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8.2 Radioactive Decay

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

8.2 Radioactive Decay

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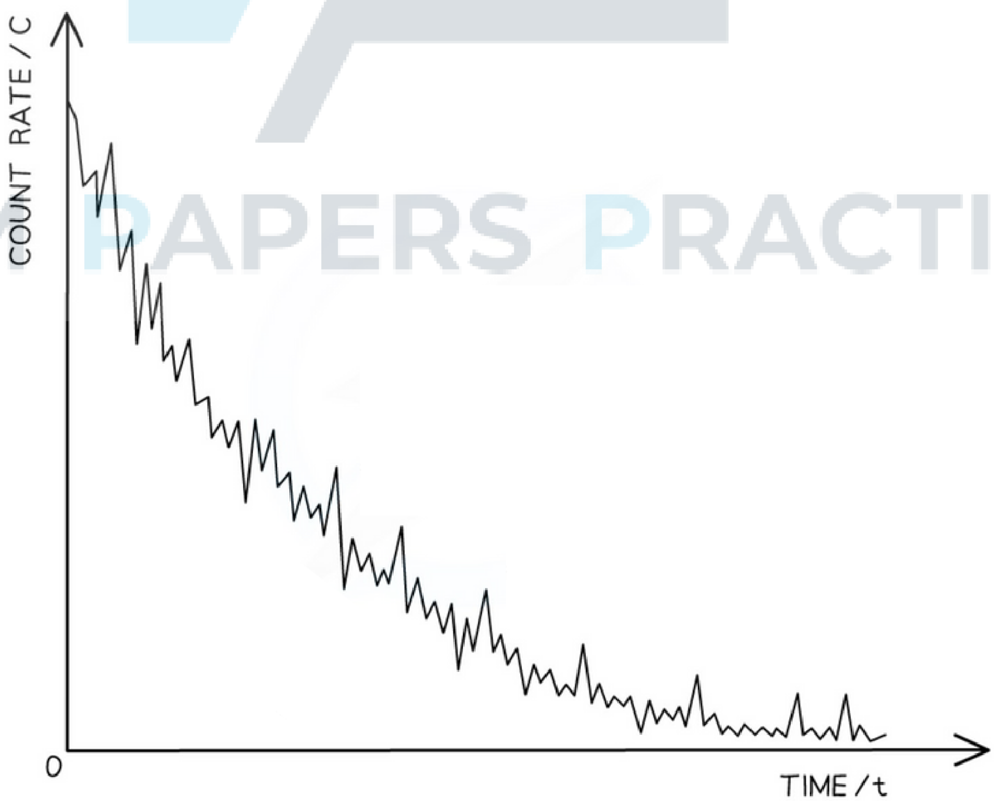
Radioactive Decay

- Radioactive decay is defined as:

The spontaneous disintegration of a nucleus to form a more stable nucleus, resulting in the emission of an alpha, beta or gamma particle

- Radioactive decay is a **random** process, this means that:
  - There is an **equal probability** of any nucleus decaying
  - It cannot be known **which particular nucleus will decay next**
  - It cannot be known **at what time a particular nucleus will decay**
  - The rate of decay is **unaffected** by the surrounding conditions
  - It is only possible to estimate the **proportion** of nuclei decaying in a given time period

- The random nature of radioactive decay can be demonstrated by observing the count rate of a Geiger-Muller (GM) tube
  - When a GM tube is placed near a radioactive source, the counts are found to be irregular and cannot be predicted
  - Each count represents a decay of an unstable nucleus
  - These fluctuations in count rate on the GM tube **provide evidence for the randomness of radioactive decay**



The variation of count rate over time of a sample radioactive gas. The fluctuations show the randomness of radioactive decay

Activity & The Decay Constant

- Since radioactive decay is spontaneous and random, it is useful to consider the average number of nuclei that are expected to decay per unit time
  - This is known as the **average decay rate**
- As a result, each radioactive element can be assigned a **decay constant**
- The decay constant λ is defined as:

The probability that an individual nucleus will decay per unit of time

- When a sample is highly radioactive, this means the number of decays per unit time is very high
  - This suggests it has a high level of **activity**
- Activity, or the number of decays per unit time can be calculated using:

$$A = -\frac{\Delta N}{\Delta t} = \lambda N$$

- Where:
  - A = activity of the sample (Bq)
  - ΔN = number of decayed nuclei
  - Δt = time interval (s)
  - λ = decay constant (s^{-1})
  - N = number of nuclei remaining in a sample
- The activity of a sample is measured in **Becquerels** (Bq)
  - An activity of 1 Bq is equal to one decay per second, or 1 s^{-1}
- This equation shows:
  - The greater the decay constant, the **greater the activity** of the sample
  - The activity depends on the number of **undecayed nuclei remaining** in the sample
  - The minus sign indicates that the number of nuclei remaining **decreases** with time

Worked Example

Radium is a radioactive element first discovered by Marie and Pierre Curie. They used the radiation emitted from radium-226 to define a unit called the Curie (Ci) which they defined as the activity of 1 gram of radium.It was found that in a 1 g sample of radium, 2.22×10^{12} atoms decayed in 1 minute.Another sample containing 3.2×10^{22} radium-226 atoms had an activity of 12 Ci.

a) Determine the value of 1 Curie

b) Determine the decay constant for radium-226

Part a)

Step 1: Write down the known quantities

- Number of atoms decayed, $\Delta N = 2.22 \times 10^{12}$
- Time, $\Delta t = 1$ minutes = 60 s

Step 2: Write down the activity equation

$$A = \frac{\Delta N}{\Delta t}$$

Step 3: Calculate the value of 1 Ci

$$A = \frac{2.22 \times 10^{12}}{60} = 3.7 \times 10^{10} \text{ decays s}^{-1}$$

Part b)

Step 1: Write down the known quantities

- Number of atoms, $N = 3.2 \times 10^{22}$
- Activity, $A = 12 \text{ Ci} = 12 \times (3.7 \times 10^{10}) = 4.44 \times 10^{11}$ Bq

Step 2: Write down the activity equation

$$A = \lambda N$$

Step 3: Calculate the decay constant of radium

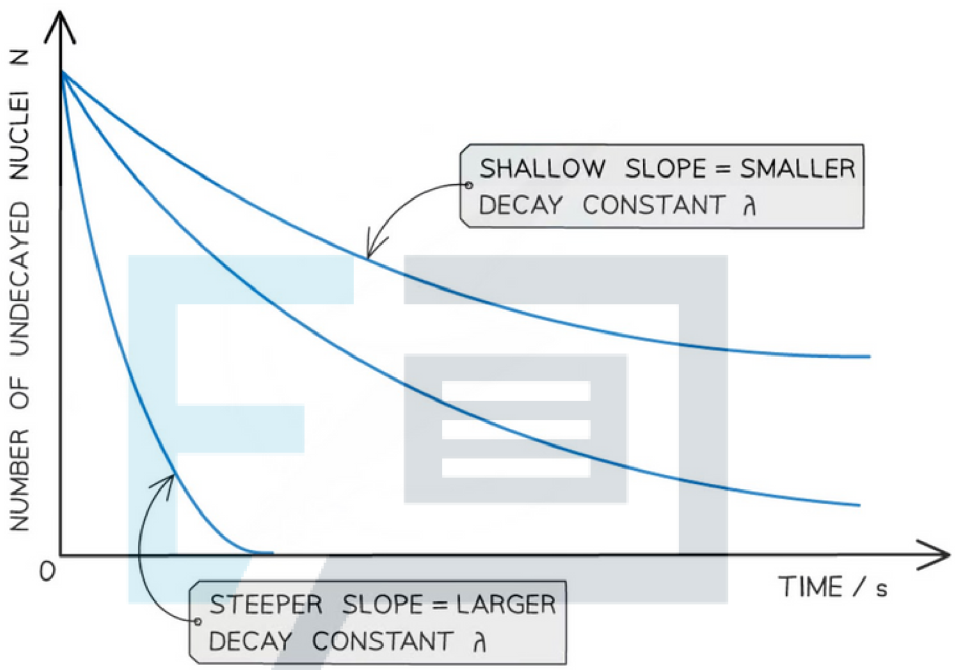
$$\lambda = \frac{A}{N} = \frac{4.44 \times 10^{11}}{3.2 \times 10^{22}} = 1.388 \times 10^{-11} \text{ s}^{-1}$$

- Therefore, the decay constant of radium-226 is 1.4×10^{-11} s^{-1}

8.2.2 Exponential Decay

Exponential Decay

- In radioactive decay, the number of undecayed nuclei falls very rapidly, without ever reaching zero
  - Such a model is known as **exponential decay**
- The graph of number of undecayed nuclei against time has a very distinctive shape:



Radioactive decay follows an exponential pattern. The graph shows three different isotopes each with a different rate of decay

- **The key features of this graph are:**
  - The steeper the slope, the larger the decay constant λ (and vice versa)
  - The decay curves always start on the y-axis at the initial number of undecayed nuclei (N_0)

Equations for Radioactive Decay

- The number of undecayed nuclei *N* can be represented in exponential form by the equation:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N_0 = the initial number of undecayed nuclei (when $t = 0$)
  - N = number of undecayed nuclei at a certain time t
  - λ = decay constant (s^{-1})
  - t = time interval (s)
- The number of nuclei can be substituted for other quantities.

- For example, the activity A is directly proportional to N, so it can also be represented in exponential form by the equation:

$$A = A_0 e^{-\lambda t}$$

- Where:
  - A = activity at a certain time t (Bq)
  - A_0 = initial activity (Bq)
- The received count rate C is related to the activity of the sample, hence it can also be represented in exponential form by the equation:

$$C = C_0 e^{-\lambda t}$$

- Where:
  - C = count rate at a certain time t (counts per minute or cpm)
  - C_0 = initial count rate (counts per minute or cpm)

The exponential function e

- The symbol e represents the exponential constant
  - It is approximately equal to e = 2.718
- On a calculator it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if $e^x = y$, then $x = \ln(y)$

Worked Example

Strontium-90 decays with the emission of a β-particle to form Yttrium-90. The decay constant of Strontium-90 is 0.025 year^{-1}. Determine the activity A of the sample after 5.0 years, expressing the answer as a fraction of the initial activity A_0.

Step 1: Write out the known quantities

- Decay constant, λ = 0.025 year^{-1}
- Time interval, t = 5.0 years
- Both quantities have the same unit, so there is no need for conversion

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange the equation for the ratio between A and A_0

$$\frac{A}{A_0} = e^{-\lambda t}$$

Step 4: Calculate the ratio A/A_0

$$\frac{A}{A_0} = e^{-(0.025 \times 5)} = 0.88$$

Therefore, the activity of Strontium-90 decreases by a factor of 0.88, or 12%, after 5 years

Using Molar Mass & The Avogadro Constant

Molar Mass

- The molar mass, or molecular mass, of a substance is the mass of a substance, in grams, in one mole
  - Its unit is **g mol^{-1}**
- The number of moles from this can be calculated using the equation:

$$\text{Number of moles} = \frac{mass\ (g)}{molar\ mass\ (g\ mol^{-1})}$$

Avogadro's Constant

- Avogadro's constant (N_A) is defined as:

The number of atoms in one mole of a substance; equal to 6.02 × 10^{23} mol^{-1}

- For example, 1 mole of sodium (Na) contains 6.02 × 10^{23} **atoms** of sodium
- The number of atoms, N, can be determined using the equation:

$$\text{Number of nuclei} = \frac{mass \times N_A}{molecular\ mass}$$

Worked Example

Americium-241 is an artificially produced radioactive element that emits α-particles.In a smoke detector, a sample of americium-241 of mass 5.1 μg is found to have an activity of 5.9 × 10^5 Bq. The supplier's website says the americium-241 in their smoke detectors initially has an activity level of 6.1 × 10^5 Bq.

a) Determine the number of nuclei in the sample of americium-241

b) Determine the decay constant of americium-241

c) Determine the age of the smoke detector in years

Part (a)

Step 1: Write down the known quantities

- Mass = 5.1 μg = 5.1 × 10^{-6} g
- Molecular mass of americium = 241

Step 2: Write down the equation relating number of nuclei, mass and molecular mass

$$number\ of\ nuclei,\ N = \frac{mass \times N_A}{molar\ mass}$$

- where N_A is the Avogadro constant

Step 3: Calculate the number of nuclei

$$\text{Number of nuclei} = \frac{(5.1 \times 10^{-6}) \times (6.02 \times 10^{23})}{241} = 1.27 \times 10^{16}$$

Part (b)

Step 1: Write down the known quantities

- Activity, $A = 5.9 \times 10^5$ Bq
- Number of nuclei, $N = 1.27 \times 10^{16}$

Step 2: Write the equation for activity

$$\text{Activity, } A = \lambda N$$

Step 3: Rearrange for decay constant λ and calculate the answer

$$\lambda = \frac{A}{N} = \frac{5.9 \times 10^5}{1.27 \times 10^{16}} = 4.65 \times 10^{-11}\ \text{s}^{-1}$$

Part (c)

Step 1: Write down the known quantities

- Activity, $A = 5.9 \times 10^5$ Bq
- Initial activity, $A_0 = 6.1 \times 10^5$ Bq
- Decay constant, $\lambda = 4.65 \times 10^{-11}\ \text{s}^{-1}$

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange for time t

$$\frac{A}{A_0} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_0}\right) = -\lambda t$$

$$t = -\frac{1}{\lambda} \ln\left(\frac{A}{A_0}\right)$$

Step 4: Calculate the age of the smoke detector and convert to years

$$t = -\frac{1}{4.65 \times 10^{-11}} \ln\left(\frac{5.9 \times 10^5}{6.1 \times 10^5}\right) = 7.169 \times 10^8\ \text{s}$$

$$t = \frac{7.169 \times 10^8}{24 \times 60 \times 60 \times 365} = 22.7\ \text{years}$$

- Therefore, the smoke detector is 22.7 years old

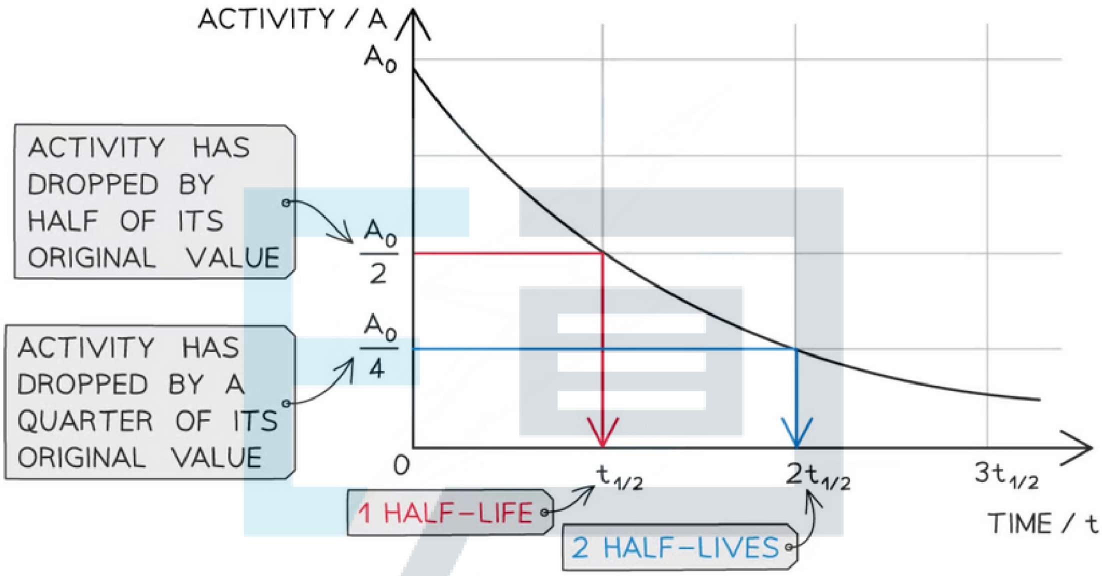
8.2.3 Half-Life

Half-Life

- Half-life is defined as:

The time taken for the initial number of nuclei to halve for a particular isotope

- This means when a time equal to the half-life has passed, the **activity** of the sample will also half
- This is because the activity is proportional to the number of undecayed nuclei, $A \propto N$



When a time equal to the half-life passes, the activity falls by half, when two half-lives pass, the activity falls by another half (which is a quarter of the initial value)

- To find an expression for half-life, start with the equation for exponential decay:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N = number of nuclei remaining in a sample
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

- When time t is equal to the half-life $t_{½}$, the activity N of the sample will be half of its original value, so $N = ½ N_0$

$$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{½}}$$

- The formula can then be derived as follows:

Divide both sides by N_0: $\frac{1}{2} = e^{-\lambda t_{½}}$

Take the natural log of both sides: $\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$

Apply properties of logarithms: $\lambda t_{1/2} = \ln(2)$

- Therefore, half-life $t_{1/2}$ can be calculated using the equation:

$$t_{1/2} = \frac{\ln 2}{\lambda} \simeq \frac{0.693}{\lambda}$$

- This equation shows that half-life $t_{1/2}$ and the radioactive decay rate constant λ are inversely proportional
  - Therefore, the **shorter** the half-life, the **larger** the decay constant and the **faster** the decay

Worked Example

Strontium-90 is a radioactive isotope with a half-life of 28.0 years. A sample of Strontium-90 has an activity of 6.4×10^9 Bq. Calculate the decay constant λ, in s^{-1}, of Strontium-90.

Step 1: Convert the half-life into seconds

- $t_{1/2} = 28 \text{ years} = 28 \times 365 \times 24 \times 60 \times 60 = 8.83 \times 10^8$ s

Step 2: Write the equation for half-life

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Step 3: Rearrange for λ and calculate

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{8.83 \times 10^8} = \mathbf{7.85 \times 10^{-10}\ s^{-1}}$$

Exam Tip

Although you may not be expected to derive the half-life equation, make sure you're comfortable with how to use it in calculations such as that in the worked example.

Half-Life from Decay Curves

- The half-life of a radioactive substance can be determined from decay curves and log graphs
- Since half-life is the **time taken for the initial number of nuclei, or activity, to reduce by half**, it can be found by
  - Drawing a line to the curve at the point where the activity has dropped to half of its original value
  - Drawing a line from the curve to the time axis, this is the half-life

Log Graphs

- Straight-line graphs tend to be more useful than curves for interpreting data
  - Nuclei decay exponentially, therefore, to achieve a straight line plot, logarithms can be used

- Take the exponential decay equation for the number of nuclei

$$N = N_0 e^{-\lambda t}$$

- Taking the natural logs of both sides

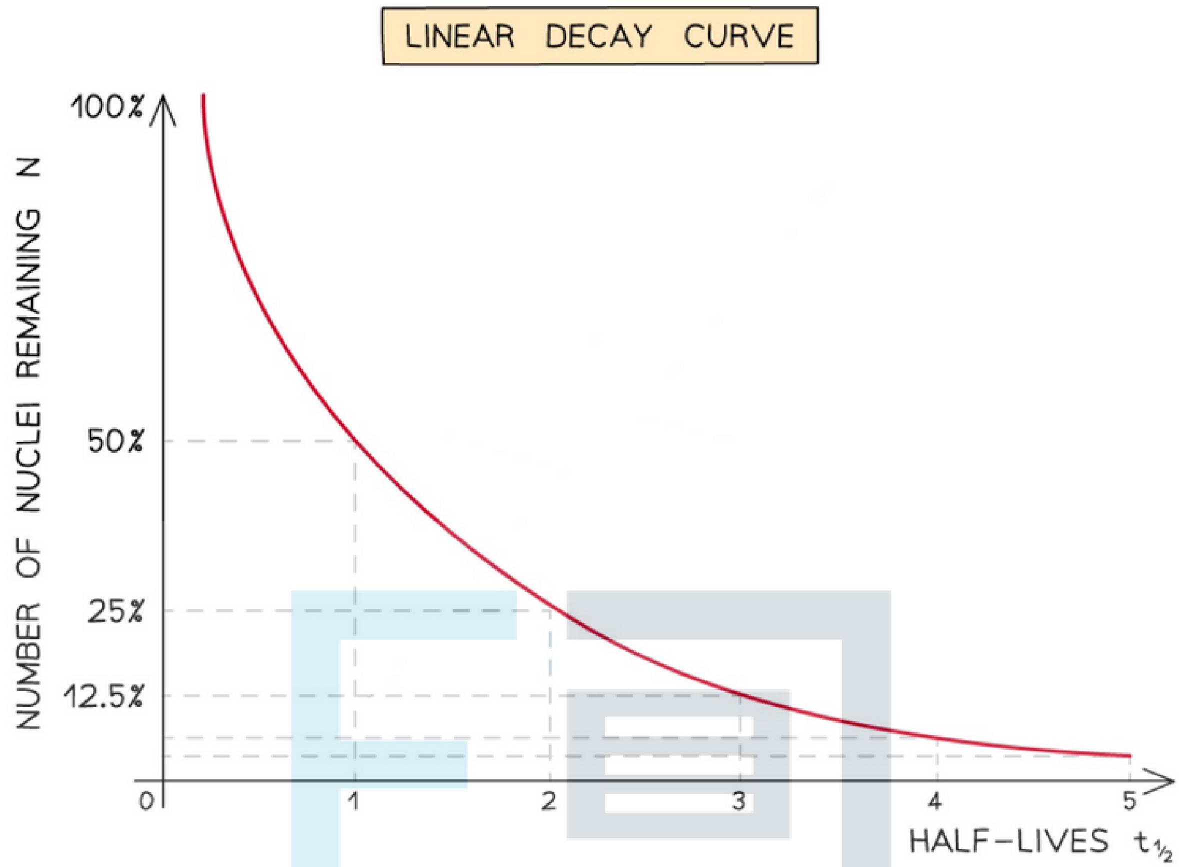
$$\ln N = \ln (N_0) - \lambda t$$

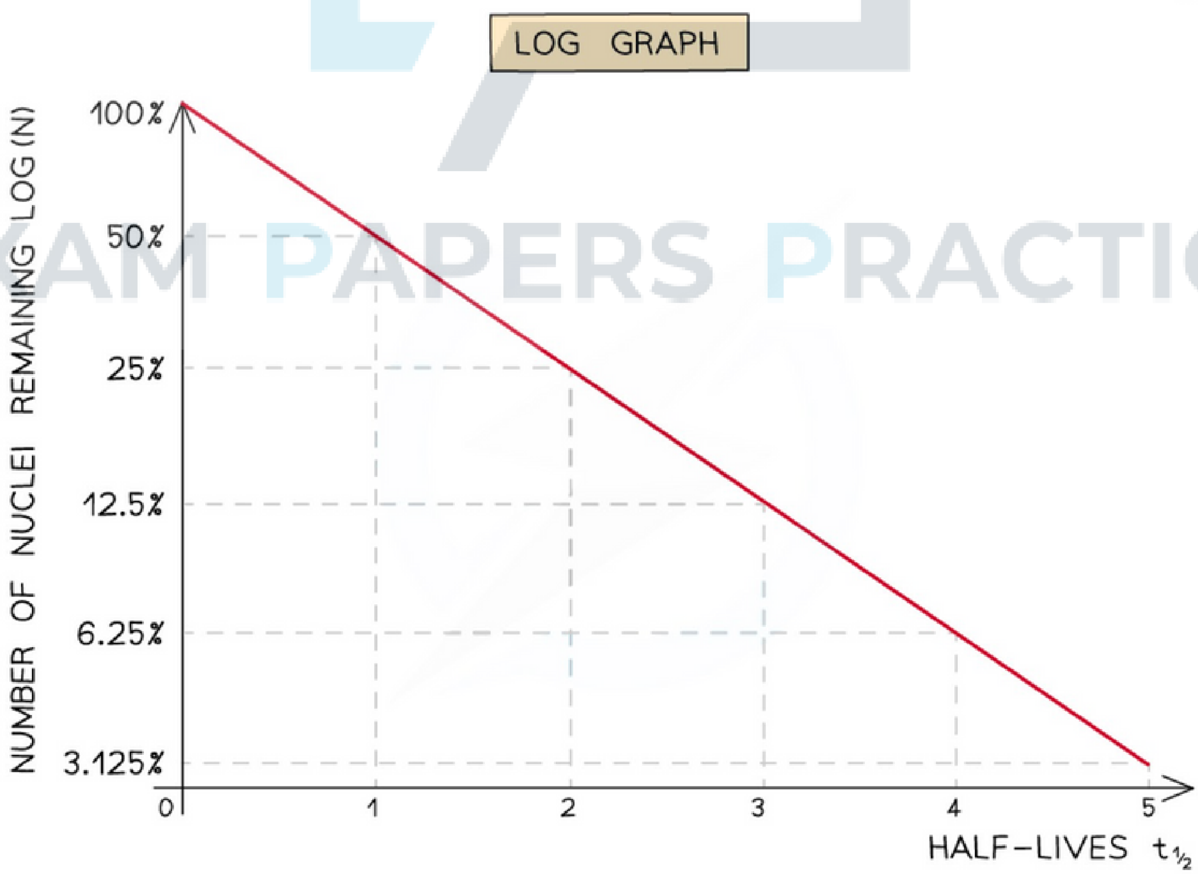
- In this form, this equation can be compared to the equation of a straight line

$$y = mx + c$$

- Where:
  - $\ln (N)$ is plotted on the y-axis
  - t is plotted on the x-axis
  - gradient = $-\lambda$
  - y-intercept = $\ln (N_0)$

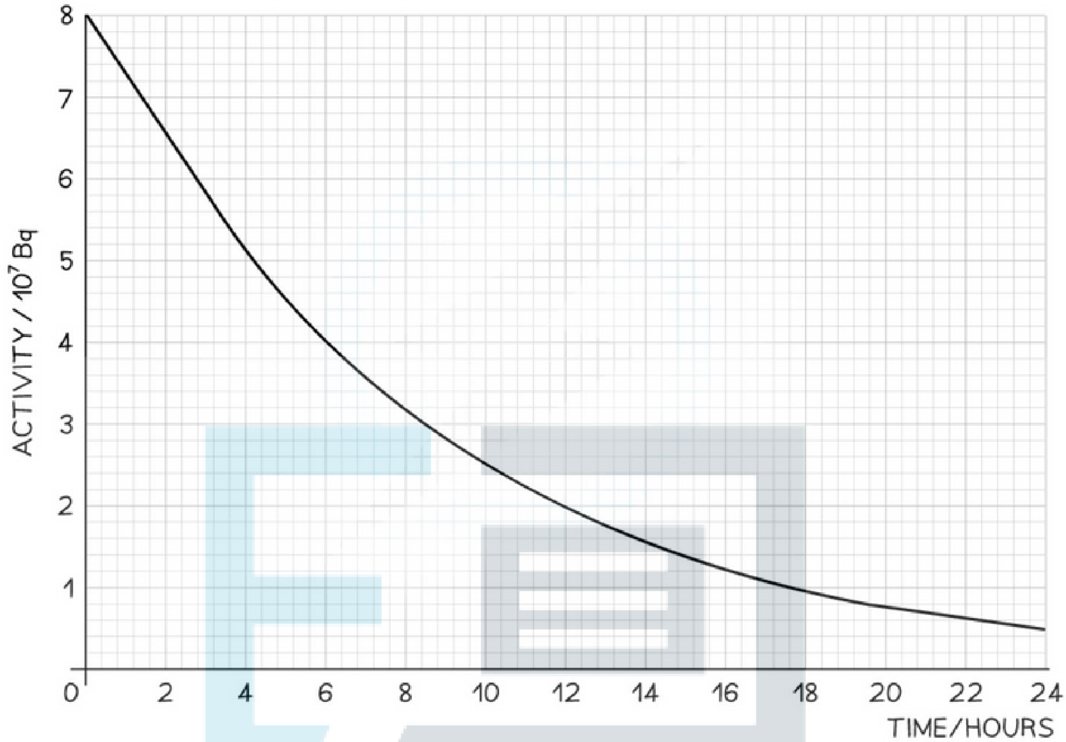
- Half-lives can be found in a similar way to the decay curve but the intervals will be regular as shown below:


LINEAR DECAY CURVE
100%
50%
25%
12.5%
NUMBER OF NUCLEI REMAINING N
0
1
2
3
4
5
HALF-LIVES $t_{1/2}$


LOG GRAPH
100%
50%
25%
12.5%
6.25%
3.125%
NUMBER OF NUCLEI REMAINING LOG (N)
0
1
2
3
4
5
HALF-LIVES $t_{1/2}$

Worked Example

The radioisotope technetium is used extensively in medicine. The graph below shows how the activity of a sample varies with time.



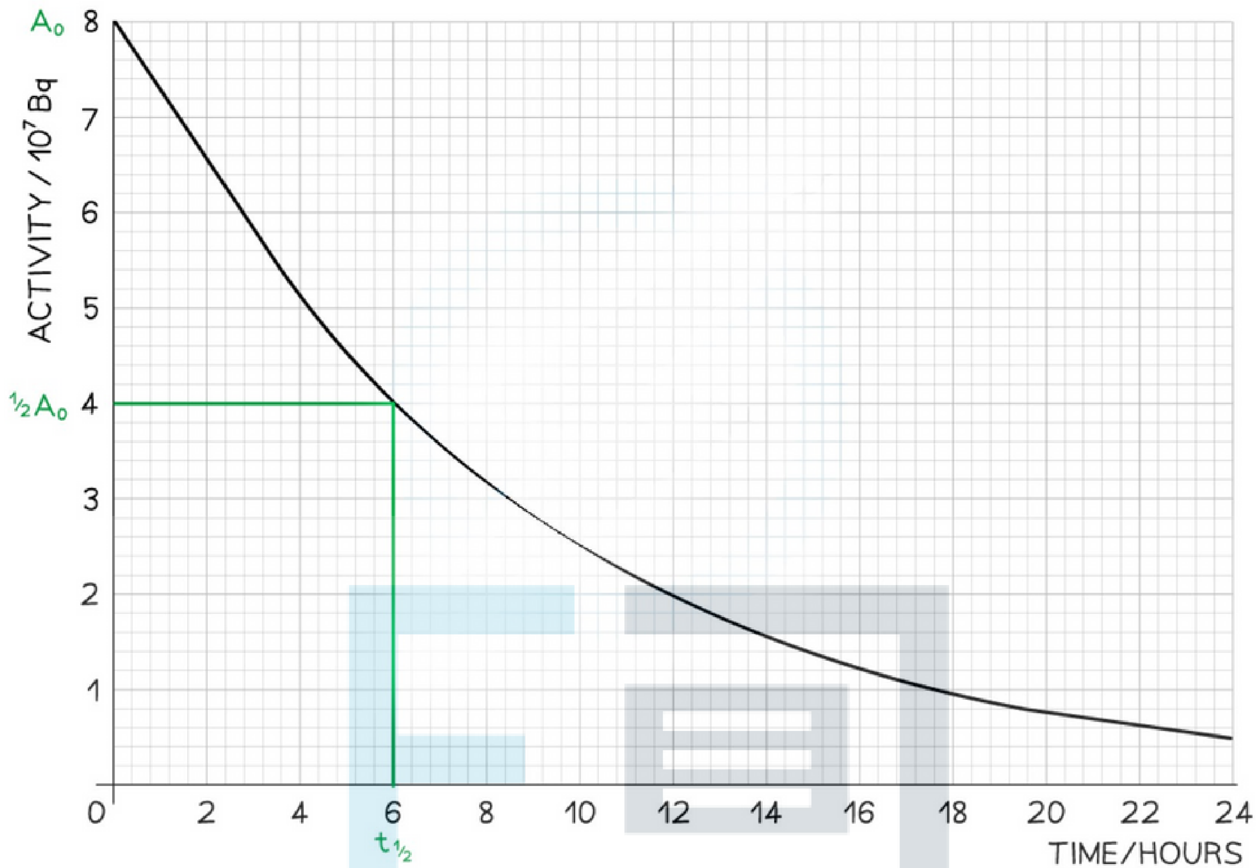
Determine:

a) The decay constant for technetium

b) The number of technetium atoms remaining in the sample after 24 hours

Part (a)

Step 1: Draw lines on the graph to determine the time it takes for technetium to drop to half of its original activity



Step 2: Read the half-life from the graph and convert to seconds

- $t_{1/2}$ = 6 hours = 6 × 60 × 60 = 21 600 s

Step 3: Write out the half life equation

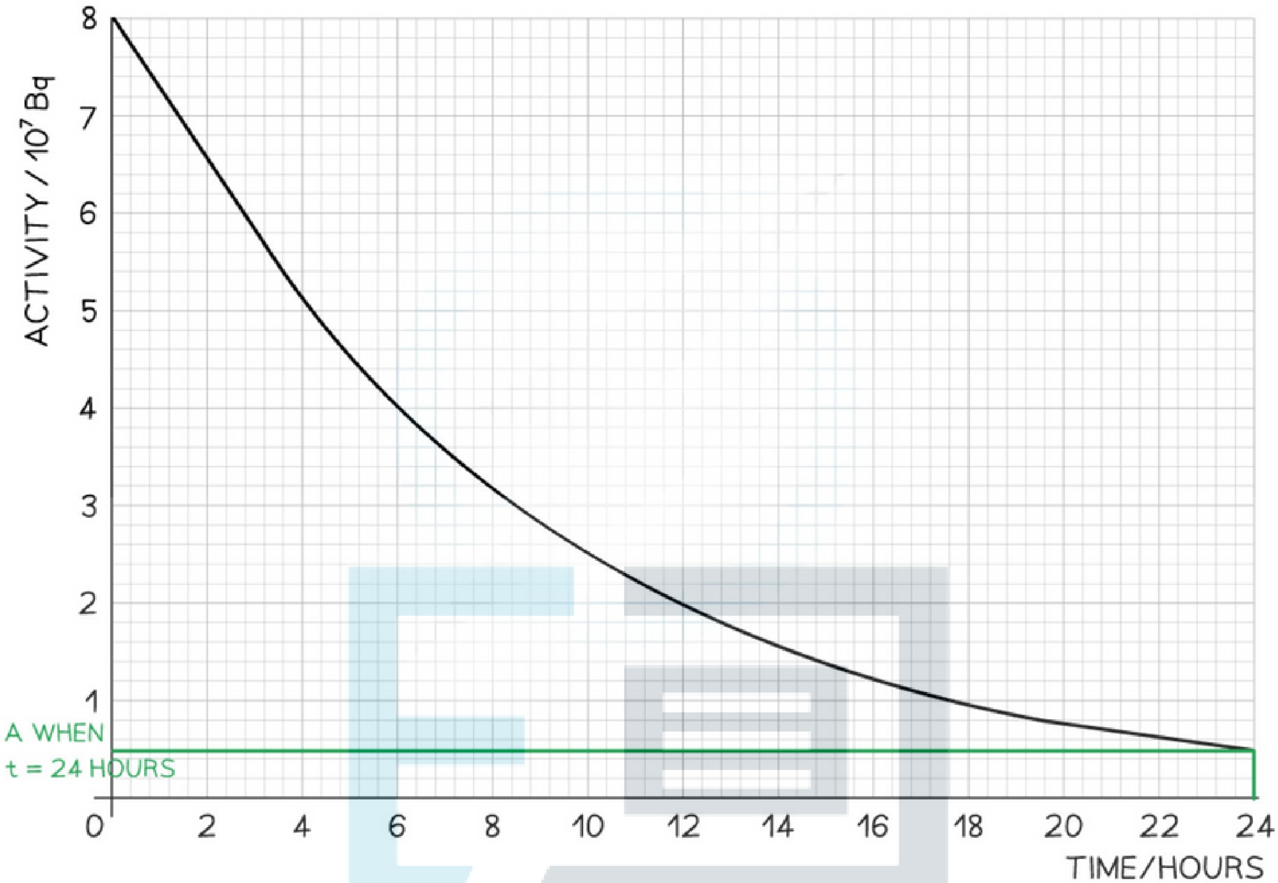
$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Step 4: Calculate the decay constant

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{21\ 600} = 3.2 \times 10^{-5}\ s^{-1}$$

Part (b)

Step 1: Draw lines on the graph to determine the activity after 24 hours



- At $t = 24$ hours, $A = 0.5 \times 10^7$ Bq

Step 2: Write out the activity equation

$$A = \lambda N$$

Step 3: Calculate the number of atoms remaining in the sample

$$N = \frac{A}{\lambda} = \frac{0.5 \times 10^7}{3.2 \times 10^{-5}} = 1.56 \times 10^{11}$$

8.2.4 Applications of Radioactivity

Radioactive Dating

- The isotope carbon-14 is commonly used in radioactive dating
- It forms as a result of cosmic rays knocking out neutrons from nuclei, which then collide with nitrogen nuclei in the air:

$$^{1}n + {}^{14}N \rightarrow {}^{14}C + {}^{1}p$$

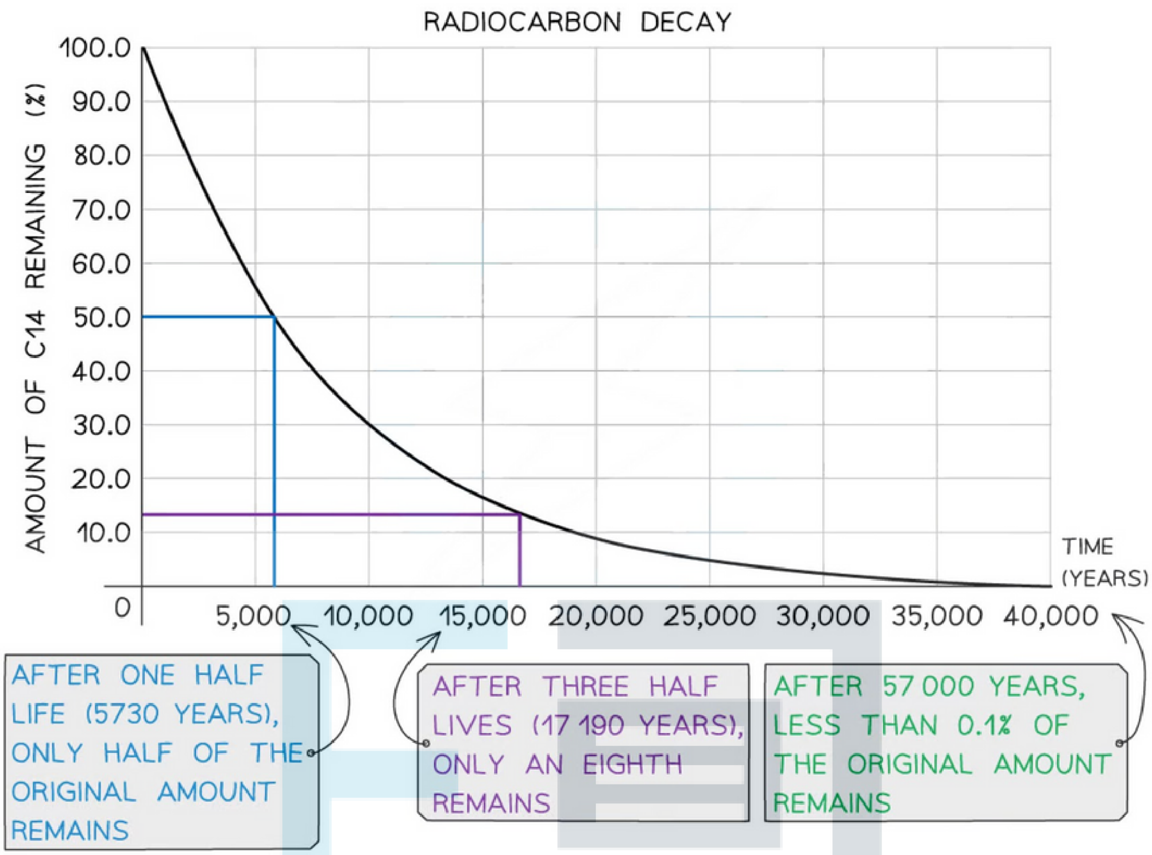
- Plants take in carbon dioxide from the atmosphere for photosynthesis, including the radioactive isotope carbon-14
- Animals and humans take in carbon-14 by eating the plants
    - Therefore, all living organisms absorb carbon-14, but after they die they do not absorb any more

- The proportion of carbon-14 is constant in living organisms as carbon is constantly being replaced during the period they are alive
- When they die, the activity of carbon-14 in the organic matter starts to **fall**, with a half-life of around 5730 years
- Samples of living material can be tested by comparing the current amount of carbon-14 in them and compared to the initial amount (which is based on the current ratio of carbon-14 to carbon-12), and hence they can be dated

Reliability of Carbon Dating

- Carbon dating is a highly reliable ageing method for samples ranging from around 1000 years old up to a limit of around 40 000 years old
    - Therefore, for very young, or very old samples, carbon dating is not the most reliable method to use

- This can be explained by looking at the decay curve of carbon-14:



Carbon-14 decay curve used for carbon dating

- If the sample is less than 1000 years old:
  - The activity of the sample will be too high
  - So, it is difficult to accurately measure the small change in activity
  - Therefore, the ratio of carbon-14 to carbon-12 will be too high to determine an accurate age

- If the sample is more than 40 000 years old:
  - The activity will be too small and have a count rate similar to that of background radiation
  - So, there will be very few carbon-14 atoms remaining, hence very few decays will occur
  - Therefore, the ratio of carbon-14 to carbon-12 will be too small to determine an accurate age

- Carbon dating uses the currently known ratio of carbon-14 to carbon-12, however, scientists cannot know the level of carbon-14 in the biosphere thousands of years ago
- Therefore, this makes it difficult to age samples which are very old

Potassium-Argon Dating

- Ancient rocks contain trapped argon gas as a result of the decay of the radioactive isotope of potassium-40
- This happens when a potassium nucleus captures an inner shell electron, also known as electron capture

$$^{40}K + e^- \rightarrow {}^{40}Ar + v_e$$

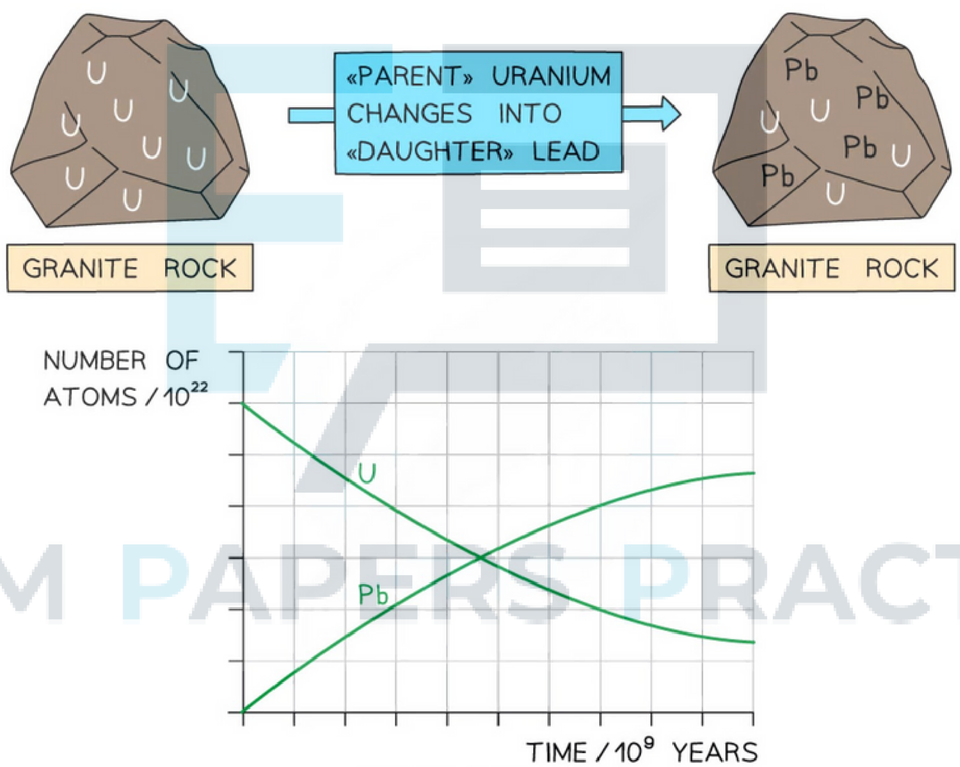
- The potassium isotope can also decay by β^- emission to form calcium-40

$$^{40}K \rightarrow {}^{40}Ca + \beta^- + \overline{v\equiv}$$

- The half-life of the potassium-40 is 1.25 billion years
- The age of the rock (when it solidified) can be calculated by measuring the proportion of argon-40 to potassium-40
- This method is accurate for dating rocks up to 100 million years old

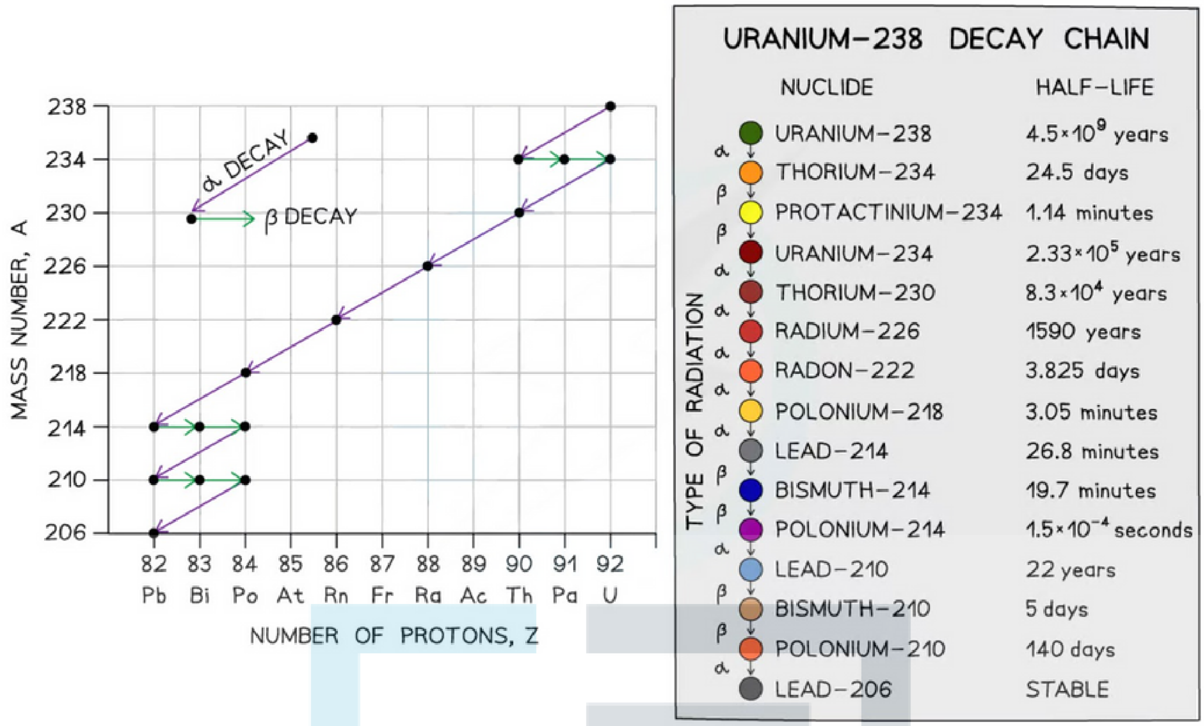
Uranium-Lead Dating

- While the potassium-argon method is best for ageing younger rocks, the uranium-lead method has been critical in dating geologic events more than 100 million years old



Uranium atoms decay whilst the number of lead atoms increases

- Initially, there is only uranium in the rock, but over time, the uranium decays via a decay chain which ends with lead-206, which is a stable isotope
- Uranium has a half-life of 4.5 billion years and over time, the ratio of lead-206 atoms to uranium-238 atoms increases
  - This ratio may be used to determine the age of a sample of rock
- Uranium is so well studied that its decay constant is much better known than other isotopes, such as potassium, making the uranium-lead dating technique the most accurate available



Uranium-238 decay chain

Storage of Radioactive Waste

- Radioactive substances can be dangerous and some substances have very long half-lives (even billions of years)
    - This means that they will be emitting harmful radiation well above background radiation for a very long time
- Waste products from nuclear power stations need to be appropriately stored for the remaining time that they are radioactive
- Common methods are **water tanks** or **sealed underground**
    - This is to prevent damage to people and the environment now and for many years into the future
    - Sealing them underground means they are less likely to be dislodged or released due to natural disasters