have a y intercept")
- that it is U shaped (or a positive parabola/quadratic or it is the derivative of a positive cubic but **not** "positive graph/curve")

Alternatively, allow a complete explanation eliminating each of the other equations.
e.g. can't be:

- $(x+1)^2-2$ as this doesn't go through the origin
- $-x(x+7)$ as this is not a U shape
- x^2-5x as this doesn't have a negative root

Or e.g.

- the curve has a turning point in quadrant 3 or is at $\left(-\frac{3}{2},-\frac{27}{4}\right)$
- one root is at the origin

If extra incorrect statements are included they can be ignored.
Note B0dB1 is not possible.

Question	Scheme	Marks	AOs
13(a)	e.g. $6=31-Ae^{-10k},\ 11=31-Ae^{-20k} \Rightarrow Ae^{-10k}=25,\ Ae^{-20k}=20$ $\Rightarrow e^{10k}=\frac{25}{20}$	M1	3.1b
	e.g. $e^{10k}=\frac{25}{20} \Rightarrow 10k=\ln\left(\frac{25}{20}\right) \Rightarrow k=\frac{1}{10}\ln\left(\frac{5}{4}\right)$	dM1	3.1a
	$k=0.0223$ or $A=31.3$	A1	1.1b
	$h=31-31.3e^{-0.0223t}$	A1	3.3
		(4)	

(a) Notes:

M1: Uses $h=31-Ae^{-kt}$ with $h=6,\ t=10$ and $h=11,\ t=20$ to create simultaneous equations and proceeds via correct work to eliminate A or k.

Condone the equations appearing as e.g. $6\,\text{m}=31-Ae^{-10k},\ 11\,\text{m}=31-Ae^{-20k}$

Look for the overall method being correct but condone slips.

<u>Examples:</u>

Eliminates A:

- $Ae^{-10k}=25,\ Ae^{-20k}=20 \Rightarrow e^{10k}=\frac{25}{20}$
- $Ae^{-10k}=25,\ Ae^{-20k}=20 \Rightarrow \ln A-10k=\ln 25,\ \ln A-20k=\ln 20 \Rightarrow 10k=\ln 25-\ln 20$

Eliminates k:

- $Ae^{-10k}=25,\ Ae^{-20k}=20 \Rightarrow A\left(e^{-10k}\right)^2=20 \Rightarrow A\left(\frac{25}{A}\right)^2=20$
- $Ae^{-10k}=25,\ Ae^{-20k}=20 \Rightarrow \ln A-10k=\ln 25,\ \ln A-20k=\ln 20 \Rightarrow \ln A=2\ln 25-\ln 20$

dM1: Proceeds to find a value for A or k using the correct order of operations but condoning slips.

A1: Achieves either k = awrt 0.0223 or A = awrt 31.3 (or A = awrt 31.2)

Note the exact values are $k=\frac{1}{10}\ln\frac{5}{4}$ or $A=\frac{125}{4}(31.25)$ which are acceptable.

A1: cso Proceeds correctly to $h=31-31.2e^{-0.0223t}$ or $h=31-31.3e^{-0.0223t}$

Exact answer is $h=31-\frac{125}{4}e^{-\frac{1}{10}\ln\frac{5}{4}t}$ o.e., e.g., $h=31-31.25e^{\frac{1}{10}\ln\frac{4}{5}t}$ which is acceptable.

Finding A and k is insufficient for this mark as the question asks for a <u>complete equation</u>, however, allow this mark if the complete correct equation is seen subsequently.

(b)	31 m	B1	2.2a
		(1)	
(c)(i)	$-0.3\,\text{m}$	B1ft	3.4
(ii)	The model is unsuitable for the early growth of the tree as it suggests that the tree had a negative height.	B1ft	3.5a
		(2)	

(b)
B1: cao 31 m and units are required. Must be seen as their answer to (b).
There must be no other values.

(c) (i)
B1ft: Accept $31-"A"$ with correct units. This must be evaluated correctly for their A.
Only penalise the omission of units **once** from an otherwise acceptable answer in (b) and (c)(i) and penalise it on the first occurrence.

(c) (ii)
B1ft: Concludes that the model is unsuitable because (the initial/early**)** **<u>height</u>** is **<u>negative</u>**
Follow through on their (c)(i) here but it must be negative.
Alternatively, allow that the model is suitable if they conclude that the depth of the seed is $"-0.3"$ m or e.g. suitable as it would be underground.

(d)(i)	$\frac{dh}{dt} = \ldots e^{-kt}$	M1	1.1b
	$\left\{\frac{dh}{dt} =\right\}$ awrt $0.698e^{-0.0223t}$	A1	1.1b
		(2)	
(d)(ii)	"$0.698e^{-0.0223t}$" $= 0.3$	B1ft	3.4
	$e^{-0.0223t} = \frac{0.3}{0.698} \Rightarrow -0.0223t = \ln\left(\frac{0.3}{0.698}\right)$	M1	3.1a
	$t = 37.9$	A1	1.1b
		(3)	

(d) (i)

M1: Achieves the form $\left\{\frac{dh}{dt} =\right\} ae^{-kt}$. It is acceptable to use k and A for this mark and allow if $k < 0$ but must be consistent with their h.

A1: $\left\{\frac{dh}{dt} =\right\}$ awrt $0.698e^{-0.0223t}$ Allow awrt $0.697e^{-0.0223t}$ or awrt $0.696e^{-0.0223t}$ and allow exact e.g. $\frac{25}{8}\ln\frac{5}{4}e^{-\frac{1}{10}\ln\frac{5}{4}t}$

(d)(ii)

B1ft: Sets their $\frac{dh}{dt} = 0.3$ where $\frac{dh}{dt}$ is not their h.

M1: Uses their $\frac{dh}{dt} = \ldots$, and proceeds from $ae^{-kt} = b$ where $ab > 0$ to $ct = \ln d$ where $d > 0$ using the correct order of operations but condoning slips.
Allow equivalent work e.g.
$0.696e^{-0.0223t} = 0.3 \Rightarrow \ln 0.696 - 0.0223t = \ln 0.3 \Rightarrow -0.0223t = \ln 0.3 - \ln 0.696$

A1: cso awrt 37.7 or 37.8 or 37.9 but note that this may be scored if k is rounded in (a).
Ignore any units if given and just look for the value.

Question	Scheme	Marks	AOs
14(a)	Attempts $\binom{8}{2}(kx)^2$	M1	1.1b
	$(p=)28k^2$	A1	1.1b
		(2)	
(b)	Attempts $a-16k=-21$ or $8ak-56k^2=-90$	M1	1.1b
	$a-16k=-21$ and $8ak-56k^2=-90$	A1ft	1.1b
	$a=16k-21\Rightarrow 8k(16k-21)-56k^2=-90\Rightarrow 12k^2-28k+15=0\Rightarrow k=...$ or $a=16k-21\Rightarrow k=\frac{a+21}{16}\Rightarrow 8a\left(\frac{a+21}{16}\right)-56\left(\frac{a+21}{16}\right)^2=-90$ $\Rightarrow 9a^2+42a-207=0\Rightarrow a=...$	dM1	2.1
	Any of $(a=)3, (k=)\frac{3}{2}, (a=)-\frac{23}{3}, (k=)\frac{5}{6}$	A1	1.1b
	$a=3, k=\frac{3}{2}$ and $a=-\frac{23}{3}, k=\frac{5}{6}$	A1	2.2a
		(5)	
		(7 marks)	

Notes:

(a) **Note that marks in (a) can be scored in (b) as long as they are not contradictory.**

M1: Correct expression for the coefficient of the third term e.g. $\binom{8}{2}k^2$

or correct expression for the third term $\binom{8}{2}(kx)^2$ but condone $\binom{8}{2}kx^2$

May be seen embedded in an expansion.

A1: cao $(p=)28k^2$ Must be extracted or e.g. underlined for this mark if seen in an expansion.

Correct answer only scores both marks. The "$p=$" is not required.
Isw after a correct answer is seen.

(b) **Note that candidates can score a maximum M1A1ftdM0A0A0 in (b) for the correct use of "p" rather than "$28k^2$"**

M1: Attempts to find an equation for the constant or for the coefficient of x
Condone slips on signs so award for $\pm a\pm 16k=\pm 21$ **or** $\pm 8ak\pm 2\times "28k^2"=\pm 90$
Allow if x's are present for this mark e.g. $\pm 8akx\pm 2\times "28k^2"x=\pm 90x$ and if awarding for the second equation, condone the omission of one x if they are present.

A1ft: Two correct equations following through on their answer for (a)
Look for $a-16k=-21$ o.e. and $8ak-2\times "28k^2"=-90$ o.e.
This may be implied but there must be no x's so e.g. $8akx-2\times "28k^2"x=-90x$ followed by $8ak-2\times "28k^2"-90=0$ would imply this mark.

dM1: Valid attempt to solve their equations simultaneously, via a 3TQ in a or k set $=0$, leading to a value for either a or k. Note that the correct quadratic for a is also $3a^2+14a-69=0$
Their 3TQ can be solved by any valid means including a calculator (you may need to check) and condone slips in reaching their 3TQ.

A1: cso Any correct value for a or k from correct work so depends on all previous marks.

A1: cso Both possible pairs of values for a and k, correctly paired and clearly identified as a and k. Depends on all previous marks.

Apply isw if they are subsequently written as coordinates e.g. $\left(3, \frac{3}{2}\right), \left(-\frac{23}{3}, \frac{5}{6}\right)$

Question	Scheme	Marks	AOs
15(a)	$n^3 + 4n$		
	Attempts $n^3 + 4n$ for any 2 natural numbers.	M1	1.1b
	$1^3 + 4 \times 1 = 5$ prime and e.g. $2^3 + 4 \times 2 = 16$ not prime $\therefore$ sometimes true.	A1	2.4
		(2)	

Condone the use of *n* for e.g. *k* for both marks.

All methods require attempting $n^3 + 4n$ when $n = 1$ for full marks.

Way 1:

M1: Attempts $n^3 + 4n$ for any 2 natural numbers.

A1: Requires:

- obtains $n^3 + 4n = 5$ when $n = 1$ and states "prime" or "true" or ✓
- correct evaluation for any other natural number and states "not prime" or "composite" or "not true" or ×
- states "sometimes true"

You can ignore any other incorrectly evaluated examples or e.g. use of negative numbers as long as these conditions are met.

Way 2 (factorisation):

M1: Attempts to factorise $n^3 + 4n = n\left(n^2 + 4\right)$

Allow for $n^3 + 4n = n\left(n^2 + ...\right)$ or $n^3 + 4n = n\left(... + 4\right)$

A1: Requires:

- uses $n = 1$ and obtains $n^3 + 4n = 5$ and states "prime" or "true"
- correct factorisation and states $n^3 + 4n = n\left(n^2 + 4\right)$ is "composite" or "not prime" or "not true" (when $n > 1$)
- states "sometimes true"

Way 3 (odd/even):

M1: Attempts to substitute $n = 2k$ oe **or** $n = 2k + 1$ oe

A1: Requires:

- uses $n = 1$ and obtains $n^3 + 4n = 5$. Here the value for $n = 1$ might be found using $k = 1$ for $n = 2k - 1$ or $k = 0$ for $n = 2k + 1$ and states "prime" or "true"
- correctly factorises any one correct form for odd or even e.g. $8k^3 + 8k = 8k\left(k^2 + 1\right)$ and concludes "not prime" or "composite" or "not true" (only one form needed here)
- states "sometimes true"

(b)	n^3+5n		
	$n^3+5n=n\left(n^2+5\right)$	M1	3.1a
	Since $1^3+5\times1=6$, n^3+5n is not prime for $n = 1$ For all other n, $n^3+5n=n\left(n^2+5\right)$ is not prime as it is the product of two other numbers not equal to 1. Hence never true.	A1	2.4
		(2)	

(4 marks)

Notes:

Condone the use of *n* for e.g. *k* for both marks.

Way 1 (Factorisation):

M1: Attempts to factorise $n^3+5n=n\left(n^2+5\right)$

Allow $n^3+5n=n\left(n^2+...\right)$ or $n^3+5n=n\left(...+5\right)$

A1: Requires:

- Correct factorisation $n^3+5n=n\left(n^2+5\right)$
- Substitution of $n=1$: $1^3+5\times1=6$
  or states $n^3+5n\neq2$ oe e.g. $n^3+5n>2$ oe
- "Never true"

Way 2 (Odd/Even):

M1: Attempts to substitute $n=2k$ **and** either $n=2k+1$ or $n=2k-1$ oe

A1: Requires:

- Correctly factorising both even and odd forms
  e.g. $8k^3+10k=2k\left(4k^2+5\right)$ **and**
  e.g. $\left(2k+1\right)^3+5\left(2k+1\right)=\left(2k+1\right)\left(\left(2k+1\right)^2+5\right)$ or e.g. $2\left(2k+1\right)\left(2k^2+2k+3\right)$
  (in this part both cases are needed)
- Substitution of $n=1$: $1^3+5\times1=6$, or e.g. $k=1$ for $n=2k-1$ or $k=0$ for $n=2k+1$
  or states $n^3+5n\neq2$ oe e.g. $n^3+5n>2$ oe
  e.g. $2k\left(4k^2+5\right)$ and $2\left(2k+1\right)\left(2k^2+2k+3\right)$ are > 2 or ≠ 2
- "Never true"

Way 3 (Odd/Even via logic):

M1: Considers n^3+5n with "odds" and "evens" e.g.

If n is **odd** then n^3+5n = odd + odd = even

or e.g. $n^3+5n=n\left(n^2+5\right)$ = odd(odd + odd) = odd×even = even

If n is **even** then n^3+5n = even + even = even

or e.g. $n^3+5n=n\left(n^2+5\right)$ = even(even + odd) = even×odd = even

A1: Requires:

- Fully correct argument for both odds and evens
- A full justification of any assumed results e.g. $n\,\text{odd}\Rightarrow n^3$ is odd via algebra or e.g. n odd means n^3 = odd×odd×odd = odd
- When $n = 1$, $n^3+5n=6$ so $n^3+5n\neq2$
- "Never true"

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