8.3 Nuclear Instability & Radius

XVIII

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

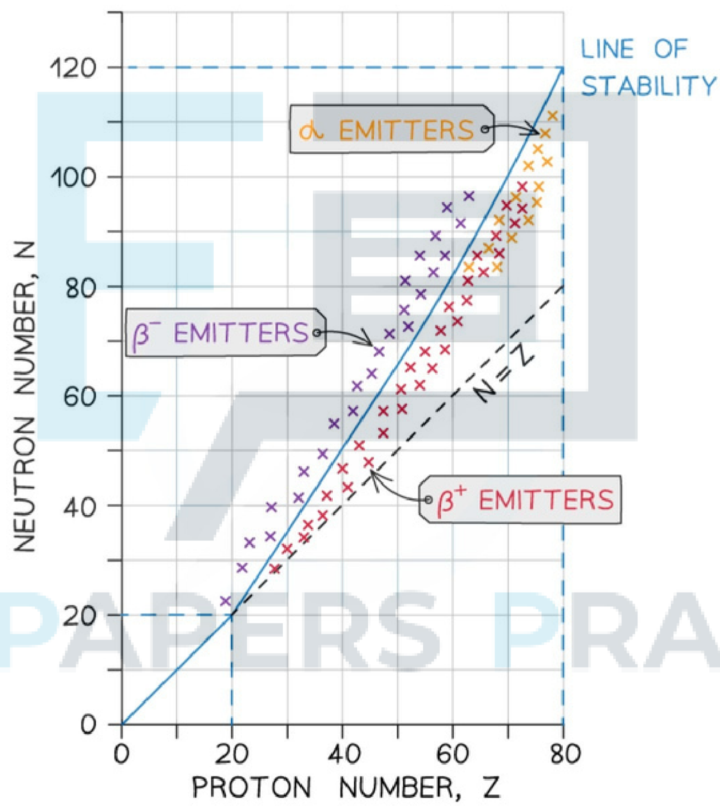
8.3 Nuclear Instability & Radius

CONTENTS

Nuclear Stability Graph

- The most common elements in the universe all tend to have values of N and Z less than 20 (plus iron which has Z = 26, N = 30)
- Where:
  - N = number of neutrons
  - Z = number of protons / atomic number

- This is because lighter elements (with fewer protons) tend to be much **more stable** than heavier ones (with many protons)
- Nuclear stability becomes vastly clearer when viewed on a graph of N against Z



This nuclear stability curve shows the line of stable isotopes and which unstable isotopes will emit alpha or beta particles

- A nucleus will be unstable if it has:
  - Too many neutrons
  - Too many protons
  - Too many nucleons ie. too heavy
  - Too much energy

- An unstable atom wants to become neutral to become stable

- For light isotopes, $Z < 20$:

    - All these nuclei tend to be very stable
    - They follow the straight-line N = Z

- For heavy isotopes, Z > 20:
    - The neutron-proton ratio increases
    - Stable nuclei must have more neutrons than protons

- This imbalance in the neutron-proton ratio is very significant to the stability of nuclei
    - At a short range (around 1-4 fm), nucleons are bound by the **strong nuclear force**
    - Below 1 fm, the strong nuclear force is **repulsive** in order to prevent the nucleus from collapsing
    - At longer ranges, the electromagnetic force acts between protons, so **more protons cause more instability**
    - Therefore, as more protons are added to the nucleus, more neutrons are needed to add distance between protons to **reduce** the electrostatic repulsion
    - Also, the extra neutrons increase the amount of binding force which helps to **bind the nucleons together**

Alpha, Beta & Electron Capture

- The graph of N against Z is useful in determining which isotopes will decay via
  - Alpha emission
  - Beta-minus (β^-) emission
  - Beta-plus (β^+) emission
  - Electron capture

- **Alpha-emitters**:
  - Occur beneath the line of stability when Z > 60 where there are **too many nucleons** in the nucleus
  - These nuclei have more protons than neutrons, but they are too large to be stable
  - This is because the strong nuclear force between the nucleons is unable to overcome the electrostatic force of repulsion between the protons

- **Beta-minus (β^-) emitters**:
  - Occur to the left of the stability line where the isotopes are **neutron-rich** compared to stable isotopes
  - A neutron is converted to a proton and emits a β^- particle (and an anti-electron neutrino)

- **Beta-plus (β^+) emitters**:
  - Occur to the right of the stability line where the isotopes are **proton-rich** compared to stable isotopes
  - A proton is converted to a neutron and emits a β^+ particle (and an electron neutrino)

- **Electron capture**:
  - When a nucleus captures one of its own orbiting electrons
  - As with β^+ decay, a proton in the nucleus is converted into a neutron, releasing a gamma-ray (and an electron neutrino)
  - Hence, this also occurs to the right of the stability line where the isotopes are **proton-rich** compared to stable isotopes

Exam Tip

To remember where the β^- and β^+ emitters are on the graph:

- Beta-minus is a **n**egative particle where a **n**eutron turns into a proton. Unstable atoms always want to go towards a roughly equal number of protons and neutrons
  - Therefore these emitters are on the **n**eutron-rich side of isotopes
- Beta-plus is a **p**ositive particle where a **p**roton turns into a neutron
  - Therefore these emitters are on the **p**roton-rich side of isotopes

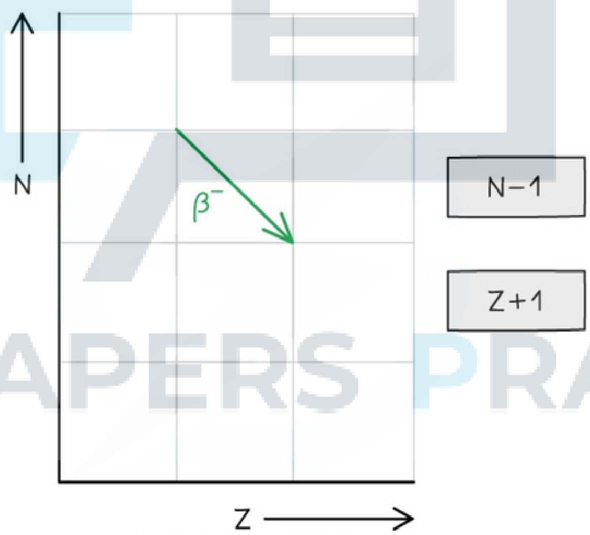
The best way to remember the nuclear stability graph is to try to draw it from memory

Changes in N and Z by Radioactive Decay

- There are four reasons why a nucleus might become unstable, and these determine which decay mode will occur

- **Too many neutrons**
  - Decays through **beta-minus (β^-) emission**
  - One of the **neutrons** in the nucleus changes into a **proton** and a β^- particle (an electron) and antineutrino is released
  - The nucleon number is constant
    - The neutron number (N) decreases by 1
    - The proton number (Z) increases by 1
  - The general decay equation for β^- emission is:

$$^{A}_{Z}X \rightarrow {}^{0}_{-1}\beta + {}^{A}_{Z+1}Y + \overline{\nu_e}$$

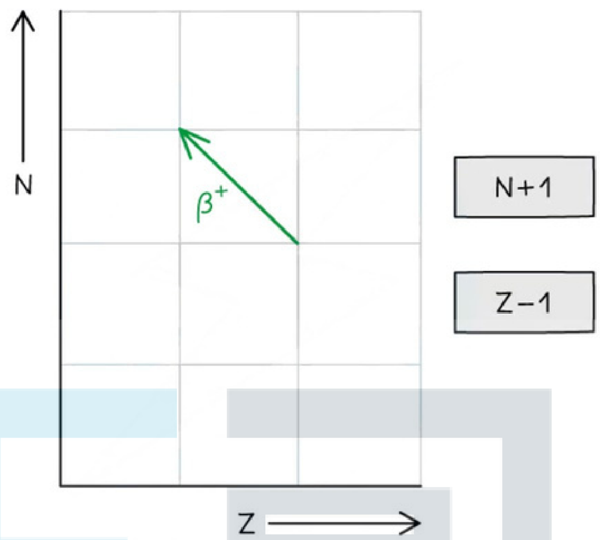


- **Too many protons**
  - Decays through **beta-plus (β^+) emission** or **electron capture**
  - In beta-plus decay, a **proton** changes into a **neutron** and a β^+ particle (a positron) and neutrino are released
  - In electron capture, an orbiting electron is taken in by the nucleus and combined with a proton causing the formation of a neutron and neutrino
  - In both types of decay, the nucleon number stays constant
    - The neutron number (N) increases by 1
    - The proton number (Z) decreases by 1
  - The general decay equation for β^+ emission is:

$$ {}^{A}_{Z}X \rightarrow {}^{0}_{+1}\beta + {}_{Z-1}^{A}Y + \nu_e $$

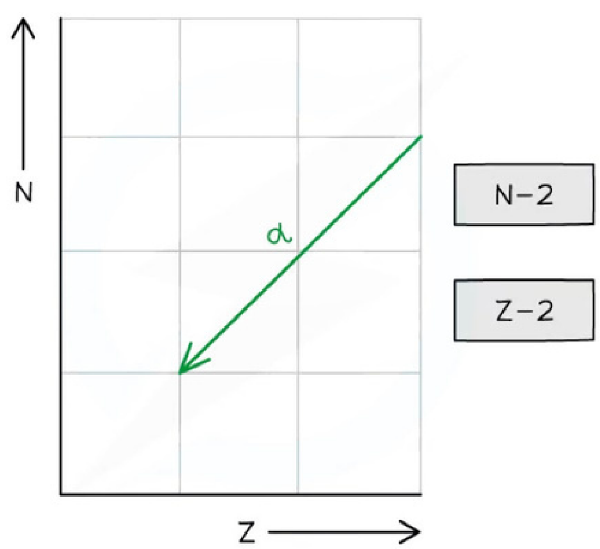
- The equation for electron capture is:

$$ {}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}_{Z-1}^{A}Y + \nu_e $$

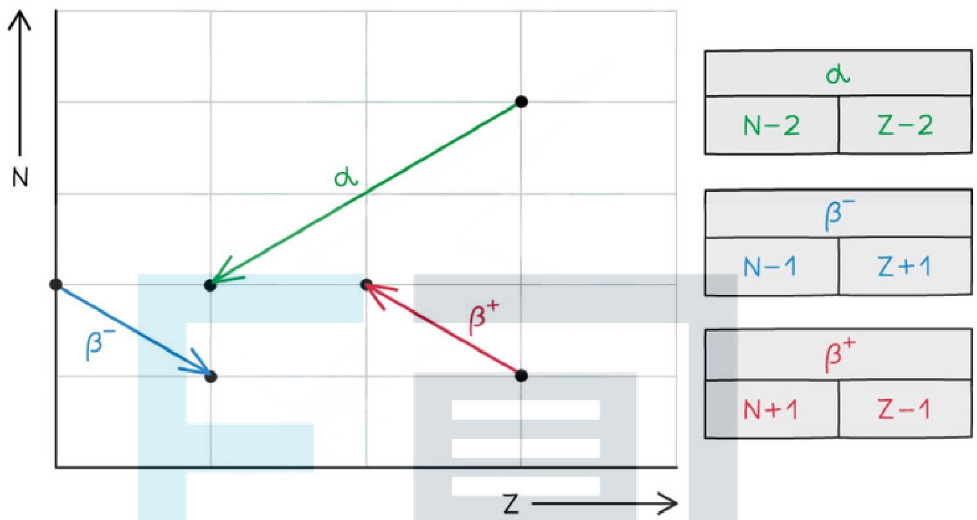


- **Too many nucleons**
  - Decays through **alpha (α) emission**
  - An α particle is a helium nucleus
  - The nucleon number decreases by 4 and the proton number decreases by 2
    - The neutron number (N) decreases by 2
    - The proton number (Z) decreases by 2
  - The general decay equation for α emission is:

$$ {}^{A}_{Z}X \rightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y $$



- **Too much energy**
  - Decays through **gamma (γ) emission**
  - A gamma particle is a high-energy electromagnetic radiation
  - This usually occurs after a different type of decay, such as alpha or beta decay
  - This is because the nucleus becomes excited and has excess energy
- In summary, alpha decay, beta decay and electron capture can be represented on an N-Z graph as follows:



Worked Example

Plutonium-239 is a radioactive isotope that contains 94 protons and emits α particles to form a radioactive isotope of uranium. This isotope of uranium emits α particles to form an isotope of thorium which is also radioactive.

a) Write two equations to represent the decay of plutonium-239 and the subsequent decay of uranium

b) Predict the decay mode of the thorium isotope

c) Draw the decay chain from plutonium-239 to the daughter product of thorium decay on an N-Z graph

Part (a)**Step 1: Write down the general equation of alpha decay**

$$^{A}_{Z}X \rightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y$$

Step 2: Write down the decay equation of plutonium into uranium

$$^{239}_{94}Pu \rightarrow {}^{4}_{2}\alpha + {}^{235}_{92}U$$

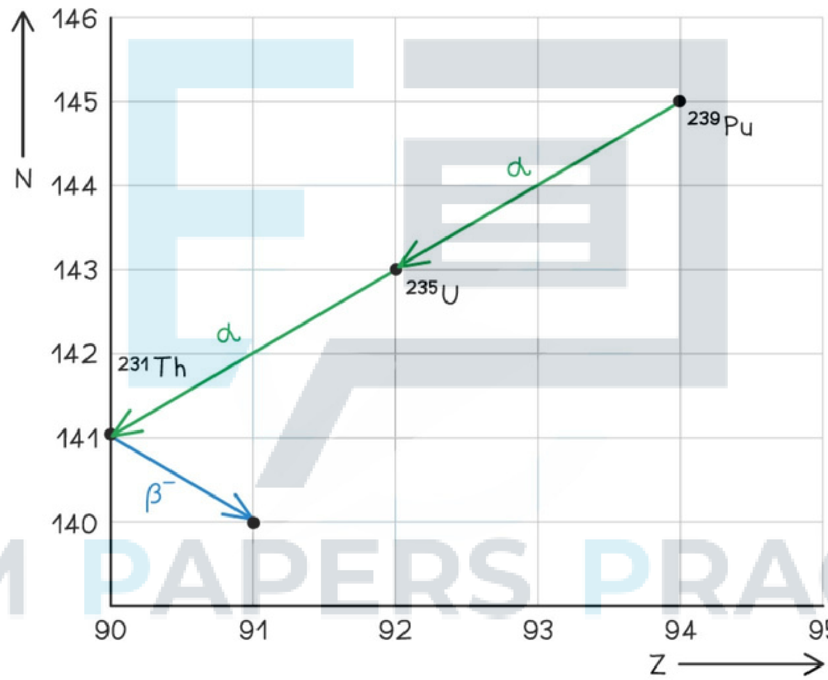
Step 3: Write down the decay equation of uranium into thorium

$$^{235}_{92}U \rightarrow {}^{4}_{2}\alpha + {}^{231}_{90}Th$$

Part (b)

- Plutonium, ^{239}Pu
  - **Number of neutrons**: 239 - 94 = 145
  - **Neutron-nucleon ratio**: 145 / 239 = 0.607
- Uranium, ^{235}U
  - **Number of neutrons:** 235 - 92 = 143
  - **Neutron-nucleon ratio**: 143 / 235 = 0.609
- Thorium, ^{231}Th
  - **Number of neutrons:** 231 - 90 = 141
  - **Neutron-nucleon ratio**: 141 / 231 = 0.610
- Thorium-231 is neutron-rich **compared** to uranium-235 and plutonium-239
- Therefore, it must be a β^- emitter

Part (c)



- **The key features to draw on an N-Z graph are:**
  - Values for neutron number (N) on the vertical axis
  - Values for proton number (Z) on the horizontal axis
  - Labels for the isotopes eg. ^{239}Pu, ^{235}U, ^{231}Th
  - Arrows showing the direction of the decay
  - Labels for the type of emission eg. α, β^-

Exam Tip

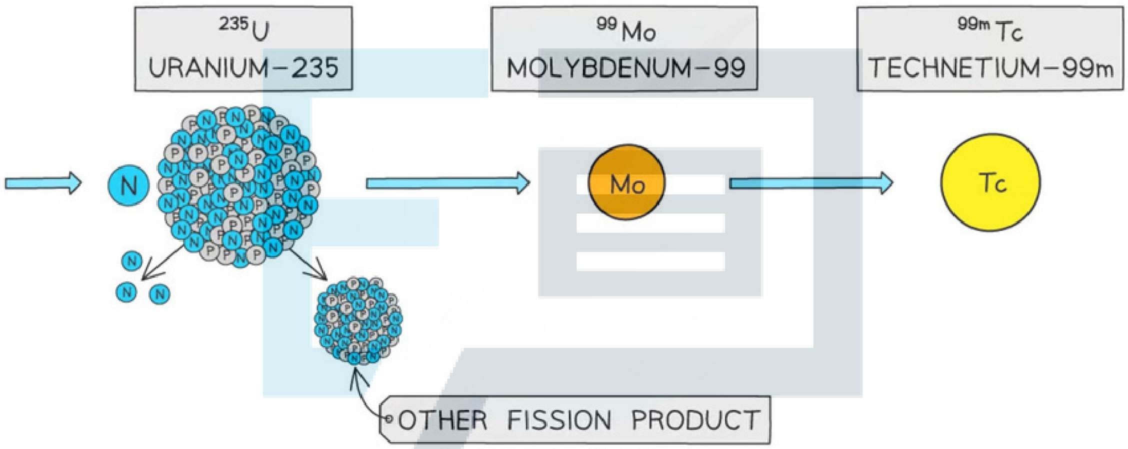
Watch out for the vertical axis for the N-Z graph. Instead of N for number of neutrons this is sometimes labelled as N for **nucleon** number (total protons and neutrons) which means the decays will be represented slightly differently.

8.3.3 Nuclear Excited States

Nuclear Excited States

- In the same way that electrons can exist in excited states, nuclei can also exist in **excited states**
- After an unstable nucleus emits an alpha particle, beta particle or undergoes electron capture, it may emit any remaining energy in the form of a **gamma photon** (γ)
    - Emission of a γ photon does not change the number of protons or neutrons in the nucleus, it only allows the nucleus to **lose energy**

- This happens when a daughter nucleus is in an excited state after a decay
- This excited state is usually very **short-lived**, and the nucleus quickly moves to its **ground state**, either directly or via one or more lower-energy excited states



- One common application of this is the use of technetium-99m as a γ source in medical diagnosis
    - The 'm' stands for **metastable** which means the nucleus exists in a particularly stable excited state

- Technetium-99m is the decay product of molybdenum-99, which can be found as a product in nuclear reactors
- The decay of molybdenum-99 is shown below:

$$^{99}_{42}Mo \rightarrow \, ^{99m}_{43}Tc + \, ^{0}_{-1}\beta + \overline{\nu_e}$$

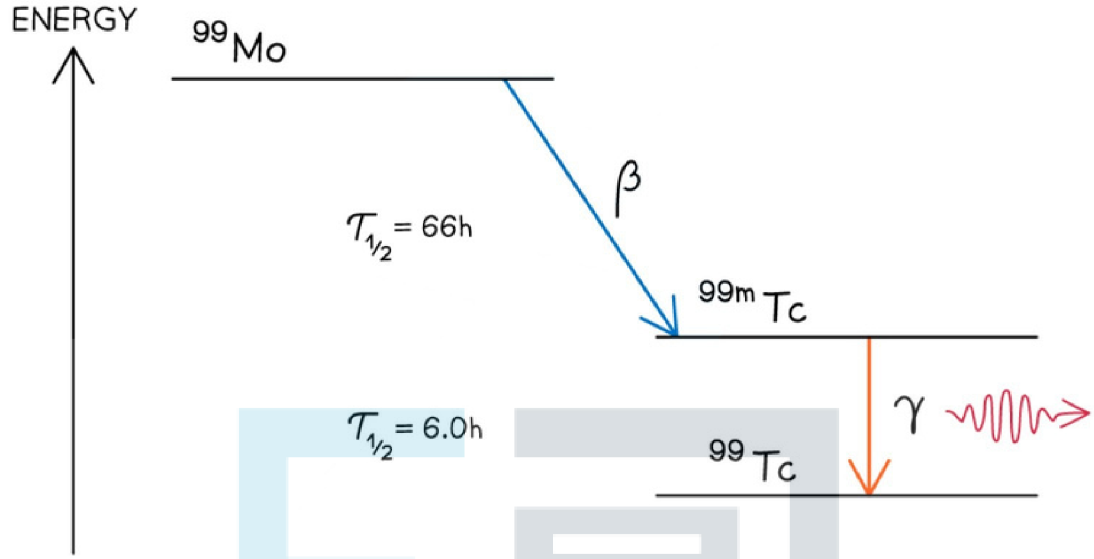
$$^{99m}_{43}Tc \rightarrow \, ^{99}_{43}Tc + \gamma$$

- The half-life of molybdenum-99 is **66 hours**
    - This is long enough for the sample to be transported to hospitals
    - Subsequently, the technetium-99m can be separated at the hospital

- Technetium-99m has a short half-life of **6 hours**
    - This is an adequate timeframe for examining a patient
    - Plus, it is short enough to minimise damage to the patient

Nuclear Energy Level Diagrams

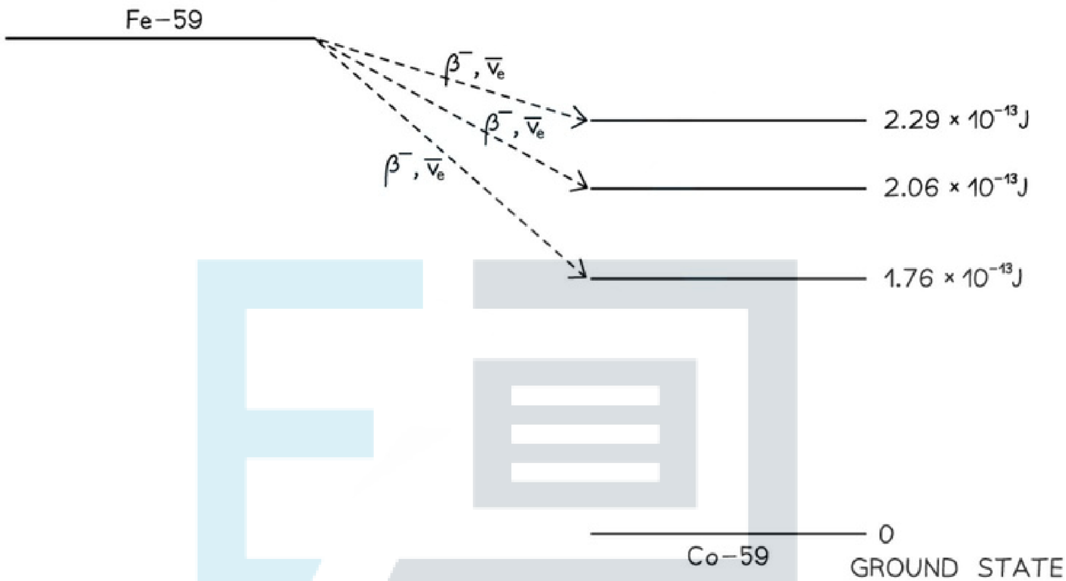
- Nuclear energy levels are similar to electron energy levels
- The nuclear energy level diagram of molybdenum-99 can be represented as follows:



- The decay mode (usually alpha or beta) is represented by a diagonal line
- The excited state, or states, are generally stacked in descending energy order to the right of the decay

Worked Example

A nucleus of iron Fe-59 decays into a stable nucleus of cobalt Co-59. It decays by β^- emission followed by the emission of γ-radiation as the Co-59 nucleus de-excites into its ground state. The total energy released when the Fe-59 nucleus decays is 2.52×10^{-13} J. The Fe-59 nucleus can decay to one of three excited states of the cobalt-59 nucleus as shown below. The energies of the excited states are shown relative to the ground state.



Following the production of excited states of Co-59, γ-radiation of discrete wavelengths is emitted.

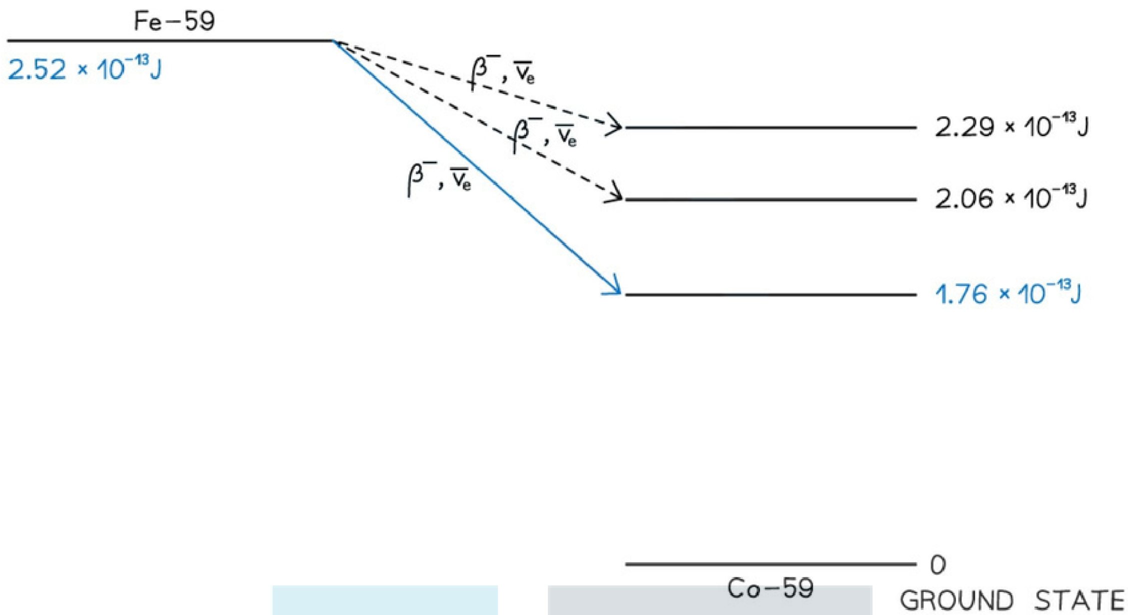
(a) Calculate the maximum possible kinetic energy of the β− particle emitted in MeV

(b) State the maximum number of discrete wavelengths that could be emitted

(c) Calculate the longest wavelength of the emitted γ-radiation

Part (a)

Step 1: Identify the beta emission with the largest energy gap



Step 2: Calculate the energy difference

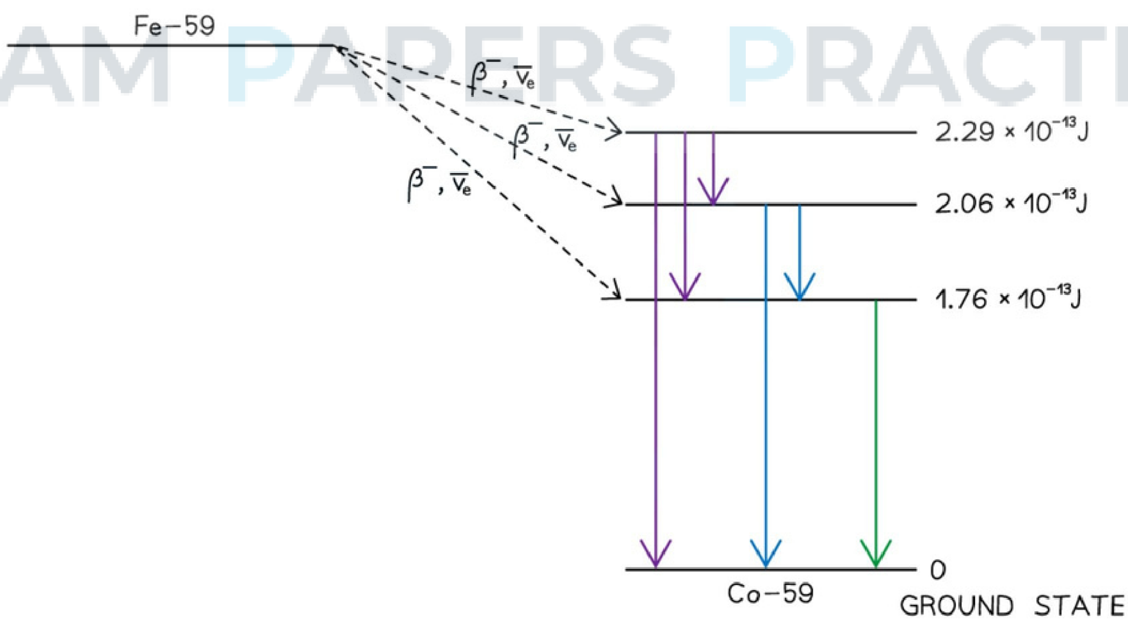
$$\Delta E = (2.52 - 1.76) \times 10^{-13} = 7.6 \times 10^{-14}\ \text{J}$$

Step 3: Convert from J to MeV

$$\Delta E = \frac{7.6 \times 10^{-14}}{1.6 \times 10^{-13}} = 0.475 = 0.48\ \text{MeV}$$

- $1\ \text{MeV} = 1.6 \times 10^{-13}$ J

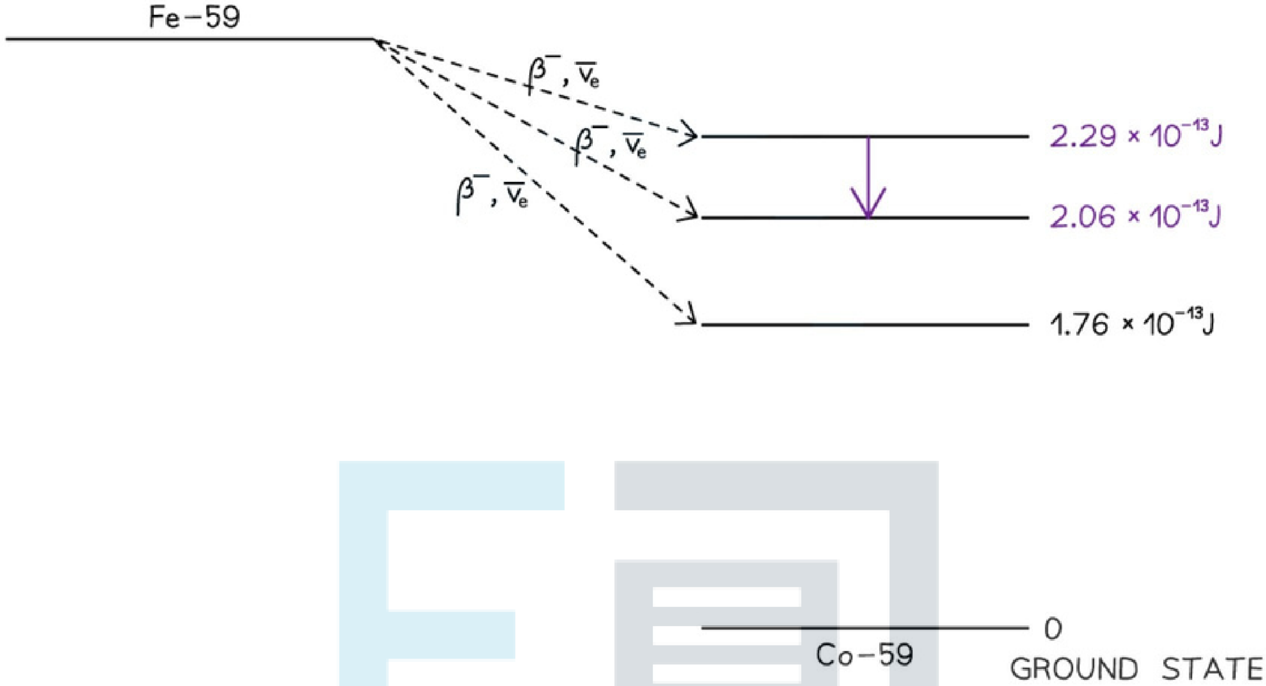
Part (b)



- There are 6 possible transitions, hence 6 discrete wavelengths could be emitted

Part (c)

Step 1: Identify the emission with the longest wavelength / smallest energy gap



- Longest wavelength = lowest frequency = smallest energy

Step 2: Calculate the energy difference

$$\Delta E = (2.29 - 2.06) \times 10^{-13} = 2.3 \times 10^{-14} \text{ J}$$

Step 3: Write down de Broglie's wavelength equation

$$E = \frac{hc}{\lambda}$$

Where:

- h = Planck's constant
- c = speed of light

Step 4: Calculate the wavelength associated with the energy change

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}) \times (3.0 \times 10^{8})}{2.3 \times 10^{-14}} = 8.6 \times 10^{-12} \text{ m}$$

8.3.4 Nuclear Radius

Estimating Nuclear Radius

Closest Approach Method

- In the Rutherford scattering experiment, alpha particles are fired at a thin gold foil
- Some of the alpha particles are found to come straight back from the gold foil
- This indicates that there is **electrostatic repulsion** between the alpha particles and the gold nucleus

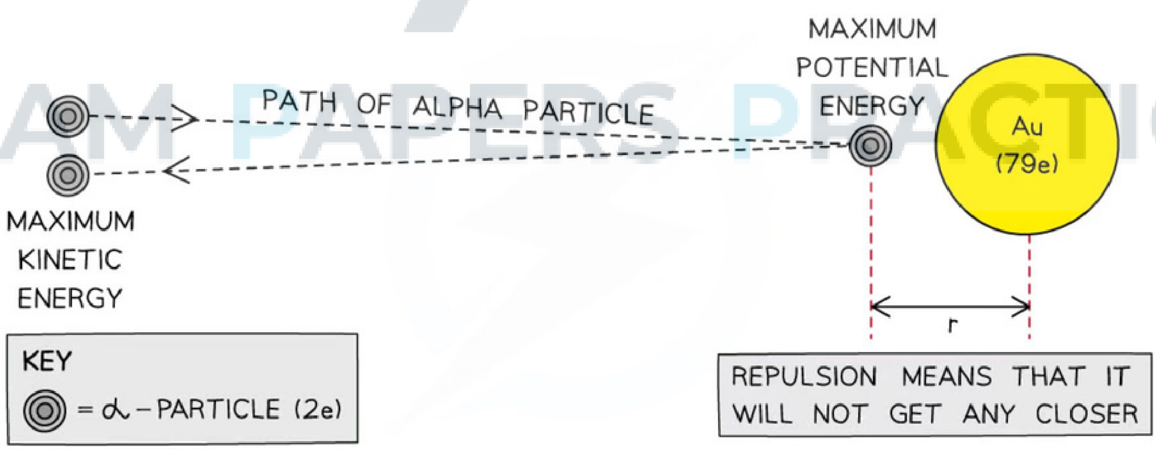
$$E_k = eV = \frac{1}{2}mv^2$$

- At the point of closest approach, *r*, the repulsive force reduces the speed of the alpha particles to zero momentarily
- At this point, the initial kinetic energy of an alpha particle, E_k, is **equal** to electric potential energy, E_p
- The radius of the closest approach can be found be equating the initial kinetic energy to the electric potential energy

$$E_p = \frac{Qq}{4\pi\varepsilon_0 r}$$

- Equating the two equations gives:

$$E_k = E_p = \frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0 r}$$



Pros & Cons of Closest Approach Method

Advantages

- Alpha scattering gives a good estimate of the upper limit for a nuclear radius
- The mathematics behind this approach are very simple
- The alpha particles are scattered only by the protons and not all the nucleons that make up the nucleus

Disadvantages

- This method does not give an accurate value for nuclear radius as it will always be an overestimate
  - This is because it measures the nearest distance the alpha particle can get to the gold nucleus, not the radius of it
- Alpha particles are hadrons, therefore, when they get close to the nucleus they are affected by the strong nuclear force and the mathematics do not account for this
- The gold nucleus will recoil as the alpha particle approaches
- Alpha particles have a finite size whereas electrons can be treated as a point mass
- It is difficult to obtain alpha particles which rebound at exactly 180°
  - In order to do this, a small collision region is required
- The alpha particles in the beam must all have the exact same initial kinetic energy
- The sample must be extremely thin to prevent multiple scattering

Electron Diffraction Method

- Electrons accelerated to close to the speed of light have wave-like properties such as the ability to diffract and have a de Broglie wavelength equal to:

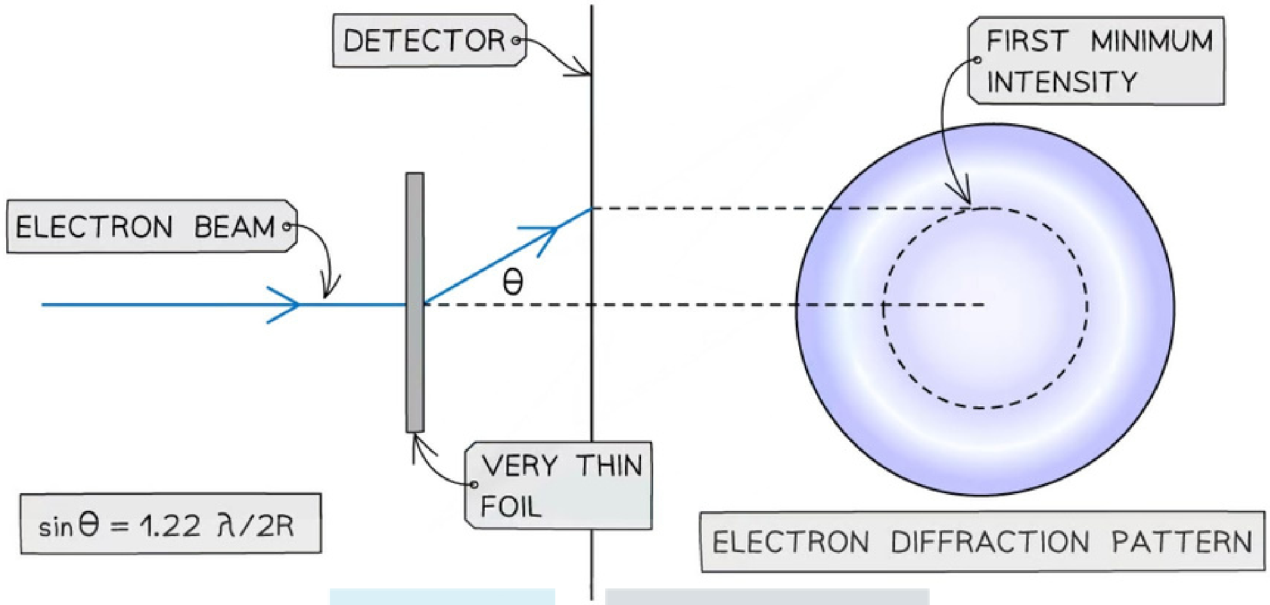
$$\lambda = \frac{h}{mv}$$

- Where:
  - h = Planck's constant
  - m = mass of an electron (kg)
  - v = speed of the electrons (m s^{-1})

- The diffraction pattern forms a central bright spot with dimmer concentric circles around it
- From this pattern, a graph of intensity against diffraction angle can be used to find the diffraction angle of the first minimum
- Using this, the size of the atomic nucleus, R, can be determined from:

$$\sin\theta = \frac{1.22\lambda}{2R}$$

- Where:
  - θ = angle of the first minimum (degrees)
  - λ = de Broglie wavelength (m)
  - R = radius of the nucleus (m)



Pros & Cons of Electron Diffraction Method

Advantages

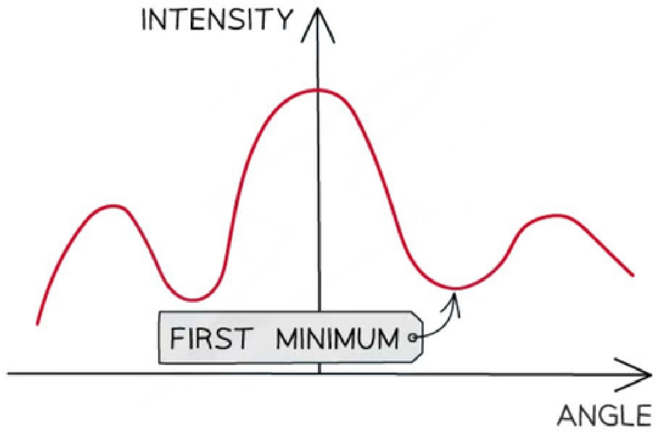
- Electron diffraction is much more accurate than the closest approach method
- This method gives a direct measurement of the radius of a nucleus
- Electrons are leptons; therefore, they will not interact with nucleons in the nucleus through the strong nuclear force as an alpha particle would

Disadvantages

- Electrons must be accelerated to very high speeds to minimise the de Broglie wavelength and increase resolution
  - This is because significant diffraction takes place when the electron wavelength is similar in size to the nuclear diameter
- Electrons can be scattered by both protons and neutrons
  - If there is an excessive amount of scattering, then the first minimum of the electron diffraction can be difficult to determine

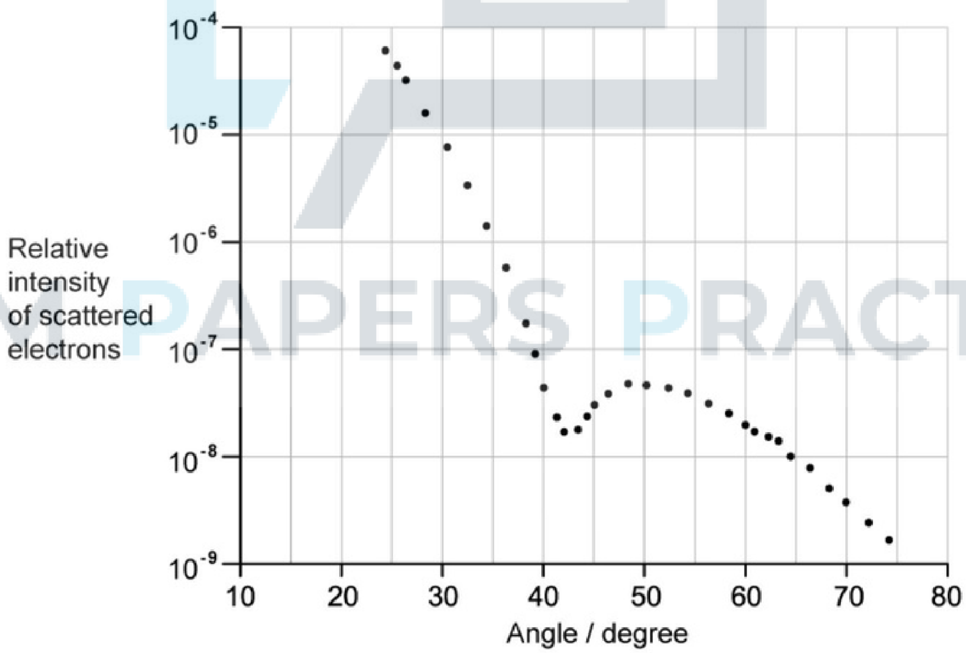
Electron Diffraction by a Nucleus

- The graph of intensity against angle obtained through electron diffraction is as follows:



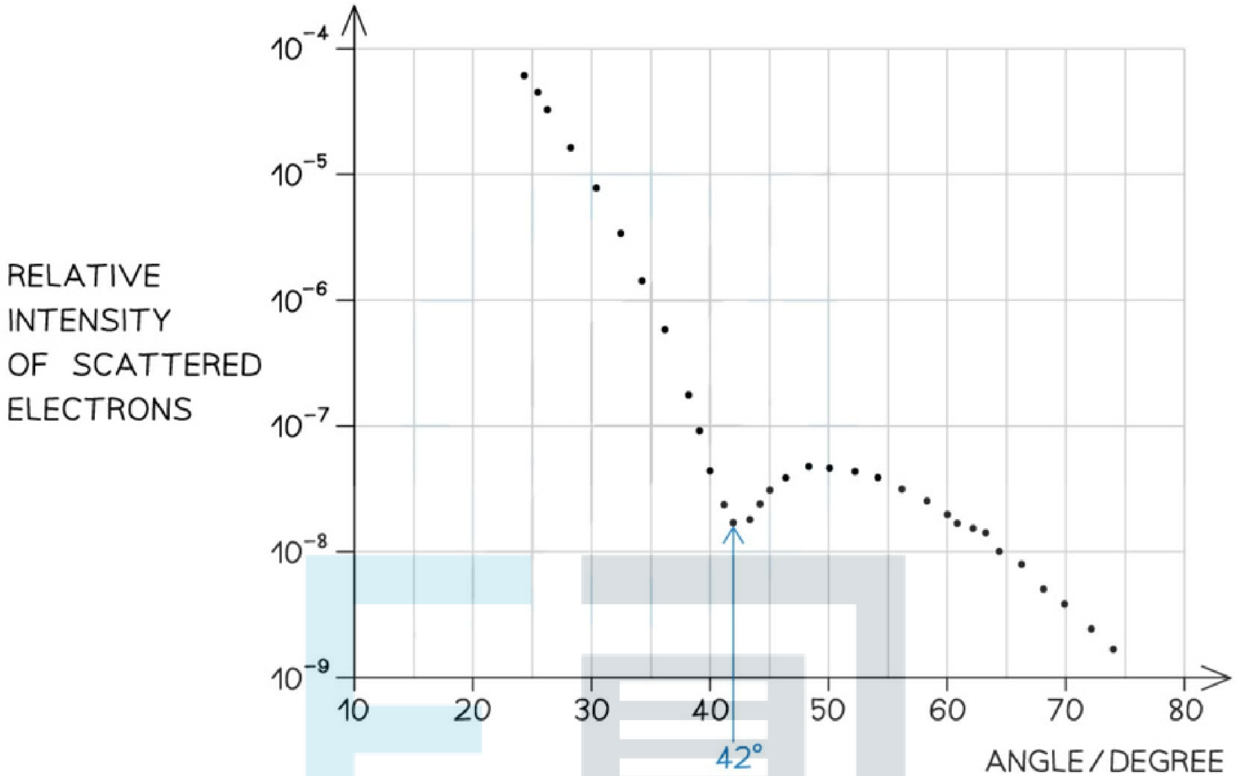
Worked Example

The graph shows how the relative intensity of the scattered electrons varies with angle due to diffraction by the oxygen-16 nuclei. The angle is measured from the original direction of the beam.



The de Broglie wavelength λ of each electron in the beam is 3.35×10^{-15} m. Calculate the radius of an oxygen-16 nucleus using information from the graph.

Step 1: Identify the first minimum from the graph



- Angle of first minimum, $\theta = 42°$

Step 2: Write out the equation relating the angle, wavelength, and nuclear radius

$$\sin\theta = \frac{0.61\lambda}{R}$$

Step 3: Calculate the nuclear radius, *R*

$$R = \frac{0.61\lambda}{\sin\theta} = \frac{0.61 \times \left(3.35 \times 10^{-15}\right)}{\sin 42°} = 3.1 \times 10^{-15} \text{ m}$$

8.3.5 Closest Approach Estimate

Closest Approach Estimate

- The Coulomb equation can be used to give an estimate for the radius of a nucleus
- When the alpha particle reaches its closest approach (to the nucleus), all of its kinetic energy has been converted to electric potential energy
- Initially, the alpha particles have kinetic energy equal to:

$$E_k = eV = \frac{1}{2}mv^2$$

- The electric potential energy between the two charges is equal to:

$$E_p = \frac{Qq}{4\pi\varepsilon_0 r}$$

- At this point, the kinetic energy E_k lost by the α particle approaching the nucleus is **equal** to the potential energy gain E_p, so:

$$E_k = E_p = \frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0 r}$$

- Where:
  - m = mass of an α particle (kg)
  - v = initial speed of the α particles (m s^{-1})
  - q = charge of an α particle (C)
  - Q = charge of the nucleus being investigated (C)
  - r = the radius of closest approach (m)
  - ε_0 = permittivity of free space

Worked Example

The first artificially produced isotope, phosphorus-30 ($_{15}$P) was formed by bombarding an aluminium-27 isotope ($_{13}$Al) with an α particle.

For the reaction to take place, the α particle must come within a distance, r, from the centre of the aluminium nucleus.

Calculate the distance, r, if the nuclear reaction occurs when the α particle is accelerated to a speed of at least 2.55×10^7 m s^{-1}.

Step 1: List the known quantities

- Mass of an α particle, $m = 4u = 4 \times (1.66 \times 10^{-27})$ kg
- Speed of the α particle, $v = 2.55 \times 10^7$ m s^{-1}
- Charge of an α particle, $q = 2e = 2 \times (1.6 \times 10^{-19})$ C
- Charge of an aluminium nucleus, $Q = 13e = 13 \times (1.6 \times 10^{-19})$ C
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

Step 2: Write down the equations for kinetic energy and electric potential energy

$$\frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0 r}$$

Step 3: Rearrange for distance, *r*

$$r = \frac{2Qq}{4\pi\varepsilon_0 mv^2}$$

Step 4: Calculate the distance, *r*

$$r = \frac{2 \times 13 \times (1.6 \times 10^{-19}) \times 2 \times (1.6 \times 10^{-19})}{4\pi \times (8.85 \times 10^{-12}) \times 4 \times (1.66 \times 10^{-27}) \times (2.55 \times 10^{7})^2}$$

$$r = 2.77 \times 10^{-15}\ \text{m}$$

Exam Tip

Make sure you're comfortable with the calculations involved with the alpha particle closest approach method, as this is a common exam question. You will be expected to remember that the charge of an α is the charge of 2 protons (2 × the charge of an electron)

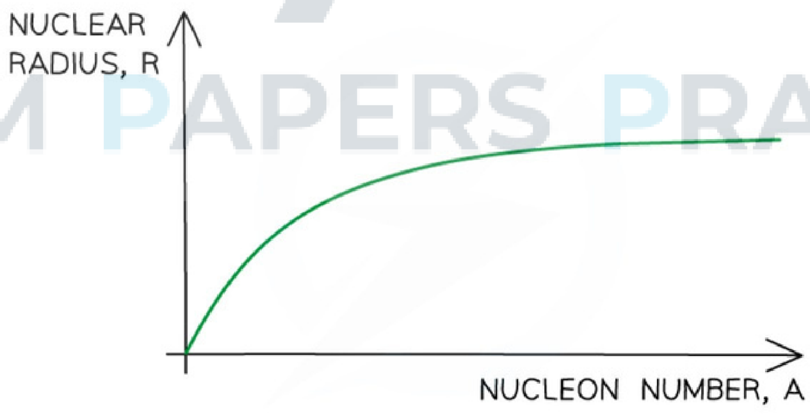
8.3.6 Nuclear Radius Equation

Nuclear Radius Values

- The radius of some nuclei are shown in the table below:

Element	Nuclear radius R / 10^{-15} m	Mass number A
Carbon	2.66	12
Silicon	3.43	28
Iron	4.35	56
Tin	5.49	120
Lead	6.66	208

- In general, nuclear radii are of the order 10^{-15} m or 1 fm
- The nuclear radius, *R*, varies with nucleon number, *A* as follows:



- **The key features of this graph are:**
  - The graph starts with a steep gradient at the origin
  - Then the gradient gradually decreases to almost horizontal
- This means that
  - As more nucleons are added to a nucleus, the nucleus gets bigger
  - However, the number of nucleons *A* is **not** proportional to its size *r*

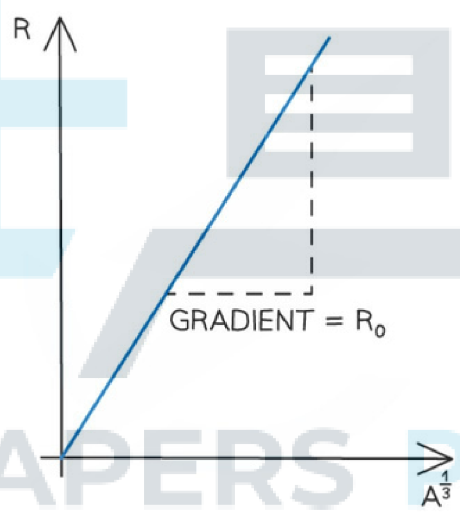
Radius v Nucleon Number

- The radius of nuclei depends on the nucleon number, *A* of the atom
- This makes sense because as more nucleons are added to a nucleus, more space is occupied by the nucleus, hence giving it a larger radius
- The exact relationship between the radius and nucleon number can be determined from experimental data
- By doing this, physicists were able to deduce the following relationship:

$$R = R_0 A^{\frac{1}{3}}$$

- Where:
  - R = nuclear radius (m)
  - A = nucleon / mass number
  - R_0 = constant of proportionality = 1.05 fm

- Plotting a graph of R against $A^{1/3}$ gives a straight line through the origin with the gradient equal to R_0



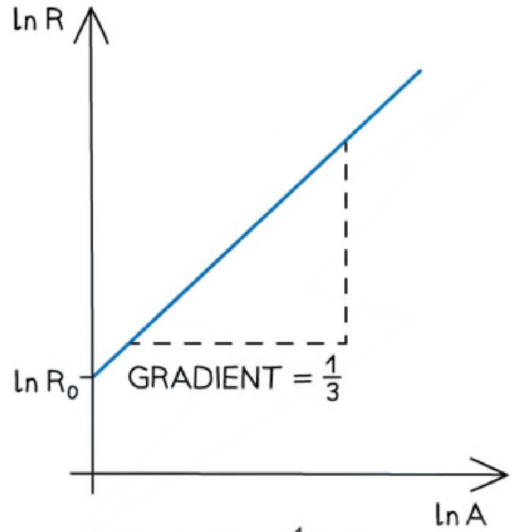
- It is also possible to plot a logarithmic graph of the relationship which can be derived as follows:

$$\ln R = \ln (R_0 A^{1/3})$$

$$\ln R = \ln R_0 + \ln (A^{1/3})$$

$$\ln R = \ln R_0 + 1/3 \ln A$$

- Therefore, a graph of ln R against ln A yields a straight line
- Comparing this to the straight-line equation: y = mx + c
  - y = lnR
  - x = lnA
  - m (the gradient) = 1/3
  - c (y-intercept) = ln R_0



$$\ln R = \ln R_0 + \frac{1}{3}\ln A$$
$$y = c + mx$$

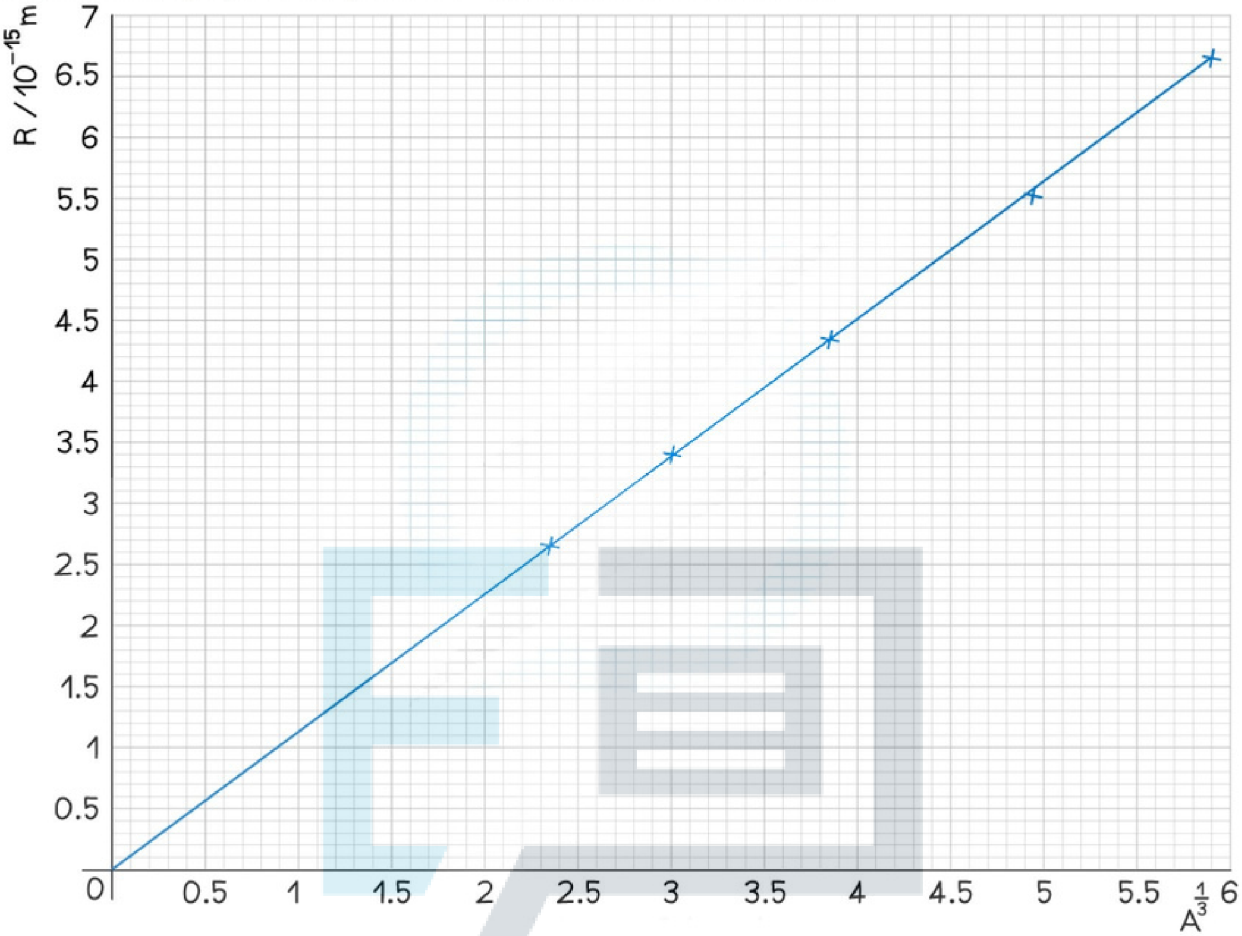
Worked Example

Verify the experimental relationship between R and A using the data from the table above and estimate a value of R_0.

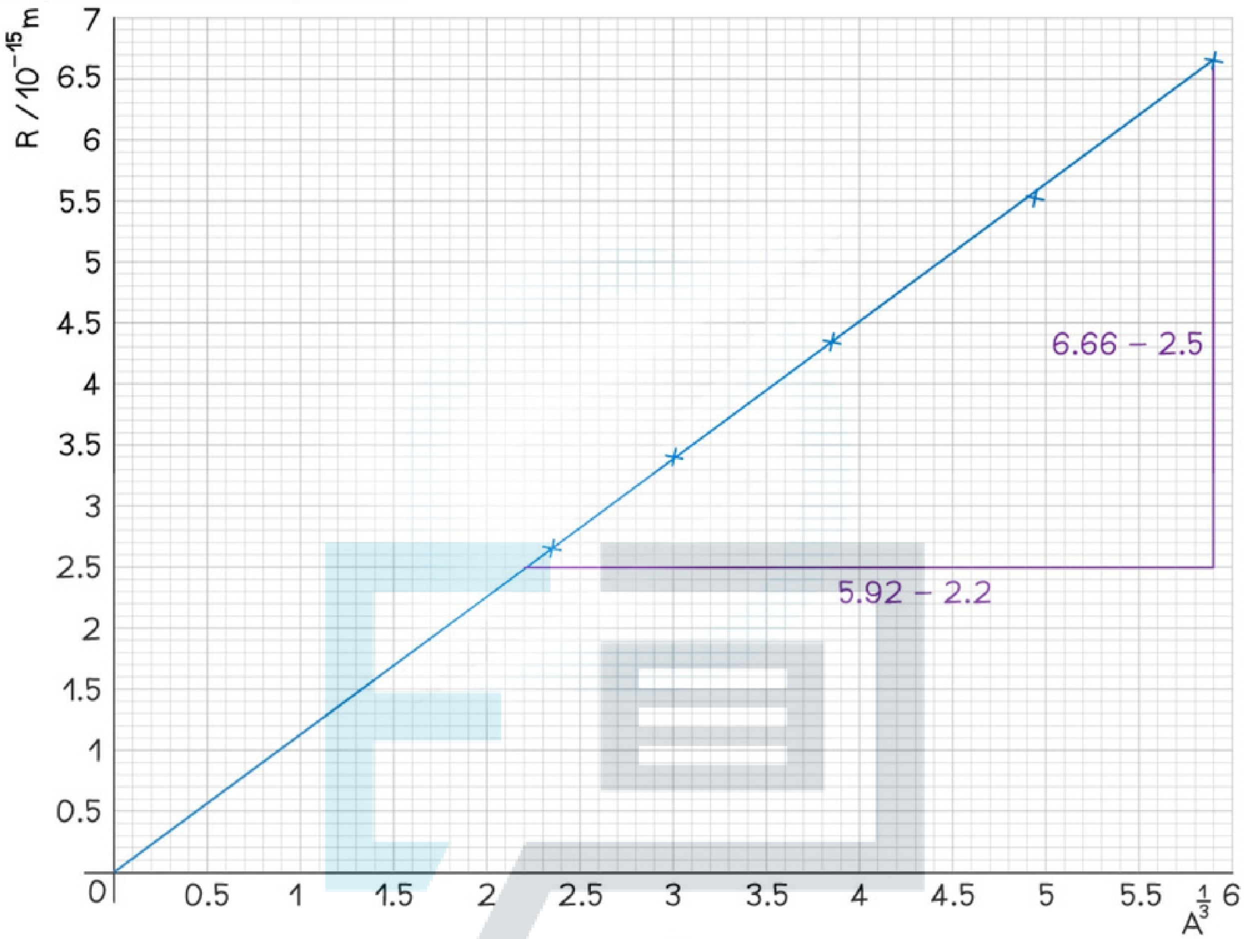
Step 1: Add a column to the table to determine the values for $A^{1/3}$

Element	Nuclear radius R / 10^{-15} m	Mass number A	Mass number $A^{1/3}$
Carbon	2.66	12	2.29
Silicon	3.43	28	3.04
Iron	4.35	56	3.83
Tin	5.49	120	4.93
Lead	6.66	208	5.92

Step 2: Plot a graph of R against $A^{1/3}$ and draw a line of best fit



Step 3: Calculate the gradient



$$\frac{\Delta y}{\Delta x} = \frac{(6.66 - 2.5) \times 10^{-15}}{5.92 - 2.2} = 1.118 \times 10^{-15} \text{ m}$$

$R_0 = 1.12$ fm

8.3.7 Nuclear Density

Constant Density of Nuclear Material

- Assuming that the nucleus is spherical, its volume is equal to:

$$V = \frac{4}{3}\pi R^3$$

- Where *R* is the nuclear radius, which is related to mass number, *A*, by the equation:

$$R = R_0 A^{\frac{1}{3}}$$

- Where R_0 is a constant of proportionality
- Combining these equations gives:

$$V = \frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi R_0^3 A$$

- Therefore, the nuclear volume, *V*, is proportional to the mass of the nucleus, *A*
- Mass (m), volume (V), and density (ρ) are related by the equation:

$$\rho = \frac{m}{V}$$

- The mass, *m*, of a nucleus is equal to:

$$m = Au$$

- Where:
  - *A* = the mass number
  - *u* = atomic mass unit
- Using the equations for mass and volume, nuclear density is equal to:

$$\rho = \frac{Au}{\frac{4}{3}\pi R_0^3 A} = \frac{3u}{4\pi R_0^3}$$

- Since the mass number *A* cancels out, the remaining quantities in the equation are all constant
- Therefore, this shows the density of the nucleus is:
  - Constant
  - Independent of the radius
- The fact that nuclear density is constant shows that nucleons are evenly separated throughout the nucleus regardless of their size

Nuclear Density

- Using the equation derived above, the density of the nucleus can be calculated:

$$\rho = \frac{3u}{4\pi R_0^3}$$

- Where:
  - Atomic mass unit, $u = 1.661 \times 10^{-27}$ kg
  - Constant of proportionality, $R_0 = 1.05 \times 10^{-15}$ m
- Substituting the values gives a density of:

$$\rho = \frac{3 \times \left(1.661 \times 10^{-27}\right)}{4\pi\left(1.05 \times 10^{-15}\right)^3} = 3.4 \times 10^{17} \text{ kg m}^{-3}$$

- The accuracy of nuclear density depends on the accuracy of the constant R_0, as a guide nuclear density should always be of the order 10^{17} kg m^{-3}
- Nuclear density is significantly larger than atomic density, this suggests:
  - The majority of the atom's mass is contained in the nucleus
  - The nucleus is very small compared to the atom
  - Atoms must be predominantly empty space